V=C: A MATHEMATICAL REVIEW ON THE ESCAPE SPEED OF A BLACKHOLE

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ABSTRACT

Blackhole are known as one of the mysterious astronomical body in the known universe. A body where gravity is infinite, time stops and space makes no sense. Blackholer are under the family of gravity and nothing seems seductive. In the last century blackhole as turned from being mathematical curiosity to real object in the cosmos, seemingly crucial for the formation of stars and nothing, not even light can escape its gravitational influence. This is what the scientific paper is specifically investigating on, if nothing can’t really escape the gravitational influence of the blackhole even light. The research also specifically investigates if the event horizon is a region or boundary within which the blackhole velocity is greater than the speed of light, or equals to the speed of light.

INTRODUCTION

John Michell in 1784 proposed that in the vicinity of compact massive objects, gravity can be strong enough that even light cannot escape at that time. The Newtonian picture of gravitation and the so called corpuscular theory of light were dominant.

In their theories, if the escape velocity of an object exceeds the speed of light, then light originating inside or from it can escape temporarily but will return.⁹

One of the best known examples of an event horizon derives from general relativity description of blackhole, celestial object so dense that no nearby matter or radiation can escape its gravitational influence.¹¹ Often this is described as the boundary within which the blackhole escape velocity is greater than the speed of light, but this is not true, from the result it shows in the
Newtonian picture that the escape velocity is not greater than the speed of light but directly equals to the speed of light.

**METHODOLOGY**

In order to explain that the escape velocity of a blackhole is directly equals to the speed of light, Newton law of universal gravitation was applied. The research derived the equation for the speed of light escape velocity of a blackhole using Newton law and went further with the law of conservation of energy, to confirm the result.

The gravitational force “f” of a blackhole cuts between two masses, mo which is the mass of the blackhole and mi which is the mass of the spacecraft. The spacecraft launched from the blackhole surface experiencing the gravitational influence of the event horizon tries to escape the gravitational field of this body but fails the attempt. This attempt put forward by the astronaut in the spacecraft fails because of the mathematical description of the escape velocity of a blackhole. The escape velocity of the blackhole in the mathematical description is not less than the speed of light $v < c$ or greater than the speed of light $v > c$, but equals to the speed of light $v = c$. If we use Newtonian law to calculate we arrive at that result.

Never being satisfied with the result Newton laws of gravitation shown, the researcher craved to use another method to solve the equations making use of the principle of conservation of energy which shows that the total energy of a closed system remains constant. In this case, the closed system consist of the two objects with the gravitational force between them and no outside energy or force affecting either object.\[1, 2\] In this case of the light escape speed when applying conservation of energy we set the equation $u_i + k_i = u_f + k_f$ for both, then set both term on the right to be zero. We want the spacecraft to barely reach infinity where the potential energy is zero.
After the calculation using the principle of conservation of energy it resulted with the same answer with that of Newtons law of universal gravitation. In conclusion both calculation pictured that the event horizon escape velocity is not greater than the speed of light but equals the speed of light.

RESULT

We define the escape velocity of a blackhole as the minimum velocity required for an object to escape the gravitational influence of it event horizon permanently.\[^{[11]}\]

We can obtain the formula for the escape velocity using the Newtons law of universal gravitation which is an inverse square law.\[^{[11]}\]

Let’s assume that “mo” is the mass of the blackhole and mi is the mass of the spacecraft.

\[
f = \frac{G m_o m_i}{r^2} \quad \text{equ 1}
\]

The distance work done in carrying a mass m from a point at a distance “r” from the centre of the blackhole to a distance so great that the gravitational field of the event horizon is negligibly weak is given by

\[
w = f \times r \quad \text{equ 2}
\]

but from equation 1  \[ f = \frac{G m_o m_i}{r^2} \]

Hence, \[ w = \frac{G m_o m_i}{r^2} \cdot r = \frac{G m_o m_i}{r} \quad \text{equ 3} \]

This work must equal the kinetic energy of the body of mass m at this point, having a velocity \( v_o \). This kinetic energy is given by \( KE = \frac{1}{2} m v_o^2 \)

\[
KE = \frac{1}{2} m v_o^2 = \frac{G m_o m_i}{r}
\]

\[
v_o^2 = \frac{2 G m_o}{r} \quad \text{equ 4}
\]

\[
vo = \sqrt{\frac{2 G m_o}{r}}
\]
If we launch the mass $mi$ of the spacecraft from the blackhole surface where $r = R$, we then have that

$$\nu_0 = \sqrt{\frac{2Gm_0}{r}}$$

hence $m_0 = \left(\frac{c^2}{2G}\right)R$

$$\nu_0 = \sqrt{\frac{2G}{R}} \frac{RC^2}{2G}$$

$$\nu_0 = \sqrt{c^2}$$

$$\nu_0 = c$$

Using the principle of conservation of energy to prove that $\nu_0 = c$.

The spacecraft has initial K.E and gravitational potential energy.

$$E_{total \ initial} = K.E \ initial + U \ initial = K.E \ initial + \left(\frac{Gm_0}{r}\right)$$

Where $r$ is the distance from the centre of mass of the blackhole to the centre of mass of rocket. If the spacecraft managed to escape the event horizon of the blackhole, its gravitational potential energy will be zero as the distance for this to occur is at infinity. Since we want to calculate the minimum initial velocity required we will assume that the spacecraft will have no K.E by the time it reaches $r = \infty$.

Thus, $E_{total \ final} = K.E \ final + U \ final = 0 + 0$

By the principle of conservation of energy

$$E_{total \ initial} = E_{total \ final}$$

$$K.E \ initial + \left(\frac{Gm_0}{r}\right) = 0$$

$$\frac{1}{2} m_0 \nu^2 = \left(\frac{Gm_0}{r}\right)$$

$$m_0 \nu^2 = \left(\frac{2Gm_0}{r}\right)$$

$$m \nu = \left(\frac{RC^2}{2G}\right)$$

$$m \nu^2 = \left(\frac{2Gm}{r} \ \frac{RC^2}{2G}\right)$$
\[
\therefore r = R \\
n_{iv}^2 \left( \frac{2Gmi}{R} \right) \left( \frac{RC^2}{2G} \right) \\
n_{iv}^2 = n_{ic}^2 \\
v = c
\]

**DISCUSSION**

Though they can be hard to imagine, blackhole are not a simple matter. In fact they continue to offer new mysteries, especially when we least expect them. Blackhole event horizon is teleological in nature, meaning that we need to know the entire future space time of the universe to determine the current location of the horizon, which is purely impossible because of the purely theoretical nature of the event horizon boundary, the travelling object does not necessarily experience strange effect and does in fact, pass through the calculating boundary in a finite and proper time.\[6\]

One may ask if the spacecraft did not experience some blackhole scenario like the hypothetical phenomenon where an object falling into a blackhole encounters high energy quanta at or near the event horizon, and also if the spacecraft did not experience the total force near the event horizon or the stretching out of an object as it comes into contact with an extreme gravitational field.

The research neglects, the firewall paradox and the noodle effect to extract a well defined mathematical expression of the escape velocity making use of the spacecraft launched from the blackhole which might not be possible because of his scary scenarios. But to give a clear view to what the escape velocity is all about the petrifying scenario must be neglected.

**CONCLUSION**
In summary, the blackhole is quite an astonishing body in the known universe with a pen and paper we can describe his behaviour in space. While the escape speed of a blackhole have been thought to be slightly greater than the speed of light, the researcher have craved a new way we understand the event horizon escape speed using different mathematical methods.

Maybe in some cases another mathematical description might give another truth about the event horizon, but for the classical picture of Newtonian physics it’s another case entirely.
REFERENCES

5. Wald 1984, pp.299-300.
7. The maximum mass of a neutron star, Bombaei, 1. 1996.
8. Hamilton, A. “Journey into a Schwarzschild blackhole”
10. Schutz, Bernard F. 2003 “Gravity from the Ground”.