Conservation of Energy and Particle Moving Towards a Mass

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Abstract

We consider a zero rest mass classical particle moving from infinity towards a point mass along a fixed line containing the mass. We show gravitation with only constants c and G with dimension does not satisfy conservation of energy.

1 Introduction

We restrict to gravitation that has only constants c and G with dimension. Units are chosen so that $c = G = 1$. Let x, y, z be coordinates of space and consider a point mass A on the x axis. Let γ be a zero rest mass particle moving along the x axis from infinity towards A. Here γ being considered as a classical particle. When γ is at infinity let A be at rest at the origin and have total energy M. Let E be the energy of γ at infinity.

2 Energy gain function

As γ moves towards A it gains energy from A. Let the function $W(M, E, h, R)$ be the amount of energy γ gains on moving from an x value of $R + h$, with $R > 0$ and $h > 0$, to an x value of R. For small E/M and M/R the amount of energy γ gains on moving from infinity to R is approximately ME/R. For small $E/M, M/R$, and h/R we have $W(M, E, h, R)$ is approximately MEh/R^2.

Since c and G are the only constants with dimension there is then a dimensionless function F of the dimensionless variables $M/R, E/R$, and h/R such that we can write

$$W(M, E, h, R) = \frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \tag{1}$$

We will assume $W(M, E, h, R)$ is an increasing function of E.

3 Bound on energy gain

By conservation of energy γ cannot gain more than an amount M of energy so

$$W(M, E, h, R) \leq M \tag{2}$$

As a consequence of this bound there is then a dimensionless function $B(M/R, h/R)$ such that

$$\sup_E \left\{ \frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \right\} = \frac{Mh}{R} B\left(\frac{M}{R}, \frac{h}{R}\right) \leq M \tag{3}$$

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For small $E/M, M/R$, and h/R since $W(M,E,h,R)$ is approximately MEh/R^2 and by the assumption that $W(M,E,h,R)$ is an increasing fuction of E we have $B(M/R,h/R) > 0$ for small M/R and h/R. Consequently we can define

$$b = \inf_{R>R_0} \left\{ B\left(\frac{M}{R},\frac{h}{R}\right)\right\} \tag{4}$$

where R_0 is chosen so that M/R_0 and h/R_0 are small. We have $b \geq 0$.

4 $b = 0$

The amount of energy γ gains on moving from $R+(N+1)h$ to R is the amount of energy γ gains on moving from $R+(N+1)h$ to $R+Nh$ plus the amount of energy γ gains on moving from $R+Nh$ to $R+(N-1)h$ and so on. For a γ having large E this is approximately

$$\sum_{n=0}^{N} \frac{Mh}{R+(N-n)h} B\left(\frac{M}{R+(N-n)h},\frac{h}{R+(N-n)h}\right) \geq \sum_{n=0}^{N} \frac{Mhb}{R+(N-n)h} \tag{5}$$

where $R > R_0$. It follows by section (3) the energy γ gains on moving from $R+(N+1)h$ to R becomes closer and closer to the left hand side of (5) as E becomes larger and larger. If $b > 0$ the right hand sum of (5) becomes unbounded as $N \to \infty$. Consequently for some N the left hand sum would become larger than M hence the energy γ gains, for large E, would be larger than M violating conservation of energy. We must have $b = 0$.

5 Contradiction

Since $B(M/R,h/R) > 0$ for $R > R_0$ and $b = 0$ it follows there must be a sequence $\{R_k\}$ where $R_k \to \infty$ as $k \to \infty$ such that $B(M/R_k,h/R_k) \to 0$ as $k \to \infty$. Define the function

$$C(M,h,R) = RB\left(\frac{M}{R},\frac{h}{R}\right) \tag{6}$$

We have $C(M_k,h_k,R) \to 0$ as $k \to \infty$ where $M_k = MR/R_k$ and $h_k = hR/R_k$. By (3) and (6)

$$\frac{MEh}{R^2} F\left(\frac{M}{R},\frac{E}{R},\frac{h}{R}\right) \leq \frac{Mh}{R} B\left(\frac{M}{R},\frac{h}{R}\right) = \frac{Mh}{R^2} C(M,h,R) \tag{7}$$

hence

$$EF\left(\frac{M}{R},\frac{E}{R},\frac{h}{R}\right) \leq C(M,h,R) \tag{8}$$

Substitute M_k for M and h_k for h in this inequality and let $k \to \infty$ gives since M_k, h_k, and $C(M_k,h_k,R)$ go to zero and $E > 0$ that

$$F\left(0,\frac{E}{R},0\right) \leq 0 \tag{9}$$

As stated before for small $E/M, M/R$, and h/R that $W(M,E,h,R)$ is approximately MEh/R^2. Comparing this with (1) we have $F(0,E/R,0)$ for small E/R is approximately one contradicting (9).

6 Conclusion

Assuming that the energy gain of γ on moving from $R+h$ to R increases as the energy γ has at infinity increases it was shown that a gravitation with only constants c and G with dimension does not satisfy conservation of energy. Other conservation of energy arguments are presented in [1] and [2].

References

[1] K. De Paepe, Physics Essays, March 2013

[2] K. De Paepe, Physics Essays, June 2017