LEPTON MASSES

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Neutrino interactions ,particle properties

Like the charged Lepton,the corresponding neutrino wave function Is assumed to be a superposition of the left and right handed components of the charged lepton with correct quantum numbers. This ensures that the neutrino obtains its mass through interaction with the same Higgs field as the corresponding charged Lepton. The charged lepton also has a γ_5 interaction with the Higgs field leading to the two masses of the neutrino and the charged lepton related

Neutrino wave function, left and right components of the charged lepton , vector and axial vector coupling constants of the charged Lepton ,electron-muon mass ratio,tau-neutrino mass.

!. INTRODUCTIONs

Of all leptons ,the neutrinos are quite difficult to understand and probably the least known particles with no sound theoretical estimates such as mass .The standard model requires that the right-handed neutrino has no interaction . This leads to the inference that the neutrino has no Dirac mass but experimentally observed oscillations strongly confirm that the neutrinos have mass like many other elementary particles. In the Weinberg-Salam (1) model, the unique handedness of the neutrinos and the particular form of Higgs fields introduced to accomplish spontaneous symmetry breaking ensure that the neutrinos have no mass. It may be noted electron wave function is split into left-and right handed pieces

it needed to be massless, (before symmetry breaking). We have

in-complete knowledge of how the Higgs field interacts with leptons.

Let the Left - handed component of the electron be e_L and the righthanded component be e_R.The wave function for the electron is given by,

$$e = e_L + e_R \quad . \tag{1}$$

It is this wave function which couples to the Higgs field ϕ which has a

a vacuum expectation value ,

$$V_0 = 246.22\, GeV \quad . \tag{2}$$

In this note the wave function of the electron -neutrino is chosen.

$$\nu = Electron\ neutrino\ wave\ function = \overline{e_L} + e_R \quad . \tag{3}$$

In the above expansion of the neutrino wave function $\overline{e_L}$ has $I_3^W = 0$ and Y^W= 2 and the charge Q= $+1$ .For e_R these three are 0, -2 ,&

-1 and so that the neutrino wave function is an expanded version of the basic entities $e_L and e_R\ of\ the\ corresponding\ charged\ lepton.$.It is this wave function which couples to the Higgs field to generate mass.

Here we recall the fact that the electric charge is a mixture of weak isospin and weak hypercharge. The electromagnetic gauge particle is

similarly a mixture of the neutral gauge particle of Weak isospin W^0

and weak hyper-charge B.

$$A = W^0 \sin\vartheta_W + B cos\vartheta_W \quad , \tag{4}$$

where the weak angle ϑ_W is the parameter which adjusts the relative proportions of the two. The remaining portions of W^0 and B also mix together to produce another gauge particle,

$$Z^0 = W^0 cos\vartheta_W - Bsin\vartheta_W \ . \tag{5}$$

By careful choice the Z^0 can be given mass while the photon remains massless. In Eq.(3) we expanded the neutrino wave function in terms of $e_L and e_R$ as there is such an expansion for the electron to account for its mass. This justifies the interaction of the electron-neutrino with the Higgs field like the corresponding charged lepton. Also it is similar to Eqs. (4) and (5) of course without the mixing angle.Let (3) be mass eigenstate of the electron- neutrino and its mass be generated through the Lagrangian,

$$\mathrm{L} = \ - \mathrm{h}\, \bar{\nu}\nu\phi \ - h\bar{e}\mathrm{e}\phi - i\ a_1\bar{e}\ \gamma_5 e\phi \ , \tag{6}$$

where, the Higgs field is ϕ with the VEV V_0 ,and h in Eq. (6), is very small as it determines the mass of the neutrino, after symmetry breaking, the electron has the same mass as its neutrino at this stage and in addition it interacts with the Higgs field through a γ_5 interaction term. We recover the standard model scenario when h is set zero and i γ_5 is replaced by one. Given a Dirac field say,ψ, the Hermitian Scalar $\bar{\psi}\psi$ and i$\bar{\psi}\gamma_5\psi$ have opposite CP and T transformation properties (In this respect they are unlike the vector and axial vector.) The CP violation is caused by the exchange of ϕ' fields. Since the coupling of Higgs Field is usually rather small, it is possible to arrange for the CP violation to be of roughly milliweak magnitude [Mohapatra 1980, Bilenkey,2 &3].

After spontaneous symmetry breaking from Eq. (6) , we note that,

$$L = -hV_0\bar{\nu}\nu - h\bar{\nu}\nu\phi' - hV_0\bar{e}e - h\bar{e}e\phi' - ia_1\bar{e}\gamma_5 eV_0 - ia_1\bar{e}\gamma_5 e\phi' .$$

$$L = -m\bar{\nu}\nu - h\bar{\nu}\nu\phi' - m\bar{e}e - h\bar{e}e\phi' - ia_1\bar{e}\gamma_5 eV_0 - ia_1\bar{e}\gamma_5 e\phi'. \quad (7)$$

Here, $m = hV_0$, is the electron neutrino mass. (8)

Let, $e = \exp\left(-\frac{1}{2}i\alpha_1\gamma_5\right)e'$, (9)

where α_1 is a real parameter. Vector and axial vector interactions are not affected by this transformation. We choose α_1 in such a way that the constant coefficient of $\bar{e}'\gamma_5 e'$ is zero. This gives,

$$-[mcos\alpha_1 + V_0a_1sin\alpha_1]\bar{e}'e' - [-imsin\alpha_1 + i\,a_1V_0cos\alpha_1]\bar{e}'\gamma_5 e'$$
$$-a_1\bar{e}'[sin\alpha_1 + i\gamma_5 cos\alpha_1]e'\phi'. \quad (10)$$

The mass of the electron is now given by,

$$m_e^2 = m^2sec^2\alpha_1 = m^2\left[1 + \frac{V_0^2a_1^2}{m^2}\right] = mV_0\left[\frac{V_0a_1^2}{m} + \frac{m}{V_0}\right]. \quad (11)$$

The very last term within the bracket is independent of the VEV. It is the sum of $\left[\frac{a_1^2}{h} + h\right]$,which are the interaction constants of the Higgs field with the electron and its neutrino . Let,

$$q_0 = \left[\frac{a_1^2}{h} + h\right]. \quad (12)$$

A similar Lagrangian can be chosen to obtain the effective mass for the muon and its neutrino through their interaction with the same Higgs field

$$L = -h_1\bar{\nu}_\mu\nu_\mu\phi - h_1\bar{\mu}\mu\phi - ia_2\bar{\mu}\gamma_5\mu\phi \quad . \tag{13}$$

In Eq. (13), $h_1 and\ a_2$ are real positive numbers and after symmetry breaking, the muon neutrino obtains the following mass:

$m_1 = h_1V_0$.The muon-neutrino wave function is chosen as, (14)

$$\nu_\mu = \overline{\mu_L} \;+\mu_R \quad . \tag{15}$$

Following the same steps as in Eq. (7, 9, and 10) for the Lagrangian (13) we readily observe that,

$$m_\mu^2 = m_1^2 sec^2\alpha_2 = m_1^2\left[1 + \frac{V_0^2 a_2^2}{m_1^2}\right] = m_1V_0\left[\frac{V_0 a_2^2}{m_1} + \frac{m_1}{V_0}\right]. \tag{16}$$

Again, we note that in Eq. (16) the parameter at the end in the brackets is independent of the VEV and is equal to $\left[\frac{a_2^2}{h_1} + h_1\right]$. Let,

$$q_1 = \left[\frac{a_2^2}{h_1} + h_1\right]. \tag{17}$$

There is another massive charged lepton, the τ lepton. Its mass is,

$$m_\tau = 1.777\ GeV \quad . \tag{18}$$

This lepton also has a neutrino and its wavefunction is

$$\nu_\tau = \overline{\tau_L} + \tau_R\ . \tag{19}$$

Like the other leptons, they start out with no intrinsic mass and obtain effective mass with their interaction to the same Higgs field with the VEV = V_0. The Lagrangian in this case is

$$L = -h_2 \overline{\nu_\tau} \upsilon_\tau \phi - h_2 \bar{\tau} \tau \phi - i a_3 \bar{\tau} \gamma_5 \tau \phi \ . \tag{20}$$

Following by now the familiar steps, after spontaneous symmetry breaking, we have,

$$m_{\upsilon\tau} = m_2 = h_2 V_0 \quad , \tag{21}$$

where m_2 is the mass of the τ -neutrino and the mass of the charged τ lepton is now given by,

$$m_\tau^2 = m_2^2 sec^2 \alpha_3 = m_2^2 \left[1 + \frac{V_0^2 a_3^2}{m_2^2}\right] = m_2 V_0 \left[\frac{V_0 a_3^2}{m_2} + \frac{m_2}{V_0}\right] . \tag{22}$$

Again, the very last factor in the brackets is independent of the VEV.

And it is given by, q_2 where,

$$q_2 = \left[\frac{a_3^2}{h_2} + h_2\right] \quad . \tag{23}$$

Even though we do not know what is the value of q, it is a real positive number. The masses of W and Z bosons are given in terms of the gauge constants g_L and g' of $SU(2)_L$ XU(1) through Higgs Mechanism. The mass generating interaction constants of the charged leptons with the Higgs boson are put by hand in the Standard model as there is no theory which will require a particular choice for these constants. In Eq. (6) the electron is coupled to the ϕ [$\bar{e}e\phi$ and $\bar{e}\gamma_5 e\phi$] in two ways, and this sort of coupling is also there with the standard Z-boson. Using this clue, we will relate the masses of the electron and muon to the interaction constants g_V and g_A of the Z boson as in Ref.(4)

2.Electron -Muon Masses.

The masses of the electron and muon are given by

$$m_e^2 = m^2 sec^2\alpha_1 = m^2\left(1+\frac{a_1^2 V_0^2}{m^2}\right), \tag{24}$$

$$m_\mu^2 = m_1^2 sec^2\alpha_2 = m_1^2\left[1+\frac{V_0^2 a_2^2}{m_1^2}\right], \tag{25}$$

In the above equations , m is proportional to h and $m_1$is proportional to h_1 .In the mass generating equation the electron is coupled to the Higgs field as $h\bar{e}$ e ϕ.The Higgs field is neutral that has VEV.The neutral Higgs field is very much related to the neutral Z -boson. The neutral Z-Boson is coupled to $\bar{e}\gamma_\mu e$ with a coupling $(g_V)_{e\mu}$ and so h is related to $(g_V)_{e\mu}$.Similarly h_1 is relatedto $(g_V)_{e\mu}$ .Hence

$m_e^2 m_\mu^2 \propto m^2 m_1^2 \propto h^2 h_1^2 \propto (g_V^2)_{e\mu}(g_V^2)_{e\mu}$,

Hence, $m_e m_\mu \propto (g_V^2)_{e\mu}$. (26)

Again the electron is also coupled to the Higgs field in (6) through $\bar{e}\gamma_5 e$

With a coupling constant a_1 and the Z-boson also has a coupling with $\bar{e}\gamma_5\gamma_\mu e$ with a coupling constant$(g_A)_{e\mu}$ so that $a_1^2 \propto 2\, g_A^2$ such that $m_e^2 = m^2 + 2g_A^2 k_1$ where k_1 is a constant, Eq. (24).In a similar way we note that $m_\mu^2 = m_1^2 + 2g_A^2 k_2$ where k_2 is some other constant. We now assume that ,$m^2 \approx m_1^2$ and this leads to,

$$m_\mu^2 + m_e^2 = 2m^2 + 2\,(g_A^2)_{e\mu} \quad \lfloor k_1 + k_2 \rfloor \quad . \tag{27}$$

The above relation leads to the following conclusion:

$$\frac{m_\mu^2}{2} + \frac{m_e^2}{2} \propto \quad (g_A^2)_{e\mu} \tag{28}$$

Where we assumed that m is very small while arriving at (28).When this relation is combined with Eq. (26) one readily observes that,

$$\frac{2m_e m_\mu}{m_{\mu+}^2 m_e^2} \propto \left(\frac{g_V^2}{g_A^2}\right)_{e\mu} \quad . \tag{29}$$

As a first approximation we take the proportionality constant equal to 1.

$$\frac{2m_e m_\mu}{m_{\mu+}^2 m_e^2} = \left(\frac{g_V^2}{g_A^2}\right)_{e\mu} = (-1 + 4sin^2\vartheta_W)^2 \tag{30}$$

In the above ,the masses are well known, and the LHS=0.009672 ,where $m_e = 0.51099\ MeV$ and $m_\mu = 105.65839\ MeV$.The RHS of Eq.(30) is Equal to 0.009683 for $sin^2\vartheta_W = 0.2254$ or 0.2746.

It is a simple matter to show that ,Ref .(4& 5),

$$m_e^2 = \quad mV_0 \frac{(g_V/g_A)_{\nu e}^4}{(g_V/g_A)_{e\mu}^4}\left[1 - \left\{1 - \left({}^{g_V}/{}_{g_A}\right)_{e\mu}^4\right\}^{1/2}\right], \tag{31}$$

$$m_\mu^2 = m_1 V_0 \frac{(g_V/g_A)_{\nu\mu}^4}{(g_V/g_A)_{e\mu}^4}\left[1 + \left\{1 - \left({}^{g_V}/{}_{g_A}\right)_{e\mu}^4\right\}^{1/2}\right]. \tag{32}$$

Whenever m$= m_1$,It is a simple matter to show that (30) results from Eqs.(31) and (32).In Eqs. (31) and (32) ,$\left({}^{g_V}/{}_{g_A}\right)_{\nu e}^4 = \left({}^{g_V}/{}_{g_A}\right)_{\nu\mu}^4 = 1$ .This factor is introduced for future generalizations of Eqs.(31&32) to quarks.

3. Neutrino Masses.

The masses of the neutrinos can now be theoretically estimated. The basic idea is that the neutrino is super- posed state of the left and right handed components of the corresponding charged lepton as in Eqs.(3,15,and 19).Let the electron- neutrino and muon- neutrino have equal mass, m where, from Eqs.(31,32),

$$m = \frac{m_e m_\mu}{V_0} \left(\frac{g_V}{g_A}\right)_{e\mu}^2 . \tag{33}$$

The above relation is based on Eq.(30).For $sin^2\vartheta_W = 0.2254$,

$m = 2.123259\ eV.$ (34)

For, $sin^2\vartheta_W = 0.23,\ \ m = 1.403373\ eV.$ (35)

The equal mass for both the neutrinos is not supported by the observed Neutrino oscillations. Moreover If these two neutrinos have equal mass Eq.(30) is exact ,and the Electro-weak model will be based on the group $SU(2)_L X\ SU(2)_R X\ U(1)$ with the mixing parameters 0.2254 and 0.2746. But experimentally the electro weak model is based on $SU(2)_L XU(1)$ with a mixing parameter 0.2254 or,0.23.Thus the two neutrinos have different masses. The Electron -neutrino mass is given by, For $sin^2\vartheta_W = 0.2254$ $m = 2.120956\ eV$, (m =2.120935 eV ,If $sin^2\vartheta_W = 0.23$) from Eq.(31).

The mass of the muon neutrino for these values of the mixing parameter Is ,$m_1 = 2.131045\ eV, (m_1 = 0.928579\ eV.)$ from Eq.(32). (36)

From Eq. (22), and Eq.(23) it just follows,

$$m_\tau^2 = m_2 V_0 q_2 = m_2 V_0 \left(\frac{g_A}{g_V}\right)^2 , \quad (37)$$

Where we took $q_2 = \left(\frac{g_A}{g_V}\right)^2.$ (38)

From Eq. (37) the mass of the Tau-neutrino is given by,

$$m_2 = \frac{m_\tau^2}{V_0}\left({g_V}/{g_A}\right)^2 \quad . \tag{39}$$

The above expression is exactly similar to Eq. (33),where in place of $m_e m_\mu$, we have m_τ^2 . If $g_V^2/g_A^2 = (-1+4sin^2\vartheta)^2$ =1 then the mixing parameter is 0.5 for the electroweak group $SU(2)_L XU(1)$ involving the Tau lepton and its neutrino .In this case ,the tau neutrino mass is,

$$m_2 = 12.825\ MeV. \tag{40}$$

If the Tau- neutrino mass turns out to be the above value the e-mu-tau universality is no longer valid.Ref.(5) &(6).

Assuming e- mu-tau universality,the tau -neutrino mass is ,fromEq.(39),

$$m_2 = 0.124184\ MeV, for\ sin^2\vartheta_W = 0.2254 \quad \text{and} \tag{41}$$

$$m_2 = 0.082080\ MeV, if\ sin^2\vartheta_W = 0.23\ . \tag{42}$$

4.CONCLUSION.

In this note the masses of all the three neutrinos are theoretically found. The ratio of $m_e/m_\mu \approx \frac{1}{2}\left({g_V}/{g_A}\right)_{e\mu}^2 = 0.004842$ when the Weinberg mixing parameter is 0.2254. More over the neutrino wave function is a superposition of the left and right handed components of the corresponding charged lepton allowing it to interact with the same Higgs to obtain mass.This expansion of the Neutrino is similar to the expansion of nucleons in terms of Quarks. Like the quarks, the left-handed and the right-handed pieces of the charged lepton can not have free existence.

Experiments should be designed to find the Tau-neutrino mass .If this mass turns out to be about 12.825 MeV ,then the e-mu- tau universality is violated. On the other -hand If this mass turns out to be about 0.12 4184 MeV, then ,e-mu-tau universality holds. The neutrinos obtain their mass through the same Higgs mechanism and there is no necessity of any see-saw mechanism. The neutrino oscillations can be explained by using two flavor mixing with a mass matrix as in Ref.(6).

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