The analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu applied to the Natario-Broeck spacetime.A very interesting approach towards a more realistic interstellar warp drive

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. However one of the major drawbacks that affects both warp drive spacetimes are the collisions with hazardous interstellar matter(asteroids,comets,interstellar dust etc)that will unavoidably occurs when a ship travels at superluminal speeds across interstellar space. The problem of collisions between a warp drive spaceship moving at superluminal velocity and the potentially dangerous particles from the Interstellar Medium IM is not new.It was first noticed in 1999 in the work of Chad Clark,Will Hiscock and Shane Larson.Later on in 2010 it appeared again in the work of Carlos Barcelo,Stefano Finazzi and Stefano Liberatti.In 2012 the same problem of collisions against hazardous IM particles would appear in the work of Brendan McMonigal,Geraint Lewis and Philip O'Byrne. Some years ago in 1999 Chris Van Den Broeck appeared with a very interesting idea.Broeck proposed a warp bubble with a large internal radius able to accommodate a ship inside while having a submicroscopic outer radius and a submicroscopic contact external surface in order to better avoid the collisions against the interstellar matter.The Broeck spacetime distortion have the shape of a bottle with 200 meters of inner diameter able to accommodate a spaceship inside the bottle but the bottleneck possesses a very small outer radius with only 10^{-15} meters 100 billion time smaller than a millimeter therefore reducing the probabilities of collisions against large objects in interstellar space.Recently a very interesting work appeared.It covers the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu applied to the Alcubierre warp drive spacetime.But the most important fact:their analysis also applies to the Natario warp drive spacetime.In this work we applied the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu to the Natario-Broeck warp drive spacetime and we arrived at the following conclusion:The analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu proves definitely that the Natario-Broeck warp drive spacetime is the best candidate for a realistic interstellar space travel.

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1 Introduction:

The warp drive as a solution of the Einstein field equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However there are 3 major drawbacks that compromises the warp drive physical integrity as a viable tool for superluminal interstellar travel.

The first drawback is the quest of large negative energy requirements enough to sustain the warp bubble. In order to travel to a "nearby" star at 20 light-years at superluminal speeds in a reasonable amount of time a ship must attain a speed of about 200 times faster than light.However the negative energy density at such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!(see [7],[8] and [9]).

Another drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons.(see [5],[7] and [8]).

The last drawback raised against the warp drive is the fact that inside the warp bubble an astronaut cannot send signals with the speed of the light to control the front of the bubble because an Horizon(causally disconnected portion of spacetime)is established between the astronaut and the warp bubble.(see [5],[7] and [8]).

We can demonstrate that the Natario warp drive can "easily" overcome these obstacles as a valid candidate for superluminal interstellar travel(see [7],[8] and [9]).

[1] do not violates Relativity

Alcubierre([12]) used the so-called 3+1 Arnowitt-Dresner-Misner(ADM) formalism using the approach of Misner-Thorne-Wheeler(MTW)([11]) to develop his warp drive theory.As a matter of fact the first equation in his warp drive paper is derived precisely from the original $3+1$ ADM formalism(see eq 2.2.4 pgs [67(b)],[82(a)] in [12], see also eq 1 pg 3 in [1])[2] and we have strong reasons to believe that Natario which followed the Alcubierre steps also used the original $3+1$ ADM formalism to develop the Natario warp drive The Natario warp drive equation that obeys the $3+1$ ADM formalism is given below:[3]

$$ds^2 = (1 - X_{rs}X^{rs} - X_\theta X^\theta)dt^2 + 2(X_{rs}drs + X_\theta d\theta)dt - drs^2 - rs^2 d\theta^2 \tag{1}$$

$$ds^2 = dt^2 - [(drs - X^{rs}dt)^2 + (rs^2)(d\theta - X^\theta dt)^2] \tag{2}$$

The problem of collisions between a warp drive spaceship moving at superluminal velocity and the potentially dangerous particles from the Interstellar Medium IM is not new.[4].

It was first noticed in 1999 in the work of Chad Clark,Will Hiscock and Shane Larson [3].Later on in 2010 it appeared again in the work of Carlos Barcelo,Stefano Finazzi and Stefano Liberatti [4].In 2012 the same problem of collisions against hazardous IM particles appeared in the work of Brendan McMonigal,Geraint Lewis and Philip O'Byrne [17].

However all these works covered the Alcubierre warp drive spacetime that do not possess negative energy in front of the ship in the equatorial plane on the x-axis to deflect incoming hazardous IM particles.[5].

From the works in [5],[7] and [8] we can see that impacts of the warp bubble against the particles of the Interstellar Medium IM (eg:asteroids,comets,space debris,supernova remnants,clouds of gas etc) are tremendously hazardous for a spaceship at superluminal velocities. However and according to the cited works we know that the negative energy density in the case of the Natario warp bubble do not vanish in the equatorial plane meaning that the repulsive gravitational behavior of the negative energy density in front of the ship can theoretically deflect the IM particles offering some degree of protection to the ship and crew members.

Although we are counting on the negative energy density in front of the ship in the case of the Natario warp drive to offer protection to the ship and the crew members we know that collisions of the warp bubble walls against IM particles are unavoidable and as large the warp bubble is [6] this means a large bubble surface exposed to heavy bombardment by the IM particles.

The ideal situation for a warp bubble in a real superluminal interstellar spaceflight would the one in which the warp bubble possesses a large internal diameter with the size enough to contains a spaceship inside the bubble but the region of the bubble in contact with the interstellar space and hence with the IM particles remains very small reducing the probabilities of dangerous collisions.

[2] see the Remarks section on our system to quote pages in bibliographic references
[3] see Appendices A and B in [16] for details
[4] see Appendices L and M
[5] see Appendix D
[6] the warp bubble must possesses size enough to contains a spaceship inside

What we need is a warp bubble with a large internal radius able to accommodate a ship inside while having a submicroscopic outer radius and a submicroscopic contact external surface in order to better avoid the collisions against the IM particles.

Some years ago in 1999 Chris Van Den Broeck appeared with this idea. Broeck introduced inside the Alcubierre warp drive metric in 1999 a new mathematical term $B(rs)$ with very interesting features:$B(rs)$ creates inside the Alcubierre warp bubble a spacetime distortion with the shape of a bottle.The bottle have an inner large radius and hence a large diameter with the size enough to contains a spaceship inside the bottle but the part of the bottle in contact with our Universe and hence with the dangerous IM particles is the bottle bottleneck wirh a very small microscopic radius and hence a small microscopic surface exposed to collisions against the IM particles protecting effectively the ship inside the bottle.Although the bottle can have an arbitrarily large size an external observer in our Universe would only see the microscopic bottleneck.

Broeck created inside an Alcubierre warp bubble with a radius R of 3×10^{-15} meters a bottle with 200 meters of inner diameter and a microscopic bottleneck radius with only 10^{-15} meters. So although a spaceship is contained(or hidden) in the inner space of a bottle with 200 meters of diameter the part of the bottle an external observer in our Universe would see would only be the bottleneck of the bottle with 10^{-15} meters and 10^{-15} meters is 10^{12} times or 100.000.000.000 times or 100 billion times smaller than a millimeter.(see pg 5 in [10]).

Effectively a surface with 10^{-15} square millimeters have less probabilities to suffer a collision than a surface of 100 square meters.And with plenty of room space with 200 meters large enough to accommodate a spaceship and hidden from our Universe and in consequence being kept isolated from the dangerous IM particles.The Broeck idea is more than welcome.[7]

The Broeck bottle provides the ideal scenario for the Natario warp drive and in this work we apply the Broeck mathematical term $B(rs)$ to the Natario warp drive equation in ADM formalism but using the original Alcubierre shape function to generate the term $B(rs)$.

Our successful approach allows ourselves to generate a Broeck bottle inside the Natario warp drive with a bottleneck radius also with 10^{-15} meters but with 200 kilometers of inner diameter.200 kilometers are 1000 times the size of the original Broeck bottle and provides a room of space large enough to contains not only a single spaceship but a large number of spaceships and with very low energy density requirements.

Recently a very interesting work appeared:[18].It covers the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu applied to the Alcubierre warp drive spacetime.But the most important fact:their analysis also applies to the Natario warp drive spacetime.We will study the section 5 pg 10 in [18] as a first approach.

According to pg 10 in [18] the major part of the incoming IM particles or photons will move towards the spaceship in a trajectory over or very close to the x-axis.These photons or particles applies over the surface of the warp bubble a pressure directly proportional to the bubble speed vs and as fast the ship or the bubble moves then the pressure becomes more and more stronger and this is a very serious problem that exists in both the Alcubierre and Natario warp drive spacetimes.

[7] see Appendices G,H and I

This is due to the fact that both bubbles when moving across interstellar space possesses Cross-Sections and the area of each Cross-Section wether in Alcubierre or Natario geometries is exposed to impacts against the hazardous IM particles and these impacts generates the mentioned pressure.As large the areas of each Cross-Section are then the surfaces exposed to impacts becomes larger too.[8]

The Natario geometry possesses negative energy in front of the ship able to deflect these IM particles but with a large surface area for the Cross-Section.The ideal scenario is provided by the Natario-Broeck spacetime in which the Cross-Section possesses a microscopically small surface area effectively protecting the ship against impacts thereby reducing the interstellar pressure.

In this work we demonstrate that the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu proves definitely that the Natario-Broeck warp drive spacetime is the best candidate for a realistic interstellar space travel.

We make extensive use of footnotes and Appendices and this may be regarded ad an exhaustive reading for experienced readers already familiarized with the ideas of Alcubierre Broeck or Natario but these Appendices and footnotes are mainly destined to students beginners or readers at an introductory level eager to assimilate these ideas.

This work may be regarded as a companion to our works in [5],[7],[8] and [16]

[8] see Appendices A,B,J and N

2 The Natario warp drive continuous shape function

Introducing here $f(rs)$ as the Alcubierre shape function that defines the Alcubierre warp drive spacetime we can construct the Natario shape function $N(rs)$ that defines the Natario warp drive spacetime using its Alcubierre counterpart.Below is presented the equation of the Alcubierre shape function.[9].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{3}$$

$$rs = \sqrt{(x - xs)^2 + y^2 + z^2} \tag{4}$$

According with Alcubierre any function $f(rs)$ that gives 1 inside the bubble and 0 outside the bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region is a valid shape function for the Alcubierre warp drive.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2]). In the Alcubierre shape function xs is the center of the warp bubble where the ship resides. R is the radius of the warp bubble and @ is the Alcubierre parameter related to the thickness.According to Alcubierre these can have arbitrary values.We outline here the fact that according to pg 4 in [1] the parameter @ can have arbitrary values.rs is the path of the so-called Eulerian observer that starts at the center of the bubble $xs = rs = 0$ and ends up outside the warp bubble $rs > R$.

The square derivative of the Alcubierre shape function is given by:

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{5}$$

According with Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < N(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive. The Natario warp drive continuous shape function can be defined by:

$$N(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{6}$$

This shape function gives the result of $N(rs) = 0$ inside the warp bubble and $N(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < N(rs) < \frac{1}{2}$ in the Natario warped region. Note that the Alcubierre shape function is being used to define its Natario shape function counterpart.For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $N(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $N(rs) = \frac{1}{2}$(exterior of the Natario bubble).

The derivative square of the Natario shape function is :

$$N'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{7}$$

The term WF in the Natario shape function is dimensionless too:it is the warp factor.It is important to outline that the warp factor $WF >> |R|$ is much greater than the modulus of the bubble radius.Note that the square derivative of the Alcubierre shape function appears in the expression of the square derivative of the Natario shape function.

[9] $tanh[@(rs + R)] = 1$,$tanh(@R) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R|$

- Numerical plot for the Alcubierre and Natario shape functions with @ = 50000 bubble radius $R = 100$ meters and warp factor with a value $WF = 200$

rs	$f(rs)$	$N(rs)$	$f'(rs)^2$	$N'(rs)^2$
$9,99970000000E+001$	1	0	$2,650396620740E-251$	0
$9,99980000000E+001$	1	0	$1,915169647489E-164$	0
$9,99990000000E+001$	1	0	$1,383896564748E-077$	0
$1,00000000000E+002$	$0,5$	$0,5$	$6,250000000000E+008$	$3,872591914849E-103$
$1,00001000000E+002$	0	$0,5$	$1,383896486082E-077$	0
$1,00002000000E+002$	0	$0,5$	$1,915169538624E-164$	0
$1,00003000000E+002$	0	$0,5$	$2,650396470082E-251$	0

According with the numerical plot above when @ = 50000 the square derivative of the Alcubierre shape function is zero[10] from the center of the bubble until $99,996$ meters.At $99,997$ meters the square derivative of the Alcubierre shape function is $2,65 \times 10^{-251}$ and starts to increase reaching the maximum value of $6,25 \times 10^{8}$ at 100 meters from the center of the bubble precisely in the bubble radius decreasing again to the minimum value of $2,65 \times 10^{-251}$ at $100,003$ meters from the center of the bubble.At 100.004 meters from the center of the bubble the square derivative of the Alcubierre shape function is again zero.Note that with respect to the distance of 100 meters from the center of the bubble exactly the bubble radius the powers of the square derivative of the Alcubierre shape function are diametrically symmetrically opposed.We have the values of 10^{-77} at $99,999$ meters and at $100,001$ meters.We have the value of 10^{-164} at $99,998$ meters and at $100,002$ meters.So the thickness of the warped region is limited or defined by the square derivatives of the shape function when these are different than zero.In the case of @ = 50000 the warped region starts at $99,997$ meters and ends up at $100,003$ meters.The thickness of the warped region is then $0,006$ meters.

Note that inside the bubble the Alcubierre shape function possesses the value of 1 and the Natario shape function possesses the value of 0 and outside the bubble the Alcubierre shape function possesses the value of 0 and the Natario shape function possesses the value of $\frac{1}{2}$ as requested.

Also while the square derivative of the Alcubierre shape function is not zero inside and outside the bubble however at the neighborhoods of the bubble radius and possesses the maximum value exactly at the bubble radius the square derivative of the Natario shape function is always zero inside and outside the bubble and possesses also a maximum value at the bubble radius however this value is extremely small when compared to its Alcubierre counterpart.

[10]not exactly zero but possesses extremely low values and we are limited by the floating-point precision of our software

- Numerical plot for the Alcubierre and Natario shape functions with $@ = 75000$ bubble radius $R = 100$ meters and warp factor with a value $WF = 200$

rs	$f(rs)$	$N(rs)$	$f'(rs)^2$	$N'(rs)^2$
$9,99980000000E+001$	1	0	$5,963392481410E-251$	0
$9,99990000000E+001$	1	0	$1,158345097767E-120$	0
$1,00000000000E+002$	$0,5$	$0,5$	$1,406250000000E+009$	$8,713331808411E-103$
$1,00001000000E+002$	0	$0,5$	$1,158344999000E-120$	0
$1,00002000000E+002$	0	$0,5$	$5,963391972940E-251$	0

According with the numerical plot above when $@ = 75000$ the square derivative of the Alcubierre shape function is zero from the center of the bubble until $99,997$ meters.At $99,998$ meters the square derivative of the Alcubierre shape function is $5,96 \times 10^{-251}$ and starts to increase reaching the maximum value of $1,4 \times 10^{9}$ at 100 meters from the center of the bubble precisely in the bubble radius decreasing again to the minimum value of $5,96 \times 10^{-251}$ at $100,002$ meters from the center of the bubble.At 100.003 meters from the center of the bubble the square derivative of the Alcubierre shape function is again zero.Note that with respect to the distance of 100 meters from the center of the bubble exactly the bubble radius the powers of the square derivative of the Alcubierre shape function are diametrically symmetrically opposed.We have the values of 10^{-120} at $99,999$ meters and at $100,001$ meters.So the thickness of the warped region is limited or defined by the square derivatives of the shape function when these are different than zero.In the case of $@ = 75000$ the warped region starts at $99,998$ meters and ends up at $100,002$ meters.The thickness of the warped region is then $0,004$ meters.

Note that inside the bubble the Alcubierre shape function possesses the value of 1 and the Natario shape function possesses the value of 0 and outside the bubble the Alcubierre shape function possesses the value of 0 and the Natario shape function possesses the value of $\frac{1}{2}$ as requested.

Also while the square derivative of the Alcubierre shape function is not zero inside and outside the bubble however at the neighborhoods of the bubble radius and possesses the maximum value exactly at the bubble radius the square derivative of the Natario shape function is always zero inside and outside the bubble and possesses also a maximum value at the bubble radius however this value is extremely small when compared to its Alcubierre counterpart.

The previous plots demonstrate the important role of the thickness parameter @ in the warp bubble geometry wether in both Alcubierre or Natario warp drive spacetimes.For a bubble of 100 meters radius $R = 100$ the regions where $1 > f(rs) > 0$(Alcubierre warped region) and $0 < N(rs) < \frac{1}{2}$(Natario warped region) becomes thicker or thinner as @ becomes higher.In the case of $@ = 50000$ the warped region starts at $99,997$ meters and ends up at $100,003$ meters.The thickness of the warped region is then $0,006$ meters and in the case of $@ = 75000$ the warped region starts at $99,998$ meters and ends up at $100,002$ meters.The thickness of the warped region is then $0,004$ meters.

Then the geometric position where both Alcubierre and Natario warped regions begins with respect to R the bubble radius is $rs = R - \epsilon < R$ and the geometric position where both Alcubierre and Natario warped regions ends with respect to R the bubble radius is $rs = R + \epsilon > R$.The thickness of the warp bubble is then $2 \times \epsilon$.As large as @ becomes as smaller ϵ becomes too.

Note from the plots of the previous pages that we really have two warped regions:

- 1)-The geometrized warped region where $1 > f(rs) > 0$(Alcubierre warped region) and $0 < N(rs) < \frac{1}{2}$ (Natario warped region).The geometrized warped region lies precisely in the bubble radius.[11]
- 2)-The energized warped region where the derivative squares of both Alcubierre and Natario shape functions are not zero.

The parameter @ affects both energized warped regions wether in Alcubierre or Natario cases but is more visible for the Alcubierre shape function because the warp factor WF in the Natario shape functions squeezes the energized warped region in a region of very small thickness centered in the bubble radius.

The negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(N'(rs))^2 \cos^2\theta + \left(N'(rs) + \frac{rs}{2} N''(rs) \right)^2 \sin^2\theta \right] \tag{8}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression[12]:

$$\rho = -\frac{c^2}{G} \frac{vs^2}{8\pi} \left[3(N'(rs))^2 \cos^2\theta + \left(N'(rs) + \frac{rs}{2} N''(rs) \right)^2 \sin^2\theta \right]. \tag{9}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for[13]:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[3(N'(rs))^2 (\frac{x}{rs})^2 + \left(N'(rs) + \frac{rs}{2} N''(rs) \right)^2 (\frac{y}{rs})^2 \right] \tag{10}$$

Considering as a simplified case the equatorial plane($1+1$ dimensional spacetime with $rs = x - xs$,$y = 0$ and center of the bubble $xs = 0$) we have:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[3(N'(rs))^2 \right] \tag{11}$$

Note that in the above expressions for the negative energy density the warp drive speed vs appears raised to a power of 2 and it is being multiplied by the square derivative of the shape function. Considering our Natario warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros!!!The planet Earth have a mass[14] of about $6 \times 10^{24} kg$

[11]In the bubble radius the presented value for the Natario shape function in the numerical plots of the previous pages is 0,5 but actually is a value between $0 < N(rs) < \frac{1}{2}$ but very close to 0,5.Again we are limited by the floating-point precision of our software

[12]see Appendix D in [16]

[13]see Appendix C

[14]see Wikipedia:The free Encyclopedia

This term is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!or better:The amount of negative energy density needed to sustain a warp bubble at a speed of 200 times faster than light requires the magnitude of the masses of 1.000.000.000.000.000.000.000.000 planet Earths!!!

Note that if the negative energy density is proportional to 10^{48} this would render the warp drive impossible but fortunately the term 10^{48} is being multiplied the square derivative of the shape function and in the Natario case the square derivative of the shape function possesses values of 10^{-102} or 10^{-103} completely obliterating the factor 10^{48} making the warp drive negative energy density more "affordable" because $10^{48} \times 10^{-102} = 10^{-54} \ \frac{Joules}{meters^3}$ a very low and affordable negative energy density.So in order to get a physically feasible Natario warp drive the square derivative of the Natario shape function must obliterate the factor 10^{48} and fortunately this is really happening with our chosen shape function .

For a detailed study of the derivatives of first and second order of the Natario shape function see pgs 10 to 41 in [16].

3 The average matter density of the Interstellar Medium(IM)

A very serious drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons.(see [5],[7] and [8])

In the previous sections we briefly resumed how the negative energy density in the Natario warp drive spacetime can be greatly lowered from 10^{48} to 10^{-55} or 10^{-54} $\frac{Joules}{meters^3}$

The warp factor WF not only squeezes the negative energy density into a very thin region almost centered over the radius of the bubble but also reduces the amount of negative energy density needed to sustain a warp bubble from impossible levels to "affordable" results.

But all we did was only a mathematical demonstration of how far can we go in the reduction of the negative energy density levels by manipulating the warp factor WF. Amounts of 10^{-55} or 10^{-54} $\frac{Joules}{meters^3}$ although desirable are completely unrealistic considering a live scenario for an interstellar travel.

The reason for the statement pointed above is the existence of the so-called Interstellar Medium[15].Interstellar Medium(IM) is mainly composed by 99 percent of gas and 1 percent of dust.[16]

For the gas 91 percent are hydrogen atoms $8,9$ percent are helium atoms and $0,1$ percent are elements heavier than hydrogen or helium.

In dense regions the IM matter is primarily in molecular form and reaches densities of 10^6 molecules per cm^3 while in diffuse regions the density is low by the order of 10^{-4} molecules per cm^3.Compare this with a density of 10^{19} molecules per cm^3 for the air at sea level or 10^{10} molecules per cm^3 for a laboratory vacuum chamber.

This means to say that the IM even in dense regions is 10^{13} times lighter than the air at sea level or better 10.000.000.000.000 times(10 trillion times) lighter than the air at sea level or 10.000 times lighter than the best vacuum chambers.

Working with cubic meters we would have for the IM the numbers of 10^{12} molecules per m^3 in dense regions and 10^2 molecules per m^3 in diffuse regions.

Since 99 percent of the IM is gas and from the gas 91 percent is hydrogen then we can use only the hydrogen atom in the following considerations and from the hydrogen atom we can use only the proton with a mass of about 10^{-27} kilograms neglecting the electron which have a much lighter mass of 10^{-31} kilograms.

Then working with mass densities of kilograms per cubic meters we would have for the IM the numbers of 10^{-15} kilograms per m^3 in dense regions and 10^{-25} kilograms per m^3 in diffuse regions.

[15] see Wikipedia the free Encyclopedia

[16] see Appendices L and M for the composition of the Interstellar Medium IM)

In terms of energy densities of Joules per cubic meters we would have for the IM the numbers of 10 Joules per m^3 in dense regions and 10^{-9} Joules per m^3 in diffuse regions.

By comparison a mass density of 1 kilogram per cubic meter means an energy density of about 10^{16} Joules per cubic meter.

The negative energy density in the Natario warp drive 3 + 1 spacetime is given by the following expressions(pg 5 in [2]):

$$\rho_{3+1} = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(N'(rs))^2\cos^2\theta + \left(N'(rs) + \frac{rs}{2}N''(rs)\right)^2\sin^2\theta\right]. \tag{12}$$

The equation above can be divided in two expressions as shown below:

$$\rho_{3+1} = \rho_1 + \rho_2 \tag{13}$$

$$\rho_1 = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(N'(rs))^2\cos^2\theta\right] \tag{14}$$

$$\rho_2 = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[\left(N'(rs) + \frac{rs}{2}N''(rs)\right)^2\sin^2\theta\right] \tag{15}$$

From [5],[7] and [8] we know that if a ship travelling at 200 times light speed collides with even a single photon in interstellar space the result would be catastrophic to the physical integrity of the ship and crew members not to mention speeds of 10.000 times faster than light.

Note this as a very important fact:The energy density in the Natario warp drive is being distributed around all the space involving the ship(above the ship $\sin\theta = 1$ and $\cos\theta = 0$ while in front of the ship $\sin\theta = 0$ and $\cos\theta = 1$).The negative energy in front of the ship must "deflect" particles or photons in order to avoid these to reach the ship inside the bubble.[17].

-)-Energy directly above the ship($y - axis$)

$$\rho_2 = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[\left(N'(rs) + \frac{rs}{2}N''(rs)\right)^2\right] \tag{16}$$

-)-Energy directly in front of the ship($x - axis$)

$$\rho_1 = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(N'(rs))^2\right] \tag{17}$$

[17] see Appendices E and F

Applying even sample Newtonian concepts we know that positive masses always attract positive masses and negative masses always attracts negative masses[18] but in interactions between positive and negative masses one repels the other.[19]

This repulsive behavior of a negative mass or a negative mass density or a negative energy density useful to deflect hazardous incoming particles from the IM is a key ingredient to protect the ship integrity and the crew members in the scenario of a real superluminal interstellar spaceflight.

The positive energy density of the IM is 10 Joules per m^3 in dense regions and 10^{-9} Joules per m^3 in diffuse regions.However in the previous section we arrived at following results of 10^{-55} or 10^{-54} $\frac{Joules}{meters^3}$ for the negative energy density of the Natario warp drive spacetime.

From above we can see that the results obtained for the Natario warp drive negative energy density are much lighter when compared to the IM energy density.A Natario warp drive with such negative energy density requirements would never be able to deflect incoming particles from the IM because in such warp drive the negative energy density is less denser or lighter than the energy density of the IM.

But remember again that all we did was only a mathematical demonstration of how far can we go in the reduction of the negative energy density levels by manipulating the warp factor WF.We used a large WF.Of course we dont need a WF of such magnitude.A smaller WF can still obliterate values of 10^{48} while providing a negative energy density denser of heavier than the density of the IM.

A denser of heavier Natario warp drive energy density when compared to the IM density would be able to deflect the incoming hazardous particles protecting the ship and the crew members.We elaborated an empirical formula to do so:

The two key ingredients in a superluminal interstellar spaceflight are the following ones:

- 1)-spaceship velocity
- 2)-IM density

As fast is the spaceship velocity or as denser is the IM the problem of impacts against hazardous particles becomes more and more serious.Considering velocities of about 200 times light speed enough to reach star systems at 20 light-years away from Earth the ideal amount of negative energy density would then be given by the empirical formula shown below:

$$\rho_{3+1} = -1 \times (|\rho_{IM}| \times |\frac{vs}{c}|) \tag{18}$$

In the formula above ρ_{3+1} is the desired negative energy density in the Natario warp drive $|\rho_{IM}|$ is the modulus of the IM density and finally $|\frac{vs}{c}|$ is the modulus of the Machian coefficient for the multiples of the light speed in the spaceship velocity.

[18]the product of two negative masses in the Newton Law of Gravitation is also positive
[19]a minus sign arises in the product of a positive mass by a negative mass in the Newton Law of Gravitation

The positive energy density of the IM is 10 Joules per m^3 in dense regions and 10^{-9} Joules per m^3 in diffuse regions.

Applying the empirical formula of the previous page considering a spaceship velocity of 10.000 times light speed we would get for the desired Natario warp drive negative energy density results the values of -10^6 Joules per m^3 in dense regions of IM and -10^{-4} Joules per m^3 in diffuse regions of IM.

Note that even in dense regions of the IM the corresponding Natario warp drive negative energy density in modulus is 10^{10} times lighter or 10.000.000.000(10 billion) times lighter than the density of 1 kilogram per cubic meter

From the statements pointed above we can take the following important conclusions:

- 1)-A negative energy density lighter or less denser in modulus when compared to the IM density will not have strength enough to deflect hazardous incoming IM particles
- 2)-The modulus of the negative energy density in the Natario warp drive in order to have strength enough to deflect incoming hazardous IM particles must be denser or heavier than the IM density and must exceed the density of the IM by a safe margin because although we used only hydrogen atoms in this study the IM is not only hydrogen but also contains space dust debris etc.[20] The multiplication of the IM density by the multiples of the light speed in the spaceship velocity provides this margin.[21]

[20]see Appendices L and M for the composition of the Interstellar Medium IM)
[21]see Appendix F for a real Natario warp drive in interstellar spaceflight)

4 The Natario-Broeck warp drive spacetime

From the previous section we know that the collisions between the outermost layers of the warp bubble and the IM particles is one of the most serious problems a warp drive spaceship must solve in the first place.

Remember that for a warp bubble with a radius R of 100 meters the total surface area is $S = 4\pi R^2$ and the front of the bubble exposed to the collisions against the IM particles have a surface area[22] of $S = 2\pi R^2$.In this case the area exposed to collisions have multiples of 100 square meters approximately 628 square meters considering $S = 2\pi R^2$ and this is a large surface area[23] suited to be heavily bombarded by the dangerous IM particles.

Of course we are counting on the negative energy in front of the spaceship with repulsive gravitational behavior to deflect these incoming IM particles but the ideal result would be the reduction of the surface area of the bubble exposed to collisions.

Our idea is to keep the surface area of the bubble exposed to collisions microscopically small avoiding the collisions with the IM particles while at the same time expanding the spatial volume inside the bubble to a size larger enough to contains a spaceship inside.

Some years ago in 1999 Broeck appeared with exactly this idea.(see pg 3 in [10]).Broeck applied to the Alcubierre original warp drive metric spatial components a new mathematical term $B(rs)$ able to do so as shown below(see eq 3 pg 3 in [10])[24]

$$ds^2 = -dt^2 + B^2(r_s)[(dx - v_s(t)f(r_s)dt)^2 + dy^2 + dz^2]. \tag{19}$$

changing the signature from $(-,+,+,+)$ to $(+,-,-,-)$ we have:

$$ds^2 = dt^2 - B^2(r_s)[(dx - v_s(t)f(r_s)dt)^2 + dy^2 + dz^2]. \tag{20}$$

Broeck created inside the warp bubble of radius R a spatial distortion of radius R_b being R_b microscopically small when seen from outside but inside the sphere generated by this R_b a large internal volume with the size enough to contains a spaceship can easily be accommodated.(see also pg 19 in [15])[25]

Applying the Broeck mathematical term $B(rs)$ to the spatial components of the Natario warp drive equation using the signature $(+,-,-,-)$ we get the following result:[26]

$$ds^2 = dt^2 - B(rs)^2[(drs - X^{rs}dt)^2 + (rs^2)(d\theta - X^\theta dt)^2] \tag{21}$$

[22] the front of the bubble is exposed to the IM particles not the rear
[23] in this case we consider $\pi = 3,14$
[24] do not confuses this term $B(rs)$ with the term B used by ourselves to differentiate the Natario shape function.
[25] see Appendix J in [16]
[26] see Appendix K

The Broeck spacetime distortion generated by the term $B(rs)$ in which the external circle surface area of the distortion seen by observers in our Universe is microscopically small while at the same time the internal spherical spatial volume inside the distortion is very large able to contains a man or s spaceship is well graphically presented aa a bottle(the Broeck bottle).[27]

According to Broeck this term $B(rs)$ have the following behavior:(see pgs 3 and 4 in [10])[28]

$$B(rs) = \begin{cases} 1+\alpha & rs < R_b \\ 1 < B(rs) \leq 1+\alpha & R_b \leq rs < R_b + \Delta_b \\ 1 & rs \geq R_b + \Delta_b \end{cases} \tag{22}$$

Considering $rs = 0$ the center of the warp bubble with radius R and R_b being the microscopically small outer radius of the Broeck bottle bottleneck circle when seen from outside the bottle but still inside the warp bubble we can analyze the expression above as follows:

In the region where $rs < R_b$ well inside the Broeck bottle the value of $B(rs)$ is very large generating the large spherical internal volume of the bottle and is given by $B(rs) = 1 + \alpha$ being α arbitrarily large.Broeck chooses for α the value of 10^{17}(see pg 5 in in [10]).We choose for α the value of 10^{27} a value 10^{10} or 10 billion times higher than the original Broeck value.Note that $B(rs)$ inside the bottle possesses always the same constant value which means to say that inside the bottle the derivatives of $B(rs)$ are always zero.

In the region where $R_b \leq rs < R_b + \Delta_b$ well exactly over the Broeck bottle bottleneck external circle and its neighborhoods the value of $B(rs)$ is given by $1 < B(rs) \leq 1 + \alpha$.This is the region where $B(rs)$ decreases from $B(rs) = 1 + \alpha$ to $B(rs) = 1$ but never reaching the value of 1 and Δ_b delimitates the thickness of this region as a thin shell in the neighborhoods of the Broeck bottle bottleneck circle.In this region the derivatives of $B(rs)$ are not zero generating an energy density given by the following equation given in Geometrized Units $c = G = 1$ as follows:(see eq 11 pg 6 in [10])

$$T_{\hat{\mu}\hat{\nu}} u^{\hat{\mu}} u^{\hat{\nu}} = T^{\hat{0}\hat{0}} = \frac{1}{8\pi} \left(\frac{1}{B^4} (\partial_r B)^2 - \frac{2}{B^3} \partial_r \partial_r B - \frac{4}{B^3} \partial_r B \frac{1}{r} \right). \tag{23}$$

Finally in the region where $rs \geq R_b + \Delta_b$ well outside the Broeck bottle bottleneck circle we recover the normal space of our Universe where the value of $B(rs)$ is always 1 and hence its derivatives are again zero.

The region where the spacetime geometry is not flat is the region around the Broeck bottle bottleneck (The Broeck bottle bottleneck is the transition region between the large inner space inside the Broeck bottle where $B(rs)$ possesses the value of $B(rs) = 1 + \alpha$ and our Universe where $B(rs)$ always possesses the value of 1) which means to say the region where $R_b \leq rs < R_b + \Delta_b$ with $B(rs)$ possessing the values of $1 < B(rs) \leq 1+\alpha$ but never reaching the value of 1 according to the Broeck criteria shown above.

We are interested in the behavior of $1 \leq B(rs) \leq 1 + \alpha$ decreasing its value from $B(rs) = 1 + \alpha$ to $B(rs) = 1$ using analytical functions[29].Note that in our redefinition of the Broeck bottle bottleneck $B(rs)$ reaches the value of 1 even inside the bottleneck.

[27]see Appendices H and I

[28]see Appendix G

[29]continuous and differentiable in every point of the domain

An elegant way to generate a continuous decrease from $B(rs) = 1 + \alpha$ to $B(rs) = 1$ can be achieved if we consider a second version of the original Alcubierre shape function redefined using the Broeck bottle bottleneck circle radius R_b as follows:[30].

$$f_b(rs) = \frac{1}{2}[1 - tanh[@(rs - R_b)] \tag{24}$$

Note that in this scenario we have two original Alcubierre shape functions:the first function $f(rs)$ was defined in section 2 to generate the Natario shape function $N(rs)$ and also the Natario warp bubble and the second function $f_b(rs)$ defined above generates the Broeck bottle with a radius R_b being R_b the microscopically small outer radius of the Broeck bottle bottleneck circle when seen from outside the bottle but still inside the warp bubble.Remember that the following condition must always be obeyed;$R_b << R$

Broeck chooses for α the value of 10^{17}(see pg 5 in in [10]).We choose for α the value of 10^{27} a value 10^{10} or 10 billion times higher than the original Broeck value.According to Broeck a value of $\alpha = 10^{17}$ for a bottle of bottleneck outer radius $R_b = 10^{-15}$ meters in a warp bubble of radius $R = 3 \times 10^{-15}$ meters can accommodate a bottle with 200 meters of inner diameter.Our $\alpha = 10^{27}$ could perfectly well accommodate a bottle with 200 kilometers of inner diameter in the same circumstances.

According with Alcubierre any function $f_b(rs)$ that gives 1 inside the bottle and 0 outside the bottle while being $1 > f_b(rs) > 0$ in the bottleneck of the bottle[31] is a valid shape function for the Broeck bottle spacetime distortion.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2]).

The analytical behavior of $1 \leq B(rs) \leq 1 + \alpha$ decreasing its value from $B(rs) = 1 + \alpha$ to $B(rs) = 1$ using analytical functions can easily be achieved if we adopt the following equation for the definition of $B(rs)$ using the second original Alcubierre shape function $f_b(rs)$.

$$B(rs) = 1 + \alpha f_b(rs) \tag{25}$$

Inside the Broeck bottle $f_b(rs) = 1$ and $B(rs) = 1 + \alpha$.Outside the Broeck bottle $f_b(rs) = 0$ and $B(rs) = 1$.In these regions the derivatives of $B(rs)$ are always 0 because the values of $B(rs)$ are always constant.[32]

The region where the derivatives of $B(rs)$ are not 0 due to the values of a variable $B(rs)$ as being $1 \leq B(rs) \leq 1 + \alpha$ which means to say the region where $1 > f_b(rs) > 0$ is the bottleneck of the Broeck bottle.

[30] $tanh[@(rs + R_b)] = 1$,$tanh(@R_b) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R_b|$

[31] Remember that in this case the second Alcubierre shape function $f_b(rs)$ is being used to generate the Broeck bottle not the warp bubble.The warp bubble is being generated by the Natario shape function $N(rs)$ using the first Alcubierre shape function $f(rs)$.Note that both $f(rs)$ and $f_b(rs)$ have mathematical structures that resembles each other.One structure gives 1 inside the bubble and 0 outside the bubble while the other structure gives 1 inside the bottle and 0 outside the bottle.

[32] The derivatives of $f_b(rs)$ in these regions are too much close of 0 and can be neglected.

The energy density in the Natario warp drive is being distributed around all the space involving the ship.The negative energy in front of the ship must "deflect" particles or photons in order to avoid these to reach the ship inside the bubble.[33].

Our idea is to have a value of $\alpha = 10^{27}$ for a Broeck bottle of bottleneck outer radius $R_b = 10^{-15}$ meters inside a Natario warp bubble of radius $R = 3 \times 10^{-15}$ meters.Our $\alpha = 10^{27}$ could perfectly well accommodate a bottle with 200 kilometers of inner diameter inside the Broeck bottle.

And a Natario warp bubble with "only" a radius R of $R = 3 \times 10^{-15}$ meters will have a surface area exposed to collisions against the IM particles much smaller than the area of a Natario warp bubble with 100 meters of radius.[34].

A detailed study of the Natario-Broeck warp drive can be found in [16].

[33] see Appendices E and F
[34] see Appendices G H and I

5 The analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu applied to the Natario-Broeck spacetime.A very interesting approach towards a more realistic interstellar warp drive

The problem of collisions between a warp drive spaceship moving at superluminal velocity and the potentially dangerous particles from the Interstellar Medium IM is not new.[35].

It was first noticed in 1999 in the work of Chad Clark,Will Hiscock and Shane Larson [3].Later on in 2010 it appeared again in the work of Carlos Barcelo,Stefano Finazzi and Stefano Liberatti [4].In 2012 the same problem of collisions against hazardous IM particles appeared in the work of Brendan McMonigal,Geraint Lewis and Philip O'Byrne [17].

However all these works covered the Alcubierre warp drive spacetime that do not possess negative energy in front of the ship in the equatorial plane on the x-axis to deflect incoming hazardous IM particles.[36].[37].[38].

On the other hand we know that the Natario warp drive spacetime possesses negative energy in front of the ship in the equatorial plane on the x-axis to deflect the dangerous IM particles protecting the ship so the restrictions covered in the works [3],[4] and [17] do not apply in the Natario case.See [5] and [7] for the Natario case. [39].

Recently a very interesting work appeared:[18].It covers the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu applied to the Alcubierre warp drive spacetime.But the most important fact:their analysis also applies to the Natario warp drive spacetime.We will study the section 5 pg 10 in [18] as a first approach.

According to pg 10 in [18] the major part of the incoming IM particles or photons will move towards the spaceship in a trajectory over or very close to the x-axis.These photons or particles applies over the surface of the warp bubble a pressure directly proportional to the bubble speed vs and as fast the ship or the bubble moves then the pressure becomes more and more stronger and this is a very serious problem that exists in both the Alcubierre and Natario warp drive spacetimes.

This is due to the fact that both bubbles when moving across interstellar space possesses Cross-Sections and the area of each Cross-Section wether in Alcubierre or Natario geometries is exposed to impacts against the hazardous IM particles and these impacts generates the mentioned pressure.As large the areas of each Cross-Section are then the surfaces exposed to impacts becomes larger too.[40]

The Natario geometry possesses negative energy in front of the ship able to deflect these IM particles but with a large surface area for the Cross-Section.The ideal scenario is provided by the Natario-Broeck spacetime in which the Cross-Section possesses a microscopically small surface area effectively protecting the ship against impacts thereby reducing the interstellar pressure.

[35]see Appendices L and M
[36]see Appendix C in [5]
[37]see Appendix K in [7]
[38]see also fig 2 pg 4 in [4]
[39]see also Appendices C and E
[40]see Appendices A,B,J and N

The pressure applied by interstellar photons over the Cross-Section of a warp bubble wether in the Alcubierre or Natario spacetimes is given by;(see eq 5.2 pg 11 in [18])

$$dp_p = \frac{E_{av}}{4} \rho_p (\Delta A) \frac{vs}{c} dt \tag{26}$$

In the eq above dp_p is the total momentum of the photons travelling towards the Cross-Section area ΔA of the warp bubble in both Alcubierre or Natario spacetimes in an infinitesimal amount of time dt,E_{av} is the average energy of each interstellar photon,ρ_p is the "surface density" or the number of impacted photons scattered across the area ΔA of the Cross-Section and vs is the speed of the bubble.[41].

The "force" F exerted by the photons over the area ΔA of the Cross-Section would be given by;[42]

$$F = \frac{dp_p}{dt} = \frac{E_{av}}{4} \rho_p (\Delta A) \frac{vs}{c} \tag{27}$$

Note that the force is directly proportional to the area ΔA of the Cross-Section and also directly proportional to the speed of the bubble vs.As large the bubble is or as fast the bubble moves then the force exerted by the interstellar photons against the Cross-Section becomes stronger and stronger.

The pressure P applied by the interstellar photons over the area ΔA of the Cross-Section would be given by;

$$P = \frac{1}{\Delta A} \frac{dp_p}{dt} = \frac{E_{av}}{4} \rho_p \frac{vs}{c} \tag{28}$$

Note that the pressure is directly proportional to the speed of the bubble vs.As fast the bubble moves then the pressure exerted by the interstellar photons against the Cross-Section becomes stronger and stronger.

The pressure applied by interstellar dust over the Cross-Section of a warp bubble wether in the Alcubierre or Natario spacetimes is given by;(see eq 5.4 pg 12 in [18])

$$dp_d = P_{av} \rho_d (\pi \Delta A) vs dt \tag{29}$$

In the eq above dp_d is the total momentum of the dust particles travelling towards the Cross-Section area ΔA of the warp bubble in both Alcubierre or Natario spacetimes in an infinitesimal amount of time dt,P_{av} is the average momentum of each interstellar dust particle,ρ_d is the "surface density" or the number of impacted dust particles scattered across the area ΔA of the Cross-Section and vs is the speed of the bubble.

The "force" F exerted by the dust particles over the area ΔA of the Cross-Section would be given by;

$$F = \frac{dp_d}{dt} = P_{av} \rho_d (\pi \Delta A) vs \tag{30}$$

Like the previous case the force is directly proportional to the area ΔA of the Cross-Section and also directly proportional to the speed of the bubble vs.

[41] see Appendices A and N

[42] we do not consider here the reflection coefficient n of the spaceship.

The pressure P applied by the interstellar dust particles over the area ΔA of the Cross-Section would be given by;

$$P = \frac{1}{\Delta A}\frac{dp_d}{dt} = P_{av}\rho_d \pi vs \tag{31}$$

Like the previous case the pressure is directly proportional to the speed of the bubble vs.

A large area ΔA of a warp bubble Cross-Section with the size able to contains a spaceship wether in the Alcubierre or Natario case and a fast speed of the bubble vs required for suitable interstellar travel will submit the bubble to high forces and pressures generated by the impacts against the IM hazardous particles. [43] [44].

The Natario case have negative energy in the front of the bubble able to theoretically deflect incoming dangerous IM particles.[45].

But even so considering a realistic interstellar spaceflight for a star at 20 light-years away with a speed 200 times faster than light collisions against IM particles would certainly occurs even in the Natario case.

The Natario-Broeck spacetime would be better suitable for a realistic interstellar space travel because a Cross-Section radius R_p of 10^{-15} meters would generate a very microscopically small area ΔA of about 10^{-30} square meters less exposed to dangerous impacts from the IM particles when compared to a Natario only spacetime with a Cross-Section radius R_p of 10^2 meters with an area ΔA of about 10^4 square meters.

The term ρ_d as the "surface density" or the number of impacted dust particles scattered across the area ΔA of the Cross-Section would also be microscopically small in the Natario-Broeck spacetime when compared to the Natario only spacetime thereby contributing to reduces the forces and pressures exerted over a bubble of microscopical size.

In a real interstellar spaceflight for a star at 20 light-years away in a reasonable amount of time a warp drive spaceship must attain a speed of about 200 times faster than light. In this case the trip is done in some months not some years.However a Natario only warp bubble with 100 meters Cross-Section radius R_p would have a Cross-Section area ΔA of about 10^4 square meters and a speed vs of about 10^{10} meters/second.The product $\Delta A \times vs$ is then a number with a magnitude of about $10^4 \times 10^{10} = 10^{14}$.A number with 14 zeros !!! or better 100.000.000.000.000 one hundred trillion times bigger than 1 square meter moving at 200 times light speed with plenty of chances to suffer impact collisions against the hazardous IM particles. However considering the Natario-Broeck bubble with a radius R_p of about 10^{-15} meters would have a very microscopically small Cross-Section area ΔA of about 10^{-30} square meters and the product $\Delta A \times vs$ would be in this case $10^{-30} \times 10^{10} = 10^{-20}$ or a number of the magnitude of $\frac{1}{10^{20}} = \frac{1}{100.000.000.000.000.000.000}$ or about one hundred "quintillion" times smaller than 1 square meter moving at 200 times light speed better protecting without shadows of doubt the ship and the crew members from the hazardous IM medium.

The analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu proves definitely that the Natario-Broeck warp drive spacetime is the best candidate for a realistic interstellar space travel.

[43] see Appendices A,B,J and N

[44] see Appendices L and M

[45] see also Appendices C and E

6 Conclusion:

In this work we applied the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu to the Natario-Broeck warp drive spacetime.

We started this work with the definition of the Natario warp drive continuous shape function and we demonstrated that the negative energy density in the equatorial plane of the Natario warp bubble do not vanish and due to the gravitational repulsive behavior of the negative energy density this can provide protection against collisions with the Interstellar Medium IM that unavoidably would occur in a real superluminal spaceflight.

We discussed the Interstellar Medium IM and we arrived at the conclusion that the negative energy density of the warp bubble walls must be higher in modulus than the positive energy density of the IM in order to allow the gravitational repulsion of the IM particles by the warp bubble walls and we introduced the empirical formula to obtain the desirable amount of negative energy density needed to deflect the IM particles multiplying the modulus of the density of the IM by the Machian coefficient of the fraction $\frac{vs}{c}$ which means to say the multiples of the light speed c in the spaceship velocity vs.The negative energy density of the Natario warp drive must exceed this product in modulus.

Collisions between the walls of the warp bubble and the IM particles would certainly occur and although the negative energy density in front of the Natario warp bubble can theoretically protect the ship we borrowed the idea of Chris Van Den Broeck proposed some years ago in 1999 in order to increase the degree of protection.

Our idea is to keep the surface area of the bubble exposed to collisions microscopically small avoiding the collisions with the IM particles while at the same time expanding the spatial volume inside the bubble to a size larger enough to contains a spaceship inside.

Some years ago in 1999 Broeck appeared with exactly this idea.Broeck applied to the Alcubierre original warp drive metric spatial components a new mathematical term $B(rs)$ able to do so.

This term $B(rs)$ creates inside the Alcubierre or Natario warp bubble a spacetime distortion with the shape of a bottle in which the large inner space of the bottle volume with a large inner radius that can contains a spaceship inside the bottle is maintained isolated from the rest of the Universe and the only contact point between the bottle and the Universe is the bottle bottleneck with a microscopically small outer radius.Broeck created a bottle with 200 meters.We redefined the Broeck mathematical term $B(rs)$ using the original Alcubierre shape function in order to create a Broeck bottle with 200 kilometers of inner diameter maintaining the submicroscopic outer radius of the bottle bottleneck and a low energy density needed to create the bottle.

A submicroscopic outer radius of the bottle bottleneck being the only part in contact with our Universe would mean a submicroscopic surface exposed to the collisions against the IM particles thereby reducing the probabilities of dangerous impacts against large objects (comets asteroids etc) enhancing the protection level of the spaceship and hence the survivability of the crew members.

Any future development for the Natario warp drive must encompass the more than welcome idea of the Broeck bottle.

Lastly we discussed the very interesting work of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu applied to the Natario-Broeck warp drive spacetime. This analysis demonstrates that the interstellar medium IM applies over any warp drive bubble a pressure and a force directly proportional to the area of the warp bubble Cross-Section and also directly proportional to the warp bubble speed independently of the chosen warp drive geometry wether in the Alcubierre or Natario warp drive spacetimes. This pressure is the results of the IM particles dangerous impacts against the Cross-Section of the warp bubble and as fast the warp bubble goes by or as large tbe warp bubble is this problem becomes very serious.

Fortunately the Natario-Broeck warp drive spacetime have a microscopically small Cross-Section area and these hazardous IM particles impacts are unlikely to occurs.The work of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu demonstrates that the Natario-Broeck warp drive spacetime is until now the best candidate for a realistic interstellar space travel.

But unfortunately although we can discuss mathematically how to reduce the negative energy density requirements to sustain a warp drive we dont know how to generate the shape function that distorts the spacetime geometry creating the warp drive effect.So unfortunately all the discussions about warp drives are still under the domain of the mathematical conjectures.

However we are confident to affirm that the Natario-Broeck warp drive will survive the passage of the Century XXI and will arrive to the Future.The Natario-Broeck warp drive as a valid candidate for faster than light interstellar space travel will arrive to the the Century $XXIV$ on-board the future starships up there in the middle of the stars helping the human race to give his first steps in the exploration of our Galaxy

Live Long And Prosper

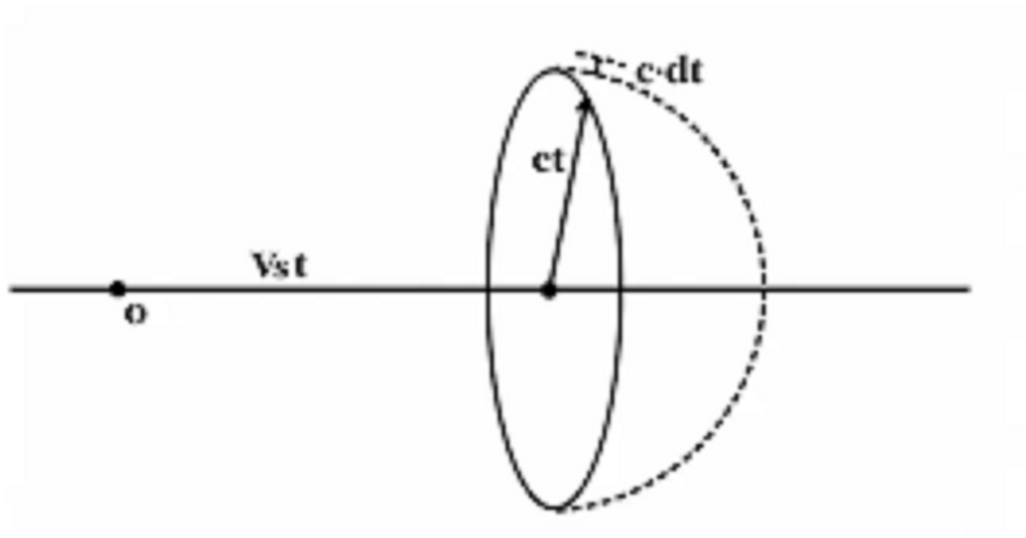

Figure 1: The Cross-Section of the warp bubble fig 9 pg 11 in [18] (Source:Internet)

7 Appendix A:The Cross-Section of the warp bubble in the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu

Above is being presented the Cross-Section of the warp bubble with the radius $R_p = ct$ and area ΔA.The point o is the center of the bubble where the spaceship is placed and the distance between the center of the shell and the center of the bubble is vst.As large the Cross-Section radius R_p[46] is then the surface area ΔA grows proportionally.

Note that the ΔA have a form of a "dish" or a "disk" and this form will suffer the collisions against the hazardous IM particles or photons.

This area ΔA is independent of the chosen warp drive geometry and it is always present wether we choose the Alcubierre or the Natario geometries.But for the Natario case the negative energy of the bubble in front if the ship deflects the dangerous IM particles.The collisions with the IM particles would generate two "streams" or flows of particles that forms a "D",or a "C" or a "U" curves.As large the area ΔA is the width of the curve grows proportionally.

But for the case of the Natario-Broeck spacetime the Cross-Section radius R_p of 10^{-15} meters would generate a very small area ΔA of about 10^{-30} square meters less exposed to dangerous impacts when compared to a Natario only spacetime with a Cross-Section radius R_p of 10^2 meters with an area ΔA of about 10^4 square meters

[46]the radius of the bubble is R

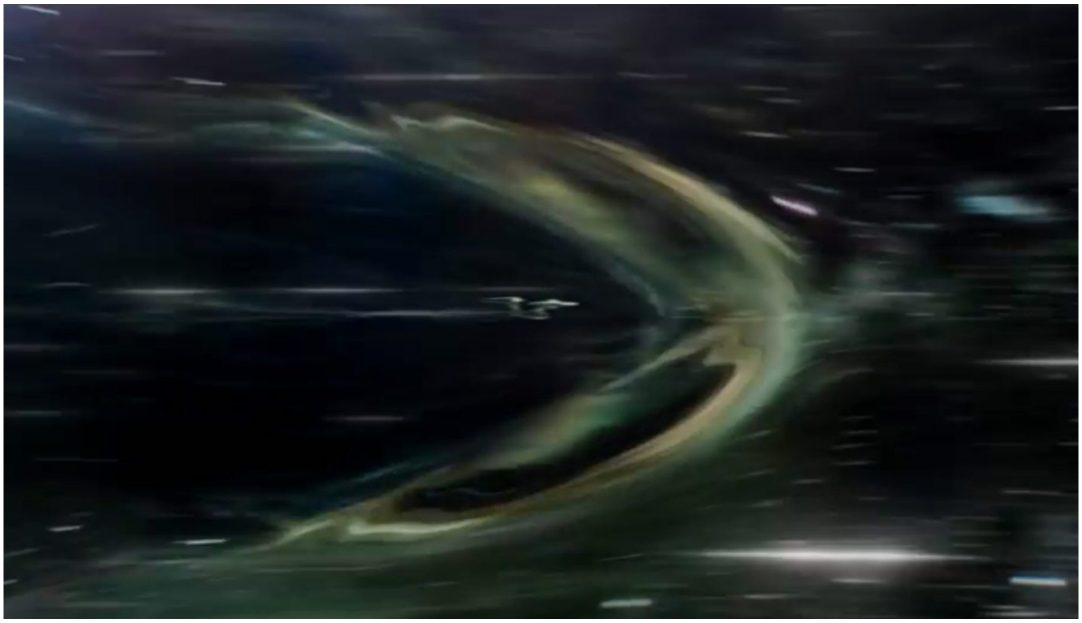

Figure 2: Artistic Presentation of a Cross-Section D curve (Source:Internet)

8 Appendix B:Artistic Presentation of a Cross-Section D curve

The image above borrowed from science-fiction depicts a spaceship in an interstellar spaceflight suffering the collision against the IM particles.As large the surface area of the Cross-Section is the width of the D curve grows proportionally and so the distance between the "upper" and "lower" particle "streams" of the D curve wether and independently from the geometries of Alcubierre or Natario.

In the Natario-Broeck spacetime the size and the width of the D curve would be microscopically small effectively protecting the ship against the hazardous IM particles.

9 Appendix C:The Natario warp drive negative energy density in Cartezian coordinates

The negative energy density according to Natario is given by(see pg 5 in [2])[47]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{32}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{33}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{34}$$

[47] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

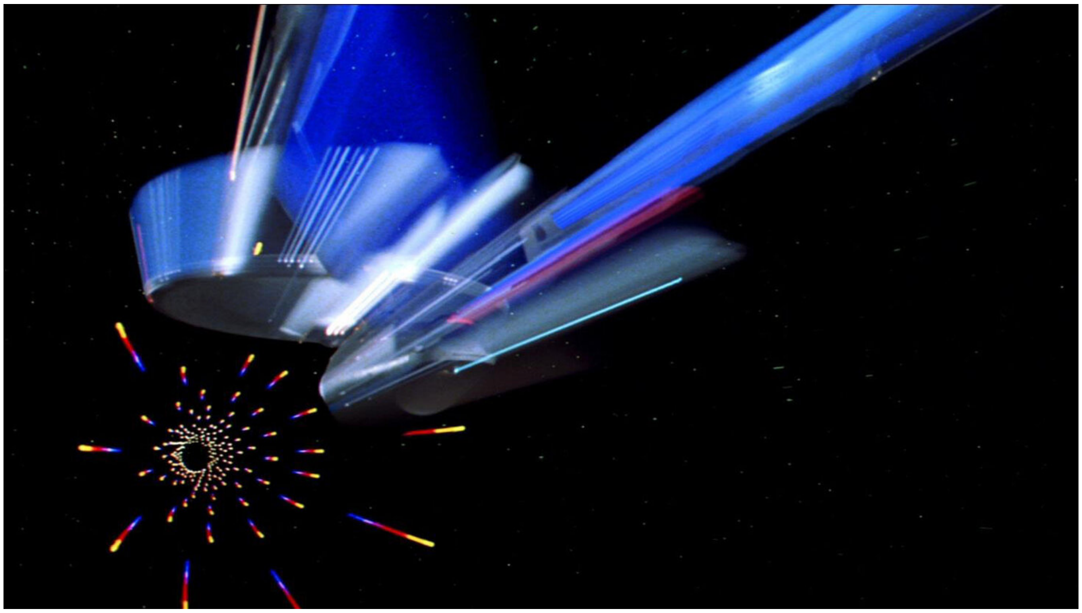

Figure 3: Artistic Presentation of a warp drive spaceship in interstellar space colliding with highly Doppler-Blueshifted photons(Source:Internet)

10 Appendix D:Artistic Presentation of a warp drive spaceship in interstellar space colliding with highly Doppler-Blueshifted Photons

The picture above borrowed from science-fiction depicts one of the most serious(and dangerous) problems a spaceship would confront in a realistic interstellar travel:Collisions with highly Doppler-Blueshifted photons.This problem was first pointed out in 1999 in the work of Chad Clark,Will Hiscock and Shane Larson [3].In 2010 it appeared again in the work of Carlos Barcelo,Stefano Finazzi and Stefano Liberatti [4].In 2012 the same problem of collisions against hazardous IM photons appeared in the work of Brendan McMonigal,Geraint Lewis and Philip O'Byrne [17].

But all these works covered the Alcubierre warp drive spacetime only and this geometry do not possess negative energy in front of the ship in the equatorial plane on the x-axis to deflect incoming hazardous IM photons.[48].[49].[50].

On the other hand we know that the Natario spacetime geometry possesses negative energy in front of the ship in the equatorial plane on the x-axis to deflect the dangerous IM photons protecting the ship so the works [3],[4] and [17] do not apply in the Natario case.See [5] and [7] for the Natario case. [51].

[48]see Appendix C in [5]
[49]see Appendix K in [7]
[50]see also fig 2 pg 4 in [4]
[51]see also Appendices C and E

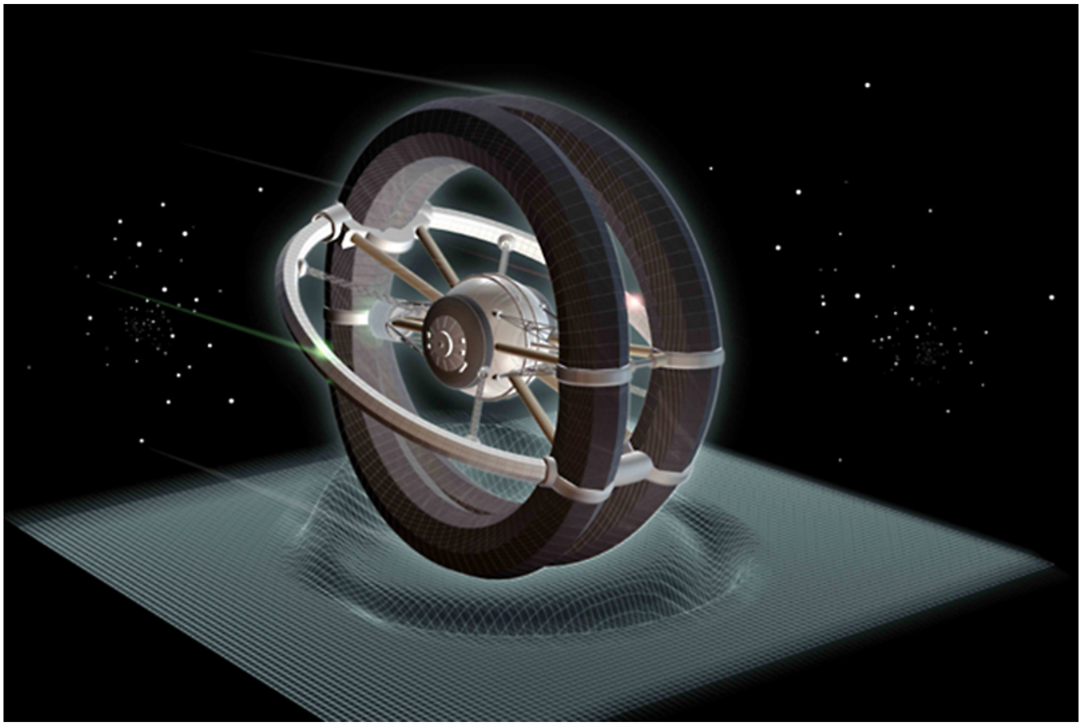

Figure 4: Artistic representation of the Natario warp drive .Note in the bottom of the figure the Alcubierre expansion of the normal volume elements .(Source:Internet)

11 Appendix E:Artistic Presentation of the Natario warp drive

According to the geometry of the Natario warp drive the spacetime contraction in one direction(radial) is balanced by the spacetime expansion in the remaining direction(perpendicular).(pg 5 in [2]).

The expansion of the normal volume elements in the Natario warp drive is given by the following expressions(pg 5 in [2]).

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r)\cos\theta \tag{35}$$

$$K_{\theta\theta} = \frac{1}{r}\frac{\partial X^\theta}{\partial\theta} + \frac{X^r}{r} = v_s n'(r)\cos\theta; \tag{36}$$

$$K_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial X^\varphi}{\partial\varphi} + \frac{X^r}{r} + \frac{X^\theta\cot\theta}{r} = v_s n'(r)\cos\theta \tag{37}$$

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{38}$$

If we expand the radial direction the perpendicular direction contracts to keep the expansion of the normal volume elements equal to zero.

This figure is a pedagogical example of the graphical presentarion of the Natario warp drive.

The "bars" in the figure were included to illustrate how the expansion in one direction can be counterbalanced by the contraction in the other directions.These "bars" keeps the expansion of the normal volume elements in the Natario warp drive equal to zero.

Note also that the graphical presentation of the Alcubierre warp drive expansion of the normal volume elements according to fig 1 pg 10 in [1] is also included

Note also that the energy density in the Natario warp drive $3+1$ spacetime being given by the following expressions(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(r))^2 \cos^2\theta + \left(n'(r) + \frac{r}{2} n''(r) \right)^2 \sin^2\theta \right]. \tag{39}$$

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2} \right)^2 \sin^2\theta \right]. \tag{40}$$

Is being distributed around all the space involving the ship(above the ship $\sin\theta = 1$ and $\cos\theta = 0$ while in front of the ship $\sin\theta = 0$ and $\cos\theta = 1$).The negative energy in front of the ship "deflect" photons or other particles so these will not reach the ship inside the bubble.The illustrated "bars" are the obstacles that deflects photons or incoming particles from outside the bubble never allowing these to reach the interior of the bubble.[52]

-)-Energy directly above the ship($y - axis$)

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[\left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2} \right)^2 \sin^2\theta \right]. \tag{41}$$

-)-Energy directly in front of the ship($x - axis$)

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta \right]. \tag{42}$$

Note also that even in a $1+1$ dimensional spacetime the Natario warp drive retains the zero expansion behavior:

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r) \cos\theta \tag{43}$$

$$K_{\theta\theta} = \frac{X^r}{r} = v_s n'(r) \cos\theta; \tag{44}$$

$$K_{\varphi\varphi} = \frac{X^r}{r} = v_s n'(r) \cos\theta \tag{45}$$

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{46}$$

In all these equations the term r is our term rs

[52] See also Appendix F

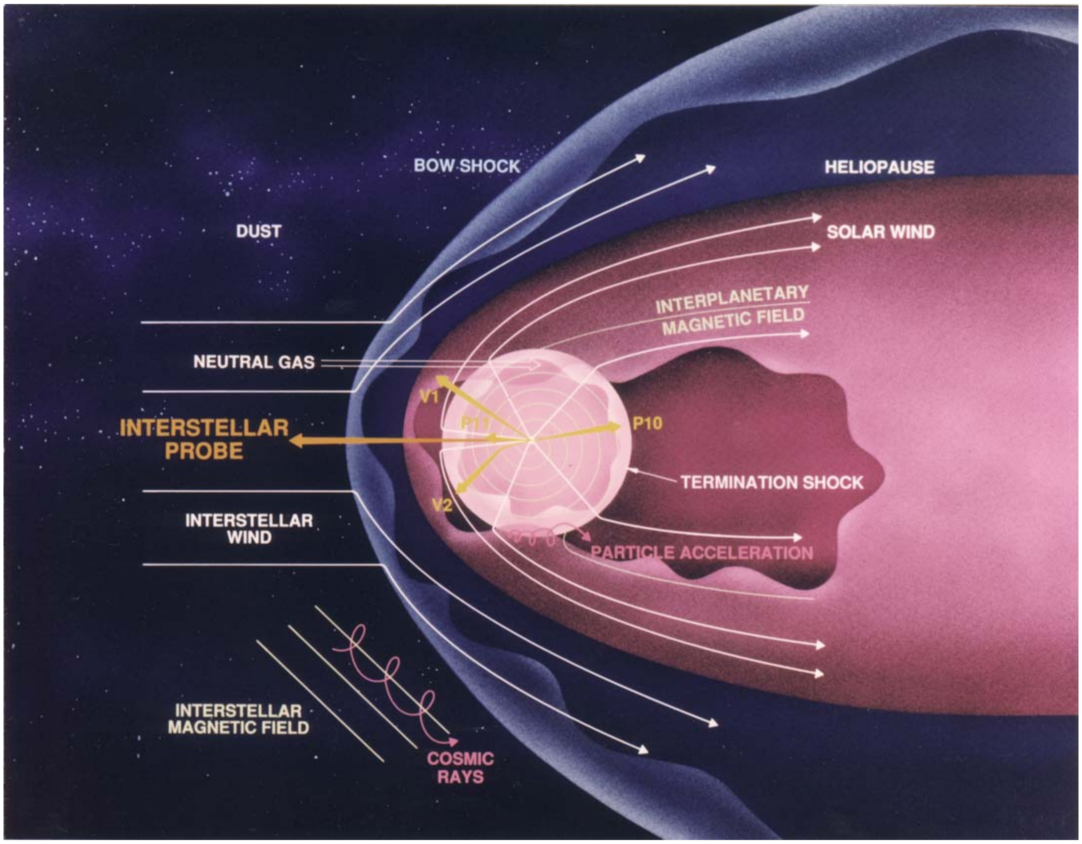

Figure 5: Artistic representation of a Natario warp wrive in a real superluminal space travel .Note the negative energy in front of the ship deflecting incoming hazardous interstellar matter(brown arrows).(Source:Internet)

12 Appendix F:Artistic Presentation of a Natario warp drive in a real faster than light interstellar spaceflight

Above is being presented the artistic presentation of a Natario warp drive in a real interstellar superluminal travel.The "ball" or the spherical shape is the Natario warp bubble with the negative energy surrounding the ship in all directions and mainly protecting the front of the bubble.[53]

The brown arrows in the front of the Natario bubble are a graphical presentation of the negative energy in front of the ship deflecting interstellar dust,neutral gases,hydrogen atoms,interstellar wind photons etc.[54]

The spaceship is at the rest and in complete safety inside the Natario bubble.

In order to allow to the negative energy density of the Natario warp drive the deflection of incoming hazardous particles from the Interstellar Medium(IM) the Natario warp drive energy density must be heavier or denser when compared to the IM density.

[53]See Appendix E
[54]see Appendices L and M for the composition of the Interstellar Medium IM)

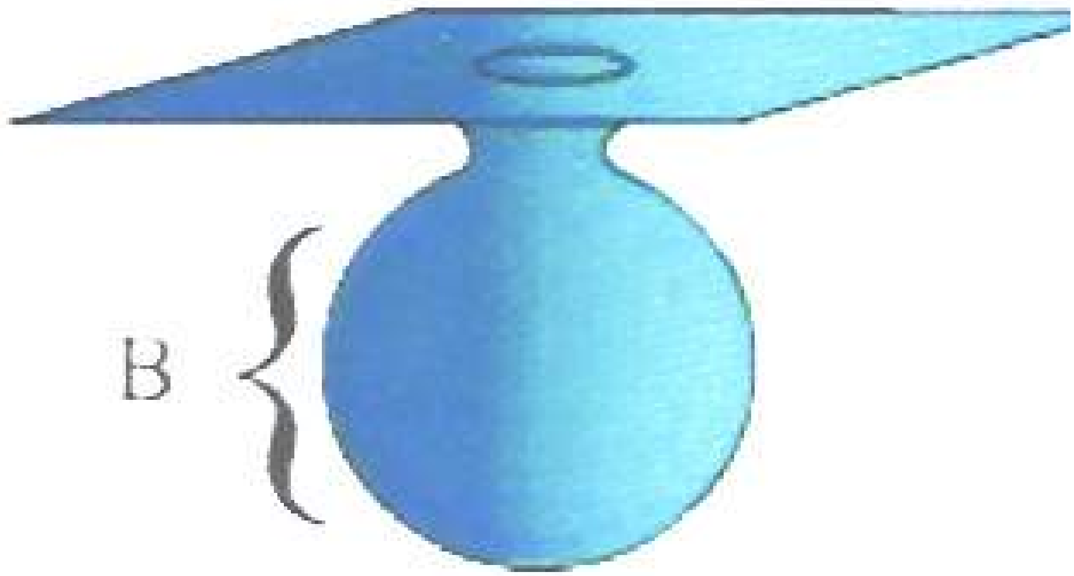

Figure 6: Artistic representation of the Broeck "pocket" or "bottle" with the Broeck coefficient $B(rs)$ shown.(Source:Internet)

13 Appendix G:Artistic Presentation of the Broeck "pocket" or "bottle"

Broeck proposed the idea to keep the surface area of the bubble microscopically small while at the same time expanding the spatial volume inside the bubble to a size larger enough to contains a spaceship inside. (see pg 3 in [10]).The "ball" in the figure above with a large internal volume is the Broeck bottle and the circle of the intersection point between the "ball" and the plane also shown in the figure is the circle of the small surface area(Broeck bottle bottleneck).Broeck created the term $B(rs)$ in order to accomplish this task. According to Broeck this term $B(rs)$ have the following behavior:(see pgs 3 and 4 in [10])

$$B(rs) = \begin{cases} 1+\alpha & rs < R_b \\ 1 < B(rs) \leq 1+\alpha & R_b \leq rs < R_b + \Delta_b \\ 1 & rs \geq R_b + \Delta_b \end{cases} \tag{47}$$

In the equation above the small outer radius R_b is the radius of the shown circle of the Broeck bottle bottleneck.This circle intersects the plane above the Broeck bottle and the plane represents our Universe.The term α according to Broeck have a large value of 10^{17}(pg 5 in [10]).We consider in this work a value of 10^{27}.

Considering the center $rs = 0$ of the bottleneck circle delimitated by the small outer radius R_b any point placed at a distance $rs < R_b$ is a point inside the Broeck bottle $B(rs) = 1+\alpha$ being α the term that generates the large internal volume of the Broeck bottle.

In the region $R_b \leq rs < R_b + \Delta_b$ the value of $B(rs)$ becomes $1 < B(rs) \leq 1 + \alpha$.This region is in the neighborhoods of the small outer radius R_b and is the region where $B(rs)$ decreases from the large value of $B(rs) = 1 + \alpha$ approaching the value of $B(rs) = 1$ but never reaching it.

The term Δ_b delimitates the thickness of the region where $B(rs)$ decreases.This region is a thin shell around the Broeck bottle bottleneck.

Finally in the region where $rs \geq R_b + \Delta_b$ far outside the Broeck bottle bottleneck we recover the normal space of our Universe (the plane above the Broeck bottle) in which $B(rs)$ always possesses the value of $B(rs) = 1$.

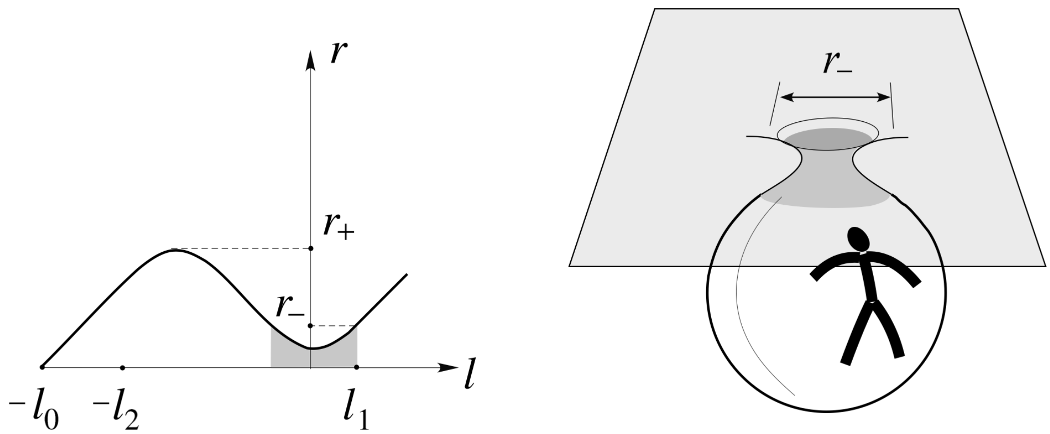

Figure 7: Alternative artistic representation of the Broeck "pocket" or "bottle" with a man shown inside the bottle. (Source:fig 5 pg 19 in [15])

14 Appendix H:Alternative Artistic Presentation of the Broeck "pocket" or "bottle"

The figure shown above represents exactly the point of view we are defending concerning the whole Broeck idea applied to the Natario warp drive in order to reduce the surface area exposed to collisions against the IM particles.

A Broeck bottle with a large internal radius r_+ large enough to contains a man os a spaceship is being graphically depicted.

This bottle intersects the bidimensional plane in the circle delimited by the outer radius r_- being this radius microscopically small.This circle is the bottleneck of the Broeck bottle.

The bidimensional plane represents our Universe and all the dangerous IM particles are contained only in this plane.

Therefore a Broeck bottle a sphere of a large internal radius r_+ able to accommodate a man or a spaceship would be seen by outside observers placed in the bidimensional plane representing our Universe as a circle with a microscopically small outer radius r_- being this circle the bottleneck of the Broeck bottle.(see pg 19 in [15]).

A microscopically small outer radius r_- the R_b in our equations delimitates a very small microscopically surface area therefore reducing the probability of collisions against the dangerous IM particles

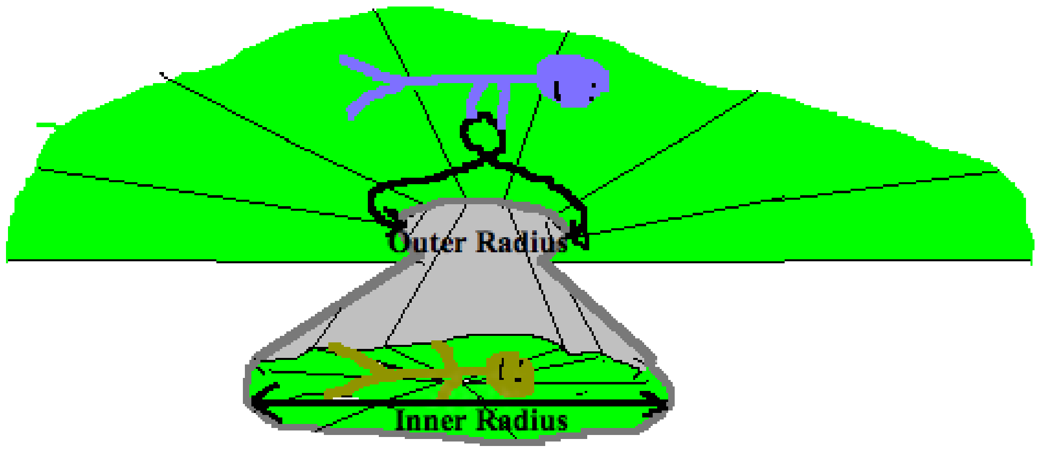

Figure 8: Alternative artistic representation of the Broeck "pocket" or "bottle" with both the inner and outer radius of the bottle shown .(Source:Internet)

15 Appendix I:Alternative Artistic Presentation of the Broeck "pocket" or "bottle"

The figure shown above also represents exactly the point of view we are defending concerning the whole Broeck idea applied to the Natario warp drive in order to reduce the surface area exposed to collisions against the IM particles.

The Broeck bottle with a large internal radius(inner radius) r_+ large enough to contains a man(the brown man) inside the bottle is depicted.

The microscopically small outer radius r_- delimitates the circle surface(bottleneck of the bottle) of the intersection points between the Btoeck bottle and out external Universe(the plane above the bottle where the blue man is placed)

The internal radius(inner radius) r_+ is much larger than the microscopically small outer radius r_-.

Therefore although the Broeck bottle can possesses a large internal volume delimitated by a large internal radius(inner radius) r_+ able to accommodate the brown man inside the Broeck bottle then the blue man in the plane representing our Universe would only see a microscopically small surface circle(bottleneck bottle) delimited by the microscopically small outer radius r_- as in pg 19 in [15].

A microscopically small outer radius r_- the R_b in our equations delimitates a very small microscopically circle surface area therefore reducing the probability of collisions against the dangerous IM particles

Figure 9: Artistic Presentation of a Cross-Section C curve(Source:Internet)

16 Appendix J:Artistic Presentation of a Cross-Section C curve

The image above borrowed from science-fiction depicts a spaceship in an interstellar spaceflight suffering the collision against the IM particles.As large the surface area of the Cross-Section is the width of the C curve grows proportionally and so the distance between the "upper" and "lower" particle "streams" of the C curve wether and independently from the geometries of Alcubierre or Natario.

In the Natario-Broeck spacetime the size and the width of the C curve would be microscopically small effectively protecting the ship against the dangerous IM particles.[55]

[55] compare this figure with the one of Appendix F

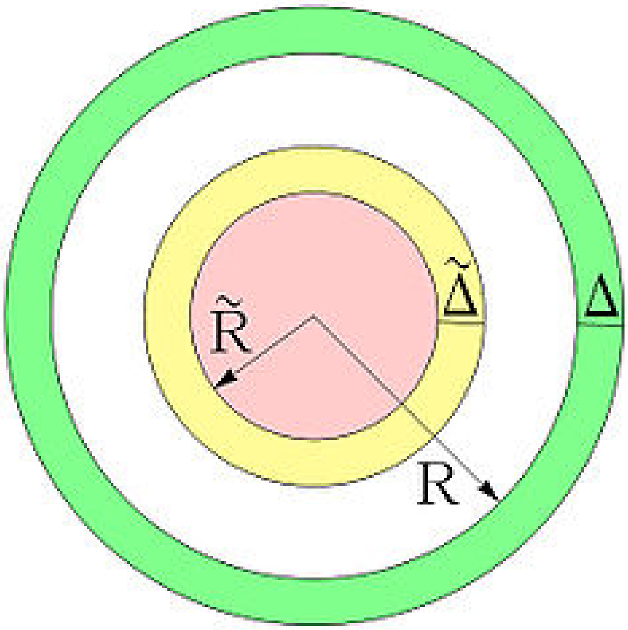

Figure 10: Artistic Presentation of the Broeck "pocket" or "bottle" inside the Natario warp drive spacetime.(Source:Internet)

17 Appendix K:Artistic Presentation of the Broeck "pocket" or "bottle" inside the Natario warp drive spacetime

Applying the Broeck mathematical term $B(rs)$ to the spatial components of the Natario warp drive equation using the signature $(+,-,-,-)$ we get the following result:

$$ds^2 = dt^2 - B(rs)^2[(drs - X^{rs}dt)^2 + (rs^2)(d\theta - X^{\theta}dt)^2] \tag{48}$$

With the contravariant shift vector components X^{rs} and X^{θ} given by:(see pg 5 in [2])[56]

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{49}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{50}$$

The term $B(rs)$ according to Broeck creates inside the Natario warp bubble of radius R a spatial distortion of radius R_b being R_b microscopically small when seen from outside but inside the sphere generated by this R_b a large internal volume with the size enough to contains a spaceship can easily be accommodated.(see also pg 19 in [15])[57]

In the figure shown above the term $\tilde{R}$ is our small outer radius R_b and the term $\tilde{\Delta}$ is our Δ_b.

[56]see also Appendices A and B in [16] for details

[57]see Appendices G,H and I

According to Broeck this term $B(rs)$ have the following behavior:(see pgs 3 and 4 in [10])[58]

$$B(rs) = \begin{cases} 1+\alpha & rs < R_b \\ 1 < B(rs) \leq 1+\alpha & R_b \leq rs < R_b + \Delta_b \\ 1 & rs \geq R_b + \Delta_b \end{cases} \quad (51)$$

-)-Considering the picture shown in the previous page:

The pink region is the Broeck bottle or "pocket" with a large inner metric defined by the region where $rs < R_b$ and $B(rs) = 1+\alpha$ being α the term that generates the large internal volume of the Broeck bottle.

The faded yellow region is the region where the bottleneck of the Broeck bottle is placed.This region is the transition region between the "blown-up" space to the "normal" space.This is the region where $R_b \leq rs < R_b + \Delta_b$ being R_b the radius of the Broeck bottle bottleneck.In this region the value of $B(rs)$ becomes $1 < B(rs) \leq 1+\alpha$ never reaching 1.The term Δ_b delimitates the thickness of the faded yellow region where $B(rs)$ decreases.This region is a thin shell around the Broeck bottle bottleneck.

The white region is the region where $rs \geq R_b + \Delta_b$ far outside the Broeck bottle bottleneck we recover the normal space of our Universe in which $B(rs)$ always possesses the value of $B(rs) = 1$.We also recover the original Natario metric.

The green region is the Natario warped region where the Natario shape function $n(rs)$ is varying from 0 to $\frac{1}{2}$.($0 < n(rs) \leq \frac{1}{2}$).According with Natario any function $n(rs)$ that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $\frac{1}{2} > n(rs) > 0$ in the Natario warped region is a valid shape function for the Natario warp drive.(see pg 5 in [2]). We define the Natario shape function as being

$$n(rs) = \begin{cases} 0 & rs < R \\ 0 < n(rs) \leq \frac{1}{2} & R \leq rs < R+\Delta \\ \frac{1}{2} & rs \geq R+\Delta \end{cases} \quad (52)$$

In the equation above R is the radius of the warp bubble and Δ is the thickness of the Natario warped region which means to say the thin shell region where $0 < n(rs) \leq \frac{1}{2}$. Remember that $R >> R_b + \Delta_b$ or $R + \Delta >> R_b + \Delta_b$

The pink,faded yellow and white regions are completely contained inside the Natario warp bubble.Note that in the pink and white regions the value of $B(rs)$ is constant which means to say that the derivatives of $B(rs)$ are zero.Also in these regions the value of $n(rs)$ is always constant hence the derivatives of $n(rs)$ are also zero.

In the faded yellow region delimitated by $R_b \leq rs < R_b + \Delta_b$ the value of $B(rs)$ is given by $1 < B(rs) \leq 1+\alpha$ and since $B(rs)$ is varying in this region then the derivatives of $B(rs)$ are different than zero.

In the green region delimitated by $R \leq rs < R+\Delta$ the value of $n(rs)$ is given by $0 < n(rs) \leq \frac{1}{2}$ and since $n(rs)$ is varying in this region then the derivatives of $n(rs)$ are different than zero.

[58]see Appendix G

Due to the terms $R >> R_b + \Delta_b$ or $R + \Delta >> R_b + \Delta_b$ the regions faded yellow and green do not "overlap" themselves.In the faded yellow region the derivatives of $B(rs)$ are non-zero but the derivatives of $n(rs)$ are zero and in the green region the derivatives of $B(rs)$ are zero but the derivatives of $n(rs)$ are non-zero.This is very important the fact that we can study both regions faded yellow and green completely separated from each other.Otherwise we would need to compute "all-the-way-round" the Christoffel symbols Riemann and Ricci tensors and the Ricci scalar in order to obtain the Einstein tensor and hence the stress-energy-momentum tensor in a long and tedious process of tensor analysis liable of occurrence of calculation errors.

Or we can use computers with programs like $Maple$ or $Mathematica$ (see pgs $[342(b)]$ or $[369(a)]$ in [11], pgs $[276(b)]$ or $[294(a)]$ in [13],pgs $[454, 457, 560(b)]$ or $[465, 468, 567(a)]$ in [14]).

Appendix C pgs $[551-555(b)]$ or $[559-563(a)]$ in [14] shows how to calculate everything until the Einstein tensor from the basic input of the covariant components of the $3+1$ spacetime metric using $Mathematica$.

The energy density for the Broeck faded yellow region in Geometrized Units $c = G = 1$ is given by the following equation:(see eq 11 pg 6 in [10])

$$T_{\hat{\mu}\hat{\nu}}u^{\hat{\mu}}u^{\hat{\nu}} = T^{\hat{0}\hat{0}} = \frac{1}{8\pi}\left(\frac{1}{B^4}(\partial_r B)^2 - \frac{2}{B^3}\partial_r\partial_r B - \frac{4}{B^3}\partial_r B\frac{1}{r}\right). \tag{53}$$

In the equation above a large $B(rs)$ from $1 < B(rs) \leq 1+\alpha$ where $R_b \leq rs < R_b+\Delta_b$ will generate very small terms $\frac{1}{B(rs)^4}$ $\frac{2}{B(rs)^3}$ and $\frac{4}{B(rs)^3}$ therefore obliterating the values of the derivatives of $B(rs)$ resulting in a very low energy density.

The Natario expressions for the negative energy density of the green region in Geometrized Units $c = G = 1$ are given by(pg 5 in [2])

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{54}$$

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(\frac{dn(rs)}{drs})^2\cos^2\theta + \left(\frac{dn(rs)}{drs} + \frac{r}{2}\frac{d^2n(rs)}{drs^2}\right)^2\sin^2\theta\right]. \tag{55}$$

Note that in the equatorial plane $\theta = 0$ $\sin(\theta) = 0$,$\cos(\theta) = 1$ the negative energy density do not vanishes protecting the ship and therefore the faded yellow region against collisions with the dangerous IM particles.(see the works in [5],[7] and [8])[59]

[59]see also Appendices E and F

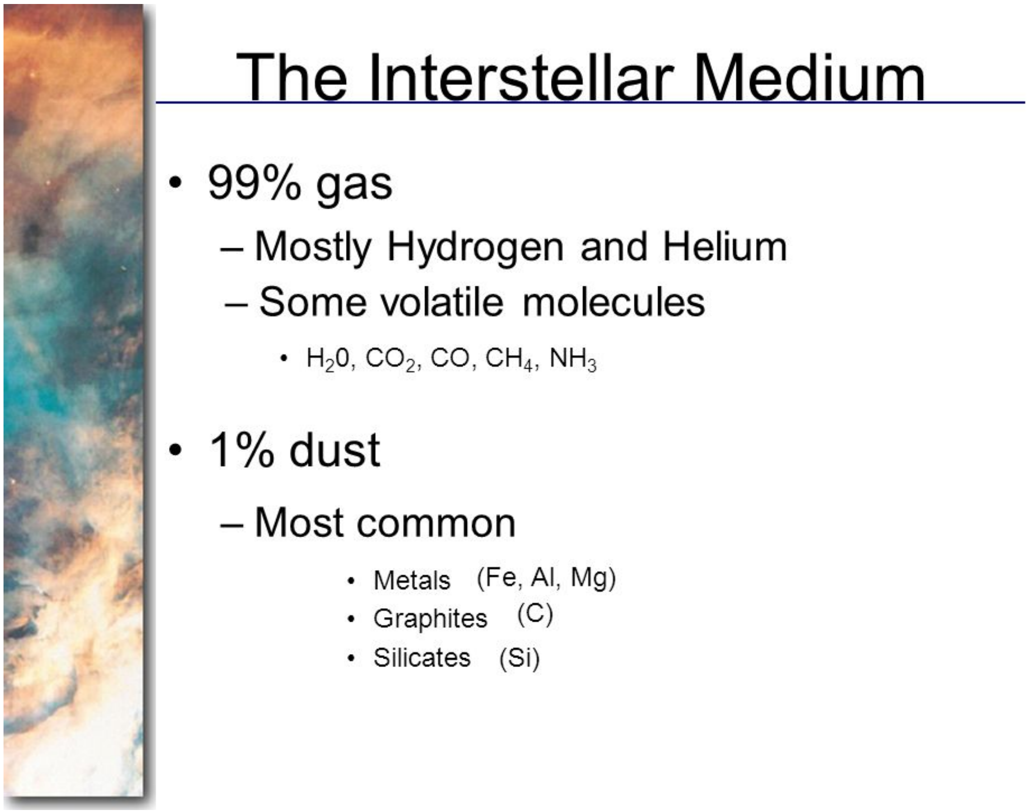

Figure 11: Composition of the Interstellar Medium IM(Source:Internet)

18 Appendix L:Composition of the Interstellar Medium IM

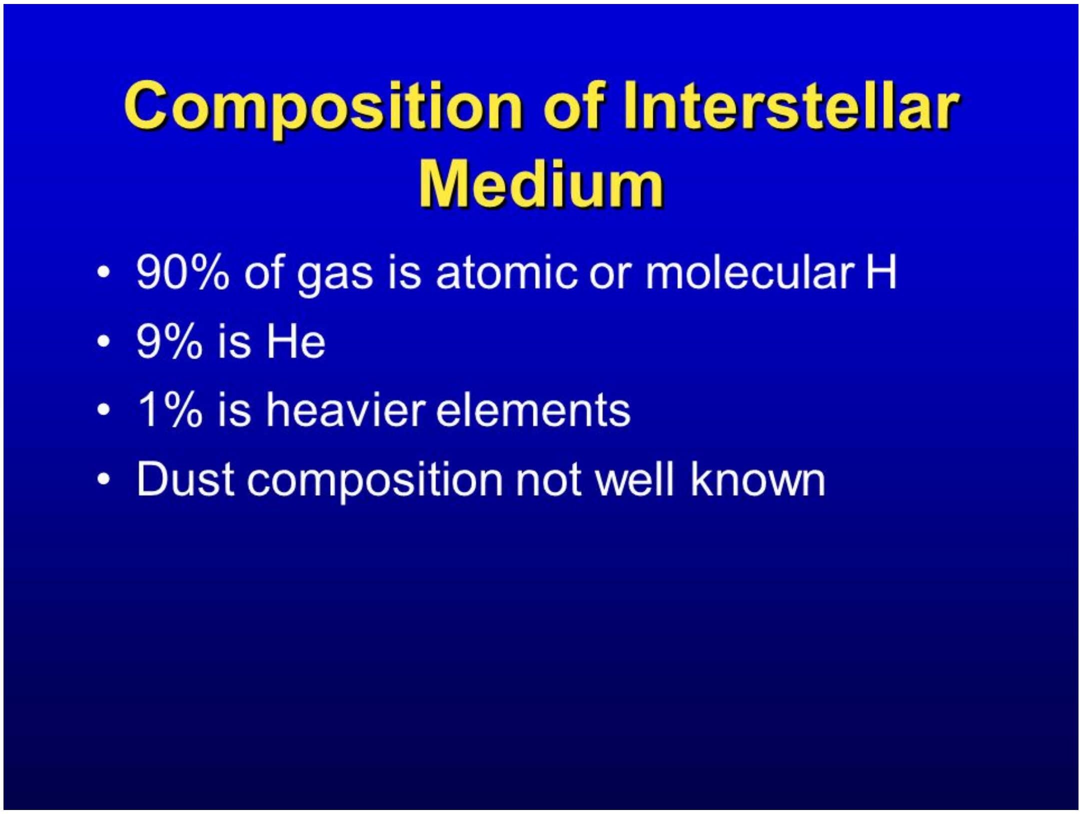

Figure 12: Composition of the Interstellar Medium IM(Source:Internet)

19 Appendix M:Composition of the Interstellar Medium IM

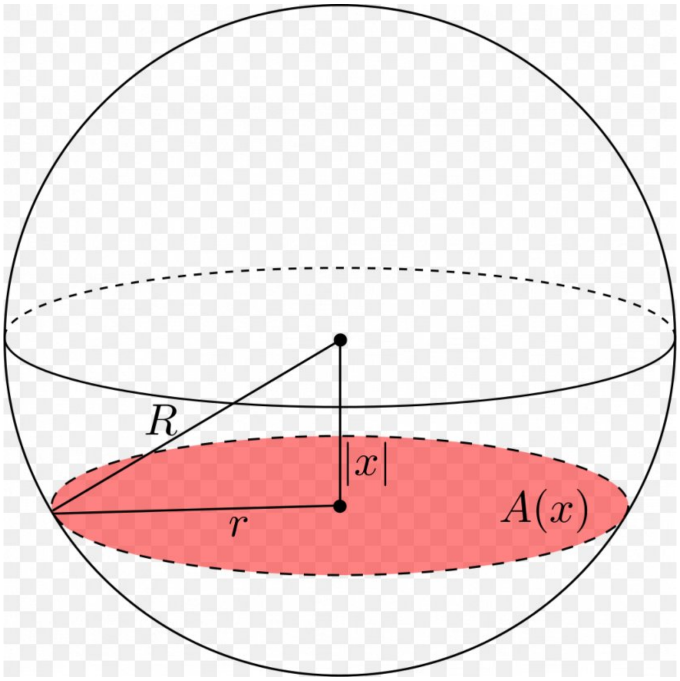

Figure 13: The Cross-Section of the warp bubble(Source:Internet)

20 Appendix N:Another(and better)Presentation for the Cross-Section of the warp bubble in the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu

Above is being presented the Cross-Section of the warp bubble with the radius r and area A.R is the radius of the bubble and the distance between the center of the shell and the center of the bubble is $|x|$.As large the Cross-Section radius r is then the surface area A grows proportionally.

Note that the A have a form of a "dish" or a "disk" and this form will suffer the collisions against the hazardous IM particles or photons.

This area A is independent of the chosen warp drive geometry and it is always present wether we choose the Alcubierre or the Natario geometries.But for the Natario case the negative energy of the bubble in front if the ship deflects the dangerous IM particles.The collisions with the IM particles would generate two "streams" or flows of particles that forms a "D",or a "C" or a "U" curves.As large the area A is the width of the curve grows proportionally.

But for the case of the Natario-Broeck spacetime the Cross-Section radius r of 10^{-15} meters would generate a very small area A of about 10^{-30} square meters less exposed to dangerous impacts when compared to a Natario only spacetime with a Cross-Section radius r of 10^{2} meters with an area A of about 10^{4} square meters.

In this case the bubble is moving from "top" to "bottom" generating a "U" curve.

21 Remarks

References [11],[12],[13] and [14] are standard textbooks used to study General Relativity and these books are available or in paper editions or in electronic editions all in Adobe PDF Acrobat Reader.

We have the electronic editions of all these books

In order to make easy the reference cross-check of pages or equations specially for the readers of the paper version of the books we adopt the following convention:when we refer for example the pages $[507, 508(b)]$ or the pages $[534, 535(a)]$ in [11] the (b) stands for the number of the pages in the paper edition while the (a) stands for the number of the same pages in the electronic edition displayed in the bottom line of the Adobe PDF Acrobat Reader

Our work was written based over the first version of [18].If future updated versions of this work appears we will adapt our work accordingly.

22 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[60]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[61][62]

[60] special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[61] "Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[62] appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

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