# Gravity and speed of light 

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#### Abstract

One of the postulates of the Special Theory of Relativity is that the speed of light is constant in all inertial reference frames regardless of the motion of the observer or source. This postulate also apply to the General Theory of Relativity. There are many experiments whose results are consistent with the assumption that gravity does not affect the speed of light. But notwithstanding all this we may argue that it has not been formally proved. One way to prove its correctness is to replace it with the opposite axiom, axiom of variable speed of light. If there is at least one experiment whose result would possibly be in contradiction with this axiom, then it must be rejected and the axiom of a constant speed of light must be accepted.


Keywords: Speed of light, GPS, Absolute time, Pound-Rebka experiment, Doppler effect, Black holes

## 1. Introduction

We measure time with a clock, but we cannot say that time is what is measured by a clock. It would be the same as saying that heat is what we measure with a thermometer or current is what we measure with a voltmeter. It has been demonstrated that atomic clocks at different altitudes will eventually show different times. The closer the clock is to the source of gravitation, the slower time passes, speeding up as the clock getting away from the source of gravitation [1].
We can explain this in two ways. This happens because "real" times at different altitudes are different. Another possibility is that the accuracy of the clock is affected by external factors, such as gravity.
As a general rule, external factors affect not only the object we are measuring but also the accuracy of the instrument we use in the measurement.

## 2. GPS and Theory of Relativity

The clocks on the satellites were ticking faster than identical clocks on the ground. In order for the GPS to be functional, it is necessary that the clocks on the satellites and the clocks on the ground be synchronized. It turned out that although the clocks were initially synchronized after some time they are out of synchronization. A calculation using general relativity predicts that the clocks in each GPS satellite should get ahead of groundbased clocks by $45,850 \mathrm{~ns} /$ day and lose $7,214 \mathrm{~ns} /$ day due to special relativity effects. These effects are added together to give $38,640 \mathrm{~ns} /$ day [2].
The satellite's clocks are slowing down before launch by $38,640 \mathrm{~ns} /$ day, so they then proceed to tick in orbit at the same rate as ground clocks. But it turned out that this was not enough to keep the synchronization between the clocks, so in addition to the Special and General Theories of Relativity, other phenomena were taken into
account, such as the Sagnac (time dilation 207.4 ns ) and Doppler effect (time dilation $7,100 \mathrm{~ns} / \mathrm{d}$ ).
All GPS satellites must transmit their data signals at the exact same time, so precise synchronization is essential and their signals are monitored constantly and adjusted as needed. Let $\Delta t_{i}$ denote the total time during one day, for which the time on the $i-t h$ clock was corrected. Unfortunately, the author could not find any information regarding the $\Delta t_{i}$. So we will consider two possibilities. If $\Delta t_{i}$ is very small with respect to $38,640 \mathrm{~ns}$ which is in line with the predictions given in STR and GTR, but if $\Delta t_{i}$ is significantly greater than 38,640 ns then some other possibilities should be considered.

## 3. Gravitational red shift

There are two postulates of special relativity:

1. The laws of the physics are the same in the all inertial systems. No preferred inertial systems exist.
2. The speed of light in free space has the same value c in all inertial systems.

These postulates also apply to the General Theory of Relativity.

## Consequences of Special Relativity:

1. There is no such thing as absolute length or absolute time.
2. A time interval (or length) measurement depends on the reference frame.

In general relativity, gravitational red shift is the phenomenon that photons travel in a direction opposite to "gravitational force" lose energy. This loss of energy corresponds to a decrease in the wave frequency and increase in the wavelength [3].

We will define some constants and parameters that will be used later.

- M-mass of the body, assumed to be a sphere
- G-gravitational constant
- c-speed of light in vacuum
- h-Planck's constant
- $r_{A}$-the distance of point $A$ from the center of the body


Figure 1: Clocks $A$ and $B$ show different times due to gravitational effect

Referring to $\operatorname{Fig}(1)$ one can define $\Delta t_{A}$ and $\Delta t_{B}$ as it follows:

- $\Delta t_{A}$ - time measured by clock $A$
- $\Delta t_{B}$ - time measured by clock $B$
- $f_{A}$ - the frequency of the light measured by the observer at point $A$
- $f_{B}$ - the frequency of the light measured by the observer at point $B$

$$
\begin{align*}
f_{A} & =\frac{1}{\Delta t_{A}}  \tag{1}\\
f_{B} & =\frac{1}{\Delta t_{B}} \tag{2}
\end{align*}
$$

Observer marked by $B$ is at an arbitrarily large distance from the massive object. A common equation used to determine gravitational time dilation is derived from the Schwarzschild metric:

$$
\begin{equation*}
\Delta t_{A}^{2}=\left(1-\frac{2 G M}{c^{2} * r_{A}}\right) \Delta t_{B}^{2} \tag{4}
\end{equation*}
$$

The time measured by observer at infinity is noted by $\Delta t_{\infty}$.

$$
\begin{gather*}
\Delta t_{B}=\Delta t_{\infty}  \tag{5}\\
\lambda_{B}=\lambda_{\infty}  \tag{6}\\
\frac{\lambda_{\infty}}{\Delta t_{\infty}}=\frac{\lambda_{A}}{\Delta t_{A}}=c  \tag{7}\\
\lambda_{\infty}=\frac{\Delta t_{\infty}}{\Delta t_{A}} \lambda_{A}  \tag{8}\\
\lambda_{\infty}=\frac{1}{\sqrt{1-\frac{2 * G M}{c^{2} r_{A}}}} * \lambda_{A} \tag{9}
\end{gather*}
$$

If we assume that $\left(1 \gg \frac{2 * G M}{c^{2} r_{A}}\right)$ then from $\mathrm{Eq}(9)$ it follows that:

$$
\begin{equation*}
\lambda_{\infty} \approx\left(1+\frac{G M}{c^{2} r_{A}}\right) * \lambda_{A} \tag{10}
\end{equation*}
$$

We will later compare this result with the result derived under the assumption that the speed of light is variable.

## 4. Vertical motion

We will assume that time and three-dimensional space are absolute. The direct consequence of this assumption is that the units for length and time are absolute and the speed of light is variable. For if, the speed of light were constant then we would come into contradiction.

Let the point $O$ represents the center and $r_{0}$ the diameter of a spherical body $\operatorname{Fig}(2)$. Suppose that at some point a photon has been emitted from point $A$ in the direction of $O A$. The initial velocity of the photon at point $A$ is denoted by $v_{0}$, and its velocity at point $B$ is denoted by $v$. From the preposition on absolute units for length and time it follows that the velocity of the photon is affected by the force of gravity. All changes in the photon velocity caused by the rotation of the body around its imaginary axis will be ignored.


Figure 2: The speed of the photon is affected due to the gravitational force

Referring to the $\operatorname{Fig}(2)$ one can define $r_{0}$ and $r$ as it follows:

$$
\begin{align*}
r_{0} & =O A  \tag{11}\\
r & =O B \tag{12}
\end{align*}
$$

We will assume that the speed of the photon noted by $v$ decreases in accordance with the laws that apply in classical physics.

$$
\begin{array}{r}
v=\frac{\delta r}{\delta t} \\
\frac{\delta v}{\delta t}=-\frac{G M}{r^{2}} \tag{14}
\end{array}
$$

It follows that

$$
\begin{array}{r}
\frac{\delta v}{\delta t}=\frac{\delta v}{\delta r} * \frac{\delta r}{\delta t}=\frac{\delta v}{\delta r} * v \\
\frac{\delta v}{\delta r} * v=-\frac{G M}{r^{2}} \\
v * \delta v=-\frac{G M}{r^{2}} * \delta r \\
\int_{v_{0}}^{v} v \delta v=-\int_{r_{0}}^{r} \frac{G M}{r^{2}} \delta r \\
\frac{v^{2}}{2}-\frac{v_{0}^{2}}{2}=-G M *\left(-\frac{1}{r}+\frac{1}{r_{0}}\right) \\
v^{2}=v_{0}^{2}-2 * G M *\left(\frac{1}{r_{0}}-\frac{1}{r}\right) \\
v=\sqrt{v_{0}^{2}-2 * G M *\left(\frac{1}{r_{0}}-\frac{1}{r}\right)} \\
\frac{v}{v_{0}}=\sqrt{1-\frac{2 * G M}{v_{0}^{2}} *\left(\frac{1}{r_{0}}-\frac{1}{r}\right)} \tag{22}
\end{array}
$$

If $\left(r \gg r_{0}\right)$ then it follows that

$$
\begin{equation*}
v \approx \sqrt{v_{0}^{2}-\frac{2 * G M}{r_{0}}}=v_{0} * \sqrt{1-\frac{2 * G M}{r_{0} * v_{0}^{2}}} \approx v_{0} *\left(1-\frac{G M}{r_{0} * v_{0}^{2}}\right) \tag{23}
\end{equation*}
$$

The Equation (20) can be written as follows:

$$
\begin{equation*}
\frac{v_{A}^{2}-v_{B}^{2}}{2}=G M *\left(\frac{1}{r_{A}}-\frac{1}{r_{B}}\right) \tag{24}
\end{equation*}
$$

where:

$$
\begin{align*}
v_{0} & =v_{A}  \tag{25}\\
v & =v_{B}  \tag{26}\\
r_{0} & =r_{A}  \tag{27}\\
r & =r_{B} \tag{28}
\end{align*}
$$

Suppose that $\left(r_{B} \rightarrow \infty\right) \wedge\left(v_{A}=c\right)$, then from Equation (24) it follows that:

$$
\begin{gather*}
v_{B}=v_{\infty}  \tag{29}\\
\frac{M}{r_{A}}=\frac{c^{2}-v_{\infty}^{2}}{2 G} \tag{30}
\end{gather*}
$$

If we assume that a photon with initial velocity denoted by $v_{B}$ is emitted from point $B$ in the direction of point $O \operatorname{Fig}(2)$, then it is easy to prove that the speed of the photon at point $A$ denoted by $v_{A}$ is given by the following equation.

$$
\begin{equation*}
v_{A}=\sqrt{v_{B}^{2}+2 * G M *\left(\frac{1}{r_{0}}-\frac{1}{r}\right)} \tag{31}
\end{equation*}
$$

And if $\left(r \gg r_{0}\right)$ then it follows that

$$
\begin{equation*}
v_{A} \approx v_{B} *\left(1+\frac{G M}{r_{0} * v_{B}^{2}}\right) \tag{32}
\end{equation*}
$$

## 5. Wavelength and wave frequency

Suppose that in the time interval $\Delta t_{A}$ from point $A$ in the direction $O A$ two photons have been emitted. The first emitted photon is denoted by point $A^{\prime}$ and the second emitted photon is denoted by point $A$. After some time the photon marked with $A^{\prime}$ will be at point $B^{\prime}$ and the photon marked with $A$ will be at point $B$. The time difference between the passage of a photon through point $B$ will be denoted by $\Delta t_{B} \operatorname{Fig}(3)$.


Figure 3: At time interval $\Delta t_{A}$ two photons were emitted from point $A$ in the direction of $O A$

Let us define the wavelengths $\lambda_{A}$ and $\lambda_{B}$ and the wave frequencies $f_{A}$ and $f_{B}$ at points $A$ and $B$ respectively:

$$
\begin{align*}
\lambda_{A} & =A A^{\prime}  \tag{33}\\
\lambda_{B} & =B B^{\prime}  \tag{34}\\
f_{A} & =\frac{1}{\Delta t_{A}}  \tag{35}\\
f_{B} & =\frac{1}{\Delta t_{B}} \tag{36}
\end{align*}
$$

We will assume that:

$$
\begin{array}{r}
\Delta t_{B}=\Delta t_{A} \\
\left(\Delta t_{B}=\Delta t_{A}\right) \Leftrightarrow\left(f_{A}=f_{B}\right) \tag{38}
\end{array}
$$

Our assumption is, as already said, that only the speed of the photon (not the time) is affected by the gravitational force, which is the opposite of the axioms valid in GTR. It therefore follows that:

$$
\begin{array}{r}
v_{A}=\frac{\lambda_{A}}{\Delta t_{A}} \\
v_{B}=\frac{\lambda_{B}}{\Delta t_{B}}=\frac{\lambda_{B}}{\Delta t_{A}} \\
\lambda_{B}=\frac{v_{B}}{v_{A}} * \lambda_{A} \tag{41}
\end{array}
$$

Then from $\operatorname{Eq}(22)$ it follows that:

$$
\begin{align*}
& r_{A}=O A  \tag{42}\\
& r_{B}=O B  \tag{43}\\
& \lambda_{B}=\sqrt{1-\frac{2 * G M}{v_{A}^{2}} *\left(\frac{1}{r_{A}}-\frac{1}{r_{B}}\right)} * \lambda_{A} \tag{44}
\end{align*}
$$

Assuming that $\left(v_{A}=c\right)$ then $\mathrm{Eq}(44)$ can be written in the following form:

$$
\begin{equation*}
\lambda_{B}=\sqrt{1-\frac{2 * G M}{c^{2}} *\left(\frac{1}{r_{A}}-\frac{1}{r_{B}}\right)} * \lambda_{A} \tag{45}
\end{equation*}
$$

If $\left(r_{B} \rightarrow \infty\right)$ then it follows

$$
\begin{array}{r}
\lambda_{B}=\lambda_{\infty} \\
\lambda_{\infty}=\sqrt{1-\frac{2 * G M}{c^{2} r_{A}}} * \lambda_{A} \approx\left(1-\frac{G M}{c^{2} r_{A}}\right) * \lambda_{A} \tag{47}
\end{array}
$$

Equation (10) derived on the basis of axioms valid in $G T R$ differs from Eq (47). Therefore, an experiment that would directly measure the change in wavelength due to gravitational force could give an answer on the constancy of the speed of light .
There are already experiments such as [4] whose results, made by analyzing $F e$ spectral lines in sunlight reflected by the Moon, are almost in perfect agreement with the theoretical value of the solar gravitational redshift predicted by the GTR (Eq. (10)). Nevertheless, our opinion is that there is one weak point in this experiment. The Doppler effect caused by the movement of the Moon and the Earth in relation to the Sun must be eliminated, since this directly affects the result of the experiment.

The energy of the photon noted by $E_{A}$ at point $A$ is given by the equation:

$$
\begin{equation*}
E_{A}=h * f_{A} \tag{48}
\end{equation*}
$$

We will define the energy of the photon at point $B$, noted by $E_{B}$, by the equation:

$$
\begin{equation*}
E_{B}=\frac{v_{B}}{v_{A}} *\left(f_{B} * h\right)=\frac{v_{B}}{v_{A}} *\left(f_{A} * h\right)=\frac{v_{B}}{v_{A}} * E_{A}=\frac{\lambda_{B}}{\lambda_{A}} * E_{A} \tag{49}
\end{equation*}
$$

Assuming that $\left(v_{A}=c\right)$, from Equation (45) it follows that:

$$
\begin{equation*}
E_{B}=\sqrt{1-\frac{2 * G M}{c^{2}} *\left(\frac{1}{r_{A}}-\frac{1}{r_{B}}\right)} * E_{A} \tag{50}
\end{equation*}
$$

if $\left(\left(r_{B} \rightarrow \infty\right) \wedge\left(\frac{G M}{r_{A} c^{2}} \ll 1\right)\right)$ then:

$$
\begin{gather*}
E_{\infty}=\sqrt{1-\frac{2 * G M}{r_{A} c^{2}}} * E_{A} \approx\left(1-\frac{G M}{r_{A} c^{2}}\right) * E_{A}  \tag{51}\\
H=r_{B}-r_{A}  \tag{52}\\
g=\frac{G M}{r_{A}^{2}} \tag{53}
\end{gather*}
$$

if $\left(H \ll r_{A}\right)$ then

$$
\begin{align*}
E_{B}=\sqrt{1-\frac{2 * G M * H}{r_{A}^{2} c^{2}}} * E_{A} \approx\left(1-\frac{G M * H}{r_{A}^{2} c^{2}}\right) * E_{A} & =\left(1-\frac{g * H}{c^{2}}\right) * E_{A}  \tag{54}\\
\frac{\Delta E}{E_{A}} & =\frac{E_{A}-E_{B}}{E_{A}}=\frac{g * H}{c^{2}} \tag{55}
\end{align*}
$$

Now suppose that the signal has been sent from point $B$ in the direction of point $A$. Assuming that $\left(v_{B}=c\right) \wedge\left(H \ll r_{A}\right)$ then $\mathrm{Eq}(54)$ can be written in the following form:

$$
\begin{align*}
E_{A} \approx \sqrt{1+\frac{2 * G M * H}{r_{A}^{2} c^{2}}} * E_{B} \approx\left(1+\frac{G M * H}{r_{A}^{2} c^{2}}\right) * E_{B} & =\left(1+\frac{g * H}{c^{2}}\right) * E_{B}  \tag{56}\\
\frac{\Delta E}{E_{B}} & =\frac{E_{A}-E_{B}}{E_{B}}=\frac{g * H}{c^{2}} \tag{57}
\end{align*}
$$

## 6. Pound-Rebka experiment

The purpose of the experiment was to prove that photons gain energy when traveling toward a gravitational filed [5]. Gamma rays were emitted from the top of a tower denoted by $(B)$ and measured by a receiver at the bottom of the tower denoted by $(A) \operatorname{Fig}(4 \mathrm{a})$.


Figure 4: Gamma rays were emitted from the top of a tower

The experiment showed that by falling through the gravitational field the photons increase the energy and pass through the detector $\operatorname{Fig}(4 \mathrm{a})$.

$$
\begin{array}{r}
H=A B \\
E_{B}=f_{B} * h \\
\Delta E=E_{A}-E_{B} \tag{60}
\end{array}
$$

where

- $E_{B}$-Photon energy at the time it was emitted
- $E_{A}$-The energy of a photon when it reaches the detector
- $f_{B}$-wave frequency at the time it was emitted

But the photon will be absorbed if its energy is equal to its energy when it was originally emitted. One way to cancel the energy received by the photon due to the gravitational field is to moves away emitter $B$ from the detector $A$ at speed $v \operatorname{Fig}(4 \mathrm{~b})$. Pound and Rebka varied the speed $v$ so that the energy that the photon loses due to Doppler-effect is equal to the energy that the photon gains due to gravity.

Referring to $\operatorname{Fig}(4 \mathrm{~b})$ it follows that:

$$
\begin{array}{r}
E_{A}=E_{B} \\
f_{B^{\prime}}=\left(1-\frac{v}{c}\right) f_{B} \\
E_{B^{\prime}}=f_{B^{\prime}} * h=\left(1-\frac{v}{c}\right) f_{B} * h=\left(1-\frac{v}{c}\right) E_{B} \\
E_{A}=E_{B^{\prime}}+\Delta E=\left(1-\frac{v}{c}\right) E_{B}+\Delta E=E_{B}-\frac{v}{c} E_{B}+\Delta E \\
\frac{\Delta E}{E_{B}}=\frac{v}{c} \tag{65}
\end{array}
$$

General relativity predicts that the gravitational field of the Earth will cause a photon emitted downwards (towards the Earth) to be blueshifted (i.e. its frequency will increase) according to the formula [3]:

$$
\begin{array}{r}
\Delta f=f_{A}-f_{B} \\
\frac{\Delta E}{E_{B}}=\frac{\Delta f}{f_{B}}=\frac{g H}{c^{2}} \tag{67}
\end{array}
$$

This result is identical to the result we got in the Eq (55).
Comparing the results obtained experimentally with the result obtained theoretically it has been found that [6]

$$
\begin{equation*}
\frac{\Delta E(\text { exp })}{\Delta E(\text { theory })}=0.999 \pm 0.0076 \tag{68}
\end{equation*}
$$

Therefore, the results obtained by the Pound-Rebka experiment experiment can be interpreted in two completely different ways:
$1^{\circ}$
(c-speed of light in vacuum is constant) $\Rightarrow$ ("proper units for length and time") $\Rightarrow\left(f_{A}>f_{B}\right) \Rightarrow$ Eq.(67) $2^{\circ}$
$\left("\right.$ common units for length and time") $\Rightarrow$ (variable speed of light) $\Rightarrow\left(f_{A}=f_{B}\right) \Rightarrow$ Eq.(55)
Equation (55) is identical to Equation (67) regardless of the fact that in the second case the speed of light $c$ is not constant, which is contrary to the first postulate $S T R$.

## 7. Black holes

The Schwarzschild radius defines the event horizon of a Schwarzschild black hole. The event horizon is a boundary beyond which events cannot affect an observer and the black hole is a region of spacetime where gravity is so strong that nothing even the electromagnetic radiation can escape from it. The Schwarzschild radius noted by $r_{0}$ is defined as follows:

$$
\begin{equation*}
r_{0}=\frac{2 G M}{c^{2}} \tag{69}
\end{equation*}
$$

We will now analyze the Equation (21) in the case where the velocity of the photon denoted by $v_{B}$ at point $B$ approaches to zero $\operatorname{Fig}(5)$.


Figure 5: The speed of the photon denoted by $v_{B}$ is approaching to zero
The Equation (20) can be written as follows:

$$
\begin{equation*}
v_{B}^{2}=v_{A}^{2}-2 * G M *\left(\frac{1}{r_{A}}-\frac{1}{r_{B}}\right) \tag{70}
\end{equation*}
$$

where:

$$
\begin{array}{r}
v_{A}=v_{0} \\
v_{B}=v \\
r_{A}=r_{0} \\
r_{B}=r \tag{74}
\end{array}
$$

We will define the variable $k$ as follows:

$$
\begin{array}{r}
k=\frac{r_{B}}{r_{A}} \\
r_{B}=k * r_{A} \quad(k>1) \tag{76}
\end{array}
$$

Suppose that $\left(v_{B} \rightarrow 0\right)$
Then from Equation (70) it follows that:

$$
\begin{align*}
& v_{A}^{2}=2 * G M * \frac{1}{r_{A}}\left(\frac{k-1}{k}\right)  \tag{77}\\
& r_{A}=2 * G M * \frac{1}{v_{A}^{2}}\left(\frac{k-1}{k}\right) \tag{78}
\end{align*}
$$

Assuming that $\left(v_{A}=c\right)$ Equation (78) can be written as follows:

$$
\begin{array}{r}
r_{A}=2 * G M * \frac{1}{c^{2}}\left(\frac{k-1}{k}\right) \\
r_{A}=\left(\frac{k-1}{k}\right) r_{0} \tag{80}
\end{array}
$$

From Equations (80) and (76) it follows that:

$$
\begin{equation*}
r_{B}=\frac{r_{0} * r_{A}}{r_{0}-r_{A}} \tag{81}
\end{equation*}
$$

From equations (80) and (81) we can conclude that if $\left(v_{B} \rightarrow 0\right)$ then the radius of the star $r_{A}$ must be less than $r_{0}\left(r_{A}<r_{0}\right)$.

We will now analyze two special cases.
$\left.1^{\circ}\left(r_{A} \rightarrow r_{0}^{-}\right) \Rightarrow\right)\left(r_{B} \rightarrow \infty\right)$
It is obvious that this interpretation differs from the existing definition of a black hole ( $r_{A}<r_{0}$ ), according to which no electromagnetic radiation such as light can escape from it.
$2^{\circ}\left(r_{A} \rightarrow 0^{+}\right) \Rightarrow\left(r_{B} \rightarrow r_{A}^{+}\right)$
It is theoretically possible that $\left(r_{A} \rightarrow 0^{+}\right)$, but we also cannot rule out the possibility that there is some upper limit value $r_{\text {min }}$, such that $r_{A} \geqslant r_{\text {min }}$.

The conclusion is that no matter how strong gravity is, it does not prevent light from being emitted. The energy of emitted photon is very small for an observer who is far enough away from the source. This can be
explained in two ways. The speed of light in the vicinity of the observer is approaching to zero or the speed of light is equal to the constant $c$ and its frequency $\left(f_{B}\right)$ is approaching to zero. But if $\left(f_{B} \rightarrow 0\right)$ then $\left(t_{B} \rightarrow \infty\right)$, which would lead to contradiction. Therefore, we will reject the second possibility, and in this case accept the definition of a black hole given at the beginning of this section.

## 8. Doppler effect

Let us define a one-dimensional coordinate system whose origin is determined by the point $O$ and $O A$ defines the positive direction of the coordinate axis.


Figure 6: At time interval $\Delta t$ two photons were emitted from point $B$ in the direction of $B O$

Suppose that at some point a photon denoted by $P_{1}$ has been emitted from point $B$. The point $B$ moves at a uniform velocity $\mathbf{u}$ with respect to a fixed point $O$, and its position after time $\Delta t$ is denoted by $B^{\prime}$. Let another photon denoted by $P_{2}$ be emitted from point $B^{\prime}$.

$$
\begin{array}{r}
\mathbf{e}=[1] \\
u_{r}=\mathbf{u} \cdot \mathbf{e} \\
B B^{\prime}=u_{r} * \Delta t \tag{84}
\end{array}
$$

Let $t$ denote the time required for photon $P_{1}$ to move from point $B$ to point $A$. Denote by $A^{\prime}$ the point such that $A^{\prime} B^{\prime}=A B$. The time required for the photon $P_{2}$ to move from point $B^{\prime}$ to point $A^{\prime}$ is equal to the time $t$. We assume that point $A$ moves at a uniform velocity $\mathbf{v}$ with respect to a fixed point $O$ and that the photons move at the same speed $c_{A}$ in the vicinity of point $A \operatorname{Fig}(6)$. With $A^{\prime \prime}$ we will denote the position of photon $P_{2}$ at the moment when photon $P_{1}$ moved to the point $A$. Thus $A A^{\prime \prime}$ denotes the distance between photons $P_{1}$ and $P_{2}$ at time $t$.

$$
\begin{array}{r}
A^{\prime} B^{\prime}=A B \\
A A^{\prime}=B B^{\prime} \\
A^{\prime} A^{\prime \prime}=\Delta t * c_{A} \\
A A^{\prime \prime}=u_{r} * \Delta t+\Delta t * c_{A}=\left(u_{r}+c_{A}\right) * \Delta t \tag{89}
\end{array}
$$

Denote by $\Delta t_{1}$ the time required for photon $P_{2}$ moving from point $A^{\prime \prime}$ to reach the signal receiver at point $A^{\prime \prime \prime}$.

$$
\begin{array}{r}
v_{r}=\mathbf{v} \cdot \mathbf{e} \\
A^{\prime \prime \prime} A^{\prime \prime}=\Delta t_{1} * c_{A} \\
A^{\prime \prime \prime} A^{\prime \prime}=A^{\prime \prime \prime} A+A A^{\prime \prime}=-\Delta t_{1} * v_{r}+\left(c_{A}+u_{r}\right) * \Delta t \\
\Delta t_{1} * c_{A}=-\Delta t_{1} * v_{r}+\left(c_{A}+u_{r}\right) * \Delta t \\
\left(c_{A}+v_{r}\right) * \Delta t_{1}=\left(c_{A}+u_{r}\right) * \Delta t \\
\Delta t_{1}=\frac{c_{A}+u_{r}}{c_{A}+v_{r}} \Delta t \tag{95}
\end{array}
$$

$$
\begin{array}{r}
t_{B}=\Delta t \\
t_{A}=\Delta t_{1} \\
t_{A}=\frac{u_{r}+c_{A}}{v_{r}+c_{A}} t_{B} \\
f_{B}=\frac{1}{\Delta t_{B}} \\
f_{A}=\frac{1}{\Delta t_{A}} \\
f_{A}=\frac{c_{A}+v_{r}}{c_{A}+u_{r}} f_{B}=\frac{1+\frac{v_{r}}{c_{A}}}{1+\frac{u_{r}}{c_{A}}} f_{B} \\
\lambda_{A} * f_{A}=c_{A} \\
\lambda_{B} * f_{B}=c_{B} \\
\frac{\lambda_{A} * f_{A}}{\lambda_{B} * f_{B}}=\frac{c_{A}}{c_{B}} \\
\lambda_{A}=\frac{c_{A}}{c_{B}} * \frac{f_{B}}{f_{A}} * \lambda_{B} \tag{105}
\end{array}
$$

From Equation (101) it follows that :

$$
\begin{equation*}
\lambda_{A}=\frac{c_{A}}{c_{B}} * \frac{1+\frac{u_{r}}{c_{A}}}{1+\frac{v_{r}}{c_{A}}} * \lambda_{B} \tag{106}
\end{equation*}
$$

If $\left(\left|\frac{u_{r}}{c_{A}}\right| \ll 1\right) \wedge\left(\left|\frac{v_{r}}{c_{A}}\right| \ll 1\right)$ then it follows that:

$$
\begin{array}{r}
f_{A}=\left(1+\frac{v_{r}}{c_{A}}\right)\left(1-\frac{u_{r}}{c_{A}}+\left(\frac{u_{r}}{c_{A}}\right)^{2}-\ldots\right) f_{B} \\
\Delta u=u_{r}-v_{r} \\
f_{A} \approx\left(1+\frac{v_{r}-u_{r}}{c_{A}}\right) f_{B}=\left(1-\frac{\Delta u}{c_{A}}\right) f_{B} \\
\lambda_{A} \approx \frac{c_{A}}{c_{B}} *\left(1+\frac{u_{r}}{c_{A}}\right)\left(1-\frac{v_{r}}{c_{A}}+\left(\frac{v_{r}}{c_{A}}\right)^{2}-\ldots\right) * \lambda_{B} \\
\lambda_{A} \approx \frac{c_{A}}{c_{B}}\left(1+\frac{u_{r}-v_{r}}{c_{A}}\right) \lambda_{B}=\frac{c_{A}}{c_{B}}\left(1+\frac{\Delta u}{c_{A}}\right) \lambda_{B} \tag{111}
\end{array}
$$

If instead of two photons we considered the emission of two successive crests of light from point $B$ then we would get identical formulas. This means that the formulas we have derived do not depend on what the nature of light is.

The photon energies denoted by $E_{B}$ and $E_{A}$ at points $B$ and $A$ respectively are given by the following equations:

$$
\begin{array}{r}
E_{B}=h * f_{B} \\
E_{A}=\frac{c_{A}}{c_{B}} * h * f_{A}=\frac{c_{A}}{c_{B}} * h * \frac{c_{A}+v_{r}}{c_{A}+u_{r}} f_{B}=\frac{c_{A}}{c_{B}} * \frac{c_{A}+v_{r}}{c_{A}+u_{r}} E_{B} \tag{113}
\end{array}
$$

## 9. A variable speed of light test

Let us define a one-dimensional coordinate system whose origin is determined by the point $Z$ and $Z A$ defines the positive direction of the coordinate axis $\operatorname{Fig}(7)$.


Figure 7: Light is emitted from point $Z$ in the direction of $Z A$
We will first assume that the speed of light is constant.
Suppose that point $A$ moves at a uniform velocity denoted by $\Delta u$, with respect to $Z$. Suppose the light is emitted from a star marked $Z$ in the direction of point $A$. Let $\Delta t_{1}$ denote the time it takes for light to travel from point $A$ to point $B$. At point $B$, light changes direction by 180 degrees. Let $\Delta t_{2}$ denote the time it takes for light to travel from point $B$ to point $A$. Suppose that the speed of light is constant.

$$
\begin{array}{r}
d=A B \\
\Delta t_{1}=\frac{d}{c-\Delta u} \\
\Delta t_{2}=\frac{d}{c+\Delta u} \\
\Delta t=\Delta t_{1}+\Delta t_{2} \\
\Delta t=\frac{2 d * c}{(c-\Delta u)(c+\Delta u)}=\frac{2 d * c}{\left(c^{2}-\Delta u^{2}\right)} \\
\Delta u^{2}=c^{2}-\frac{2 d * c}{\Delta t} \tag{119}
\end{array}
$$

In order for $\mathrm{Eq}(119)$ to have real solutions, the following condition must be met:

$$
\begin{gather*}
\left(c^{2}-\frac{2 d * c}{\Delta t} \geqslant 0\right) \Rightarrow\left(c \geq \frac{2 d}{\Delta t}\right)  \tag{120}\\
\Delta u=\sqrt{c^{2}-\frac{2 d * c}{\Delta t}}  \tag{121}\\
\Delta u_{1}=\Delta u \tag{122}
\end{gather*}
$$

The relativistic Doppler effect wavelength equation is given by following equation:

$$
\begin{equation*}
\lambda_{1}=\sqrt{\frac{1+\gamma}{1-\gamma}} \lambda \tag{123}
\end{equation*}
$$

where $\gamma=\frac{\Delta u}{c}, \lambda_{1}-$ observed wavelength, $\lambda-$ source wavelength

$$
\begin{array}{r}
\lambda_{1}^{2}(1-\gamma)=\lambda^{2}(1+\gamma) \\
\gamma=\frac{\lambda_{1}^{2}-\lambda^{2}}{\lambda_{1}^{2}+\lambda^{2}} \\
\Delta u=\frac{\lambda_{1}^{2}-\lambda^{2}}{\lambda_{1}^{2}+\lambda^{2}} c=\left|\left|\lambda_{1}-\lambda\right| \ll \lambda\right| \approx \frac{\left(\lambda_{1}-\lambda\right) * 2 \lambda}{2 * \lambda^{2}} \approx \frac{\Delta \lambda}{\lambda} * c \\
\Delta u_{2}=\Delta u \tag{127}
\end{array}
$$

We can consider two possibilities:
$1^{\circ}\left(\Delta u_{1}=\Delta u_{2}\right)$
This result is consistent with the axiom of constancy of the speed of light.
$2^{\circ}\left(\Delta u_{1} \# \Delta u_{2}\right)$
This result is in contradiction with the axiom of constancy of the speed of light.
This means that the result obtained by the described experiment should be explained in a different way.
We will now assume that the speed of light is not constant.
Let define a one-dimensional coordinate system whose origin is determined by the some fixed point $O$ (we will assume that there is such a point) and let $O A$ defines the positive direction of the coordinate axis $\operatorname{Fig}(8)$.


Figure 8: Light is emitted from point $Z$ in the direction of $Z A$

Suppose that point $A$ moves at a uniform velocity denoted by $v_{r}$, with respect to $O$. The motion of the star $Z$ in relation to the point $(O)$ will be neglected since it does not affect the speed of light emitted from the star $Z$.
Suppose the light is emitted from the star marked $Z$ in the direction $Z A$. Let $\Delta t_{1}$ denote the time it takes for light to travel from point $A$ to point $B$. At point $B$, light changes direction by 180 degrees. Let $\Delta t_{2}$ denote the time it takes for light to travel from point $B$ to point $A$. Denote by $c_{A}$ the speed of light in the vicinity of point $A$.

$$
\begin{array}{r}
d=A B \\
\Delta t_{1}=\frac{d}{c_{A}-v_{r}} \\
\Delta t_{2}=\frac{d}{c_{A}+v_{r}} \\
\Delta t=\Delta t_{1}+\Delta t_{2} \\
\Delta t=\frac{2 d * c_{A}}{\left(c_{A}-v_{r}\right)\left(c_{A}+v_{r}\right)}=\frac{2 d * c_{A}}{\left(c_{A}^{2}-v_{r}^{2}\right)} \\
c_{A}^{2}-v_{r}^{2}=\frac{2 d}{\Delta t} * c_{A} \\
c_{A}-\frac{v_{r}^{2}}{c_{A}}=\frac{2 d}{\Delta t} \tag{134}
\end{array}
$$

We will consider three possibilities:
$1^{\circ}\left(v_{r}^{2} \ll c_{A}\right) \Leftrightarrow\left(c_{A} \approx \frac{2 d}{\Delta t}\right)$
In this case, it would be possible to approximately determine the speed of light in the vicinity of point $A$.
$2^{\circ}\left(v_{r}>c_{A}\right)$
Point $A$ moves away from point $O$ at a speed $v_{r}$ that is greater than $c_{A}$, the speed of light in the vicinity of point $A$, so we can conclude that light emitted from the star $Z$ will never reach point $A$.
$3^{\circ}\left(-v_{r}>c_{A}\right) \quad \Rightarrow \quad\left(\Delta t_{2} \longrightarrow \infty\right)$
Point $A$ moves towards the point O at speed $-v_{r}$ that is greater than $c_{A}$. Light emitted from the star $Z$ will be registered at point $A$, but after being reflected at point $B$ it will never reach point $A$. In other words $\left(\Delta t_{2} \longrightarrow \infty\right)$.

## 10. Conclusion

New interpretations of several well-known experiments and theories are given, which so far could only be explained within the theory of relativity. In addition, the new experiments have been proposed that could help us test the postulate of a constant speed of light.

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