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# A Balancing Method for Constructing Super Perfect Magic Square

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Abstract: Super perfect magic square refers to a magic square with multiple changes and mutual conversion. In this paper, a construction method of magic square, called balancing method, is created, which can quickly construct even order super perfect magic square. The characteristic of this method is that the larger the order of magic square, the more selectivity, and the easier it is to construct.
Key words: magic square, super perfect magic square, magic square construction, even magic square, balancing construction method

Magic squares can be divided into odd, single even and double even number orders according to the number order. In the construction of magic square, the magic square of single even order is the most difficult. Super perfect magic square [1] refers to a magic square with multiple changes and mutual conversion.

There are many methods to construct magic squares [2,3,4,5]. The simple and fast construction method is the continuous allocation method, which is suitable for the construction of odd number order magic squares; Symmetry method and diagonal method are suitable for double even number magic squares. Single even number magic square has not been able to find a fast and effective construction method for a long time. This paper creates a construction method of even numbermagic square, called balancing method, which can quickly and conveniently construct high-order even number super perfect magic square. The results are reported as follows:

## 1. Construct the subgraph of magic square

Let n be an integer, the magic square of order 2n can be regarded as composed of n×n regions, each region is composed of subgraphs composed of 2×2 numbers, and the regions can be divided into three parts: corner, edge and center .

The construction of subgraph consists of four consecutive natural numbers, namely m,m+1,m+2,m+3,and the subgrphs are numbered from small to large, k = 1,2,..., $n^2$, and m = 4 (k-1) + 1.

Deviation value: it is defined that the difference between the sum of two numbers in the row or column of the subgraph and the average value of the sum of two numbers in the row or column of the subgraph is the deviation value of the row or column of the subgraph.

The deviation values of various combinations of subgraphs can be divided into five cases, namely: - 2, - 1, 0, + 1, + 2. For example, the following subgraph

| | |
|---|---|
| 5 | 7 |
| 6 | 8 |

The sum of the two numbers in the first row is 12, and the sum of the two numbers in the second row is 14. The average of the two sums is 13. The differences between the two sums and the average value are - 1 and + 1 respectively; Similarly, the deviation values of the subgraph column are - 2 and + 2 respectively.

## 2. Construct n-order magic squares of subgraph sequence numbers

When n is an odd number, the continuous allocation method is used; When n is an even number, is the symmetry method [2]. For example, when n is an even number of 4, in order to construct an eighth order super perfect magic square, first apply the subgraph sequence number to construct a fourth order sequence number magic square, as shown in Figure 1.

| | | | |
|---|---|---|---|
| (1) | (15) | (14) | (4) |
| (12) | (6) | (7) | (9) |
| (8) | (10) | (11) | (5) |
| (13) | (3) | (2) | (16) |

Figure 1. Magic square of sequence number of fourth-order subgraph.

## 3. Construct subgraph on magic square diagonal

The subgraphs corresponding to the subgraph sequence numbers are arranged on the diagonals of the magic square of the sequence number, which meets the structural requirements of the super perfect magic square, that is, the sum of the corresponding numbers symmetrically centered along the diagonal is equal, which is $(2n)^2 + 1$. The diagonal filling process of magic square is shown in Figure 2.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 3 | | | | | 16 | 13 |
| 2 | 4 | | | | | 14 | 15 |
| | | 22 | 24 | 25 | 26 | | |
| | | 21 | 23 | 27 | 28 | | |
| | | 40 | 38 | 42 | 41 | | |
| | | 39 | 37 | 44 | 43 | | |
| 49 | 51 | | | | | 61 | 62 |
| 52 | 50 | | | | | 63 | 64 |

Figure 2. Diagonal subgraph meeting the requirements of super perfect magic square.

## 4. Construct the subgraphs on the edge region of the magic square to complete the whole magic square

Calculate the deviation values of each row and column of the subgraphs on the magic square diagonals. According to the deviation values of the subgraphs on the diagonal, select the edge subgraphs corresponding to the subgraph sequence numbers, and balance the values according to the requirements of the super perfect magic square, so that the total deviation value of each row and column of the magic square is zero.

For example, the deviation values of the first and second columns of the subgraph in the upper left corner of Figure 2 are - 2 and + 2, and the deviation values of the first and second columns of the subgraph in the lower left corner are + 1 and - 1. Therefore, the sum of the deviation values of the first and second columns of the two edge subgraphs should be + 1 and - 1, and its combination can be + 2 and - 2 combined with - 1 and + 1, or - 1 and + 1 combined with + 2 and - 2.

The example is a super perfect magic square of order 8. See Figure 3 for details.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 59 | 57 | 55 | 56 | 16 | 13 |
| 2 | 4 | 58 | 60 | 54 | 53 | 14 | 15 |
| 48 | 45 | 22 | 24 | 25 | 26 | 36 | 34 |
| 47 | 46 | 21 | 23 | 27 | 28 | 33 | 35 |
| 29 | 32 | 40 | 38 | 42 | 41 | 20 | 18 |
| 31 | 30 | 39 | 37 | 44 | 43 | 17 | 19 |
| 50 | 51 | 12 | 10 | 6 | 8 | 61 | 62 |
| 52 | 49 | 9 | 11 | 7 | 5 | 63 | 64 |

Figure 3. Super perfect magic square of order 8.

## 5. Dynamic changes of super perfect 8th Order Magic Square

For the convenience of narration, the corresponding numbers before and after the transformation are placed in two brackets respectively, which are corresponding and interchanged one by one according to the position. For example: (1, 2, 3) - > (6, 7, 8) indicates that 1 and 6 are interchangeable, 2 and 7 are interchangeable, 3 and 8 are interchangeable, etc.

The following is the transformation of the 8th order super perfect magic square when the numbers on the diagonal remain unchanged.

(1) row and column exchanges

For example:

(1， 3， 59， 57， 55， 56， 16， 13) -> (52， 49， 9， 11， 7， 5， 63， 64)

(1， 2， 48， 47， 29， 31， 50， 52) -> (13， 15， 34， 35， 18， 19， 62， 64)

(2) exchanges within a single edge area

For example:

(48， 45) -> (46， 47)

(59， 58) -> (57， 60)

(3) exchanges within the central area

For example:

(22， 21， 40， 39) -> (24， 23， 38， 37)

(24， 40， 44， 28) -> (21， 37， 41， 25)

(4) exchanges of two pairs of edge areas simultaneously
For example:
(59， 9) -> (57， 11)
(36， 17) -> (34， 19)
(29， 20， 18) -> (31， 17， 19)

(5) exchanges of both in the central area and in the edge area simultaneously
For example:
(22， 24， 25， 26， 36， 34) -> (21， 23， 27， 28， 33， 35)
(42， 44， 6) -> (41， 43， 8)
(40， 38， 42， 41， 32) -> (39， 37， 44， 43， 30)

(6) exchanges of both in the corner area and in the edge area simultaneously
For example:
(3， 31， 59， 55， 57) -> (2， 30， 58， 54， 60)
(63, 55) - >(62, 56), and at the same time rotate (8, 6, 7, 5) at 90 degrees clockwise
(50, 18) - > (49, 19), and at the same time rotate (32, 30, 31, 29) at 90 degrees clockwise

When the numbers on the diagonal remains unchanged, it can be seen that the super perfect magic square can carry out a variety of transformations and generate a large number of magic squares.

Since there are 880 basic patterns in the fourth-order magic square composed of subgraph sequence numbers[2] , 144 basic patterns by changing the diagonal numbers of the outer layer of the super perfect eighth-order magic square [1], and similarly, 144 basic patterns by changing the number of diagonal lines in the inner layer, the cumulative number of basic patterns available is 18,247,680. Therefore, a simple derivation shows that the total number of magic squares generated by super perfect order 8 magic squares is astronomical.

# 6. Construction of magic square of order 10 of single even number

In the same way, the magic square of order 10 of single even number can be constructed, as shown in Figure 4.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 65 | 66 | 95 | 96 | 1 | 4 | 30 | 32 | 59 | 57 |
| 67 | 68 | 94 | 93 | 3 | 2 | 31 | 29 | 60 | 58 |
| 90 | 91 | 20 | 19 | 28 | 25 | 54 | 53 | 61 | 64 |
| 92 | 89 | 17 | 18 | 27 | 26 | 56 | 55 | 62 | 63 |
| 14 | 13 | 21 | 23 | 51 | 52 | 79 | 78 | 86 | 88 |
| 15 | 16 | 24 | 22 | 49 | 50 | 77 | 80 | 87 | 85 |
| 37 | 40 | 47 | 45 | 73 | 75 | 83 | 84 | 11 | 10 |
| 39 | 38 | 48 | 46 | 76 | 74 | 82 | 81 | 12 | 9 |
| 42 | 41 | 69 | 72 | 99 | 100 | 6 | 8 | 33 | 35 |
| 44 | 43 | 70 | 71 | 98 | 97 | 7 | 5 | 34 | 36 |

Figure 4. Super perfect 10th order magic square.

## 7. Conclusion and discussion

It can be seen from the results shown above that the even order magic square is simple in structure by using the balancing method designed in this paper. No matter how the order of magic square increases, the amount of data calculation is very small and easy to operate manually. In particular, there is no simpler method for single even order magic square at present. The second feature is that the constructed magic square has a dynamic change mode, which can generate a large number of magic squares, which is intuitive and visible, and it is easy to calculate the total number of such magic squares. In addition, another feature is that the larger the magic square order, the more the combination number of edge subgraphs and the greater the selectivity, the easier it is to complete the construction.

## 8. References

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