Prolegomena to Any Kinematics of Quantum Gravity

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It is known that existence of Planck length and time is in contradiction with fundamental results of special relativity, i.e. length contraction and time dilation[1, 2, 3]. In a previous attempt[4] I approached the problem from a blindly-formal perspective, but Nature can be more subtle than what formal reasoning can achieve. Although that attempt of mine always had a little value for myself, I (and for that matter nobody) has still come up with a completely satisfactory solution, leaving [4] the only work that at least gets some transformations. In this note I sketch the outlines of another approach that is completely satisfactory, but much more difficult to work out. Although I am not able to get any final result, I write this work to merely share the raw idea; maybe someone can build on this.

There are three crucial requirements for kinematics of any proper theory of quantum gravity

1. Invariance of Planck Length
2. Invariance of Planck Time
3. Invariance of Planck Mass

under any group of transformations. It is improbable for a theory that focuses on only one of these requirements to be able to satisfy others as well; the conditions must be satisfied simultaneously. This is one reason that the existing approaches of *Doubly Special Relativity* are not satisfactory.

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The first two requirements are purely kinematical, hence it is perfectly conceivable that a merely kinematical theory can satisfy them. The third requirement on the other hand, is a *dynamical* one, at least as far as special relativity is concerned, for special relativity talks about energy, mass and momentum only when it enters laws of motion; not in its kinematics. As our current goal is kinematics, we only attempt at satisfying the first two requirements.
The essential contradiction of special relativity and existence of Planck length and time is that special relativity via its length contraction

$$L' = \gamma L,$$

does not leave

$$l_P = \sqrt{\frac{\hbar G}{c^3}}$$

invariant. Similarly via time dilation in

$$T' = \gamma T,$$

Planck time

$$t_P = \sqrt{\frac{\hbar G}{c^5}}$$

is not left invariant.
There is but one way to resolve this contradiction: we must modify length contraction and time dilation in the following manner

$$\begin{cases} L' = L\gamma\zeta(L/l_P), \text{ s.t. } \zeta(1) = 1/\gamma \\ T' = T\gamma\xi(T/t_P), \text{ s.t. } \xi(1) = 1/\gamma, \end{cases} \tag{1}$$

for unknown smooth functions $\zeta, \xi : \mathbb{R} \to \mathbb{R}$. We expect for lengths $L \gg l_P$ and time durations $T \gg t_P$ to recover the familiar expressions of special relativity. Therefore

$$\begin{cases} \lim_{x\to\infty} \zeta(x) = 1 \\ \lim_{x\to\infty} \xi(x) = 1. \end{cases} \tag{2}$$

Our task now is to find functions ζ and ξ. It might be expected that determining coordinate transformations will determine ζ and ξ as well, but it turns out that coordinate transformations give little information about these functions. To see this, we assume that the new coordinate transformations are given by

$$\begin{cases} x' = f(x,t) \\ t' = g(x,t), \end{cases} \tag{3}$$

where f, g are arbitrary functions and contrary to special relativity, not necessarily linear. Now to apply (1), we find

$$L' = x'_2 - x'_1 = f(x_2, t_2) - f(x_1, t_1),$$

$$T' = t_2' - t_1' = g(x_2, t_2) - g(x_1, t_1),$$

Application of (1) now yields

$$\begin{cases} f(x_2, t_2) - f(x_1, t_1) = \gamma(x_2 - x_1)\zeta(x_2 - x_1) \\ g(x_2, t_2) - g(x_1, t_1) = \gamma(t_2 - t_1)\xi(t_2 - t_1) \end{cases} \tag{4}$$

In first equation let $x_2 \to x_1$ and in the second, $t_2 \to t_1$,

$$\lim_{x_2 \to x_1} \frac{f(x_2, t_2) - f(x_1, t_1)}{x_2 - x_1} = \gamma\zeta(0)$$

$$\lim_{t_2 \to t_1} \frac{g(x_2, t_2) - g(x_1, t_1)}{t_2 - t_1} = \gamma\xi(0)$$

giving

$$\frac{\partial f(x,t)}{\partial x} = \gamma\zeta(0) \tag{5}$$

and

$$\frac{\partial g(x,t)}{\partial t} = \gamma\xi(0). \tag{6}$$

We now apply the principle of constancy of velocity of light,

$$\frac{dx}{dt} = \frac{dx'}{dt'} = c, \tag{7}$$

since

$$\begin{cases} dx' = f_x dx + f_t dt \\ dt' = g_x dx + g_t dt, \end{cases} \tag{8}$$

$$\frac{dx'}{dt'} = \frac{\gamma\zeta(0)dx + f_t dt}{g_x dx + \gamma\xi(0)dt} = \frac{\gamma\zeta(0)cdt + f_t dt}{g_x cdt + \gamma\xi(0)dt} = \frac{\gamma\zeta(0)c + f_t}{g_x c + \gamma\xi(0)} = c,$$

we arrive at

$$\gamma\zeta(0)c + f_t = g_x c^2 + \gamma\xi(0)c. \tag{9}$$

As we see, functions ζ and ξ cannot be determined from coordinate transformations. *There must be some condition on (1) itself, using which one can determine ζ and* ξ, and then, coordinate transformations.
This is the farthest that I could come so far.

References

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