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Geometrical Constraints for Quantum Mechanics, General Relativity and Cosmology in an N-Dimensional Topology

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Abstract

Geometrical constraints for general relativity and quantum mechanics are formulated in a multidimensional Cartesian space. A fundamental relationship between these correspondences and complementarity constraints directs us towards a new understanding of the fundamental relationship between relativity and quantum theory. The set of geometrical constraints of this n-dimensional topology are expressed in terms of a hyper-dimensional Minkowski metric, M_n for n > 4 which yields naturally closed cosmological solutions to Einstein's field equations which also yields compatibility between Einstein's field equations and the current big bang model without Guth's inflationary model and its possible Googol ~10^{100} solutions as related to string theory. A comprehensive group theoretical approach to the model of the Cartesian space incorporates dark energy and dark matter which results from the model in a natural manner. A lemma is formulated for the relationship of the maximum invariance for an n-dimensional Cartesian space and the dimensions of that space. Group multiplication tables for the M_n geometry are formulated for a Cartesian 10 and 11 dimensional spaces.

Keywords: Cartesian space, Cosmology, General relativity, Minkowski metric, N-Dimensional Topology, Quantum mechanics

1. Introduction

A group theoretical prescription is made for a geometrical interpretation of the spacetime manifold as an extension, modification, and reinterpretation of Wheeler's *wormhole* topology [1,2]. In the procedure presented a set of physical dimensions of quantized quantities that act as dimensions in the Cartesian manifold termed extended dimensions (ED) are uniquely expressed in terms of geometric constraints. These constraints or conditions are re-expressed in terms of conditions on the manifold of the universal constants.

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By use of the ED interpretation, a set of generalized Heisenberg uncertainty relations are developed, as a generalized set of complementarities formulated canonically conjugate variables obeying commutation relations expressible in terms of the ED. A group theoretical formulation is constructed for the generalized complementarity relations on a generalized metrical space, M_n. The extended dimensions are interpreted as a representation of the geometrical structure of the spacetime manifold and are manifest in classical mechanics, electromagnetics, strong and weak interactions and cosmology and also act as a unifying element of this group theoretic approach.

Since the advent of Einstein's theory of special relativity in 1915, his general theory of relativity in 1905, and development of the quantum concept of Planck near the turn of the last century, and the further development of the quantum theory in the 1920's and 1930's, physicists have been attempting to reconcile these theories into a comprehensive and consistent framework with the long existing foundations of classical mechanics.

Attempts have been made with the introduction of the correspondence principle between classical and quantum mechanics, based on a scale correspondence to Planck's constant, and Bohr's principle of complementarity expressed in the Heisenberg uncertainty principle, and the Dirac formulation of relativistic quantum field theory. Complementarity and phase space relations are based on the neoclassical Hamilton-Jacobi theory.

We present five propositions acting to unify the various branches of physics. These propositions are:

(1) The fundamental role in physics of constancy as expressed by the universal constants, and also constraints on dynamic processes, conservation principles and group symmetry relations.

(2) The fundamental significance of canonically conjugate relations as an expression of a basic set of dual variables in Nature.

(3) A geometrical interpretation of the space-time manifold in terms of a discrete set of quantities which comprise dimensional manifold space vectors.

(4) The introduction of a new quantization procedure in terms of these fundamental discrete dimensional vectors and scalars as elements of a group theory.

(5) Introduction of a ten-and 11D Cartesian space which represents, in group theoretical terms, micro and macro generalized correspondences and universal complementarity. The metrical expression is performed for the Cartesian space for *EnD* where nis the number of extended dimensions of the space

These propositions lead to the following new concepts when the universal constants, in Planck unit formulation, are placed on a fundamental theoretical basis. A set of physical variables, uniquely expressed in terms of universal constants, termed extended dimensions or *EnD*, is developed [3,4]. All physical variables can be expressed in extended dimensional form. These discrete entities represent the geometrical constraints of the spacetime manifold and obey a set of canonically conjugate relations, which are like Heisenberg uncertainty relations in terms of generalized phase space conditions [5]. A group theoretic interpretation yields a common basis for cosmology and the quantum theory. In this conceptual framework, a detailed discussion of the group theoretical formalism is given in [6] and the detailed structure of the generalized Minkowski metrical space is given in [7]. This approach appears to lead to a unification of relativity and quantum theories.

2. The N-dimensional Cartesian Space Geometry

Rene Descartes in 1619, suggested, in addition to his three spatial rectilinear coordinates, that time, velocity, energy, and momentum, etc. be considered as coordinates [8]. Spinoza, in his treatise on ethics [9], suggested the concepts of space, time, and substance (matter). Einstein, who wrote a preface for the republication of Spinoza's work, was influenced by both authors' consideration of additional dimensions to the coordinates of space, such as time, which led to his consideration of the Minkowski four-spacetime [10]. Dimensional quantities similar to those of Planck were developed by G.N. Lewis [11]. Our multidimensional space, in terms of group theory, [6] and an extended Minkowski metric, [7] denoted as M_n for $n > 4$ and usually $n < 11$, we term the *Cartesian geometry.*

In an earlier paper, we examined the manner in which a set of geometrical constraints yield closed cosmological solutions to Einstein's field equations [12,13]. These constraints are expressed as a set of quantities or extended physical dimensions which are uniquely defined in terms of universal constants [1,4-7,12]. We have also presented a set of canonically conjugate relations of these physical variables expressed in terms of extended dimensions, *EnD* [6]. It was demonstrated that the quantized variables have operator representations [7], and a generalized form of the Schrodinger wave equation was developed in terms of these operators [13].

The set of geometrical constraints expressed in *n*-dimensional topology of the Cartesian geometry comprise a multidimensional space called the Cartesian space. [3,6,12] This multidimensional geometry was given in terms of invariant extended dimensions termed the *EnD* [4]. The metric for invariant dimensions is called the generalized Minkowski metric, M_n [7]. In this paper we demonstrate the manner in n which the *EnD* geometrical constraints act in quantum theory and in relativistic physics, and the fundamental relationship of these two formalisms. The motivation is to determine a fundamental basis of the relativity and quantum theories in the context of quantum gravity [3,4].

3. The Basis of the Cartesian Space Theoretical Approach

M. Planck introduced a set of units which are physical variables unequally expressed in terms of universal constants [14]. A similar, more limited approach was considered by G.N. Lewis and M. Randall [5,11]. Wheeler and others explained the use of the *EnD* quantities in geometrodynamic cosmological models [13,15]. Some of the *EnD*'s relevant here are:

Planck's length or Wheeler's *wormhole* length, $\ell = \sqrt{G/c^3}$; time, $t = \sqrt{G\hbar/c^5}$; momentum

$p = \sqrt{c^3\hbar / G}$; energy, $E = \sqrt{c^5\hbar / G}$ and the Planck's density $\rho = c^5 / G^2\hbar.$ In these equations, $\hbar, G$ and *c* denote, respectively, Planck's constant, the universal gravitational constant, and the velocity of light. Other extended dimensions, or *EnD*'s, are given in Table 1 of Ref. [7].

These quantities can represent geometrodynamic quantities or physical variables as well as *EnD*'s. That is, each *EnD* has an associated physical variable [6,7]. Note the somewhat different terminology in Refs. [3-7,12,16]. In relation to our terminology, EnD's or quantized variables, we define two distinct quantization procedures, primary in terms of the quantized variables, and the standard quantization procedure [3,5,6,7]. The distinguishing characteristic between these two procedures is the classical, quantum, or relativistic domain of physics [3,5-7,12,13]; but in all these procedures there exists a set of canonically conjugate variables, termed the generalized Heisenberg relations [5,7]. It can be demonstrated that these two procedures give equivalent results. We form sets of pair relations as relativistic invariant "four-vectors" [7]. In the next section, we will develop the relationship between the canonically conjugate formalism of the *EnD*'s and the set of invariant relations that make up the generalized Minkowski metric space, M_n.

4. Cartesian Space Geometrical Constraints in Relativity and Quantum Mechanics

The set of canonically conjugate relations, are given in Fig. 1. We have the two usual relations, $(x, p) \geq \hbar$ and $(E,t) \geq \hbar,$ also four new relations [6]. We shall denote possible pair representations as the generalized pair $(\rho_{i\varepsilon}, \upsilon_{i\varepsilon}),$ where index *i* runs 1 to 3 for vector variables and *i* = 1 for scalar variables. In Fig. 1, we consider only one component of each vector; for example, $x = x_1 = x_2 = x_3.$

In this notation, for the pair $(\rho_\varepsilon, \upsilon_\varepsilon),$ if $\varepsilon = 1,$ have the canonical conjugate (cc) pair $(x, p) \geq \hbar,$ and for $\varepsilon = 2,$ we have the cc pair $(E,t) \geq \hbar.$ The four new relations (4) are for $\varepsilon = 3, (x, E) \geq c\hbar;$ $\varepsilon = 4, (p,t) \geq \hbar / c; \varepsilon = 5, (x,t) \geq \hbar / F;$ and $\varepsilon = 6, (p, E) \geq \hbar F,$ where F is the universal force, $F = c^4 / G.$ Extensive literature exists which discusses the interpretation of the scalar Heisenberg relation $(E,t) \geq \hbar.$ Time operators have been presented by several authors [5,17,18]. H. Eberly and L. P. S. Singh [18] develop an unambiguous and non-singular statement of the energy-time uncertainty relationship. We develop a time operator (as well as a space operator) in conjunction with the development of the generalized Schrödinger equation [13], which V. S. Olkhousky and E. Recami also discuss [19]. Eberly and Singh use the density matrix formalism to develop time operators and their uncertainty principle with Hamiltonian operators.

We can form an invariant generalized line element in terms of a universal constant, or combinations of universal constants, between any two variables, $\mu_{i\kappa}$ and $\eta_{i\kappa}$, where again the index *i* runs 1 to 3 for vector variables and *i* = 1 for scalar variables and the index κ runs 1 to *n* where *n* is defined as the dimensionality of the Cartesian space D_n. Considering one component vector quantities, for example $x = x_1$ and $p = p_1$, we can define a Cartesian 4-space as $\{x_k\} = \{x, t, p, E\}$. For example, we have the usual invariant relation:

$$s_1^2 = x^2 - c^2t^2 \tag{1}$$

for one component of *x*; for an isotropic subspace $x = x_1 = x_2 = x_3$ in Eq. (1) and for metrical signature $(+,+,+,-)$. There are six invariant line-elements for a Cartesian 4-space [4,7]. We define a generalized 4-vector invariant. The usual definition of a 4-vector, for invariance relations, is in terms of a spatial vector quantity and a temporal scalar quantity which form an invariant variable pair relation in terms of the invariance of the universal constant, c.

The usual case is Eq. (1) and now we have:

$$S_{\delta=2}^2 = p^2 - \frac{1}{c^2}E^2. \tag{2}$$

Again, we consider one component of the momentum vector only: $p = p_1 = p_2 = p_3$. We show six invariant relations for a Cartesian 4-space in Table 1. We have the usual relations for $\delta = 1$, $\delta = 2$ and for $\delta = 3$. We have one of the new relations in terms of the invariance of the force, F.

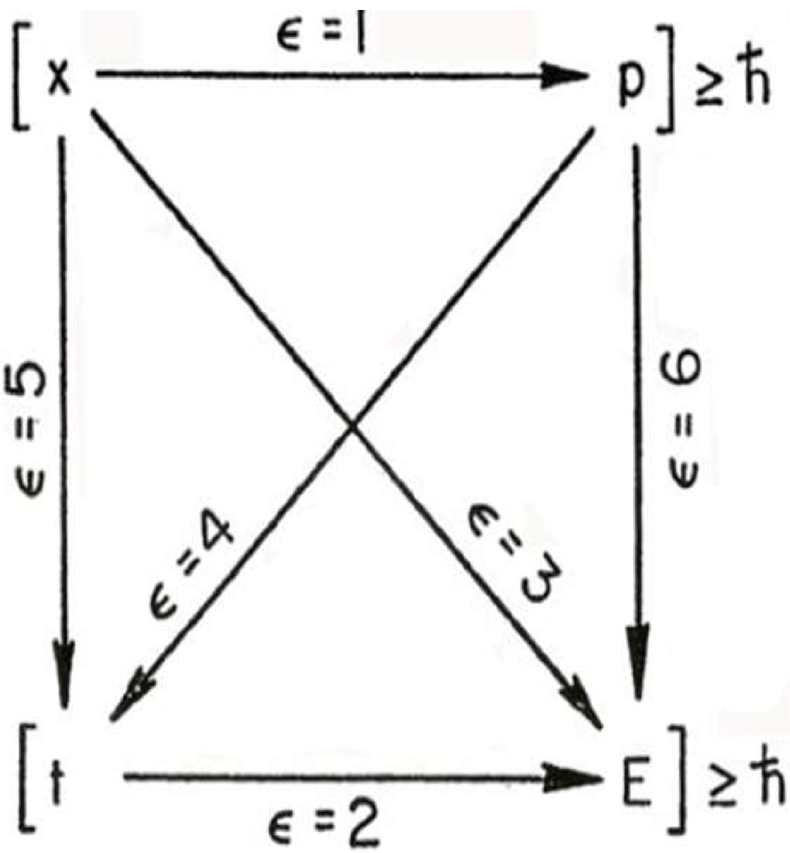

Figure 1. Pair variable relations represented schematically as the generalized Heisenberg Relations, where ε denotes a particular variable pair; for example, $\varepsilon = 4$ denotes the pair $(\rho_\varepsilon = 4, \upsilon_\varepsilon = 4) = (p,t) \geq \hbar / c$.

$$S_{\delta=3}^2 = x^2 - \frac{1}{F^2}E^2 \tag{3}$$

The universal cosmological force is uniquely expressed in terms of the universal constants c and G as $F = c^4/G$ [4,13]; thus, the invariance of the expression in Eq. (3) is dependent on the invariance of the universal constants c and G. For $\delta = 5$ in Table 1, we have a 6-subspace for the (x,p) variable pair and for $\delta = 6$, we have a 2-subspace for the (E,t) variable pair. Using the one component forms, as in Eqs. (1) and (2), each subspace is then a 2-space.

The generalized invariant 4-vector (which can also be a 6 or 2-vector space) can be formed in terms of any two variables in terms of the invariance of many of the universal constants, $\hbar$, G and c or combinations of them. Using 1-component vector quantities, the set of generalized invariant relations are generalized 2-subspaces. A generalized invariant expression for a multidimensional Cartesian space is given in Ref. [7], both for the 4-space and a 10-space in terms of 1-component vector and scalar coordinates, $\{x_\kappa\} = \{x, t, p, E, m, c, a, P, L\}$ where m is mass, a is acceleration, P is power and L is angular momentum; other quantities are defined previously. In Ref. [4] a higher-order Cartesian space of as many as thirty dimensions is presented which includes electromagnetic and thermodynamic coordinates
[4,12,19-21].

The generalized form of an invariant variable pair is:

$$s_\delta^2 = \mu_\kappa^2 + m_{\lambda\kappa}\eta_\kappa^2 \tag{4}$$

for any two variables μ_{ik} and η_{ik}, where λ and κ run from 1 to n which is the dimensionality of the Cartesian space being considered and the index δ runs 1 to I (I is the number of invariant relations for a Cartesian space of n dimensions). As before i runs 1 to 3 for vectors and i = 1 for scalars. The invariance relation between any two variables is expressed in terms of the metrical elements $m_{\lambda\kappa}$ which are expressed in terms of universal constants or combinations of universal constants. The constant elements form a non-diagonal matrix termed the generalized Minkowski metric, M_n [4,7].

A generalized invariant, for all n dimensions of the space, can be expressed in terms of the diagonal form of the Minkowski metric. The diagonal form of M_4 is an analytic expression in a form that can be simplified in a linearized formalism [22]. In this approximation, the formalism is quite useful. In the full analytic form, there are a number of interesting implications of the theory [15]. In [4,7] we detail the group theoretical formulation of the Cartesian space for an n-dimensional representation. In [4], we consider D_n for n as a 10 and 11D space. In the *EnD* model, the group generators are considered to form a finite algebra and a mapping is performed to the SU_n special unitary groups, having infinitesimal group generators. The details of the group structure and multiplication tables are given in [4]. The Heisenberg uncertainties fall naturally out of the D_n group formalism.

In forming the invariants for a particular Cartesian space, of dimensionality n, the maximum number of invariants that can be formed is given by the following Lemma.

Lemma: Given: a Cartesian space of dimensionality n; the number of invariants of the Cartesian space of one less dimension (n - 1); plus the number of dimensions of the n - 1 dimensional space.

Let I_n be the maximum number of invariants for a Cartesian space of n dimensions and let I be the maximum number of invariants for a Cartesian space of $n' \equiv n-1$ dimensions, then:

$$I_n = I_n + n'. \tag{5}$$

A rigorous proof of this new Lemma is given in Ref. [7].

In developing the generalized Heisenberg relations and generalized invariance in a multidimensional geometry, in which each dimensioned physical variable is considered on an equal footing, we see that physical variables can be paired in uncertainty relations, such as those in Fig. 1. Also, paired variables can form invariant relations in terms of universal constants, as in Table 1. The relationship between these two pair relationships, as given in-more detail in Refs. [6] and [7], is presented in Table 2. For example, the index $\varepsilon = 1$ denotes the Heisenberg pair (x,p); the index $\delta = 5$, denotes the same pair (x,p) in an invariant relation. In the notation in Table 2, $\varepsilon = \delta = 3$ and $\varepsilon = \delta = 4$, both denote the same variable pair related in a Heisenberg relation and as a relativistic invariant. In this manner, and with the assumption of *equal footing* of physical variables, we see a way in which quantum mechanics and relativistic invariance can be tied together by a geometrical model of the manifold.

Invariant, S_k	Variable pair μ_k,	Component of the Minkowski metric
$S_\delta^2 = 1$	$\mu_1 = x, \eta_1 = t$	c^2
$S_\delta^2 = 2$	$\mu_2 = p, \eta_2 = E$	$c^{\frac{1}{2}}$
$S_\delta^2 = 3$	$\mu_3 = x, \eta_3 = E$	$\frac{1}{F^2}$
$S_\delta^2 = 4$	$\mu_4 = p, \eta_4 = t$	F^2
$S_\delta^2 = 5$	$\mu_5 = x, \eta_5 = p$	$\frac{c^2}{F^2}$
$S_\delta^2 = 6$	$\mu_6 = t, \eta_6 = E$	c^2F^2

Table 1. Pair-variable relations as invariants in terms of the elements of the generalized Minkowski metric. Six such invariant pair relations can be formed for a 4-dimensional Cartesian space.

Note that all invariant expressions in this paper are special relativistic invariants. General relativistic invariants are discussed in [7] as are light cone relations for the generalized special relativistic invariants. In [4], the quantized variable geometrical constraints are applied in general relativity and closed cosmological solutions are found for Einstein's field equations. Experimental evidence for closed cosmologies are sited in [4,12,15].

In [4,7,12], we formulate the manner in which we solve Einstein's field equations in the n-dimensional Cartesian space which leads to uniquely closed cosmological solutions. In [4] we formulate in detail quantum cosmogenesis and evolution of the current universe. Self-consistency between a modified big bang model and Einstein's field equations is attained without Guth's inflationary model, [23] which is extraneous in this picture. Also, our model is consistent with nucleon abundances [12]. Note that Guth's model may require a velocity of 6×10^3 the velocity of light. Dark matter comes out naturally from this new, self-consistent model, and modifications to the cosmic evolution allow us to account for dark matter and dark energy. Current Hubble's constant values, and critical densities, appear consistent with observed densities and matter content of the universe, and may accommodate a cosmological constant value Λ near zero, and yet still accommodate an apparent dark energy component [24].

Generalized Heisenberg Pairs	Generalized invariant pair variables	
$\epsilon = 1$, (x, p)	$\delta = 1$, $s_\delta = 1$	(x, t)
$\epsilon = 2$, (t, E)	$\delta = 2$, $s_\delta = 2$	(p, E)
$\epsilon = 3$ (x, E)	$\delta = 3$, $s_\delta = 3$	(x, E)
$\epsilon = 4$, (p, t)	$\delta = 4$, $s_\delta = 4$	(p, t)
$\epsilon = 5$, (x, t)	$\delta = 5$, $s_\delta = 5$	(x, p)
$\epsilon = 6$, (p, E)	$\delta = 6$, $s_\delta = 6$	(t, E)

Table 2. We can represent the relation between a pair of physical variables in two different formalisms. We have the uncertainty relation between two variables $(p_\delta, \upsilon_\varepsilon)$ where the index ε denotes a particular variable pair. We can also represent an invariant relation between two variables as $(\mu_\kappa, \eta_\kappa)$ and the index δ denotes a particular invariant relation. For $\varepsilon = 1$ and 2 we have the usual quantum mechanical relation and for $\delta = 1$ and 2 we have the usual invariant 4-vector relations. Table 2 depicts the manner in which these two representations of paired variables relate to each other.

5. Closed Cosmologies, Missing Matter, and Energy Without Inflation

Einstein originally introduced the cosmological constant to create a *Steady State* universal model in 1915. Sadly, this all changed in the early 1920's when Hubble's discovery of red shift was erroneously interpreted as a Doppler expansion of the universe; Hubble discovered redshift not a Doppler expansion of the Universe. In current thinking however, the Hubble constant is the ratio of the velocity of recession over the distance to the stellar, galactic or other cosmological structures for:

$$H = \frac{V}{R} = \frac{R/t}{R} \propto \frac{1}{t_0} \tag{6}$$

where R is the distance to the nearest galactic or stellar source, $V = \dot{R}$ is the velocity of recession and t_0 is the age of the universe. The Doppler red shift is given as $z = \Delta\lambda / \lambda$ where λ is the rest frame wavelength emitted from the observed cosmological object.

Einstein stated that his inclusion of the cosmological constant Λ was the greatest blunder of his career, when in actuality, his greatest mistake was abandoning what he knew to be correct. Currently, astrophysicists have partly corrected this error by reinstating Λ as an explanation of very distant galactic acceleration [25]. We write Einstein's field equations as:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R_{\mu\mu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{7}$$

where $T_{\mu\nu}$ is the Riemann-Christoffel curvature mass-energy tensor, and $F = c^4 / G = \frac{8\pi}{F} T_{\mu\nu}$ is the Rauscher cosmological acceleration force. The term $R_{\mu\mu}$ is the Ricci Curvature tensor and $R_{\mu\nu}$ is the space time curvature.

One of the most promising explanations of non-gravitational and non-electromagnetic acceleration of the universe is due to the vacuum energy which is implied as a source of the acceleration. This manifestation of the vacuum energy may be expressed as the need to reintroduce Einstein's cosmological constant Λ, where, rather than ensuring a static universe, it forms a more dynamic one with an acceleration parameter as a function of R, the cosmological distance. Thus, Λ may become an effective gravitational repulsion term (especially when crossing from the Hubble Universe to a *Holographic Anthropic Multiverse*). The additional term $\Lambda g_{\mu\nu}$ can be related to the vacuum in the following manner. We express the right side of Einstein's field equations in terms of the stress energy tensor as $G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$ and write this symbolically as two terms, $G_{\mu\nu} = -\frac{8\pi G}{c^4}\left[T_{\mu\nu} + T^{vac}_{\mu\nu}\right]$, where $c = 1 = G$ and $T^{(vac)}$ represents the stress energy of the vacuum and can be written as $T^{(vac)} = \frac{\Lambda}{8\pi} g$, that is, the stress energy density expressed as the cosmological constant term contributes to the vacuum energy density; the vacuum density $\rho^{vac} = T_{00}^{(vac)} = \Lambda / 8\pi.$ In standard approaches $\rho^{(vac)}$ is considered to be small or $\rho^{(vac)} < \rho^{(matter)} \sim 10^{-29}\, gm / cm^3$; but in fact, the structure of the vacuum of $\rho \sim 10^{93}\, gm / cm^3$ may well determine the structure and form of observable matter. Note that the limits for $\Lambda \neq 0$ are units of cm^{-2}. Acceleration appears to occur at the edge of the known universe. The accelerating universe, apparent magnitude vs. red shift, z from z_0 to $z > 1$ is statistically significant. Galactic red shifts will eventually vanish beyond the event horizon where $z > 6$. On the Schwarzschild universe see [4,12,15] and the next section.

We examine the concept of the action of the vacuum as a dynamic system that may explain the recently observed anomalous expansion rates indicated by high *z*. Most recently, the Sloan Digital Sky Survey telescope at Apache Point, New Mexico, has identified a very high red shift quasar at *z* = 4.75. This red shift indicates that this cosmological object ought to be 13.75 billion years old, formed when the universe was less than one billion years old. Note in the standard view, the gravitational red shift of
the sum is:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_r - \lambda_e}{\lambda_e} = 2 \times 10^{-6}. \tag{8}$$

The Sloan Digital Sky survey's main task is to measure red shifts and they have identified several new quasars, the most distant ever observed [26]. Their most distant quasar has a *z* = 6.28 and another quasar has a *z* = 5.73 which was discovered last year by the Sloan survey. In fact, observable red shifts have been identified for four quasars in this survey: *z* = 5.80, *z* = 5.82, *z* = 5.99and *z* = 6.28, which are the highest *z* quasars yet observed. It is believed that 20 more quasars with *z* > 6 will be found [27]. The so-termed *missing mass*, first suggested by Fritz Zwicky in 1933, is considered to be different from any ordinary matter in the universe in that there are zero detectable emissions or absorption of light in any known or ordinary manner. The rate of expansion of the big bang universe from a black hole indicates the conditions for a closed universe, and throughout the evolution of the universe [4,15]. Currently, it is thought that the observed mass of the universe indicates that 94% of the matter needed for a just closed universe is *missing* [15].

The critical density for a just closed universe is $\rho_c = \frac{3H^2}{8\pi G}$. Hubble's constant is given as $H = \frac{\dot{R}}{R}$, where *R* is the distance scale, and $\dot{R}$ the velocity as determined by red shift, z. We then deduce that $\rho_c \propto \frac{1}{R^2}$ and thus, also, the time scale of the universe, $\frac{1}{H} \sim t_0$ is affected. Variations in the red shift and the distance scale estimates affect the most likely value of ρ_c. Nuclear abundance and *x*-ray data are also relevant to determining the value of ρ_c [12].

The value is about $\rho_c \sim 2.4 \times 10^{-29}\, gm / cm^3$ derived from $H = 73.8 Km / \sec / MPC$. These density estimates are very sensitive to the (Deuterium/Hydrogen interstellar ratios). These densities only account for a few percent of the value needed to fit with the current big bang Schwarzschild universe which has about 94% of the closed universe as *missing* mass. The form of the *missing* mass is hypothesized to be, in part, in the form of black holes, interstellar plasma, or vacuum state energy [28]. Some astrophysicists consider the missing mass may be *cold dark matter* (CDM). Three proposed pictures emerge. Dark matter is said to only weakly interact with ordinary matter. Three models have been put forward:

(1) The concept of WIMP's, or weakly interacting massive particles, that create vibrations and bursts of light and heat, (perhaps as quasars which rotate around the center of the galaxy);

(2) MACHO's or massive compact halo objects, which may relate to the clumping of galaxies; and

(3) HPVS's or hyperspace resonance vacuum structures, which relates to both of the above models and expands on them. If the vacuum energy is structured and, black holes, and galactic matter, it may act to guide the evolution of universal features such as galactic clumping, since galaxy formation and evolution is thought to have started and from a relatively uniform distribution during cosmogenesis.

All three of these models fail to yield phenomena that are directly observable by even the most sophisticated telescopes; but their properties are reflected in the form of matter–energy we observe, ranging from particles, to galaxies, to galactic clusters, and beyond. Some of these models have within them empirical implications of a matter-energy and entropy evolution occurring throughout cosmogenesis and cosmology [4]. This, in turn, leads to a reconciliation of the Hoyle-Narlikar matter creation model with the big bang cosmology through the formulation of the *little whimper* model [15,29]. This picture leads to a just closed cosmology throughout the evolution of the universe.

6. Quantum Cosmology and The Evolution of a Closed Schwarzschild Universe

The initial condition constraints that characterize cosmogenesis are chosen to be the quantum gravity level vacuum containing the Planck length of $\ell \sim 10^{-33}\,cm$, and quantized time of $t \sim 10^{-44}$ sec [4,17]. E.R. Harrison discusses early universe quantities, which are Planck-like units related to the vacuum energy [30]. They are the initial conditions of the universe, in our model, and act as a set of constraints throughout the evolution of the universe. E.R. Harrison, as we do, discusses the role of the vacuum in quantum gravity and mechanics and such units as length, time and energy, and also the thermodynamic properties of cosmogenesis [4,15,30]. See the previous section on the discussion of the vacuum state contributions to the stress energy term.

In Table 3, we list some characteristic early universe physical quantities as quantized variables and compare these to characteristic present day universe values which obey the Schwarzschild condition $R_s = \frac{2Gm}{c^2}$. The length $\ell \sim 10^{-33}\,cm$ can be interpreted as the limit of length in the manifold, and $\ell \sim 10^{-44}$ sec the corresponding characteristic time for $c = \ell / t$, where *c*, the velocity of light is taken to be the characteristic signal propagation velocity in the manifold. The present-day magnitude of the physical variable of length is the size of the universe, $R \sim 10^{27}\,cm$ and the age of the universe is $t_0 \sim 10^{17}$ sec, and *c* the velocity of light, $c \sim 10^{10}\,cm/\sec$.

The fundamental vacuum velocity is the velocity of maximal real mass signal propagation in the manifold, $v \le c.$ The approximate critical density for a just closed universe is $\rho_c = 3H^2 / 8\pi G \sim 10^{-29} gm / cm^3,$ in this case [2,4] where Λ is taken as zero and $\kappa = +1.$ In the previous section we considered cases where $\Lambda \ne 0.$ The ratio $\dot{R} / R = H$ is Hubble's constant, and, for $t_0 \le 1/ H, H = 2.4 \times 10^{-18} cm / \sec/ cm,$ where t_0 is the initial time condition of the universe and H is Hubble's constant. Note that for the present universe $\dot{R} = HR = 10^{-18} cm / \sec/ cm \times 10^{27}$ giving $\dot{R} \sim 10^{10} cm / \sec,$ or velocity of light. We have examined the limits set by deuterium abundance as a constraint on H in Ref. [15].

We calculate the evolution from initial to current characteristic values of physical variables during cosmogenesis to the present-day universe. In Table 4 we tabulate the initial values of physical variables and their values throughout the evolution of the universe. These values are plotted in Figure 2 (ordinate of physical variables in their respective units) versus time (abscissa in seconds) for various physical variables. The physical variable values in Table 4 are obtained as follows: At time, $t = 10^{-44} \sec,$ all physical variables assume their Planck unit values as initial constraints on the universe. We assume that the characteristic velocity of signal propagation of the universe is $v = c,$ the velocity of light; therefore, the radius at any time is $R = vt.$ Utilizing the Schwarzschild condition, we obtain the mass as $m_s = c^2 R_s / 2G.$

The corresponding density for the Schwarzschild condition is given by $\rho = m_s / R_s^3$ where the subscript *s* denotes the Schwarzschild criterion. In Table 3 and Figure 2 we denote this density as ρ_2 which is compared to R. Omnes' [31] values (ρ_1) and E. R. Harrison's values, (ρ_3) [30,32]. The expression for Hubble's constant for a *now* or present-day universe applies as $\rho_c = 3H^2 / 8\pi G$ for $\rho_c \sim 10^{-29} gm / cm^3.$ For the initial conditions of the universe, $\rho_i \sim 10^{93} gm / cm^3,$ where we utilize $\rho_i = 3H^2 / 8\pi G.$ If we take G as a constant, then $H^2 \cong 8\pi G / 3 \times 10^{93}$ or $H \sim 10^{43} cm / \sec = \dot{R} / RH \sim 10^{10} cm / \sec.$ We can still use the present-day value of Hubble's constant, $H \sim 10^{-18} cm / \sec/ cm$ to define $t_0 \le H,$ as $1.37 \times 10^{17} \sec,$ or the age of the universe. We must note that there is still some controversy over the value of H but the D abundance does set constraints on H [15]. We also make the implied assumption that the microscopic (nuclear and atomic) and macroscopic (electromagnetic and gravitational) physics, are valid throughout the evolution of the universe.

The total energy of the universe is given by $E_s = m_s(t)c^2,$ where again the subscript denotes the Schwarzschild mass at any time, *t*, and $T_s = E_s / k$ is the energy particle. Comparison can be made to E.R. Harrison [30,32]. The relationship $T_s = E_s / k$ holds for initial conditions only. In general, throughout the evolution of the universe, $T_s \propto 1/ E.$ The increase in energy (and mass) has to be reconciled with the cooling off or decrease in temperature. We will now omit the subscript *s*. The entropy under the Schwarzschild condition is given as $S = m_s(t) \times S_I / m_i,$ where S_I and $m_i,$ are the quantized variable initial conditions of the universe and $m_i(t)$ is the Schwarzschild mass as a function of time. The initial entropy is $S \cong k = 10^{-16} erg / \deg,$ where *k* is the Boltzmann constant.

In Table 3 and Figure 2 we denote this entropy as S_i.

Physical Quantity	Symbol	Early Universe	Present
Length	ℓ	10^{-33} cm	10^{27} cm
Time	t	10^{-44} sec	10^{17} sec
Mass	m	10^{-5} gm	10^{56} gm
Velocity	c	10^{10} cm / sec	10^{10}cm/sec^2
Acceleration	a	10^{53} cm / sec ^{2}	10^{-7} cm / sec^2
Force	F	10^{49} dynes	10^{49} dynes
Power	P	10^{59} dynes cm / sec	10^{57}dynes cm/sec
Pressure	p	10^{114} dynes / cm^2	10^{-7} dynes / cm ^{2}
Energy	E	10^{16} erg	10^{75} erg
Density	ρ	10^{94} gm/cm^3	10^{-29} gm/cm^3
Frequency	ω	10^{43} cycles/sec	10^{-17} cycles/sec
Temperature	T	10^{32} degrees (K)	3 degrees (K)
Entropy	S	10^{-16} erg/degree	10^{45} erg/degree

Table 3. The early universe physical quantities are the Planck units which are compared to some present-day characteristic values of physical variables on the scale of the universe.

We compare this to the entropy, denoted S_2, obtained from the third law of thermodynamics, $S \sim \Delta E / T$ where $\Delta E = E_f(t) - E_i(t)$, and for temperature, $T = T(t)$. We can use the approximation $\Delta E \sim E_f(t)$ since for $t \geq 10^{-30}$ sec, $E_f \gg E_i$. For $t = t_0$ (present age of universe), $S_2 =$ $(10^{76} - 10^{-16}) erg / 3°K \cong 10^{75} erg / °K$, which agrees with E. R. Harrison's value [30,32]. The Force is given by $F = E_s(t) / R_s(t)$ with the ratio being constant and given by the universal force, $F = c^4 / G$. The power is given by $p = vF$ and for $v = c, P = cF$, which is the quantized power. Interpretation of the quantized Force and quantized Power for an isotropic homogeneous universe with a Robertson metric is given in Rauscher E A[4,15] In Table 3, the present day Power, $p \sim 10^{57}\, dynes \times cm / \sec$ is just the power to expand the universe to its present day configuration, which is a factor of 100 less than the usual power value. The Pressure, $p \sim 10^{-7}\, dynes \times cm^{2}$ is the present density pressure for a critical density, $\rho_{c} \sim 10^{-29}\, gm \times cm^{3}$. The acceleration is given by $a_{s}(t) = F / m_{s}(t)$, and $m_{s}(t)$ is the Schwarzschild mass. As the *big bang* expansion continues, the rate of expansion slows down. The results in Table 4 and Figure 3 are consistent with this calculation.

The frequency ω is given as $\omega \propto 1/t$. There have been some attempts to calculate the total rotation of the universe [15]. This rotation may possibly be interpreted in a Machian sense [3]. The theory of general relativity is compatible with Mach's principle but is left out of the usual interpretation of frames of reference. Indeed, Mach's principle is consistent with matter creation and may even be explained by the continuous creation of matter (matter-energy) [4]. In the multidimensional geometrical model, we may be able to formulate matter-energy creation in terms of a coordinate transformation. Very simply, we can consider a model in which "stretching" field lines "pops" particles into existence from the vacuum, i.e., space-time transforms into matter-energy by a generalized rotation of the dual Riemannian sheets of the Cartesian topology [4]. It is shown by E.A. Rauscher that the generalized Minkowski metric has this property, if rotations in the Cartesian space are considered [6,7].

x ↓	↑Y t	↑R	↑v	↑m	↑ρ_1	↑ρ_2	↑ρ_3	↑E	↑T	↑S_1	↑S_2	↑F	↑P	↑a	↑ω
	10^{-44}	10^{-33}	10^{10}	10^{-5}	10^{94}	10^{94}	10^{43}	10^{16}	10^{32}	10^{-16}	10^{-16}	10^{49}	10^{59}	10^{59}	10^{43}
	10^{-20}	10^{-10}	10^{10}	10^{18}	—	10^{48}	—	10^{38}	10^{7}	10^{7}	—	10^{49}	10^{59}	10^{31}	10^{20}
	10^{-10}	1	10^{10}	10^{28}	—	10^{28}	--	10^{48}	—	10^{17}	—	10^{49}	10^{59}	10^{21}	10^{10}
	10^{-4}	10^{7}	10^{10}	10^{35}	10^{35}	10^{14}	10^{14}	10^{55}	10^{12}	10^{24}	10^{43}	10^{49}	10^{59}	10^{14}	10^{4}
	1	10^{10}	10^{10}	10^{38}	10^{13}	10^{8}	10^{4}	10^{58}	10^{9}	10^{27}	10^{49}	10^{49}	10^{59}	10^{11}	1
	10^{4}	10^{15}	10^{10}	10^{43}	10^{11}	10^{-2}	10^{-21}	10^{63}	10^{4}	10^{32}	10^{59}	10^{49}	10^{59}	10^{6}	10^{-9}
	10^{10}	10^{20}	10^{10}	10^{48}	—	10^{-12}	—	10^{68}	—	10^{37}	—	10^{49}	10^{59}	10^{-1}	10^{-10}
now	10^{7}	10^{28}	10^{10}	10^{56}	10^{-29}	10^{-29}	10^{-29}	10^{76}	3	10^{45}	10^{75}	10^{49}	10^{59}	10^{-7}	10^{-17}
unit	sec	cm	cm/sec	gm	gm/cm^3	gm/cm^3	gm/cm^3	ergs	deg.K	erg/°	erg/°	dynes	dynes-cm/sec	cm/sec^2	cycles/sec

Table 4. Schwarzschild Evolution of the Universe.

The remarkable fact then emerges that in order to maintain the Schwarzschild condition, as an initial and present condition, we must evoke matter creation (at a macroscopic constant rate) to make this model self-consistent between Einstein's field equations and the big bang, which precludes the need for the inflationary model. It appears that we may be able to reconcile the continuous creation and big bang cosmologies in our "little whimper" model, in which the universe, under initial conditions, “explodes” as a “mini-black hole” and then larger scale black holes, with their surrounding plasma fields, [33] evolve with the creation of matter by a continuous process of matter influx from the vacuum energy, which may be, in part, detected as Hawking radiation from black holes in the universe. It is interesting to speculate as to the role of matter “creation”, from geometry, and matter “destruction” into geometry, in terms of a multitude of black holes, and in terms of available vacuum energy. One may formulate the one as the converse of the other in terms of time reversal. The issue of time reversal invariance and/or, more completely, CPT invariance, requires consideration in this picture [4]. An active vacuum, in excited plasma states in the vicinity of black holes, may account, at least in part, for some of the missing mass [4,28,33].

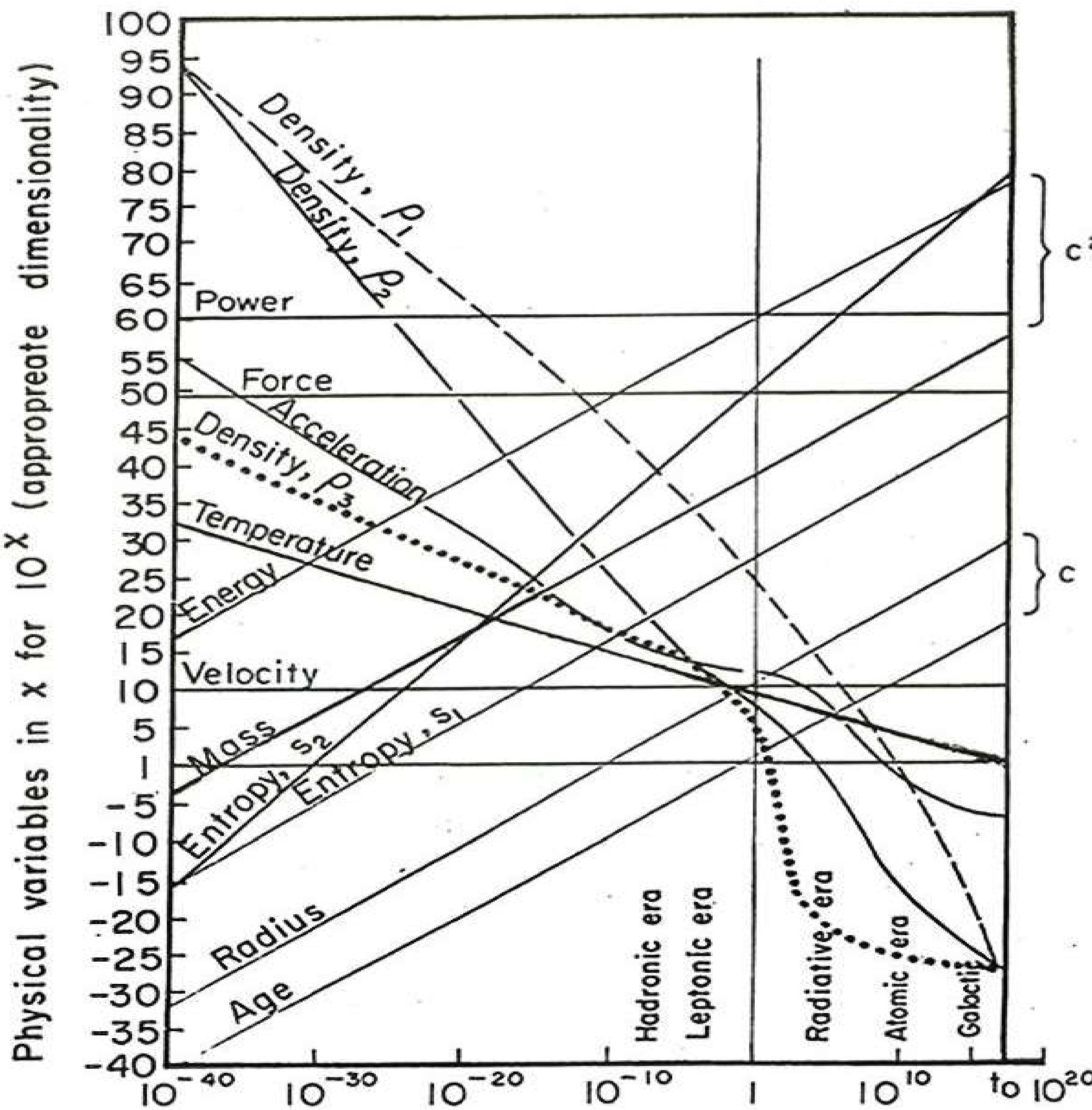

Figure 2. Physical Variables in the Time-Evolution of a Schwarzschild Universe.

By imposing the generalized Schwarzschild condition initially, presently, and throughout the evolution of the universe, which acts as boundary conditions, we have found that matter creation is necessary in a model that is self-consistent with Einstein's field equations (in the expansion phase of the universe) and the big bang model. This condition led to the *little whimper* closed cosmologies. We look now at *the ashes and cinders* of which A. G. Lemaitre spoke, and ask, are they ashes of a big bang, in which all the matter of the universe *instantaneously*, in $t \sim 10^{-44}$ sec, exploded, or are these the ashes of a developing, on-going exploding growth [4]? When we consider rotations in our Cartesian space [4], stretching or distorting the generalized metric, such as the outer reaches of the cosmos, we may be able to interpret this as a disruption of the mechanism whereby the radiation field is converted into particulate matter. One important experimental factor in determining closed versus open cosmologies is the present-day deuterium abundance. In [15] we discuss the possible reconciliation of our model with the observed deuterium abundance and the implied value of Hubble's constant (See Table 4 and Figure 2).

7. Discussion

The details of a unification scheme between quantum mechanics and relativity is a work in progress [3,4] but a start has been made based on demonstrating the common roots of the Hamilton-Jacobi classical mechanics, non-relativistic and relativistic quantum mechanics and the Field Equations. Unifying quantum mechanics and general relativity is one thing; it is quite another to demonstrate the "common roots" of the formalism. A relativistic quantum field theory, or a "quantized" gravitational theory, may relate to an aspect of a unifying theory, but more is needed to attain a fundamental unifying theory. However, quantized gravitational theories may shed some light on the path toward a fundamental relation between quantum and relativistic physics. For example, the problems of reconciling non-linearities in general relativistic fields and the linear super-position principle in quantum mechanics, are well pointed up by some of the workers in quantized gravitational theories [3]. B. S. De Witt [34,35] in an extensive treatise, discusses some of the difficulties of non-linearities of the metric tensor and the quantum superposition principle. Attempts have been made to develop a linear theory of the massless, spin-2 field [22]. Relating to the fundamental structure of the "roots" of the canonical formalism is fundamental to a unifying of quantum mechanics and relativity [3]. Since both quantum mechanics (primarily relating to micro-phenomena) and relativity (relating to macro-phenomena) are so successful in elegantly describing physical phenomena, a unifying aspect for these fields of physics must necessarily involve the interrelation of diverse aspects of reality.

It also seems apparent that every aspect of reality depends on every other aspect, as expressed clearly by H. Stapp [36], "Every part of the universe depends on every other part". The bootstrap model of elementary particles developed by G. Chew [37] is another statement of this proposition. A unified theory of reality which must intimately bring together quantum and relativistic phenomena must necessarily be complete. The concept of completeness in quantum theory is discussed in the classic paper by A. Einstein, B. Podolski and N. Rosen [3,38] and subsequent papers inspired by this work [3] and references therein.

The extended dimensions of Cartesian space, which are expressible uniquely in terms of universal constants, are manifest in quantum mechanics and special relativistic invariants and general relativity, as well as thermodynamics [21], and electromagnetic theory [4]. According to B. N. Taylor, W. H. Parker, and D. W. Langenberg [39], the "universal constants are an important link in the chain of physical theory which binds all the diverse branches of physics together". To paraphrase J. A. Wheeler: "*spacetime is not just a passive arena for doing physics, it is the physics"* [1] and, we might add, matter, energy, momentum, spin, etc. as in Cartesian geometry.

In 1921, Einstein discussed a fundamental aspect of reality: "*It was formerly believed that if all material things disappeared out of the universe, time and space would be left; according to the relativity theory, however, time and space disappear together with the things*" [25]. In a Cartesian space, should any 1D physical variable vanish, then all the rest of the universe would also vanish. It is the interactive whole that comprises the universe.

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