The Vacuum Energy Fractal, the Amazing Quantum Vacuum

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Abstract

In this letter we analyze the vacuum energy as a simple fractal. With elementary mathematics and a novel approach, we will study its properties.

Keywords: Vacuum energy, compact dimensions, relative fractal dimension, transition of dimensions, hypothetical quantum generalization

1 Introduction

The existence of Planck's quantum of action turns Newton's classical and deterministic universe into a quantum universe, with Heisenberg's uncertainty principle. The vacuum is filled with a zero-point energy (ZPE) with a higher value the smaller the distance considered. The minimum length considered, called Planck's length (lp), is associated with a maximum energy called Planck's energy (Ep). For a distance n (lp) the associated energy is (Ep)/n, where “n” is a natural number. This property, conserved at all known scales, will help us analyze this fractal.

2 Fractal dimension, study of Brownian motion and the Koch snowflake

The fractal dimension is made up of two addends, the topological dimension and a dimensional coefficient (topol_dim + dimens_coef.). The more irregular the fractal, the higher the dimensional coefficient. For our study, it is interesting to analyze simple fractals such as the fractal path of Brownian motion, of topological dimension 1.

Brownian motion (britannica.com, December 23, 2021), also called Brownian movement, any of various physical phenomena in which some quantity is constantly undergoing small, random fluctuations. It was named for the Scottish botanist Robert Brown, the first to study such fluctuations (1827).

For a particle, moving with a Brownian motion, to move away N effective steps, it must take N^2 total steps. The N effective steps are considered in a straight line, in one dimension. The N^2 steps occur in a space of two or more dimensions. The relation log (N^2) / log (N) = 2 gives us the value of its fractal dimension (basic property of fractal lines) [1]. The topological dimension is 1 and the dimensional coefficient is also 1. *The value 2 of the fractal dimension indicates that a linear movement, of topological dimension 1, can fill a plane, of topological dimension 2.*

In Brownian motion, and in general, $\boxed{\textbf{fractal value} = N^2 \textbf{ = distance}^{\textbf{fractal\_dimension}}}$**.**

This can also be observed in the Koch curve, in figure 1. In the first iteration, the side that measures 3 segments becomes 4 segments. The fractal dimension is log 4 / log 3 = 1,26186. In one dimension 3 segments, they become 4 segments in two dimensions (the plane):$4= 3^{1,26186}$, $4=3^{\text{fractal\_dimension}}$ (**Mandelbrot, 1987**).

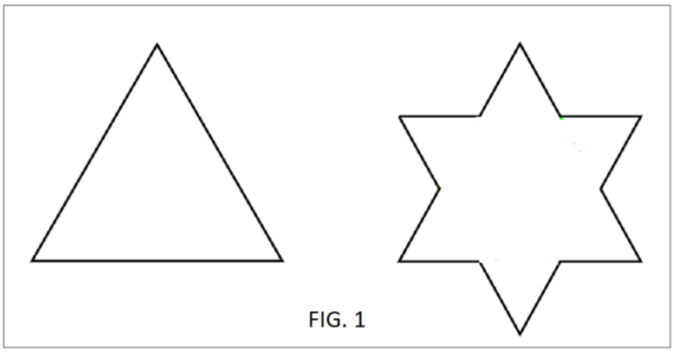

FIG.1 Koch curve, first iteration.

3 Fractal dimension of vacuum energy

We know the dependence of the vacuum energy with the distance: En = Ep / n = (Ep) (distance^{-1}). We assume that we live in hyperspace (string theory) , and we know the dependence of the vacuum energy with distance. Let $En_{(hyper)}$ be the value of the energy in hyperspace, then:

Log ($En_{(hyper)}$) / log (En)= -1. This implies that vacuum energy is proportional to distance in hyperspace. Although the energy has no topological dimension 1, the quotient of the two logarithms behaves the same as in the case of the Brownian motion. When comparing two energies, the topological dimension no longer matters because the result is a relative fractal dimension:

Relative fractal dimension = (topol_dim. + dimens_coef.)/(topol_dim.). To simplify we will write:

$\boxed{\text{Relat\_fr\_dim.} = (\delta+\varepsilon)/\delta}$ **(1).**

So, we have: $\boxed{\text{Relat\_fr\_dim} = \text{Log}(En_{(hyper)}) / \log(En) = -1 = (\delta+\varepsilon)/\delta}$.

The -1 value reminds us of the compacted dimensions of the string theory, since while a positive dimensional coefficient indicates that the fractal occupies a space greater than its topological dimension, a negative dimensional coefficient indicates dimension compaction (**Ruiz-Fargueta, 2004**). The situation indicates a transition of dimensions such that: $\boxed{T:\ \delta \dashrightarrow \delta-\varepsilon}$

The expression (1), with this transition becomes: $\boxed{\delta/(\delta-\varepsilon)}$ **(2)**.
If the dimensional coefficient is the same as the number of compact dimensions.

Expression (2) is consistent with the value -1, since for d = 3 it gives us the value -6 for the number of compact dimensions, which coincides with the value predicted by string theory. Applying these values to the expression (1):

$\boxed{(\delta+\varepsilon) / \delta = (3+6)/3= 3}$. *3 is the relative fractal dimension of the vacuum energy, 9 its true fractal dimension*.

The same result is found in the following equivalent transformations:

$T1_1$: $1/n \rightarrow n$ } log(n)/log(1/n) = -1. Apparent result in relative fractal dimension.

$T1_2$: $n \rightarrow n^3$ } $\log(n^3)/\log(n) = 3$. True result in relative fractal dimension.

The $T1_1$ transformation gives us the apparent result -1. But the transformation $T1_2$ gives us the true result 3.

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4 Generalization and possible transition of dimensions

The value -1 is the result of En, as a function of distance, in the expression $\boxed{\text{(En) (n) <Constant}}$, where we have replaced the time (energy-time uncertainty principle) by the space (n) traveled by the light in that time. If in this expression we add a fictitious coefficient f, we will have:

$\boxed{(En)\ (n^f) < Constant}$ **(3)** (*Hypothetical quantum generalization)*

Now the transformations $T1_1$ and $T1_2$ will be:

$T1_1$: $1/n^f \rightarrow n$ }

$T1_2$: $n \rightarrow n^{2+f}$ }

The true generalized result of the relative fractal dimension is
$\log(n^{2+f})/\log(n) = 2+f$, with the expression (1): $\boxed{(\delta+\varepsilon)/\delta = 2+f}$ **(4)**

During the transition of dimensions, the value of the fictitious coefficient f, associated with the very nature of the quantum (hypothetically), was defined. We will analyze the transition of dimensions combining expressions (3) and (4), for $\varepsilon=9-\delta$.

$\boxed{(En)\ (n^{(\varepsilon-\delta)/\delta}) < Constant}$ multiplying and dividing by n^{δ} which is the generalized volume to ordinary dimensions δ:

(Energy_density) (n^{ϕ})<Constant. The value of $\phi = (\delta2-2\delta+9)/\delta$, and is represented in figure 2.

Ordinary dimensions	Distance exponent
1	8
2	4,5
3	4
4	4,25
5	4,8
6	5,5
7	6,285714
8	7,125
9	8

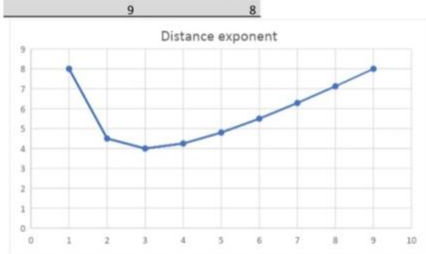

FIG. 2

FIG.2 Distance exponent.

For δ = 3 there is a minimum that corresponds to a maximum in energy density. For δ = 0, the value is infinite and corresponds to a minimum density equal to zero. The transition of dimensions from δ = 0, ordinary dimensions, to δ = 3, ordinary dimensions, takes us from a vacuum energy equal to zero to a maximum value. “In particular, our laws of physics arise from the geometry of the extra dimensions. Understanding this geometry ties string theory to some of the most interesting questions in modern mathematics, and has shed new light on them, such as mirror symmetry” (**Polchinski, 2015**)

5 Conclusion

Possibly, there was a transition of dimensions that maximized the energy density of the vacuum for $\delta=3$ and $\varepsilon=6$ (δ= ordinary dimensions, ε= compact dimensions). The nature of the quantum of action is tied to these values of δ and ε.

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