Discussion of cosmological acceleration and dark energy

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Abstract

The title of this workshop is: "What comes beyond standard models?". Standard models are based on Poincare invariant quantum theory. However, as shown in the famous Dyson's paper "Missed Opportunities" and in my publications, such a theory is a special degenerate case of de Sitter invariant quantum theory. I argue that the phenomenon of cosmological acceleration has a natural explanation as a consequence of quantum de Sitter symmetry in semiclassical approximation. The explanation is based only on universally recognized results of physics and does not involve models and/or assumptions the validity of which has not been unambiguously proved yet (e.g., dark energy and quintessence). I also explain that the cosmological constant problem and the problem why the cosmological constant is as is do not arise.

Keywords: quantum de Sitter symmetry; cosmological acceleration; irreducible representations; dark energy

1 Introduction

The title of this workshop is: "What comes beyond standard models?". Standard models are based on Poincare invariant quantum theory. However, as shown in the famous Dyson's paper "Missed Opportunities" and in my publications, such a theory is a special degenerate case of de Sitter invariant quantum theory.

The problem of cosmological acceleration is an example where the approach based on de Sitter symmetry solves the problem proceeding only from universally recognized results of physics without involving models and/or assumptions the validity of which has not been unambiguously proved yet (e.g., dark energy and quintessence). This problem was considered in my papers published in known journals, and in the book recently published by Springer.

My publications are based on large calculations. To understand them, the readers must be experts not only in quantum theory, but also in the theory of representations of Lie algebras in Hilbert spaces. Therefore, understanding my results can be a challenge for many physicists. Since the problem of cosmological acceleration is very important and my approach considerably differs from approaches of other authors, in this presentation to the 25th Bled workshop I outline only the ideas of my approach without calculations.

2 History of dark energy

This history is well-known. First Einstein introduced the cosmological constant Λ because he believed that the universe was stationary and his equations can ensure this only if $\Lambda \neq 0$. But when Friedman found his solutions of equations of General Relativity (GR) with $\Lambda = 0$, and Hubble found that the universe was expanding, Einstein said (according to Gamow's memories) that introducing $\Lambda \neq 0$ was the biggest blunder of his life. After that, the statement that Λ must be zero was advocated even in textbooks.

The explanation was that, according to the philosophy of GR, matter creates a curvature of space-time, so when matter is absent, there should be no curvature, i.e., space-time should be the flat Minkowski space. That is why when in 1998 it was realized that the data on supernovae could be described only with $\Lambda \neq 0$, the impression was that it was a shock of something fundamental. However, the term with Λ in the Einstein equations has been moved from the left hand side to the right hand one, it was declared that in fact $\Lambda = 0$, but the impression that $\Lambda \neq 0$ was the manifestation of a hypothetical field which, depending on the model, was called dark energy or quintessence. In spite of the fact that, as noted in wide publications (see e.g., [1] and references therein), their physical nature remains a mystery, the most publications on the problem of cosmological acceleration involve those concepts.

Several authors criticized this approach from the following considerations. GR without the contribution of Λ has been confirmed with a good accuracy in experiments in the Solar System. If Λ is as small as it has been observed, then it can have a significant effect only at cosmological distances while for experiments in the Solar System the role of such a small value is negligible. The authors of [2] titled "Why All These Prejudices Against a Constant?" note that it is not clear why we should think that only a special case $\Lambda = 0$ is allowed. If we accept the theory containing the gravitational constant G, which cannot be calculated and is taken from outside, then why can't we accept a theory containing two independent constants?

Let us note that currently there is no physical theory which works under all conditions. For example, it is not correct to extrapolate nonrelativistic theory to the cases when speeds are comparable to c, and it is not correct to extrapolate classical physics for describing energy levels of the hydrogen atom. GR is a successful classical (i.e., non-quantum) theory for describing macroscopic phenomena where large masses are present, but extrapolation of GR to the case when matter disappears is not physical. One of the principles of physics is that a definition of a physical quantity is a description how this quantity should be measured. The concepts of space and its curvature are pure mathematical. Their aim is to describe the motion of real bodies. But the concepts of empty space and its curvature should not be used in physics because nothing can be measured in a space which exists only in our imagination. Indeed, in the limit of GR when matter disappears, space remains and has a curvature (zero curvature when $\Lambda = 0$, positive curvature when $\Lambda > 0$ and negative curvature when $\Lambda < 0$ while, since space is only a mathematical concept for describing matter, a reasonable approach should be such that in this limit space should disappear too.

A common principle of physics is that when a new phenomenon is discovered, physicists should try to first explain it proceeding from the existing science. Only if all such efforts fail, something exotic can be involved. But in the case of cosmological acceleration, an opposite approach was adopted: exotic explanations with dark energy or quintessence were accepted without serious efforts to explain the data in the framework of existing science.

3 Elementary particles in relativistic and de Sitterinvariant theories

In the problem of cosmological acceleration, only large macroscopic bodies are involved and that is why one might think that for considering this problem, there is no need to involve quantum theory. Most works on this problem proceed from GR with additional assumptions the validity of which has not been unambiguously proved yet (see e.g. [1] and references therein).

However, ideally, the results for every classical (i.e., non-quantum) problem should be obtained from quantum theory in semiclassical approximation. We will see that considering the problem of cosmological acceleration from the point of view of quantum theory, sheds a new light on understanding this problem. Standard particle theory and standard models are based on Poincare symmetry where elementary particles are described by irreducible representations (IRs) of the Poincare group or its Lie algebra. The representation generators of the Poincare algebra commute according to the commutation relations

$$[P^{\mu}, P^{\nu}] = 0, \quad [P^{\mu}, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^{\nu} - \eta^{\mu\nu}P^{\rho}), \\ [M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma})$$
(1)

where $\mu, \nu = 0, 1, 2, 3, P^{\mu}$ are the operators of the four-momentum, $M^{\mu\nu}$ are the operators of Lorentz angular momenta and $\eta^{\mu\nu}$ is such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and $\eta^{\mu\nu} = 0$ if $\mu \neq \nu$.

Although the Poincare group is the group of motions of Minkowski space, the description in terms of relations (1) does not involve Minkowski space at all. It involves only representation operators of the Poincare algebra, and those relations can be treated as a definition of relativistic invariance on quantum level (see the discussion in [3, 4]). In particular, the fact that $\eta^{\mu\nu}$ formally coincides with the metric tensor in Minkowski space does not imply that this space is involved.

In classical field theories, the background space (e.g., Minkowski space) is an auxiliary mathematical concept for describing real fields and bodies. In quantum theory, any physical quantity should be described by an operator, but there is no operator corresponding to the coordinate x of the background space. In quantum field theory, Minkowski space is an auxiliary mathematical concept for describing interacting fields. Here a local Lagrangian L(x) is used, and x is only an integration parameter. The goal of the theory is to construct the S-matrix in momentum space, and, when this construction has been accomplished, one can forget about space-time background. This is in the spirit of the Heisenberg S-matrix program according to which in quantum theory one can describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$.

The fact that the S-matrix is the operator in momentum space does not exclude a possibility that, in semiclassical approximation, it is possible to have a space-time description with some accuracy but not with absolute accuracy (see e.g., [4] for a detailed discussion). For example, if **p** is the momentum operator of a particle then, in the nonrelativistic approximation, the position operator of this particle in momentum representation *can be defined* as $\mathbf{r} = i\hbar\partial/\partial \mathbf{p}$. In this case, **r** is a physical quantity characterizing a given particle and is different for different particles.

In relativistic quantum mechanics, for considering a system of noninteracting particles, there is no need to involve Minkowski space. A description of a single particle is fully defined by its IR by the operators commuting according to Eq. (1) while the representation describing several particles is the tensor product of the corresponding single-particle IRs. This implies that the four-momentum and Lorenz angular momenta operators for a system are sums of the corresponding single-particle operators. In the general case, representations describing systems with interaction are not tensor products of single-particle IRs, but there is no law that the construction of such representations should necessarily involve a background space-time.

In his famous paper "Missed Opportunities" [5] Dyson notes that de Sitter (dS) and anti-de Sitter (AdS) theories are more general (fundamental) than Poincare one even from pure mathematical considerations because dS and AdS groups are more symmetric than Poincare one. The transition from the former to the latter is described by a procedure called contraction when a parameter R (see below) goes to infinity. At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

The paper [5] appeared in 1972 (i.e., more than 50 years ago) and, in view of Dyson's results, a question arises why general theories of elementary particles (QED, electroweak theory and QCD) are still based on Poincare symmetry and not dS or AdS ones. Probably, physicists believe that, since the parameter R is much greater than even sizes of stars, dS and AdS symmetries can play an important role only in cosmology and there is no need to use them for describing elementary particles. We believe that this argument is not consistent because usually more general theories shed a new light on standard concepts. The discussion in our publications and, in particular, in this paper is a good illustration of this point.

By analogy with relativistic quantum theory, the definition of quantum dS symmetry should not involve dS space. If M^{ab} $(a, b = 0, 1, 2, 3, 4, M^{ab} = -M^{ba})$ are the operators describing the system under consideration, then, by definition of dS symmetry on quantum level, they should satisfy the commutation relations of the dS Lie algebra so(1,4), *i.e.*,

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad})$$
(2)

where η^{ab} is such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$ and $\eta^{ab} = 0$ if $a \neq b$. The *definition* of AdS symmetry on quantum level is given by the same equations but $\eta^{44} = 1$.

The procedure of contraction from dS and AdS symmetries to Poincare one is defined as follows. If we *define* the operators P^{μ} as $P^{\mu} = M^{4\mu}/R$ where R is a parameter with the dimension *length* then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^{μ} are finite, Eqs. (2) become Eqs. (1). This procedure is the same for the dS and AdS symmetries and it has nothing to do with the relation between the Minkowski and dS/AdS spaces.

In [4, 6] it has been proposed the following

Definition: Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that, with any desired accuracy, theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken, one cannot return to theory A, and theory B cannot reproduce all results of theory A. Then theory A is more general than theory B and theory B is a special degenerate case of theory A.

As argued in [4, 6], in contrast to Dyson's approach based on Lie groups, the approach to symmetry on quantum level should be based on Lie algebras. Then it has been proved that, on quantum level, dS and AdS symmetries are more general (fundamental) than Poincare symmetry, and this fact has nothing to do with the comparison of dS and AdS spaces with Minkowski space. It has been also proved that classical theory is a special degenerate case of quantum one in the formal limit $\hbar \to 0$, and nonrelativistic theory is a special degenerate case of relativistic one in the formal limit $c \to \infty$. In the literature the above facts are explained from physical considerations but, as shown in [4, 6], they can be proved mathematically by using properties of Lie algebras.

Physicists usually understand that physics cannot (and should not) derive that $c \approx 3 \cdot 10^8 \text{m/s}$ and $\hbar \approx 1.054 \cdot 10^{-34} \text{kg} \cdot \text{m}^2/\text{s}$. At the same time, they usually believe that physics should derive the value of Λ , and that the solution of the dark energy problem depends on this value. However, background space in GR is only a classical concept, while on quantum level symmetry is defined by a Lie algebra of basic operators.

The parameters (c, \hbar, R) are on equal footing because each of them is the parameter of contraction from a more general Lie algebra to a less general one, and therefore those parameters must be finite. In particular, the formal case $c = \infty$ corresponds to the situation when the Poincare algebra does not exist because it becomes the Galilei algebra, and the formal case $R = \infty$ corresponds to the situation when the de Sitter algebras do not exist because they become the Poincare algebra.

Quantum de Sitter theories do not need the dimensionful parameters (c, \hbar, R) at all. They arise in less general theories, and the question why they are as are does not arise because the answer is: \hbar is as is because people want to measure angular momenta in kg·m²/s, c is as is because people want to measure velocities in m/s, and R is as is because people want to measure distances in meters. The values of the parameters (c, \hbar, R)

in (kg, m, s) have arisen from people's macroscopic experience, and there is no guaranty that those values will be the same during the whole history of the universe (see e.g., [4] for a more detailed discussion). The fact that particle theories do not need the quantities (c, \hbar) is often explained such that the system of units $c = \hbar = 1$ is used. However, the concept of systems of units is purely classical and is not needed in quantum theory.

It is difficult to imagine standard particle theories without IRs of the Poincare algebra. Therefore, when Poincare symmetry is replaced by a more general dS one, dS particle theories should be based on IRs of the dS algebra. However, as a rule, physicists are not familiar with such IRs. The mathematical literature on such IRs is wide but there are only a few papers where such IRs are described for physicists. For example, an excellent Mensky's book [7] exists only in Russian.

4 Explanation of cosmological acceleration

In this section we explain that, as follows from quantum theory, the value of Λ in classical theory must be non-zero and the question why Λ is as is does not arise.

Consider a system of free macroscopic bodies, i.e., we do not consider gravitational, electromagnetic and other interactions between the bodies. Suppose that distances between the bodies are much greater than their sizes. Then the motion of each body as a whole can be formally described in the same way as the motion of an elementary particle with the same mass. In semiclassical approximation, the spin effects can be neglected, and we can consider our system in the framework of dS quantum mechanics of free particles.

The explicit expressions for the operators M^{ab} in IRs of the dS Lie algebra have been derived in [8] (see also [4, 6, 9]). In contrast to standard quantum theory where the mass m of a particle is dimensionful, in dS quantum theory, the mass m_{dS} of a particle is dimensionless. In the approximation when Poincare symmetry works with a high accuracy, these masses in units $c = \hbar = 1$ are related as $m_{dS} = Rm$. Also, in dS quantum theory, the Hilbert space of functions in IRs is the space of functions depending not on momenta but on four-velocities $v = (v_0, \mathbf{v})$ where $v_0 = (1 + \mathbf{v}^2)^{1/2}$. Then in the spinless case, the explicit expressions for the operators M^{ab} are (see e.g., Eq. (3.16) in [4]):

$$\mathbf{J} = l(\mathbf{v}), \quad \mathbf{N} = -iv_0 \frac{\partial}{\partial \mathbf{v}}, \quad \mathcal{E} = m_{dS} v_0 + iv_0 (\mathbf{v} \frac{\partial}{\partial \mathbf{v}} + \frac{3}{2})$$
$$\mathbf{B} = m_{dS} \mathbf{v} + i [\frac{\partial}{\partial \mathbf{v}} + \mathbf{v} (\mathbf{v} \frac{\partial}{\partial \mathbf{v}}) + \frac{3}{2} \mathbf{v}]$$
(3)

where $\mathbf{J} = \{M^{23}, M^{31}, M^{12}\}, \mathbf{N} = \{M^{01}, M^{02}, M^{03}\}, \mathbf{B} = \{M^{41}, M^{42}, M^{43}\}, \mathbf{l}(\mathbf{v}) = -i\mathbf{v} \times \partial/\partial \mathbf{v}$ and $\mathcal{E} = M^{40}$. The important observation is that, at this stage, we have no coordinates yet. For describing the motion of the particle in terms of coordinates, we must define the position operator. If Poincare symmetry works with a high accuracy, the momentum of the particle can be defined as $\mathbf{p} = m\mathbf{v}$ and, as noted above, the position operator can be defined as $\mathbf{r} = i\hbar\partial/\partial \mathbf{p}$.

In semiclassical approximation, we can treat \mathbf{p} and \mathbf{r} as usual vectors. Then, if $E = \mathcal{E}/R$, $\mathbf{P} = \mathbf{B}/R$ and the classical nonrelativistic Hamiltonian is defined as $H = E - mc^2$, it follows from Eq. (3) that

$$H(\mathbf{P}, \mathbf{r}) = \frac{\mathbf{P}^2}{2m} - \frac{mc^2\mathbf{r}^2}{2R^2}$$
(4)

Here the last term is the dS correction to the non-relativistic Hamiltonian.

The representation describing a free N-body system is a tensor product of the corresponding single-particle IRs. This means that every N-body operator M^{ab} is a sum of the corresponding single-particle operators.

Consider a system of two free particles described by the quantities \mathbf{P}_j and \mathbf{r}_j (j = 1, 2). Define standard nonrelativistic variables

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2, \quad \mathbf{q} = (m_2 \mathbf{P}_1 - m_1 \mathbf{P}_2) / (m_1 + m_2)$$
$$\mathbf{R} = (m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2) / (m_1 + m_2), \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$
(5)

Here **P** and **R** are the momentum and position of the system as a whole, and **q** and **r** are the relative momentum and relative radius-vector, respectively. Then as follows from Eqs. (4) and (5), the internal two-body Hamiltonian is

$$H_{nr}(\mathbf{r}, \mathbf{q}) = \frac{\mathbf{q}^2}{2m_{12}} - \frac{m_{12}c^2\mathbf{r}^2}{2R^2}$$
(6)

where m_{12} is the reduced two-particle mass. Then, as follows from the Hamilton equations, in semiclassical approximation the relative acceleration is given by

$$\mathbf{a} = \mathbf{r}c^2/R^2 \tag{7}$$

where \mathbf{a} and \mathbf{r} are the relative acceleration and relative radius vector of the bodies, respectively.

The fact that the relative acceleration of noninteracting bodies is not zero does not contradict the law of inertia, because this law is valid only in the case of Galilei and Poincare symmetries. At the same time, in the case of dS symmetry, noninteracting bodies necessarily repulse each other. In the formal limit $R \to \infty$, the acceleration becomes zero as it should be.

Equations of relative motion derived from Eq. (6) are the same as those derived from GR if $\Lambda \neq 0$. In particular, the result (7) coincides with that

in GR if the curvature of dS space equals $\Lambda = 3/R^2$, where R is the radius of this space. Therefore the cosmological constant has a physical meaning only on classical level, the parameter of contraction from dS symmetry to Poincare one coincides with R and, as noted above, a question why R is as is does not arise.

In GR, the result (7) does not depend on how Λ is interpreted, as the curvature of empty space or as the manifestation of dark energy or quintessence. However, in quantum theory, there is no freedom of interpretation. Here R is the parameter of contraction from the dS Lie algebra to the Poincare one, it has nothing to do with dark energy or quintessence and it must be finite because dS symmetry is more general than Poincare one.

Every dimensionful parameter cannot have the same numerical values during the whole history of the universe. For example, at early stages of the universe such parameters do not have a physical meaning because semiclassical approximation does not work at those stages. In particular, the terms "cosmological constant" and "gravitational constant" can be misleading. General Relativity successfully describes many data in the approximation when Λ and G are constants but this does not mean that those quantities have the same numerical values during the whole history of the universe.

5 Discussion and conclusion

In view of the problem of cosmological acceleration, the cosmological constant problem is widely discussed in the literature. This problem arises as follows.

One starts from Poincare invariant quantum field theory (QFT) of gravity defined on Minkowski space. This theory contains only one phenomenological parameter — the gravitational constant G, and the cosmological constant Λ is defined by the vacuum expectation value of the energy-momentum tensor. The theory contains strong divergencies which cannot be eliminated because the theory is not renormalizable. Therefore, the results for divergent integrals can be made finite only with a choice of the cutoff parameter. Since G is the only parameter in the theory, a reasonable choice of the cutoff parameter in momentum space is the Planck momentum \hbar/l_P where l_P is the Plank length. In units $\hbar = c = 1$, G has the dimension $1/length^2$ and Λ has the dimension $length^2$. Therefore, the value of Λ obtained in this approach is of the order of 1/G. However, this value is more than 120 orders of magnitude greater than the experimental one.

In view of this situation, the following remarks can be made. As ex-

plained in Sec. 3, Poincare symmetry is a special degenerate case of dS symmetry in the formal limit $R \to \infty$. Here R is a parameter of contraction from dS algebra to Poincare one. This parameter has nothing to do with the relation between Poincare and dS spaces. The problem why R is as is does not arise by analogy with the problem why c and \hbar are as are. As explained in Sec. 4, the cosmological constant Λ has a physical meaning only in semiclassical approximation and here it equals $3/R^2$. Therefore the cosmological constant problem why the cosmological constant is as is do not arise.

As noted in Sec. 3, the background space-time is only a mathematical concept which has a physical meaning only in classical theory. This concept turned out to be successful in QED. In particular, the results for the electron and muon magnetic moments agree with experiments with the accuracy of eight decimal digits. However, QED works only in perturbation theory because the fine structure constant is small. There is no law that the ultimate quantum theory will necessarily involve the concept of background space-time. QFTs of gravity (for example, Loop Quantum Gravity) usually assume that in semiclassical approximation, the background space in those theories should become the background space in GR. However, in Sec. 4, the result for the cosmological acceleration in semiclassical approximation has been obtained without space-time background and this result is the same as that obtained in GR.

Although the physical nature of dark energy remains a mystery, there exists a wide literature where the authors propose QFT models of dark energy. These models are based on Poincare symmetry with the background Minkowski space. So, the authors do not take into account the fact that de Sitter symmetry is more general (fundamental) than Poincare symmetry and that the background space is only a classical concept. While in most publications, only proposals about future discovery of dark energy are considered, the authors of [1] argue that dark energy has been already discovered by the XENON1T collaboration. In June 2020, this collaboration reported an excess of electron recoils: 285 events, 53 more than the expected 232 with a statistical significance of 3.5σ . However, in July 2022, a new analysis by the XENONnT collaboration discarded the excess [10].

As shown in Sec. 4, the result (7) has been derived without using dS space and its geometry (metric and connection). It is simply a consequence of dS quantum mechanics in semiclassical approximation. We believe that this result is more important than the result of GR because any classical result should be a consequence of quantum theory in semiclassical approximation.

Therefore, the phenomenon of cosmological acceleration has nothing to do with dark energy or other artificial reasons. This phenomenon is purely a kinematical consequence of dS quantum mechanics in semiclassical approximation.

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