Common recognition of Metric Tensor for Special and General Relativity Theory

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Abstract

Regard to metric tensor, it is necessary to have common concept and calculation for Special and General Relativity Theory. It is tried here based on the result of research for both.

1. Introduction

Metric tensor for Special Relativity Theory is ([1] (5.100))

$$\eta_{\mu\nu}=\begin{Bmatrix} -\frac{1}{c^2}, & 0, & 0, & 0 \\ 0, & -\frac{1}{c^2}, & 0, & 0 \\ 0, & 0, & -\frac{1}{c^2}, & 0 \\ 0, & 0, & 0, & 1 \end{Bmatrix}. \tag{1}$$

The definition of metric tensor is ([1] p64)

$$g_{il}=\frac{dx_k}{d\xi^i}\frac{dx_k}{d\xi^l}. \tag{2}$$

(1)is different from the metric tensor calculated from Lorentz transformation and definition (2).

It is necessary to get metric tensor for Special Relativity Theory on general definition.

2. Unit of Time dimension

There are space and time dimensions in Minkowski world. These four dimensions need to have same unit to handle as equivalent dimension.

Lorentz transformation is ([1] (4.13)

$$x^*=\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y^*=y$$

$$z^*=z \tag{3}$$

$$t^*=\frac{t-\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

For convenience, following variables are changed as follows.

$$x_1 = x$$
$$x_2 = y \tag{4}$$
$$x_3 = z$$

For convenience and consideration of succeeded calculation, following variable change and unit change is done.

$$x_4 = ct \tag{5}$$

c:light speed

On these change, (3) becomes

$$x^*{}_1 = \frac{x_1 - \frac{v}{c}x_4}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$x^*{}_2 = x_2 \tag{6}$$
$$x^*{}_3 = x_3$$
$$x^*{}_4 = \frac{-\frac{v}{c}x_1 + x_4}{\sqrt{1 - \frac{v^2}{c^2}}}$$

3. Metric tensor

Based on definition (2) and Lorentz transformation (6), metric tensor becomes

$$\eta_{\mu\nu} = \left\{ \begin{matrix} \frac{1+\frac{v}{c^2}}{1-\frac{v^2}{c^2}}, & 0, & 0, & -\frac{2\frac{v}{c}}{1-\frac{v^2}{c^2}} \\ 0, & 1, & 0, & 0 \\ 0, & 0, & 1, & 0 \\ -\frac{2\frac{v}{c}}{1-\frac{v^2}{c^2}}, & 0, & 0, & \frac{1+\frac{v}{c^2}}{1-\frac{v^2}{c^2}} \end{matrix} \right\}. \tag{7}$$

This is different from (1) which is known that it is consistent with facts.

Same result as (1) except unit value can be calculated if following recognition is used in calculation process.

"About metric tensor $\frac{dx_k}{dx^*{}_i}\frac{dx_k}{dx^*{}_l}$, if k is 1,2 or 3, these terms should be multiplied by minus one. This is that square of space distance has minus effect to total square of distance." (8)

On this calculation, we can get following metric tensor.

$$\eta_{\mu\nu}=\begin{Bmatrix} -1, & 0, & 0, & 0 \\ 0, & -1, & 0, & 0 \\ 0, & 0, & -1, & 0 \\ 0, & 0, & 0, & 1 \end{Bmatrix} \tag{9}$$

4. Conclusion

Metric tensor of Special Relativity Theory cannot be calculated from general metric tensor definition and Lorentz transformation.

This is resolved if space dimension distance contribution to total space-time distance is changed in metric tensor definition.

Also on a unit definition of time dimension, metric tensor can be simpler.

5. Consideration

The assumption of constant light speed for all inertia systems made Lorentz transformation.

The formula also represents following relation. ([1] (4.30))

$$\tau_{12}^2 = (x_{4_1} - x_{4_2})^2 - \{(x_{1_1} - x_{1_2})^2 + (x_{2_1} - x_{2_2})^2 + (x_{3_1} - x_{3_2})^2\} \tag{10}$$

This value is invariant regarding to Lorentz transformation varying v.

(7) should be correct as a metric tensor because it is created on definition.

But it or time-space distance is expanded according to v. [2]

Formula (10) adjust expanded time-space distance to original static length.

Then (9) is handled like metric tensor because value (10) from recognition (8) is invariant for all frame of reference.

So at least (9) should be handled with the recognition this is not metric tensor accurately.

Reference

[1] Peter Gabriel Bergmann, *Introduction to the Theory of Relativity*, (Dover Publication, INC 1976),p19

[2] viXra:1611.0077