Model of Collision DART with Dimorphos

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Abstract – A digital model of the collision of DART with Dimorphos, the 'moon' of the asteroid Didymos, shows that the predicted long-term effect of the collision on the orbit of this 'moon' will not occur at all.

I Introduction

Reference [1] provides sufficient background information on the planned collision of satellite DART with the 'moon' Dimorphos to build a digital model of this collision and to judge the predicted orbits of that 'moon' after such a collision by the scientists concerned.

II Description of the applied model

The centre of Didydos (Did) has been fixed as a point mass in the origin of a coordinate system in which Dimorphos (Dim) its orbit is described by the coordinates d_x, d_y, the velocities v_x, v_y and the accelerations a_x, a_y. The distance between the objects is presented by r, so $r = \sqrt{(d_x^2 + d_y^2)}$.

The gravitational force between the objects is GMm/r^2, with M the mass of Did en m of Dim and G the gravitational constant. This force leads to an acceleration a of Dim, with $m \cdot a = GMm/r^2$. So $a = GM/r^2$, proving that the mass of the orbiting object does not play any role in the model, nor in reality!

Remark: If m has a velocity component v perpendicular to m-M the equation $m \cdot a = GMm/r^2$ represents the balance between centrifugal force $m \cdot v^2/r$ and centripetal force GMm/r^2, both applied to m.

The following calculations are carried out.
Initially m is positioned at $d_x = d_{x0}$, $d_y = d_{y0}$, resulting in: $a_{x0} = -a \cdot (d_{x0}/r)$ and $a_{y0} = -a \cdot (d_{y0}/r)$. The initial velocity components are v_{x0} , v_{y0}. Given $a(t) = GM/r^2(t)$, velocities and positions are now calculated as:

$$v_x(t) = v_{x0} + \int a_x(t)\,dt \qquad v_y(t) = v_{y0} + \int a_y(t)\,dt$$

$$d_x(t) = d_{x0} + \int v_x(t)\,dt \qquad d_y(t) = d_{y0} + \int v_y(t)\,dt$$

starting again with: $a_x(t) = -a(t) \cdot (d_x(t)/r(t))$ $\qquad$ $a_y(t) = -a(t) \cdot (d_y(t)/r(t))$

with which the calculation circle is completed and the orbit created. Most likely it is possible to find the mathematical expression for the orbit, because the result is always an ellipse, or a perfect circle in the most extreme situation. The author has chosen for a digital simulation. The background of the following calculations has been presented in the appendix.

$$v_x(nT) = v_x(nT-T) + \{a_x(nT)+a_x(nT-T)\} \cdot T/2, \quad v_y(nT) = v_y(nT-T) + \{a_y(nT)+a_y(nT-T)\} \cdot T/2$$

$$d_x(nT) = d_x(nT-T) + \{v_x(nT)+v_x(nT-T)\} \cdot T/2, \quad d_y(nT) = d_y(nT-T) + \{v_y(nT)+v_y(nT-T)\} \cdot T/2$$

At time nT = 0, $v_{x,y}(-T)$ and $d_{x,y}(-T)$ are equal to the respective mentioned initial values of these variables.

III Test of the applied model

III.1 Input parameters

The following values have been used, given in reference [1].

Mass of Didymos	$5.28 \cdot 10^{11}$	kg
Only for the calculation of the effect on the velocity of Dimorphos after collision:		
Mass of Dimorphos	$4.8 \cdot 10^9$	kg
Velocity of Dimorphos	0.17	m/s
Mass of DART	500	kg
Velocity of DART	6600	m/s

According to reference [2]:		
Semi-major axis orbit Dimorphos	1190 ± 300	m
Eccentricity	< 0.05	
Orbital period	11.93 ± 0.01	hr

Taking an eccentricity of 0.05 the semi-minor axis becomes 1189 m. For the test the orbit is assumed to be a perfect circle. Its radius is chosen to be 1181 m, in order to fulfil the criterion that the orbital period has to be 11.93 ± 0.01 hr, with this value of r being 11.933 hr / 42957.8 s.

III.2 Test

The initial test is aimed at the creation of an orbit as a perfect circle with r = 1181 m. The launch will be at the coordinates: $d_{x0} = -1181$ m, $d_{y0} = 0$. The initial velocity $v_{x0} = 0$. v_{y0} is calculated as follows.
In case of a perfect circle centrifugal force equals *continuously* gravitational force, so: $mv^2/r = GMm/r^2$, resulting in $v^2 = GM/r$.

Given the values above and $G = 6.7 \cdot 10^{-11}$ Nm^2kg^{-2}, v = 0,17274 m/s. This value is applied for v_{y0}, simulating a launch "at 9 o'clock" upwards.

Yet one parameter of which the value has to be determined is left: the sample time T.
A digital integration is an approximation of the analog integration (as happens in reality). The accuracy of a digital integrator can infinitely be increased, as long as the sample time can infinitely be decreased. Initially, the accuracy is sufficient when the calculated trajectory matches the launch coordinates so well that a difference is not visible in the corresponding graph. However such a criterion doesn't work in this situation, because the effect of the collision on the orbit is extremely small. The final criterion resulted in such a small sample time that the applied Excel program is just still operable.
The simulated orbit is shown in figure 1. The deviation at the position of the launch is 0.44 m towards the left on the x-axis. The orbital period is 42969.4 instead of the theoretical 42957.8 s.

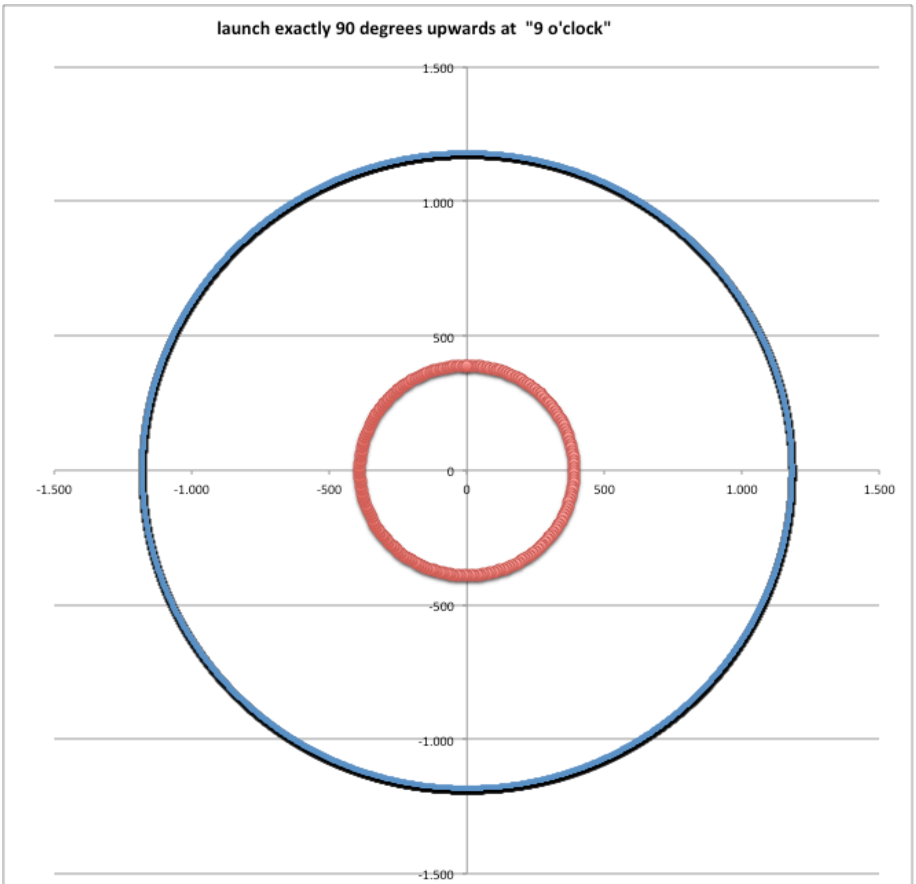

Figure 1. Orbit in case of a perfect circle

The other test is a launch at the same position and initial velocity directed to "northeast" (45°). The first impression of the graph is that such an orbit is impossible in reality. That is correct. However the mass of Didymos is simulated as a point mass. The following remarkable properties have been observed:

The orbit closes perfectly at the position of the launch, at least in the graph.
The direction of the major axis of the ellipse equals the launch direction.

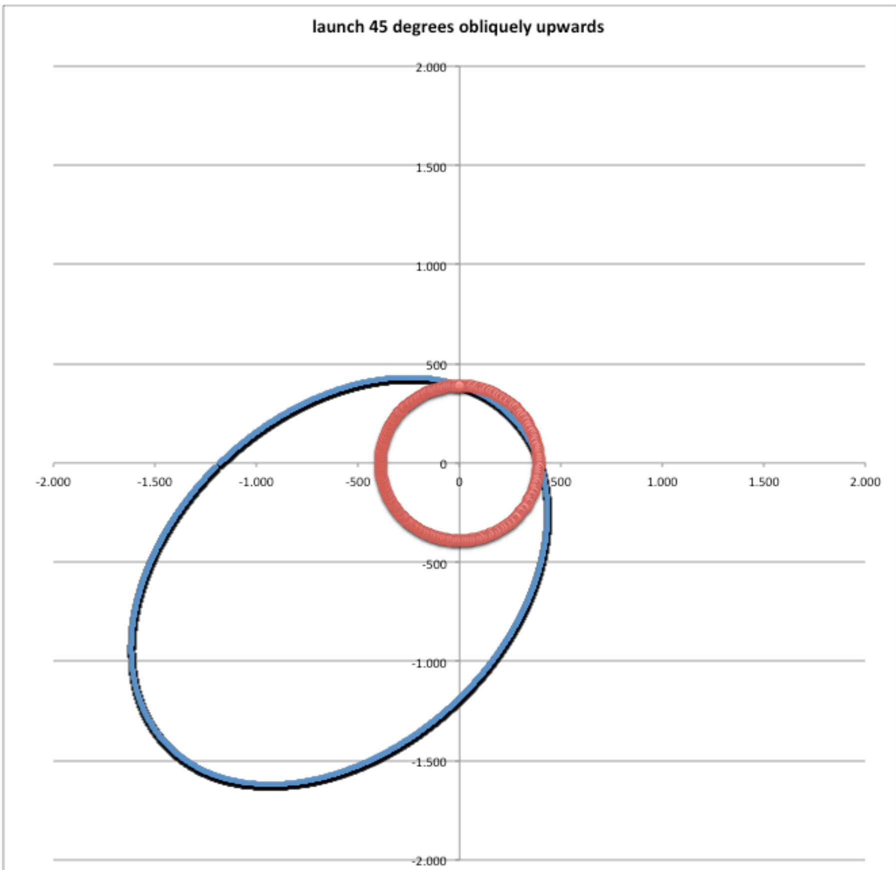

Figure 2 Orbit in case of a pronounced ellipse

IV Repeatability of the circular orbit

Reference [2] says, under "Mission/impact":

> "It is estimated that the impact of the 500 kg DART at 6.6 km/s will produce a velocity change on the order of 0.4 mm/s, which leads to a small change in trajectory of the asteroid system, *but over time, it leads to a large shift of path. Over a span of years, the cumulative trajectory change from such a small change in velocity could mitigate the risk of a hypothetical Earth-bound asteroid hitting Earth. The impact will target the center of figure of Dimorphos and should decrease the orbital period, currently 11.92 hours, by roughly 10 minutes."*

The text, italicized by the author, speaks clearly: the prediction is that the orbit period will increase for many years, in fact meaning forever, after collision.

It will be demonstrated that the results of the model have most likely been misleading for the scientists concerned with regard to the long-term effect of the collision.

As mentioned above, the digital integrators applied in this model have a restricted accuracy that in principle can be controlled by the value of the sample time. Up to now a sample time of 0.2 sec. has been used, leading the ~40000/0.2 = 200 thousand rows in the Excel sheet.

The inaccuracy of this configuration, due to the sample time, will be shown in the following experiment. After each completed orbit the values of the variables, at the time the orbit most closely approximates the start of the previous orbit, are copied and used as the initial values for the next orbit. The reason to apply this method is that an Excel sheet of significantly more than 200000 rows is not operable anymore.

The criterion applied for the best fit of the initial variables is concentrated only on the value of d_y. The moment this value is most close to zero, the values at this moment are copied. Two variables, at the end of each orbit, are chosen to present the accuracy: d_x and the orbital period O_p.
In the two tables below the acceleration components are taken out, in order to make the tables readable. Table I proves that the two deviations ΔO_P and Δd_x are always "about exactly" the same and cause a series of increasing spiral orbits. Forever!

				T_s	0.2			
orbit	r	v_x	v_y	d_x	d_y	O_p	ΔO_p	Δd_x
0	1181,00	1,058E-17	1,727E-01	-1181,00	0,0000	42957,8	theoretical	
1	1181,43	-1,170E-06	1,727E-01	-1181,43	-0,0081	42969,4	11,6	-0,43
2	1181,87	5,118E-07	1,727E-01	-1181,87	0,0034	42993,2	23,8	-0,43
3	1182,30	-3,713E-09	1,726E-01	-1182,30	-0,0002	43016,8	23,6	-0,43
4	1182,74	2,327E-06	1,726E-01	-1182,74	0,0157	43040,6	23,8	-0,43
5	1183,17	2,463E-06	1,726E-01	-1183,17	0,0166	43064,2	23,6	-0,43
6	1183,60	4,087E-07	1,725E-01	-1183,60	0,0025	43087,8	23,6	-0,43
7	1184,04	1,197E-06	1,725E-01	-1184,04	0,0078	43111,6	23,8	-0,43

Table I

Table II proves that if the sample time is made two times as large, the variables ΔO_P and Δd_x also become two times as large. The logical conclusion is that for a zero sample time, which represents the real situation, these deviations will also be zero.

				T_s	0.4			
orbit	r	vx	vy	dx	dy	Op	ΔOp	Δdx
0	1181,00	1,058E-17	1,727E-01	-1181,00	0,0E+00	42957,8	theoretical	
1	1181,87	2,699E-06	1,727E-01	-1181,87	1,8E-02	42981	23,4	-0,87
2	1182,74	1,005E-06	1,726E-01	-1182,74	6,4E-03	43029	47,6	-0,87
3	1183,60	4,997E-06	1,725E-01	-1183,60	3,4E-02	43076	47,2	-0,87
4	1184,47	4,605E-06	1,725E-01	-1184,47	3,1E-02	43124	47,6	-0,87
5	1185,34	-1,524E-07	1,724E-01	-1185,34	-2,2E-03	43170	46,8	-0,87
6	1186,20	7,614E-07	1,724E-01	-1186,20	3,9E-03	43218	48,0	-0,87
7	1187,07	-2,681E-06	1,723E-01	-1187,07	-2,0E-02	43265	46,8	-0,87
8	1187,94	-4,733E-07	1,722E-01	-1187,94	-5,1E-03	43313	48,0	-0,87

Table II

The results thus clearly show that the ‘large shift’ in: “*....over time, it leads to a large shift of path”* is only caused by the inaccuracy of the applied digital integrators. In whatever digital model!

V The impact of the collision on the velocity of Dimorphos.

At collision the kinetic energy of DART is $1.1 \cdot 10^{10}$ J and of Dimorphos $7.2 \cdot 10^{7}$ J.
The momenta of these objects are: $M_1 = 3.3 \cdot 10^6$ kg·m/s, resp. $M_2 = 8.8 \cdot 10^8$ kg·m/s
These values show that the impact of the collision on the velocity of Dimorphos can certainly not be calculated using de variable ‘kinetic energy’.
Taking the momenta such an influence can mathematically be expressed by: $m_2 \cdot v_2 - m_1 \cdot v_1 = m_2 \cdot v_2'$, with v_2' Dimorphos’ velocity after the collision. This leads to: $v_2' = v_2 - m_1/m_2 \cdot v_1$.
For $m_1 = 500$ kg, $m_2 = 4.8 \cdot 10^9$ kg, $v_1 = 6600$ m/s and $v_2 = 0.17274$ m/s, the result is: $v_2' = 0.17205$ m/s, so $\Delta v_2 = -0.0007$ m/s = -0.7 mm/s.

The fact that reference [2] presents -0.4 mm/s can most likely be explained by the argumentation that DART is destroyed during the collision. So effectively its mass reduces during the collision. Given the result, apparently half of DART’s original mass has been taken in such a calculation.

VI Modelling of the collision

If the launch direction would be chosen 89.97° an ellipse is created roughly like the present orbit of Dimorphos: a semi-minor axis being ~1.7 m shorter than the semi-major axis.

As has been shown above the orientation of the major axis of the ellipse coincides with the launch direction. In this case therefore almost equal to the y-axis.

The collision is simulated by starting with this direction of the velocity, decreased by 0.4 mm/s. It causes the following changes:

v_{mean}:	0.1727 m/s	–>	v_{mean}:	0.1723 m/s
orbital period:	42969 s	–>	orbital period:	42673 s
orbital length:	7421 m	–>	orbital length:	7353 m

Conclusions

The modelling of the collision of DART with Dimorphos has learned that a digitally simulated orbit has the following three remarkable properties:

1 Its mean velocity equals the initial velocity.

2 Its shape and orientation is predetermined, based on the starting position and based on the start speed, in terms of amplitude and direction.

3 The orbit will, after launch, end at the initial position with an accuracy depending on the applied sample time, but with an infinite accuracy in reality.

The orbital period of Dimorphos decreases after collision from 42969 to 42673 s, being ~5 minutes shorter. This decrease will be maintained forever. The predicted 10 minutes, *after many years*, is not correct, given the above mentioned properties.

References

[1] https://en.wikipedia.org/wiki/Double_Asteroid_Redirection_Test

[2] https://en.wikipedia.org/wiki/Dimorphos

Appendix Formula for digital integration

If F(s) is the Laplace transformation of f(t), then F(s)/s is that of ∫f(t)dt and e^{-sT} that of f(t-T).
The expression e^{-sT} F(s) thus can be written as $z^{-1}\cdot F(z)$ by applying $z^{-1} = e^{-sT}$, with F(z) the sequence of (digital) numbers representing f(t) at time intervals T. T is the so-called sample time.

The function e^{-sT} can be approximated by 1-sT or by (1-sT/2)/(1+sT/2). The second one is more accurate.

Given the relation $z^{-1} \approx (1-sT/2)/(1+sT/2)$, it follows that $s \approx 2(1-z^{-1}) / (1+z^{-1})$ T.

Applying this approximation of s in the Laplace transformation G(s) = F(s)/s results in the digital integrator $G(z) = F(z) \cdot (1+z^{-1})\, T / 2(1-z^{-1})$, leading to:

$2(G-Gz^{-1}) = (f+fz^{-1})\,T$ –> $G = Gz^{-1} + (f + fz^{-1}) \cdot T/2$ –> $G(nT) = G(nT-T) + \{f(nT) + f(nT-T)\} \cdot T/2$

So in the model the following expressions have been programmed:

$v_x(nT) = v_x(nT-T) + \{a_x(nT)+a_x(nT-T)\} \cdot T/2$, $v_y(nT) = v_y(nT-T) + \{a_y(nT)+a_y(nT-T)\} \cdot T/2$

$d_x(nT) = d_x(nT-T) + \{v_x(nT)+v_x(nT-T)\} \cdot T/2$, $d_y(nT) = d_y(nT-T) + \{v_y(nT)+v_y(nT-T)\} \cdot T/2$

At time nT = 0 the initial values: $v_x(-T) = v_{x0}$, $v_y(-T) = v_{x0}$, $d_x(-T) = d_{x0}$, $d_y(-T) = d_{y0}$ are used.