Detection of the continuous gravitational wave of HM Cancri

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HM Cancri is expected to be be one of the brightest sources of gravitational waves in our galaxy. Despite its known frequency, the radiation could not be detected so far. A novel technique can compensate for phase modulation and detect this GW in the records of superconducting gravimeters. This new observational window will allow a deeper understanding of the enigmatic stellar system.

1 Introduction

23 years ago a sequence of X-ray pulses was discovered in the data series of the ROSAT satellite, repeating every 321 seconds. The source is thought to be two white dwarfs orbiting each other. The pair HM Cancri (RX J0806.3+1527) is difficult to find in the optical range, but besides X-rays it also generates a gravitational wave (GW) of twice the frequency. The theory states that a GW slightly changes lengths of all objects around in the same rhythm. In the year 2016 researchers succeeded for the first time to measure the final stage of a GW - the last seconds of a merging double star system [1].

Astronomers would like to observe the properties of a GW at an early stage, but previous attempts of reception [2] failed. Is it due to insufficiently long antennas or suitable analysis programs? In order to clarify both questions, the longest scale existing on earth - the earth itself - was chosen as antenna. The change in diameter is measured with gravimeters and the signal evaluation is done with proven standard methods of communications engineering. The recorded data of 17 instruments provide clear evidence that the GW of the nearby source HMC affects sensitive gravimeters.

2 Instrumentation and noise

The expected frequency of the GW is within the measurement range of superconducting gravimeters [3]. In the frequency range around 10 mHz there are no comparable sensitive instruments that could react to GW. Every second, the very low-noise Gravimeters measure the vertical component of the acceleration due to gravity (about 9.81 m/s^2). Despite the frequent disturbances by earthquakes, the Power Spectral Density (PSD) in the range around 10 mHz is smaller than $10^{-18}\,\frac{m^2}{s^4 Hz}$ [4] [5]. Signals always pass through filters during their processing to reduce noise. The measurable noise amplitude after the filter is calculated according to the equation

$$A_{noise} = \sqrt{PSD \cdot BW} \tag{1}$$

where BW is the bandwidth. One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because that increases the necessary time span T_{min} that the filter needs to settle. This relationship was first formulated by Küpfmüller [6] and is reminiscent of the Heisenberg uncertainty principle.

$$T_{min} \cdot BW \geq 0.5 \tag{2}$$

If one looks for weak GW signals in the recordings of gravimeters, this means: If one wants to reduce the amplitude of the interfering noise to 10^{-14} m/s^2, the bandwidth of the filters must not exceed 1 nHz (equation (1)). Because of (2), gravimeters must be operated for at least 15 years and the frequency of the GW must not vary by more than 0.5 nHz during the entire period to keep the signal within the filter range.

3 The reception of an idealized GW

Let us assume that a binary star system [7] generates a GW of constant frequency and the distance to the Earth remains constant. When the GW passes the Earth, the diameter L oscillates in the same rhythm and sinusoidal with the maximum amplitude ΔL. Because the diameter of the earth is much smaller than the assumed wavelength of the GW ($\approx 5 \times 10^{10}$ m), the strain h is calculated with the approach

$$h = \Delta L/L = h_0 \cdot sin(\omega t) \tag{3}$$

Previous estimates give values between $h_0 \approx 10^{-25}$ for rotating pulsars and $h_0 \approx 10^{-19}$ for binary systems in our Galaxy. In the following calculations we assume that a close double star system generates a GW of constant frequency $\omega = 0.2\ s^{-1}$ and the strain h_0 here on Earth has the value 10^{-19}. Thus the change of the local gravity $\ddot{L}$ near the Earth surface is

$$\ddot{L} = L \cdot \omega^2 \cdot h_0 = 12.7 \times 10^6\ m \cdot (0.2\ \frac{1}{s})^2 \cdot 10^{-19} \approx 5 \times 10^{-14}\ \frac{m}{s^2} \tag{4}$$

As this value exceeds the $10^{-14}\ \frac{m}{s^2}$ limit of common gravimeters calculated above, the GW of nearby binary systems should be readily measurable.

So far, gravimeters are instruments of geophysics and therefore the data pre-processing is optimized for the frequency range of tides ($f \approx 11\ \mu$Hz) and natural resonances of the earth ($f \approx 5$ mHz). Earthquakes produce short-term disturbances in the lower frequency range, which can be easily removed and do not interfere with the search at high frequencies above 10 mHz. A sound-insulating mounting of the gravimeters and an optimization of the data pre-processing would simplify the search for further GW.

4 The reception of a real GW

The high orbital speed around the sun causes a periodic Doppler shift and changes the reception frequency on an annual basis. This manifests itself as a phase modulation of the signal with $f_{year} = 31.69$ nHz and is a signature of any GW. This modulation produces sidebands, which is why the signal appears in the spectrum no longer as an isolated line, but as a bundle of closely spaced lines (Figure 1). The modulation index η determines how many lines with which individual amplitudes can be expected. A typical spectrum hardly differs from noise and is difficult to identify. The method described below solves this problem without knowledge of the spectrum.

A GW can be successfully detected if the S/N is sufficiently high. The proportion $S/N \propto h_0\sqrt{T/BW}$ provides hints on how to improve the S/N. h_0 means the amplitude of the GW signal, T is the observation period, and BW is the bandwidth of the receiver. Superconducting gravimeters have been recording minute changes in gravity for years and are therefore excellent sources of long-term data. A significant reduction in bandwidth can only be achieved if both phase modulation and frequency drift are completely eliminated.

5 Frequency or phase modulation (PM)

In communications engineering, phase-modulated signals are used to transmit data. The method is so closely related to frequency modulation that the two are often confused. The advantage is that the respective sets of equations can be converted into each other [8].

Although HM Cancri does not transmit data, we still receive a phase modulated signal because the earth moves in the GW wave field (compare figure 3). If this PM occurs with the frequency f_{year} and a not too large modulation index η (see equation 5), the source is probably not in our solar system.

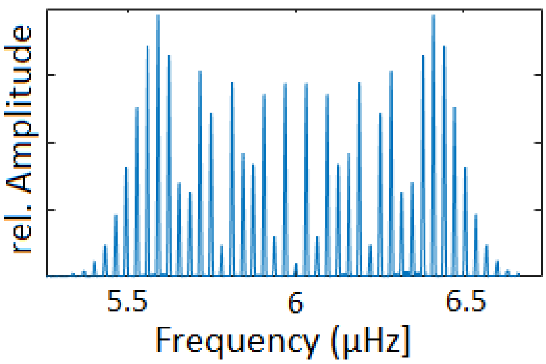

Figure 1): *The spectrum of a phase modulated oscillation of sufficiently long duration with $\eta = 10$ fills a wide band. The modulation frequency determines the spacing of the lines.*

The PM equations require that the duration of the record spans several years. Then, the modulation frequency is equal to the reciprocal of the orbital period around the Sun and the spectrum resembles a "picket fence" of many lines with mutual spacing f_{year} (Fig 1). The following equation defines the modulation index η, which determines the amplitudes of the spectral lines:

$$\eta = \frac{\Delta f}{f_{modulation}} = \frac{\Delta f}{f_{year}} \tag{5}$$

The maximum frequency deviation Δf results from the relativistic Doppler effect due to the orbital speed of the Earth. Knowing this value, it is possible to calculate the maximum frequency deviation Δf of the GW source HM Cancri, which is located almost in the plane of the ecliptic. On April 24 and October 27, the orbital velocities of the Earth are opposite and have the value 29630 m/s [9]. This allows the Doppler shift to be calculated.

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c + v_{Earth}}{c - v_{Earth}}} - 1 \right) \approx 614 \; nHz \tag{6}$$

The modulation index η reaches the surprisingly high value 20 and the corresponding spectrum claims the Carson bandwidth of 1.4 μHz [10]. This bandwidth must not be undercut when processing an FM signal in order to avoid distortion of the modulation. It does not seem very promising to search in the noise for a set of about 50 spectral lines with unknown amplitude distribution. In addition, for certain values of η the amplitude of the carrier frequency disappears (zeros of the Bessel function).

Every terrestrial sensor orbits the earth's axis daily, which is why the receiving frequency is phase modulated with $f_{day} = 11.57$ μHz. The small peripheral speed at the equator causes a tiny frequency deviation of only

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c + v_{equator}}{c - v_{equator}}} - 1 \right) = 9.6 \; nHz \tag{7}$$

and ensures the extremely small modulation index

$$\eta = \frac{\Delta f}{f_{day}} = 8.3 \times 10^{-4}. \tag{8}$$

In communications engineering, such a small value is called phase noise and is neglected. The amplitude of the two sideband frequencies are smaller than the amplitude of f_{GW} by a factor

$$\frac{A_{sideband}}{A_{carrier}} = \frac{J_1(\eta)}{J_0(\eta)} = 0.0004 \tag{9}$$

and can be detected in the spectrum only with an extremely high $S/N > 6 \times 10^6$.

6 The modified superhet principle (MSH)

In radio engineering, a high receive frequency f_E is reduced to a lower value f_{IF} by mixing it with a locally generated frequency f_{OSZ}, because it can be investigated more advantageously. For the frequencies, $f_{IF} = |f_E - f_{OSZ}|$ is valid.

Usually, the value f_{OSZ} is constant in order not to change the modulation content of the signal. In the search for GW, the opposite is true: one must remove the known but unwanted PM in order to reduce the bandwidth. Therefore, one modulates the frequency f_{OSZ} with the goal of obtaining a *constant* diffence frequency f_{IF}. When the modulation of the received signal and the oscillator coincide, the "picket fence-like" spectrum turns into a single high-amplitude spectral line. Illustratively speaking: The many spectral lines that are adjacent to each other in Figure 1 are rearranged so that they add up to a large total length. Then the following statements are valid:

- With a broadband FM signal, the total energy is distributed over many spectral lines, each with low amplitudes. For an unmodulated signal, the total energy is concentrated on a single line of high amplitude.
- A constant frequency appears in the spectrum as an isolated line that can be easily and unambiguously identified in the noise.
- If it is possible to compensate the phase modulation and the frequency drift completely, one may reduce the receive bandwidth strongly to improve the S/N. In the present case, the bandwidth may be reduced from 1.4 μHz to about 1.5 nHz.

7 The data basis of the investigations

Gravitational data have been recorded for decades and the IGETS Potsdam [11] stores correspondingly long data series. So far, the gravimeters have been used for earthquake research, are mounted directly on the ground, and therefore respond to distant, minute ground motions. Stronger earthquakes overload the sensors and cause data gaps of several minutes. These irregular tremors of the ground affect the quality of all data sets. Prior to release, the signals from the sensors pass through low-pass filters of cutoff frequency 8.3 mHz, which is why the data series from the gravimeters include the range in which the GW from HM Cancri is expected.

The gravimeters are not identical and the data sets differ because the raw data are apparently processed by different methods: For a few, the noise floor in the frequency range of interest around 6 mHz is too large, others contain data gaps that are too wide, and occasionally experiments were performed on the gravimeter during the recording period. Some data errors can be mitigated by high-pass filters. The inconsistent pre-processing of the sensor data does not allow the amplitude of the GW to be determined.

The IGETS stores gravity data in a variety of formats. When searching for GW in the frequency range above 1 mHz, one can ignore the influence of the variable air mass above the gravimeter and focus on the gapless data series in the ninth column. After concatenation to a single file, a high-pass filter removes the interfering tides. Last, we reduce the frequency from 6.22 mHz to 6 μHz and the bandwidth to 7 μHz in two steps. This does not change the modulation content of the signal (phase modulation and drift), but has the advantage that we can extend the sampling interval by a factor of 500. This shortens the computing time of the following iterations because the file lengths are reduced by the same factor.

8 Comparison of previous data

In 1999 the periodic changes of the X-ray emission of the stellar system HM Cancri were discovered [12] in the records of the satellite ROSAT. A first estimation gave the period duration 321.25 s ± 0.25 s. The system is hardly visible in the optical range, so measurements of the period are difficult. Further investigations followed:

- Initial evaluations of the observations in the X-ray region [13] yielded period lengths of 321.5393 ± 0.0004 s and 321.5465 ± 0.0004 s, respectively. This corresponds to GW frequencies of 6.220079474 mHz and 6.219940195 mHz. These 2001 results may have been obtained using short record lengths and should therefore be viewed with caution.
- From optical observations [14] the period 321.5304 s follows. In conjunction with ROSAT data the slightly different value 321.53033 s is calculated. The frequency of the radiated GW is 6.220251647 mHz or 6.22025307 mHz.
- In the following year, the probably most exact value 3.11013824 mHz ± 0.17 nHz was determined from the Chandra data [15]. At that time the GW had the frequency 6.22027648 mHz.

From X-ray data, Strohmayer [15] obtained the value $\frac{df}{dt} = (3.63 \pm 0.06) \times 10^{-16} \frac{Hz}{s} = 11.5 \frac{nHz}{year}$ for the drift of the orbital frequency. For the GW one has to double this value because $f_{GW} = 2 \cdot f_{orbit}$. The aim of this work is to identify the GW in the records of as many gravimeters as possible, whose characteristic values correspond very exactly to these specifications.

9 Methodology of measurement of a GW

The following investigations focus on the frequency range 6220 μHz to 6220.6 μHz. In this narrow search range, there are apparently several GW that can be distinguished based on their phases and modulation indices. To identify the signals of HM Cancri, one needs accurate initial values. Every year on July 22, Earth - Sun - HMC lie approximately on a straight line. On this day and with sufficient S/N, one could measure the undistorted frequency of the GW (The small ecliptic latitude -4.7° of HMC shall be neglected for the moment). Three months later, on October 27, the Earth is heading toward HMC at about 29 km/s. Six months later, on April 25, it moves away from HMC [9] at about the same speed.

The usual approach for a phase modulated oscillation with constant frequency is

$$y = sin(2\pi t \cdot f_{GW} + \phi_{modulation}) \tag{10}$$

The simple equation (10) contains two parameters f_{GW} and ϕ which must be adapted to the problem: The frequency f_{GW} of HMC is not constant, but increases proportionally to the time. The attempt to model the drift by a 2nd degree polynomial failed because of the poor S/N of the gravitational data. The phase $\phi_{modulation}$ indicates the position of the earth on the way around the sun. After determining the times and amplitude of maximum phase shift, it is possible to calculate the ecliptic position of the source HMC. Translated into mathematical language it becomes:

$$y = sin(2\pi t(f_{GW} + t \cdot k_{drift}) + \eta \cdot sin(2\pi t f_{year} + \varphi)) \tag{11}$$

The equation (11) contains all necessary parameters to describe the receivable GW. The initial values for HM Cancri are:
$f_{GW} \approx$ 6,22 mHz (frequency of GW; depends on the year)
$k_{drift} \approx$ 23 nHz per year, twice the value from [15]
$\eta \approx 20$ (Modulation index, follows from the ecliptic latitude of the source)
$f_{year} =$ 31,688 nHz (constant orbital frequency of the earth)
$\varphi \approx$ 1,37 (Gravimeter data recordings start on January 1 of each year; astronomers use the vernal equinox of March 21 as a phase reference.)

10 Iterative analysis of gravity data

Data basis were the records of 17 gravimeters, which are characterized by a low noise level in the vicinity of 6 mHz and were in operation almost without gaps in the period 1997 to 2020: BF1+2 (Germany), ST (France), BH (Germany), CO (Austria), DJ (Benin), MC (Italy), MO1+2 (Germany), PE (Czech Republic), SU1+2 (South Africa), OS (Sweden), WE (Germany), YS (Spain), CB (Australia), MB (Belgium).

Records from these instruments are used to form 32 series, each starting on January 1 of a year between 1997 and 2012 and spanning ten years. Each data series is treated in the same way:

1. The center frequency of a narrow range near 6220 μHz is reduced in two steps to the intermediate frequency 6 μHz. This corresponds to a superhet of usual type.

2. A phase modulated auxiliary oscillator reduces the intermediate frequency to 1 μHz. The frequency drift of this oscillator (initial value 23 μHz per year [15]) can be adjusted. This stage corresponds to a modified superhet (MSH).

3. One iterates the phase φ, the modulation index η and the drift of the auxiliary oscillator until the amplitude of a spectral line near 1 μHz reaches a maximum. This causes the amplitudes of all other spectral lines of the phase modulated signal to decrease. Their energy flows into the central spectral line. A small frequency deviation ($\Delta f < 10$ nHz) is adjusted. If this limit is exceeded, the iteration stops and can be restarted with changed parameters.

4. As soon as a reproducible result with high S/N is obtained, the parameters of the phase-modulated auxiliary oscillator match the characteristics of the GW. These are tabulated with the initial date of the record.

Applying equation (11) to the data from all 32 records yields the following results: $\varphi = 1.319 \pm 0.030$, $\eta = 19.510 \pm 0.163$, and $k_{drift} = 24.356 \pm 0.279$ nHz per year. The error bars were calculated using the jackknife method.

The frequency drift agrees well with the slope calculated from the starting points alone (Fig 2) and the value given by Strohmayer [15].

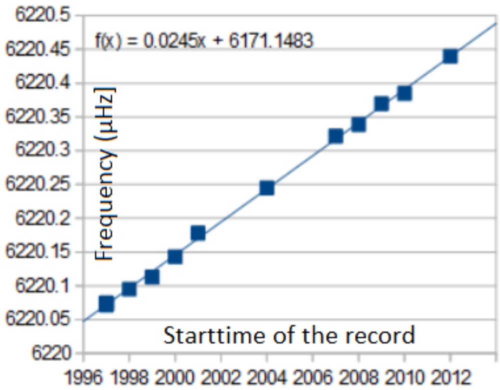

Figure 2): *The start frequencies of all 32 records as a function of the start date. The results differ so little that the differences are barely noticeable. The last data points extend till 2020. The frequency of the GW increases by 24.5 nHz per year.*

11 Where is the source of GW?

The abstract results of phase modulation must be translated into the language of astronomers.

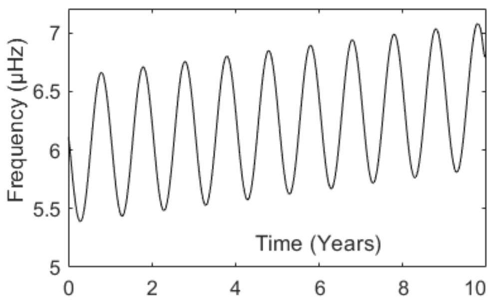

Figure 3): *The frequency of the auxiliary oscillator as a function of time. The frequency changes and the time points of the extrema apply unchanged also to the GW. The initial frequency of the GW (6220 μHz) was reduced to 6 μHz before processing to improve relative frequency resolution $\Delta f/f$.*

Figure 3 illustrates that the instantaneous frequency of the GW fluctuates around a mean value which increases slowly. The closer the source of the GW is to the poles of the ecliptic, the smaller is the maximum frequency deviation Δf. If this PM signature disappears, it becomes difficult to prove that a received signal is a GW. Here, the opposite is true: Calculating the frequency deviation with equation (5), the result

$$\Delta f = \eta \cdot f_{year} = 618.224 \pm 5.165 \; nHz \tag{12}$$

corresponds approximately to the relativistic maximum value according to equation (6). This means that the source of GW is very close to the plane of the ecliptic. The actual ecliptic latitude of HM Cancri is -4.7°. If this signal really comes from HMC, this result also confirms the basic assumption of equation (6): GW propagate at the speed of light.

In addition to the ecliptic latitude, the ecliptic longitude of the source is also of interest. For an MSH, the parameters of the phase-modulated auxiliary oscillator are identical to the characteristics of the GW, which remains hidden in the noise.

The value of the phase shift φ determines the times of the extreme values of redshift and blueshift, which alternate at half-yearly intervals. From figure 3 follows:

On every 105th day of a year the frequency of the oscillator is minimum, and on the 289th day of each year it is maximum (measured with gravimeters).

With Chandra one obtains for HM Cancri the values [9]: On every 110th day in the year the frequency of the GW is minimum, on the 295th day of each year it is maximum. The agreement could hardly be better.

12 Discussion

All previous attempts to detect continuous GW use the three LIGO interferometers as antennas. Surprisingly without success, although the interferometers are considerably more sensitive than gravimeters – but only for frequencies above 20 Hz. This cannot only be due to the many data gaps in the LIGO records. More serious might be that the main effect of the PM is not considered: GWs are always phase modulated and each PM is characterized by a very broadband spectrum consisting of many thousands of spectral lines (at high frequencies). Each individual line carries a fraction of the total energy. One must either treat the total spectrum because one cannot reconstruct the original signal from a subset. Unfortunately, this fact is hardly considered in the previous literature, mostly one looks for single spectral lines. Or one compensates the PM to force a single spectral line. This seems to be the more successful way.

In addition: Many gravimeters have produced gigantic amounts of data, run in continuous operation and were very rarely modified. Gravimeters are ideal antennas for a promising search for the important signature PM in an annual rhythm. They are particularly sensitive in the frequency range between 1 μHz and 1 Hz – exactly where the GW of binary star systems are expected to be.

The previous investigations have shown that in the very narrow frequency range 6220 μHz to 6220.6 μHz there are several candidates for GW, which can be well separated because of different PM (different ecliptic directions). Without the tool *Compensation of PM with MSH* it would be extremely difficult to distinguish these closely neighboring spectral lines.

In this work, it was demonstrated that:

- *Each* superconducting gravimeter is sufficiently sensitive and stable over time to measure the GW of HM Cancri, although (currently) no instrument is acoustically isolated from the ground and therefore picks up numerous disturbances. This type of mounting needs improvement.
- This GW is phase modulated with f_{year} and therefore has a broad spectrum of many individual lines which disappear in the noise. The GW can only be detected if a) multi-year records are chosen and b) the phase modulation and drift are compensated.
- The modified superhet method MSH can compensate the PM and amplify the central spectral line sufficiently to make the GW measurable.
- The frequency and frequency drift of all measurements are in very good agreement with X-ray astronomy measurements [15] for the HM Cancri stellar system.
- MSH also provides the coordinates of the GW source. Considering the frequent disturbances of the gravimeters by earthquakes, the agreement with the astronomical coordinates is very satisfactory.

13 Technical details of the data reduction

Superconducting gravimeters measure every second. The records are dominated by tides with frequencies around 11 μHz [16], whose amplitudes are at least a factor of 10^6 higher than the amplitude of the searched GW. Prior to publication by IGETS, data gaps are filled by synthetic tidal waves, then an initial decimation by a factor of 60 is performed to reduce the data size. Attached data such as barometric pressure do not contain useful information to detect GW with frequencies above about 1 mHz. Only at lower frequencies, the influence of the air mass must be taken into account.

The intermediate frequency must be higher than the Carson bandwidth [10] of the GW signal of about 1.4 μHz to process the FM signal without distortion (compare Fig 1). The value of the Carson bandwidth follows from the highest modulation index η given by the relativistic Doppler shift.

Using the commonly used superhet principle, the receive and oscillator frequencies differ by the value of the intermediate frequency : $f_{IF} = f_E \pm f_{OSZ}$. This ambiguity leads to the problem of the mirror frequency, which is not known by the so-called IQ-procedure [17] and which was therefore used in all investigations. A more detailed account of the procedure is beyond the scope of this article.

The equation (11) is valid for a circular orbit of the earth. Due to the poor S/N of the gravitational data, the implementation of a more precise model did not lead to satisfactory results.

The poor S/N did not allow a reproducible investigation of possible amplitude modulation of HM Cancri. After eliminating the PM by the MSH method, the only spectral line reaches the value $S/N \approx 3$; Without this measure, none of the approximately 50 spectral lines can be identified in the noise.

14 Data availability

The recordings of all gravimeters can be downloaded free of charge from GFZ Potsdam [11]. A detailed description of the IGETS data base, the IGETS products and the registration procedure can be found in [18]. The raw data are formatted as ASCII files and cover one month each. They may be requested as MATLAB files from the author.

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