LIGHT CONES DIFFERENTIAL MANIFOLDS QUANTUM GRAVITY

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ABSTRACT. I will present a way of making gravity quantum by using light cones differential manifolds.

1. Light cone manifold

All units used here are Planck units equal to one. For given light cone i can separate it into two regions- positive light cone $\mathscr{L}^{+}(X)$ and negative light cone $\mathscr{L}^{-}(X)$, where there both functions of space-time that is denoted (X). Manifold for given point for flat space-time is defined as field with both light cones and a space only manifold $M^{P}(X)$:

$$M_{p+q}(X) := \left(\mathscr{L}^{-}_{p+q}(X), M_{P}(X), \mathscr{L}^{+}_{p+q}(X)\right) \tag{1.1}$$

Where light cone has p space dimensions and q time dimensions. Where each part of manifold has to obey simple equality:

$$\mathscr{L}^{-}_{p+q}(X) \leq M_{P}(X) \leq \mathscr{L}^{+}_{p+q}(X) \tag{1.2}$$

That is just a way of saying that past light cone is in past so it has negative value, manifold of space is at center point and future light cone is positive value so it's in the future.

2. Light cone manifold in present of energy

If there is any matter present or equally energy light cone manifold changes to more complex equation, i use nabla operator for covariant derivative:

$$\sum_{k=p+1}^{p+q} \nabla_k M_{p+q}(X) := \left(\int_{X_-} \mathscr{F}(X) M_P(X), 0, \int_{X_+} \mathscr{F}(X) M_P(X) \right) \tag{2.1}$$

I will integrate over two regions, first region goes from negative value of space coordinates to positive and from negative time coordinates to zero, second one goes from same with space coordinates and from zero to positive time coordinate value.

$$\int_{X_-} = \int_{-x^1}^{x^1} ... \int_{-x^p}^{x^p} \int_{-x^{p+1}}^{0} ... \int_{-x^{p+q}}^{0} d^{p+q} X \tag{2.2}$$

$$\int_{X_+} = \int_{-x^1}^{x^1} ... \int_{-x^p}^{x^p} \int_{0}^{x^{p+1}} ... \int_{0}^{x^{p+q}} d^{p+q} X \tag{2.3}$$

Energy field tensor with respect to two vector fields where i again use nabla notation for covariant derivative is equal to, where K represents energy in system:

$$\mathscr{F}(X) = \nabla_i V_i(X) \nabla_j V_j(X) + \nabla_i \nabla_j K \left(V_i(X), V_j(X) \right) \tag{2.4}$$

That energy tensor field is key to light cones where there is energy present and not always equality of light cones (1.2) is still valid.

3. Rotation symmetry of gravity field

Spin is essential property in quantum physics, if i take a light cone and rotate it around space axis depending on spin i will get results depending on that spin. Simplest case is spin one if i denote rotation operator as $R\left(X,\alpha\right)$ where α is rotation angle i can write for spin one acting on manifold:

$$\left(R\left(X,\frac{\alpha}{2}\right)\mathscr{L}_{p+q}^{-}\left(X\right),R\left(X,\alpha\right)M_{P}\left(X\right),R\left(X,\frac{\alpha}{2}\right)\mathscr{L}_{p+q}^{+}\left(X\right)\right) \tag{3.1}$$

Where general rule is that manifold rotation is equal to multiplication of rotations of future and past light cones. For spin one half i can have two possible state of rotation:

$$\left(R\left(X,\frac{\alpha}{2}\right)\mathscr{L}_{p+q}^{-}\left(X\right),R\left(X,\frac{\alpha}{2}\right)M_{P}\left(X\right),\mathscr{L}_{p+q}^{+}\left(X\right)\right) \tag{3.2}$$

$$\left(\mathscr{L}_{p+q}^{-}\left(X\right),R\left(X,\frac{\alpha}{2}\right)M_{P}\left(X\right),R\left(X,\frac{\alpha}{2}\right)\mathscr{L}_{p+q}^{+}\left(X\right)\right) \tag{3.3}$$

Where both of them give same results. For spin two and gravity field i will get one more light cone. That light cone represents additional rotation symmetry and it will be pointed perpendicular to normal light cone. I will denote it as $\dot{\mathscr{L}}_{p+q}^{+}$ where dot does not mean change with respect to time. But that its second light cone perpendicular to first one:

$$\left(R\left(X,\frac{\alpha}{2}\right)\dot{\mathscr{L}}_{p+q}^{-},R\left(X,\frac{\alpha}{2}\right)\mathscr{L}_{p+q}^{-}\left(X\right),R\left(X,2\alpha\right)M_{P}\left(X\right)\right.$$
$$\left.,R\left(X,\frac{\alpha}{2}\right)\mathscr{L}_{p+q}^{+}\left(X\right),R\left(X,\frac{\alpha}{2}\right)\dot{\mathscr{L}}_{p+q}^{+}\right) \tag{3.4}$$

4. Massive particles movement

Light cones represent movement of massless particles, but if particle is massive i need to add extra term to manifold equation that term is how manifold of space changes in time, and it's equal to value of it's energy tensor:

$$\sum_{k=p+1}^{p+q} \nabla_k M_{p+q}(X) := \left(\int_{X_-} \mathscr{F}(X) \sum_{k=p+1}^{p+q} \nabla_k M_P(X), 0, \int_{X_+} \mathscr{F}(X) \sum_{k=p+1}^{p+q} \nabla_k M_P(X)\right) \tag{4.1}$$

$$\sum_{k=p+1}^{p+q} \nabla_k M_P(X) = \sqrt{\nabla_i \nabla_j K\left(V_i(X), V_j(X)\right) \cdot \nabla_i \nabla_j K\left(V_i(X), V_j(X)\right)} \tag{4.2}$$

For massless particles i will have that change always equal to manifold itself:

$$\sum_{k=p+1}^{p+q} \nabla_k M_P(X) = M_P(X) \tag{4.3}$$

Now what is physical interpretation of that all light cones? Object moves in all possible ways it can move, creating a cone that represents a possible trajectory it can take.

5. Measurement

Measurement is key component of a physical theory, if i have all possible trajectories that a object can take in space-time that is it's cone or for massless particles it's light cone so it is manifold itself, i can take integral over whole manifold - now i take integral over some trajectory $t_p(X)$ and divide it by integral over whole manifold and i use on both of them a dot product so i get probability squared of that trajectory that can be written as:

$$\rho^2(X) = \frac{\int_{t_p(X)} M_{p+q}(X)\,dX \cdot \int_{t_p(X)} M_{p+q}(X)\,dX}{\int_{M_{p+q}(X)} M_{p+q}(X)\,dX \cdot \int_{M_{p+q}(X)} M_{p+q}(X)\,dX} \tag{5.1}$$

So when I do measurement i take one path of all possible paths that object can fallow so manifold itself. And object did follow this trajectory with probability $\rho^2(X)$, but when measured it follows only that trajectory, before it follows all possible paths so it's equal to manifold.

6. Simplest case solutions

Simplest case solutions are ones that are spherical , unchanging with mass that agree energy potential agrees with general relativity solutions. First i write that potential that depends only on radius:

$$\mathscr{F}\left(r\right)=\nabla_{r}r\nabla_{r}r+\nabla_{r}\nabla_{r}\frac{2M}{r}\left(1-\log\left(r\right)\right) \tag{6.1}$$

$$\mathscr{F}\left(r\right)=1-\frac{2M}{r} \tag{6.2}$$

Now from it can calculate all changes first of space manifold that do not move with speed of light:

$$\nabla_{4}M_{r,\phi,\theta}\left(r,\phi,\theta,x^{4}\right)=\sqrt{\left(-\frac{2M}{r}\right)\left(-\frac{2M}{r}\right)}=\frac{2M}{r} \tag{6.3}$$

From it i can calculate change of light cones, positive and negative ones:

$$\nabla_{4}M^{-}_{r,\phi,\theta,x^{4}}=\int_{-x^{4}}^{0}\int_{1}^{r}\int_{0}^{\pi}\int_{0}^{2\pi}\left(1-\frac{2M}{r}\right)M_{r,\phi,\theta}\left(r,\phi,\theta,x^{4}\right)drd\phi d\theta dx^{4} \tag{6.4}$$

$$\nabla_{4}M^{+}_{r,\phi,\theta,x^{4}}=\int_{0}^{x^{4}}\int_{1}^{r}\int_{0}^{\pi}\int_{0}^{2\pi}\left(1-\frac{2M}{r}\right)M_{r,\phi,\theta}\left(r,\phi,\theta,x^{4}\right)drd\phi d\theta dx^{4} \tag{6.5}$$

And same for massive particles:

$$\nabla_{4}M^{-}_{r,\phi,\theta,x^{4}}=\int_{-x^{4}}^{0}\int_{1}^{r}\int_{0}^{\pi}\int_{0}^{2\pi}\frac{2M}{r}\left(1-\frac{2M}{r}\right)M_{r,\phi,\theta}\left(r,\phi,\theta,x^{4}\right)drd\phi d\theta dx^{4} \tag{6.6}$$

$$\nabla_{4}M^{+}_{r,\phi,\theta,x^{4}}=\int_{0}^{x^{4}}\int_{1}^{r}\int_{0}^{\pi}\int_{0}^{2\pi}\frac{2M}{r}\left(1-\frac{2M}{r}\right)M_{r,\phi,\theta}\left(r,\phi,\theta,x^{4}\right)drd\phi d\theta dx^{4} \tag{6.7}$$

Those are simplest case solutions that depend on manifold of space for spherical space-time.