Meaning of speed of light in the FLRW Universe

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The ΛCDM is frequently referred to as the standard model of Big Bang cosmology because it is the simplest model that provides a reasonably good account of most cosmological observations. This model is based on the assumption of the cosmological principle, which states that the universe looks the same from all positions in space at a particular time and that all directions in space at any point are equivalent. One can define the surface of simultaneity of the local Lorentz frame (LLF) with a global proper time in the Friedmann–Lemaître–Robertson–Walker (FLRW) universe. The Lorentz invariance is *locally* exact along the worldline and well defined on the hypersurface at a given time $t_{\mathfrak{k}}$. However, it is meaningless to argue the validity of the local Lorentz invariance along the geodesic in the manifold. We show that the speed of light on each hypersurface (LLF) is constant but its value should be the function of a scale factor (*i.e.*, cosmological redshift) to define the null interval consistently. This means the varying speed of light as a function of cosmic time. Also, the entropy of the Universe should be conserved to preserve the homogeneity and isotropy of the Universe. This adiabatic expansion condition induces the cosmic evolutions of other physical constants including the Planck constant. We conclude that the conventional assumption of the constant speed of light in the FLRW universe should be abandoned to obtain a consistent and accurate interpretation of cosmological measurement.

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I. LOCAL LORENTZ FRAME

A point in the Minkowski spacetime is a time and spatial position called an "event", or sometimes the four-position, described in some reference frame by a set of four coordinates

$$x^{\mu} = \left(ct\,, x^{i}\right)\,, \tag{1}$$

where c is the speed of light, t denotes the coordinate time along a worldline, and x^i is the three-dimensional space position vector on the Minkowski spacetime (*i.e.*, Lorentz frame). The path followed by an object in spacetime is termed the worldline for the object. If x^i is a function of coordinate time t in the same frame, (*i.e.*, $x^i = x^i[t]$), this corresponds to a sequence of events as t varies. One can define the differential four-position on a worldline dx^{μ}

$$dx^{\mu} = \left(dx^{0}\,, dx^{i}\right) = \left(cdt\,, dx\,, dy\,, dz\right)\,, \tag{2}$$

where we adopt the assumption that c is a constant and $dx \equiv x[t+dt] - x[t]$. In Minkowski spacetime, the square of the infinitesimal distance ds^2 between two points (*i.e.*, the line element) is given by

$$ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu} = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \equiv -c^2 dt^2 + dl^2\,. \tag{3}$$

The line element (3) defines a cone. The lightcone is a 3-dimensional surface in the 4-dimensional spacetime. If $ds^2 = 0$, then the interval is called lightlike (or null).

$$ds^2 = -c^2 dt^2 + dl^2 = 0 \quad \Rightarrow \quad \frac{dl}{dt} \equiv v = c\,. \tag{4}$$

This means that events in Minkowski space separated by a null interval are connected by signals moving at light velocity. The tangent to the worldline of any particle at a point defines the local velocity of the particle at that point and constant velocity implies straight worldlines.

All observers in Newtonian mechanics share a universal (or absolute) time, and in special relativity (SR), the time has a well-defined meaning within an inertial reference frame (IRF). However, it is impossible to define a global time in general relativity (GR) due to the absence of global IRF. Nevertheless, it is well known that a global time for the Universe can be defined when a following set of requirements is satisfied, and a metric embodying the cosmological principle meets those requirements. Then, one can define a global time by a foliation of spacetime as a sequence of non-intersecting spacelike three-dimensional (3D) surfaces [1–3]. We briefly review this in the following.

All galaxies are assumed to lie on a hypersurface so that the surface of simultaneity for the local Lorentz frame (LF) of each galaxy coincides locally with the hypersurface. Thus, one may regard a hypersurface as consisting of the smoothly meshed LF of all the galaxies, with the four-velocity of each galaxy orthogonal to the hypersurface. One may label this series of hypersurfaces by a parameter t that can be regarded as the proper time of any galaxy defining a universal time.

It is assumed that the worldlines of galaxies are a bundle of geodesics in spacetime. The bundle of geodesics possesses a set of spacelike hypersurfaces orthogonal to them. The proper time along a geodesic is chosen as a parameter τ such that each of these hypersurfaces corresponds to $\tau = \tau_{\mathfrak{t}} =$ constant. Thus, the worldline of a galaxy is given by

$$x^{\mu}\left[\tau_{\mathfrak{t}}\right] = \left(x^0 = c\tau_{\mathfrak{t}}\,, x^1 = \text{const}\,, x^2 = \text{const}\,, x^3 = \text{const}\right)\,, \tag{5}$$

where $\tau_{\mathfrak{k}}$ is the proper time along with the galaxy. We emphasize that the conventional *assumption* that the speed of light is constant on every hypersurface is used in this equation. Under these conditions, one can write the metric as

$$ds^2_{\mathfrak{k}} = -c^2 d\tau^2 = -c^2 dt^2 + h_{ij}[t_{\mathfrak{k}}, x^k] dx^i dx^j \,. \tag{6}$$

The proper time $\tau_{\mathfrak{k}}$ along the galaxy is equal to the coordinate time $t_{\mathfrak{k}}$ because $dx^i = 0$ along the worldline (*i.e.*, $ds_{\mathfrak{k}} = c d\tau = c dt$). One can understand this easily from the fact that a vector along the worldline given as $A^\mu = (cdt\,, 0\,, 0\,, 0)$ and the vector $B^\mu = (0\,, dx^1\,, dx^2\,, dx^3)$ lying in the hypersurface $t = t_{\mathfrak{k}}$ are orthogonal. Thus, one can regard the worldline at each $t_{\mathfrak{k}}$ as a local LF. Additionally, the worldline x^μ satisfies the geodesic equation

$$\frac{d}{dx^0}\frac{dx^\mu}{dx^0} + \Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{dx^0}\frac{dx^\lambda}{dx^0} = 0 \quad \text{where} \quad \frac{dx^\mu}{dx^0} = \frac{dx^\mu}{ds} = (1, 0, 0, 0)\,. \tag{7}$$

However, the metric (6) does not incorporate the cosmological principle of space. By adopting the postulate of homogeneity and isotropy, one can write the spatial separation on the same hypersurface $t = t_{\mathfrak{k}}$ of two nearby galaxies at coordinates (x^1, x^2, x^3) and $(x^1 + dx^1, x^2 + dx^2, x^3 + dx^3)$ as

$$d\sigma^2_{\mathfrak{k}} \equiv a^2[t_{\mathfrak{k}}]\gamma_{ij} dx^i dx^j \,. \tag{8}$$

Thus, one can define the standard form of the FLRW metric as

$$ds^2_{\mathfrak{k}} = -c^2 dt^2 + a^2[t_{\mathfrak{k}}] \left(\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right) \,. \tag{9}$$

One can conclude that a global cosmic time for all galaxies on the hypersurface, but only if the space is homogeneous and isotropic (which implies further that the spatial curvature is constant). Cosmic time is the time measured by a comoving observer who sees the Universe expanding uniformly around her. This is shown in Fig. 1. The upper surface is a hypersurface at the cosmic time t_0 and the lower one is a one at t_i. Each hypersurface is an LF with a constant value of $c[a_i]$ on that surface.

To proceed further we focus on the radial direction ($d\theta = d\phi = 0$) for the flat universe ($k = 0$). Then the square of the infinitesimal distance $ds^2_{\mathfrak{k}}$ between two worldlines at that time, $x^\mu[t_{\mathfrak{k}}]$ and $x^\mu[t_{\mathfrak{k}} + dt]$ is given by

$$ds^2_{\mathfrak{k}} = -c^2 dt^2 + a^2_{\mathfrak{k}} dr^2 \equiv -c^2 dt^2 + a^2_{\mathfrak{k}} dl^2 \equiv -c^2 dt^2 + d\sigma^2_{\mathfrak{k}} \,, \tag{10}$$

where r is the circumferential radius defined by $r \equiv \sqrt{x^2 + y^2 + z^2}$. If one regards a null ray on the FLRW metric (*i.e.*, $ds_{\mathfrak{k}} = 0$), then one obtains

$$v_{\mathfrak{k}} \equiv \frac{d\sigma_{\mathfrak{k}}}{dt} = a_{\mathfrak{k}}\frac{dl}{dt} \stackrel{?}{=} c\,. \tag{11}$$

This result leads to the conclusion that events on an FLRW metric at a given time, $t_{\mathfrak{k}}$ separated by a null interval are connected by signals moving at light velocity, $v_{\mathfrak{k}} = c$. However, one needs to be careful about quantities in the above equation (11). dl in Eq.(10) is the spatial interval on the hypersurfaces at $t = t_{\mathfrak{k}}$. Contrary to this, $d\sigma_{\mathfrak{k}}$ is the so-called physical spatial interval and its magnitude is different at different epoch by a factor, $a_{\mathfrak{k}}$. Thus, $v_{\mathfrak{k}}$ in Eq. (11) can be different from each other at each epoch. This contradicts the universality of the speed of light when one uses the coordinate cosmic time, t. One might solve this problem by rescaling the time interval $dt = a_{\mathfrak{k}} d\eta$ [4, 5].

$$v_{\mathfrak{k}} \equiv \frac{d\sigma_{\mathfrak{k}}}{dt} = \frac{d\sigma_{\mathfrak{k}}}{a_{\mathfrak{k}} d\eta} = \frac{dl}{d\eta} = c\,. \tag{12}$$

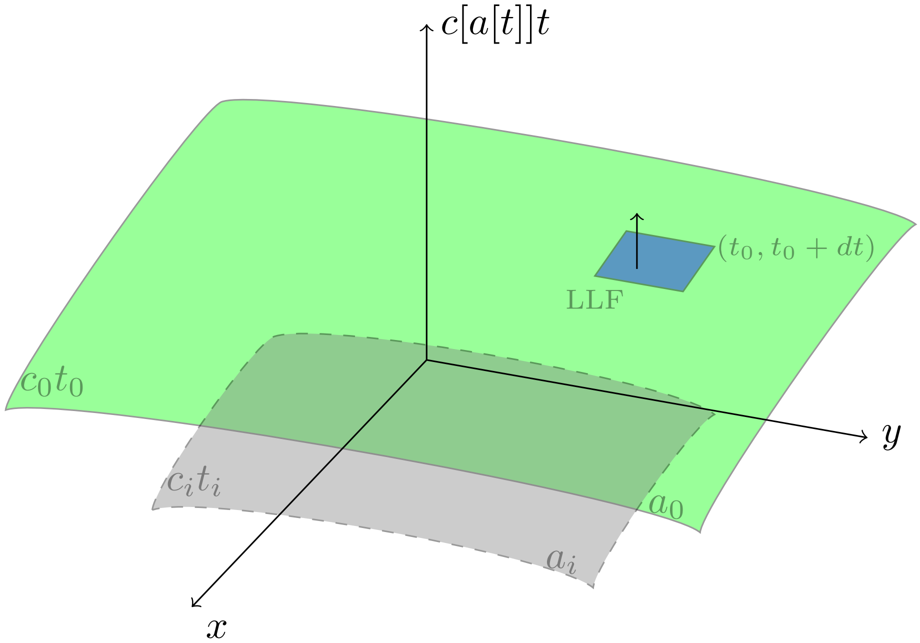

FIG. 1: The surface of simultaneity of local Lorentz frame (LLF) on the spacelike hypersurface. The time interval along the geodesic between two hypersurfaces given at t_i and $t_0 = t_i + dt$ is given by dt.

However, one should notice that $d\eta = dt/a_{\mathfrak{e}}$ is not the conformal time ($d\eta = dt/a$) but just the rescaling of the time interval.

The cosmic time t is defined universally in the FLRW metric. Thus, after one fixes the cosmic time at one moment, it should be used without rescaling it. Also, one usually uses the coordinate cosmic time when one expresses the cosmological distances including comoving distances [6]. Thus, one needs to worry about the consistency problem when the metric is written by Eq. (10).

One can cure this contradiction in Eq. (11) by abandoning the *assumption* that speeds of light written by the coordinate cosmic time are the same on all hypersurfaces. Then, one can redefine x^0 as

$$x^0_{\mathfrak{e}} = c[a_{\mathfrak{e}}]t_{\mathfrak{e}} \,, \tag{13}$$

by allowing the evolution of speed of light as a function of coordinate cosmic time, $c[a(t_{\mathfrak{e}})]$. Then one obtains

$$dx^0_{\mathfrak{e}} = \left(\frac{d\ln c}{d\ln a}\Big|_{\mathfrak{e}} H_{\mathfrak{e}} t_{\mathfrak{e}} + 1 \right) c[a(t_{\mathfrak{e}})]dt \equiv \tilde{c}_{\mathfrak{e}} dt \,. \tag{14}$$

By using this, one can rewrite Eq. (10) as

$$ds^2_{\mathfrak{e}} = -\tilde{c}^2_{\mathfrak{e}} dt^2 + a^2_{\mathfrak{e}} \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \equiv -\tilde{c}^2_{\mathfrak{e}} dt^2 + a^2_{\mathfrak{e}} dl^2 \equiv -\tilde{c}^2_{\mathfrak{e}} dt^2 + d\sigma^2_{\mathfrak{e}} \,, \tag{15}$$

to obtain

$$v_{\mathfrak{e}} \equiv \frac{d\sigma_{\mathfrak{e}}}{dt} = a_{\mathfrak{e}} \frac{dl}{dt} = \tilde{c}_{\mathfrak{e}} \,. \tag{16}$$

We should emphasize that Eq. (15) is satisfied at one specific time $t_{\mathfrak{t}}$. From equation (16), one can conclude that events in an FLRW universe separated by a null interval are connected by signals moving at the value of the speed of light $v_{\mathfrak{t}} = c_{\mathfrak{t}}$ written in terms of the coordinate cosmic time. It is a constant value at each $t_{\mathfrak{t}}$. However, its value varies as $a_{\mathfrak{t}}$ does.

One can generalize the infinitesimal interval on a given hypersurface (*i.e.*, $t = t_{\mathfrak{t}}$) given in Eq. (15) into the one along the geodesic on the entire manifold as

$$ds^2 = -\tilde{c}^2[a[t]]dt^2 + a^2[t]\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \equiv -\tilde{c}^2[t]dt^2 + a^2[t]\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) , \tag{17}$$

where t is the cosmic time defined along the geodesic on the manifold.

One can obtain a consistent conclusion of LLF for the null signal by abandoning the conventional assumption of the constant speed of light on the FLRW metric. Thus, one can conclude that $\tilde{c}$ in Eq.(14) is a function of a scale factor $a[t]$. Recently, we propose the so-called minimally extended varying speed of light (meVSL) model where $\tilde{c}$ is given by [7]

$$\tilde{c}[a] = \tilde{c}_0 a^{b/4} \quad \text{where} \quad \tilde{c}_0 \equiv \tilde{c}[a_0] \text{ and } b = \text{const} . \tag{18}$$

However, this is not the end of the story. One should guarantee the homogeneity and the isotropy of the hypersurface to keep the validity of the cosmic time through the foliation of the hypersurface. If one permits the cosmic evolution of the speed of light, then homogeneity and isotropy can be broken. Next, we show the necessary condition to keep this cosmic principle in the VSL model.

II. ADIABATICITY

The first law of thermodynamics (law of conservation of energy), $dQ = dE + PdV$ where dQ is the heat flow into or out of a volume, dE denotes the energy change, P is the pressure, and dV means the change of a volume. If the Universe is perfectly homogeneous, then for any volume $dQ = 0$, that is, there is no bulk flow of heat. Processes for which $dQ = 0$ are known as adiabatic processes. A homogeneous, isotropic expansion of the universe does not increase the universe's entropy [8].

Adiabaticity is a necessary condition to maintain the homogeneity and isotropy of the Universe. Net flux of energy would falsify the isotropy if there is a preferential energy flow direction of homogeneity if the outward (inward) flux is isotropic.

One infers that the early Universe was in the *local* thermal equilibrium from the perfect blackbody spectrum of the cosmic microwave background (CMB). Also, because the photon is the dominant component to contribute the entropy of the Universe, we consider both the energy density and the pressure of photon obtained from statistical mechanics [7]

$$\varepsilon_\gamma = \frac{\pi^2}{15}\frac{(k_{\mathrm{B}}T_\gamma)^4}{(\hbar\tilde{c})^3} \equiv \tilde{\sigma}_\gamma T_\gamma^4 \quad , \quad P_\gamma = \frac{1}{3}\varepsilon_\gamma , \tag{19}$$

where k_{B} is the Boltzmann constant, $\hbar$ is the Planck constant, and $\tilde{\sigma}_\gamma$ is the so-called blackbody constant. They are local quantities and the cosmological evolutions of them are embedded in that of the photon temperature T_γ and physical constants including $\tilde{c}$. To use the first law of thermodynamics, we use the following relations

$$E_\gamma = \varepsilon_\gamma V \quad , \quad T_\gamma = T_{\gamma 0} a^{-1} \quad , \quad V = V_0 a^3 , \tag{20}$$

where subscript 0 denotes the corresponding value at the present epoch (*i.e.*, $a_0 = 1$). From Eqs.(18)-(20), one writes the entropy change as

$$dQ = E_\gamma \left(d \ln \tilde{\sigma}_\gamma + 4 d \ln T_\gamma + \frac{4}{3} d \ln V \right) = E_\gamma d \ln \tilde{\sigma}_\gamma \,, \tag{21}$$

where we use $d \ln T_\gamma = -d \ln V/3 = -d \ln a$. If one just adopts the cosmic evolution of the speed of light without allowing that of the Planck constant, then the entropy of the Universe is not conserved as shown in the above Eq.(21). Thus, to satisfy the adiabaticity expansion of the Universe, $\tilde{\sigma}_\gamma$ should be constant as a cosmic time. One of the possible choices to satisfy this condition is

$$\tilde{\hbar} = \tilde{\hbar}_0 a^{-b/4} \quad , \quad \tilde{k}_{\mathrm{B}} = \tilde{k}_{\mathrm{B}0} \equiv k_{\mathrm{B}} \,, \tag{22}$$

as determined in meVSL model [7].

III. SPEED OF LIGHT

In Sec. I, we introduce a series of non-intersecting space-like hypersurfaces, that is, surfaces any two points of which can be connected by a curve lying entirely in the hypersurface which is space-like everywhere. We assume that all galaxies lie on such a hypersurface in such a manner that the surface of simultaneity of the LLF of any galaxy coincides locally with the hypersurface. Thus the four-velocity of a galaxy is orthogonal to the hypersurface. The proper time, τ along the galaxy is, in fact, equal to the coordinate time t. This is because the spatial infinitesimal interval $dx^i = 0$ along with the worldline yields $ds = \tilde{\tilde{c}} d\tau = \tilde{c} dt$, so that $\tau = t$.

The speed induced in Sec. I can be interpreted as the speed of signals connecting two events in the LLF. In this sense, it is more accurate to say that it is the speed of the massless particle mediating the gravitational force (*i.e.*, graviton) [4]. However, $c_{\mathfrak{t}}$ is the speed of light on the hypersurface at $t_{\mathfrak{t}}$. This speed is defined on the local Lorentz transformation to be used in Maxwell's equation. Thus, one might identify the speed of a massless particle with that of a photon. Under this assumption, one may obtain the cosmic evolutions of permittivity, $\tilde{\epsilon}$ and permeability, $\tilde{\mu}$ of vacuum

$$\tilde{\epsilon} = \tilde{\epsilon}_0 a^{-b/4} \quad , \quad \tilde{\mu} = \tilde{\mu}_0 a^{-b/4} \,, \tag{23}$$

as shown in meVSL model [7].

IV. CONCLUSIONS

Consideration of the time variations of dimensional constants is occasionally claimed to be meaningless because they might be just human-constructed values coming from the different choices of units [9]. However, the possibility of the cosmic time evolutions of dimensional constants has meaning only when the principle of locality of the given theory is considered. Einstein's general relativity (GR) is a *local theory*. A solution of Einstein's field equations is local if the underlying equations are covariant. Thus, all classical physics laws by using dimensional constants at one given cosmic time, t_1, can be rewritten by using the same dimensional constants at another cosmic time, t_0. Then, time evolutions of dimensionless quantities related to the ratio of dimensional constants can be obtained by admitting cosmic evolutions of dimensional constants. For example, the gravitational constants at different cosmic times $G(t_1)$ and $G(t_0)$ can be written by

$$\frac{G_0}{G_1} = \left(\frac{a_0}{a_1}\right)^b . \tag{24}$$

With this principle, we recently proposed the so-called meVSL model [7]. With these relations, we can establish thermodynamics, electromagnetism, and special relativity which are consistent with those obtained from general relativity. The cosmological evolutions of physical quantities are determined by the value of b. And one can constrain the value of b from both cosmological observations and blackhole thermodynamics [10–13].

Most of the current cosmological observations are measured under ΛCDM cosmological model. This is based on the FLRW metric and it is quite important to clarify the meaning and the consistent form of the speed of light in that model. We show that the speed of light with cosmic time evolution is consistent in the FLRW Universe. To keep the homogeneity and isotropy of the Universe, other physical constants including the Planck constant also should evolve as a function of cosmic time.

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Appendix: Speed of light

As pointed out in [5], the central issue relating to the varying speed of light is that one has distance and time units set up to measure the speed of light. Generally, there are no global inertial frames in general relativity. However, one can still define a moment in general relativity which is valid globally if the homogeneity and isotropy of the Universe are satisfied [1–3].

One can write the square of radial infinitesimal interval on a given hypersurface at $t = t_{\mathfrak{k}}$ as in (17)

$$ds_{\mathfrak{k}}^2 = -c_{\mathfrak{k}}^2 dt^2 + a^2[t_{\mathfrak{k}}] dr^2 . \tag{A.1}$$

where we focus on the flat space $k = 0$. The scale factor in this equation is the one at one specific hypersurface $a[t_{\mathfrak{k}}]$ instead of $a[t]$. This point has been missed in the pioneering work, *E07* [5]. The squared infinitesimal interval at two different epochs are given in *E07* as

$$ds^2 = -c_0^2 dt^2 + a^2[t] dl^2 = -c_1^2 dt^2 + a^2[t] dl^2 . \tag{A.2}$$

It is claimed that one obtains

$$a[t]\frac{dl}{dt} = c_0 \quad \text{or} \quad c_1 , \tag{A.3}$$

for the null ray. It is claimed that the difference is just due to the different choice of a unit on t and they become the same if one changes the coordinate $t \to (c_0/c_1)t$. The confusion stems from the missing fact that each hypersurface is a local Lorentz frame and the infinitesimal interval is given by Eqs. (A.1) instead of Eq. (A.2). We summarize this in table. I.

defined on	null interval	time	Lorentz invariance	consequence
LLF	$ds_{\mathfrak{t}}^2 = -\tilde{c}_{\mathfrak{t}}^2 dt^2 + a_{\mathfrak{t}}^2 \gamma_{ij} dx^i dx^j = 0$	at a $t = t_{\mathfrak{t}}$	locally $\tilde{c}_{\mathfrak{t}} =$ const	$\tilde{c}_{\mathfrak{t}} = a_{\mathfrak{t}} dr/dt$ on a hypersurface
manifold	$ds^2 = -\tilde{c}^2[t] dt^2 + a^2[t] \gamma_{ij} dx^i dx^j = 0$	at any t along geodesic	meaningless	comoving distance: $\int_0^t (\tilde{c}/a) dt$

TABLE I: LLF denotes the local Lorentz frame at a given moment $t = t_{\mathfrak{t}}$. The Lorentz invariance is locally exact on each hypersurface defined at a given time $t_{\mathfrak{t}}$.

[1] H. Weyl, Phys. Z. **24**, 230 (1923) (English translation: H. Weyl, "Republication of: On the general relativity theory" Gen. Rel. Grav. **41**, 1661 (2009)).

[2] J. N. Islam, *An Introduction to Mathematical Cosmology* (Cambridge University Press, 2001).

[3] M. Guidry, *Modern General Relativity: Black Holes, Gravitational Waves, and Cosmology* (Cambridge University Press, 2019).

[4] G. F. R. Ellis and J. P. Uzan, Am. J. Phys. **73**, 240-247 (2005) doi:10.1119/1.1819929 [arXiv:gr-qc/0305099 [gr-qc]].

[5] G. F. R. Ellis, Gen. Rel. Grav. **39**, 511-520 (2007) doi:10.1007/s10714-007-0396-4 [arXiv:astro-ph/0703751 [astro-ph]].

[6] T. M. Davis and C. H. Lineweaver, Publ. Astron. Soc. Austral. **21**, 97 (2004) doi:10.1071/AS03040 [arXiv:astro-ph/0310808 [astro-ph]].

[7] S. Lee, JCAP **08**, 054 (2021) doi:10.1088/1475-7516/2021/08/054 [arXiv:2011.09274 [astro-ph.CO]].

[8] P. Coles and F. Lucchin, *Cosmology: The Origin and Evolution of Cosmic Structure, 2nd Edition* (Wiley, 2002).

[9] M. J. Duff, Contemp. Phys. **56**, no.1, 35-47 (2015) doi:10.1080/00107514.2014.980093 [arXiv:1412.2040 [hep-th]].

[10] S. Lee, [arXiv:2110.08809 [gr-qc]].

[11] S. Lee, [arXiv:2108.06043 [astro-ph.CO]].

[12] S. Lee, [arXiv:2104.09690 [astro-ph.CO]].

[13] S. Lee, [arXiv:2101.09862 [astro-ph.CO]].