# Quantum entanglement: Bell's inequality trivially violated also classically

Arto Annila and Mårten Wikström*
Institute of Biotechnology, University of Helsinki, Helsinki, Finland

Abstract
Quantum entanglement manifests in the perfect correlation between particles or photons separated by space and time beyond causal interference. However, classical covariance between vectorial properties, such as spin, is also found following the observed sinusoidal form, attributed to quantum phenomena, instead of following the linear form, ascribed to hypothetical hidden variables. Thus, the concept of quantum entanglement is not necessary; the classical correlation between paired spins suffices. Since pairing, the spins are correlated, e.g., antiparallel; still, their direction remains undefined until put into a frame of reference. Thus, the measurement of one does not determine the correlated property of the other but discloses it in the given frame.

Keywords: entanglement, non-locality, phase, polarization, probability, wavefunction

## Introduction

Quantum entanglement is perceived as a phenomenon without classical correspondence. However, as we show below, the same sinusoidal form of correlation between particle spins or photon phases results from classical derivation, thus, questioning the need for the concept of entanglement. Moreover, we maintain that since pairing the correlated property exists, only its value in reference to an external frame remains indefinite until measured.

The perfect correlation between two particles, e.g., photons,[1-3] neutrinos,[4] electrons,[5] and molecules [6], separated by a distance in space or time[7] outside causal influence, is not the quantum hallmark *per se*. It is thought to be the sinusoidal form of the correlation because the linear form is ascribed to the hidden variable hypothesis.[8-10]

Thus, the issue is not Rosen, Podolsky, and Einstein challenging Bohr and Heisenberg, whether the particle property, such as spin, has or has not a definite value until measured[11] because the value depends on the frame of reference. The issue is that instead of being sinusoidal, the classical correlation between vectors[12] has been erroneously thought to be linear, in line with the hidden-variable postulate.[9] We believe this matter is worth clarifying, considering the foundations of modern physics and expectations of quantum computing using qubits.

## The phase concept

Let us think through the classical correlation between vectorial properties for two particles paired antiparallel. As Bohr and Heisenberg posited, we cannot say, for instance, that one spin is *up* and the other is *down*, as long as we have not specified a reference coordinate system, such as a laboratory frame. In other words, even though the spins are definitely antiparallel, their direction remains undefined relative to an external reference until measured.

Suppose we fix the frame of reference and detect that one spin is parallel with the detector axis pointing upward while the other is antiparallel pointing downward. However, our choice for *up* and *down* depends on how we set up our detector axis. For instance, had we pivoted the detector by 90°, one spin would have pointed to the right while the other left. In other words, the spin orientation is a property relative to the frame of reference rather than an absolute property of a particle, let alone a hidden variable defining the outcome of measurement already at the onset.

## The correlation concept

Experimentally, the correlation $E = (N_{xx} - N_{xy} - N_{yx} + N_{yy})/N$ is worked out from a large number of coincident counts $N = N_{xx} + N_{xy} + N_{yx} + N_{yy}$ of the diametrically opposite but randomly pointing spin pairs. For example, $N_{xy}$ denotes the number of correlated counts on the A detector's x-axis and the B detector's y-axis (Fig. 1). Since the angle $\phi$ between the spin and the detector axis varies randomly from pair to pair, classical correlation is calculated from the four combinations, i.e., the spins' projections on the detector axes, as an integral over all directions

$$E = \frac{1}{2\pi}\int_0^{2\pi} [\cos\phi\cos(\phi+\pi-\theta) - \cos\phi\sin(\phi+\pi-\theta) - \sin\phi\cos(\phi+\pi-\theta) + \sin\phi\sin(\phi+\pi-\theta)]\,d\phi$$
$$= \frac{-1}{2\pi}\int_0^{2\pi} [\cos\theta + \sin(\theta-2\phi)]\,d\phi = -\cos\theta$$

where $\theta$ is the angle between the corresponding axes of the two detectors. In the case of photons emerging pairwise from spontaneous parametric down-conversion (SPDC) with orthogonal polarizations, the cosine function shifts by $\pi/2$.[13] Accordingly, a polarization rotator placed on a photon path shifts the correlation by an angle of choice $\delta$.

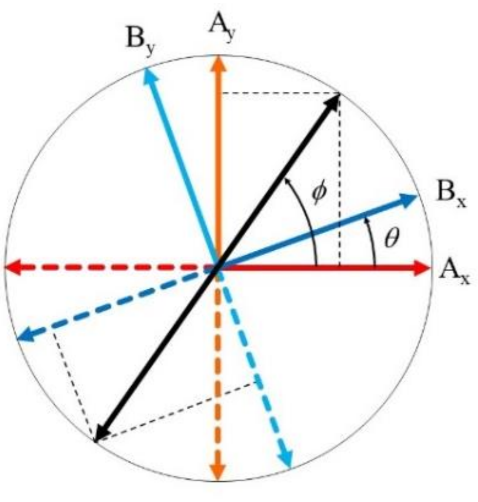


**Fig. 1.** Spins of two particles point in opposite directions (black arrows). Each particle is captured by a two-channel detector (A and B) whose axes are perpendicular ($A_x \perp A_y$ and $B_x \perp B_y$). The corresponding axes ($A_x$ & $B_x$ and $B_y$ & $A_y$) of the two detectors are at an angle $\theta$ relative to each other. Thus, the probability of one particle entering $A_x$ is $\cos\phi$ and $A_y$ $\sin\phi$, and that of the other particle with the opposite orientation $\phi+\pi$ entering $B_x$ is $\cos(\phi+\pi-\theta)$ and $B_y$ $\sin(\phi+\pi-\theta)$.

The above analysis shows that not only quantum but also classical covariance between vectors is the inner product $\mathbf{a}\cdot\mathbf{b} = |a||b|\cos\theta$ of the two detector axes, say, $a$ for one detector setting (Alice) and $b$ for the other (Bob), as has been pointed out.[12]

By classical physics, the paired spins, like two clocks running at the same rate but set off by 12 hours, are perfectly correlated without causal connection; checking one phasor does not determine but discloses the other in the chosen frame of reference. In other words, the clock faces are not without phasors but digits until the timezone is defined. (1)

It is worth emphasizing that the probability $P$ of a particle entering one of the two channels is proportional to the spin's projection on that detector axis. For example, for $\phi = 45°$, both $P_x$ and $P_y$ are $\cos(45°) \approx 0.71$. Although it is equally likely for the particle to take either one of the two options, $P_x = P_y \approx 0.71$, not 0.50. Conversely, for $\phi = 0°$, $P_x = \cos(0°) = 1$ and $P_y = 0$. Surely, each detector counts the particles with 100% efficiency irrespective of $\phi$, i.e., $\cos^2\phi + \sin^2\phi = 1$. Still, the probability of a particle going through one channel or the other depends on the spin's projection.

From the classical physics viewpoint, the particle enters one or the other phase-sensitive channel of a detector in the same way one would go through one or the other door opening on either side of a corner. For example, when viewed from the 45° angle, about 71%, not 50%, of each opening is visible, whereas when viewed straight ahead, one opening is fully visible (100%) while the other is not visible at all (0%). Thus, when one detector is rotated relative to the other, the phase-sensitive area open for particles to enter varies sinusoidally, not linearly. The fact that percentages do not add up to unity over all orientations is not an issue. After all, one would also go only through one opening, not all of them.

Quadrature detection of spinning and decohering nuclei is a matter of routine in correlation spectroscopy.[14] However, when the photon polarization instead of its phase $\phi$ is detected, $\phi$ is indistinguishable from $\phi \pm \pi$. Then the classical correlation varies at a double rate with half counts, i.e., $\frac{1}{4}(\cos 2\theta + 1) = \frac{1}{2}\cos^2\theta$. The same result follows from quantum mechanics.

The functional form of correlation between vectors is familiar from Malus' law: pivoting one polarizer about the other tells the angle between the two, whereas the photon energy, i.e., intensity proportional to $\cos^2\theta$, triggers the counter. In other words, one should not mistake the correlation coefficient, $r = \cos\theta$, denoting covariance in the coincident counts between the two detectors, for the expectation value, i.e., the coefficient of determination, $r^2$, defining variance in the counts of one detector that is predictable from the counts of the other.

**Discussion**

From the classical physics perspective, the spins of paired particles, just as the phases of paired photons, are diametrically opposite to each other ever since pairing at the breakup but unrelated to the detector axes until detected. Likewise, the SPDC photons have a fixed relative orientation of 90° ever since exiting a non-linear crystal but none relative to the detector axes until detected. The experiment's outcome is thus contingent on the detector phases, the frame of reference, so to say, the background. This self-evident fact is an essential point, for quantum mechanics is a background-dependent theory.

While the paired spins exist with definite directions relative to each other, the correlation across space and time is not sustained with any physical substance. Hence the correlation-representing wave function can, so to say, collapse instantaneously at the detection when the frame of reference is imposed. Thus, utmost care is taken to keep the paired spins or photons from decohering.[15]

Also, the macroscopic entanglement of two oscillators[16,17] can be understood as classically correlated oscillators. Eventually, the correlation is lost because of noise. After decohering, one oscillator is no longer in step with the other.

So, in hindsight, it seems somewhat of a contrived idea from Schrödinger to refer to correlated states as entangled, "the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought".[18]

In short, the common-sense comprehension about the correlation between vectors agrees with measurements. Even when far apart in space and time, the spins retain opposite phases unless perturbed. Thus, the phases are defined relative to each other but undefined against an external reference until detected.[19,20] Spooky or superluminal action is out of the question.

Perhaps there has been confusion about the correlation because particle orientations or photon polarizations are not detected directly; instead, the particles or photons that make it through the phase-sensitive analyzers to the receivers are counted. Even in the case probabilities are equal for the two analyzer channels, the probability of a particle, or a photon, entering one or the other channel is proportional to the spin or the phase projection, i.e., to the inner product of two vectors, and not to a scalar count.

Thus, as we show, the classical outcome for vectors could not but violate Bell's inequality. The correlation between vectors is not equal to the correlation between scalars. In other words, Bell's frame-independent definition of correlation does not pertain to vectorial quantities. Momentum, angular momentum, spin, and phase, have no meaning without any frame of reference, internal or external.[21,22] Thus, there are no grounds to reject the classical explanation of the observed correlation.

Of course, also conserved scalar quantities can be correlated. For example, net neutrality of pair production (of electron and positron) allows inferring the charge of one particle from that observed of the other. In that case, too, since the breakup, the pair of charges exists, but the result of a measurement, labeled as negative or positive, depends on the reference frame, i.e., convention.

In conclusion, the miscomprehension, dating back to the Einstein–Bohr debate, about entanglement as an exclusive quantum phenomenon is cleared. After all, classical physics makes perfect sense of the correlation between spins.

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