On entanglement swapping and teleportation with local hidden variables

Eugen Muchowski [1]

[1] Independent researcher, formerly University Karlsruhe and UC Berkely, now retired, Vaterstetten, Germany, eugen@muchowski.de

Keywords: Bell's theorem, entanglement swapping, teleportation, hidden variables, EPR

Abstract

A model with local hidden variables is presented, which describes phenomena such as entanglement swapping and teleportation and also reproduces the quantum mechanical expectation values for the measurement of entangled photons. It refutes Bell's theorem and at the same time extends our physical understanding of entangled states, since it can also explain the phenomena mentioned above. The manuscript is thus a step forward towards a complete theory describing quantum physical reality as thought possible by Einstein, Podolsky and Rosen. According to the model presented, the quantum mechanical correlations on entangled photons can be explained only if one assumes that the polarization states of photons are already defined before a measurement and do not arise during the measurement. Therefore, there are reasonable doubts that photons can exist in several physical states at the same time.

Introduction

Entanglement swapping is a protocol that allows particles that were not previously in contact to become entangled. This entanglement can be accomplished using a Bell state measurement [1]. Quite a few physicists are convinced that this process is non-local. This conviction is ultimately based on the assumption of the validity of Bell's theorem [2]. It states that quantum mechanics cannot be local because it cannot be described by local realistic models with hidden variables. A detailed description of the literature and arguments around Bell's theorem can be found in [3]. Not long ago, Bell's theorem has been refuted by a local contextual model with hidden variables [4] that correctly predicts the quantum mechanical expectation values with polarization entangled particles. This model is based on the fact that the two members of an entangled pair are connected by a common hidden parameter.

However, assuming a common value of a hidden parameter for the members of an entangled pair, as also proposed by Bell [5], cannot explain phenomena such as entanglement swapping and teleportation [6-9]. When photons that did not interact before become entangled by entanglement swapping they cannot have a common parameter with a statistical distribution.

With the model [4], Bell's theorem was formally refuted by presenting a locally realistic model that correctly reproduces the quantum mechanical expectation values of entangled particles. However, it remains unsatisfactory that well-known phenomena such as entanglement swapping and teleportation cannot be explained in this way. The added value of the model presented in this paper is that this shortcoming has been addressed. With a model that not only predicts the quantum mechanical correlations with entangled photons, but also explains teleportation and entanglement swapping, the understanding of the physical correlations also grows. To circumvent the difficulties mentioned above, we introduce here a model in which the indistinguishability of the entangled photons explains the physical states, as in [4], but in which the photon pairs do not share the value of a statistical parameter. Instead of a common statistical parameter for both partners in a pair, a weaker coupling is introduced. Then the question arises how the photons on side B get information about the position of the polarizer on side A without communication. This information comes firstly from the mixing ratio of the horizontally and vertically polarized photons from the constituent initial states (see model assumption MA2), which contribute to a selection, and secondly from the initial conditions (see model assumption MA3), which couple both side.

The reader is assumed to be familiar with the Bell states *Ψ+*, Φ-, *Φ+ and Φ-* the definition of which can be found in the literature [1]. For each Bell state, we will show what a selection of photons by a polarizer on one side means for the state on the other side. The resulting states are listed in Table 1. From this, expectation values for correlation measurements on entangled photons and the states for entanglement swapping and teleportation are derived.

A new model for polarization entangled photons with weakly coupled hidden variables

Model overview

In polarization measurements, photons can choose one of two perpendicular exits of the polarizer. A model with hidden variable must describe which of these two possible exits a photon will take. Four model assumptions are introduced, which are outlined and then described in italics:

MA1 introduces the statistical parameter λ which controls the polarizer exit that a photon will take. This model assumption is the same as MA1 in [4].

MA2 describes the polarization of a selection of photons from an entangled pair. This is a new model assumption.

MA3 describes how photons from the entangled pairs are coupled together. This is a new model assumption.

MA4 states that photons carry the complete set of the hidden variable after a measurement. This model assumption is the same as MA4 in [4].

Figure 1 shows the coordinate systems and nomenclature of the experiments with polarization entangled photons.

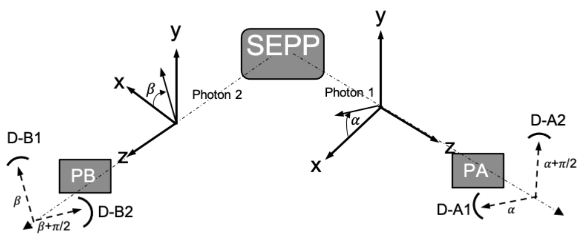

Figure 1: The SEPP (source of entangled photon pairs) emits entangled photons propagating towards the adjustable polarizers PA and PB and detectors DA-1 and DA-2 on wing A and DB-1 and DB-2 on wing B. A coincidence measuring device (not seen in the picture) encounters matching events. The polarization angles are defined in the x–y-plane, which is perpendicular to the propagation direction of the photons. The coordinate systems are left-handed with the z-axis in propagation direction for each wing, with the x-axis in horizontal and the y-axis in vertical direction.

Model assumptions

Model assumption MA1: *The statistical parameter λ, uniformly distributed between 0 and 1, controls which of the two polarizer exits the photon will take. Given the polarizer setting α and the photon polarization φ we define $\delta = \alpha - \varphi$ as the difference between the polarizer setting and the polarization of the photon. The function $A(\delta,\lambda)$ indicates which polarizer exit the photon will take.*

$A(\delta,\lambda)$ can have values +1 and -1. For $0 \le \delta < \pi/2$, we define
$A(\delta,\lambda) = +1$ for $0 \le \lambda \le \cos^2(\delta)$, (1)
meaning the photon takes polarizer exit α and
$A(\delta,\lambda) = -1$ for $\cos^2(\delta) < \lambda \le 1$, (2)
meaning the photon takes polarizer exit $\alpha + \pi/2$. MA1 is valid for single photons as well as for each wing of entangled photons.

The case $\pi/2 \le \delta < \pi$ is covered referring to the other exit of the polarizer. Then equation (2) applies and the range of values of λ for positive results is $\cos^2(\delta) < \lambda \le 1$. The case $\delta < 0$ is covered by reversing the polarizer direction by 180°. Thus, $-\pi \le \delta < -\pi/2$ is equivalent to $0 \le \delta < \pi/2$ and $-\pi/2 \le \delta < 0$ is equivalent to $\pi/2 \le \delta < \pi$.

Thus $A(\delta,\lambda) = +1$

1. for $0 \le \delta < \pi/2$ and $0 \le \lambda \le \cos^2(\delta)$, (3.1)
2. for $-\pi/2 \le \delta < 0$ and $\cos^2(\delta) < \lambda \le 1$, (3.2)
3. for $\pi/2 \le \delta < \pi$ and $\cos^2(\delta) < \lambda \le 1$, (3.3)
4. for $-\pi \le \delta < -\pi/2$, and $0 \le \lambda \le \cos^2(\delta)$ and (3.4)

$A(\delta,\lambda) = -1$ otherwise. (3.5)

Model assumption MA2: *If the fractions of horizontally and vertically polarized photons from the entangled state contributing to a photon stream selected by a polarizer are $\cos^2(\alpha)$ and $\sin^2(\alpha)$ respectively, obtain a common polarization of α or $-\alpha$, because of the indistinguishability of the photons.*

The fractions of horizontally and vertically polarized photons which leave a polarizer exit α are $\cos^2(\alpha)$ and $\sin^2(\alpha)$ respectively. This makes up the common polarization. A selection comprises all photons which take the same polarizer exit. Photons with polarization α and $\alpha+\pi/2$ come in equal shares, due to symmetry. MA2 accounts for the fact that the polarization of photons from the entangled state is undefined due to their indistinguishability, but is changed and re-defined by the entanglement. Thus, the photons of a selection cannot be distinguished by their polarization. This argument was already made in [4] but only for photon pairs with common hidden variables. MA2 is a contextual assumption, as the polarization of a selection coincides with the setting of a polarizer. However, it is a local realistic assumption, since it assigns a real value to the physical quantity polarization. MA2 leaves open whether the polarization of a selection is positive or negative. To distinguish this, we define as the initial conditions

Model assumption MA3: *Each Bell state is a mixture of indistinguishable constituent photon pairs in equal shares whose components have the same polarization 0° or 90° for Φ+ and Φ- and an offset of $\pi/2$ for Ψ+ and Ψ-. The constituent photon pairs make up the initial state.*

The coupling of a selection on wing A with polarization α and the corresponding selection of the partner photons on wing B with polarization β is a relation between the normalized signs of the polarizations on both sides and is given
for Ψ+ and Φ+ as $\mathrm{sign}'(\alpha)_1 = \mathrm{sign}'(\beta)_2$, and (4.1)
for Ψ- and Φ- as $\mathrm{sign}'(\alpha)_1 = -\mathrm{sign}'(\beta)_2$, (4.2)
where the normalized sign' is given by
$\mathrm{sign}'(\alpha) = \mathrm{sign}(\alpha)$ for $-\pi/2 \le \alpha \le \pi/2$ and
$\mathrm{sign}'(\alpha) = \mathrm{sign}(\alpha-\pi)$ for $\pi/2 \le \alpha \le 3\pi/2$ and
$\mathrm{sign}'(\alpha) = \mathrm{sign}(\alpha-2\pi)$ for $3\pi/2 \le \alpha \le 2\pi$, (5)
with α and $\alpha-\pi$ denoting the same polarization and the suffixes 1 and 2 denote the sides.

From equation (5) we obtain

$$\mathrm{sign}'(\alpha) = -\mathrm{sign}'(\alpha+\pi/2) = \mathrm{sign}'(-\alpha-\pi/2). \quad (6)$$

Model assumption MA4: *Photons having left a polarizer exit α have polarization α with λ evenly distributed in the range $0 \leq \lambda \leq 1$.*

MA4 emphasizes the fact that photons carry the full set of hidden variables after leaving a polarizer.

Predicting measurement results for single photons

Using equations (3.1 or 3.4), a photon with polarization φ is found behind the exit α of a polarizer with probability

$$P_\delta = \int_0^{\cos^2(\delta)} d\lambda = \cos^2(\delta), \qquad (7)$$

where $\delta = \alpha - \varphi$ with $0 \leq \delta < \pi/2$ or $0 \leq \lambda \leq \cos^2(\delta)$.

With equations (3.2 or 3.3) for $-\pi/2 \leq \delta < 0$ or $\pi/2 \leq \delta < \pi$ we refer to the other exit of the polarizer and have, with $\vartheta^* = \delta - \pi/2$

$$P_\delta = \int_{\cos^2(\delta*)}^{1} d\lambda = 1 - \cos^2(\vartheta^*) = \cos^2(\vartheta), \text{ as well.} \qquad (8)$$

With $\delta = \alpha - \varphi$ we obtain the same P_δ for a photon in state $\cos(\varphi)*|H\rangle + \sin(\varphi)*|V\rangle$ by projection onto $\cos(\alpha)*\langle H| + \sin(\alpha)*\langle V|$, according to QM (i.e., Born's rule).

Conclusions from the model assumptions

MA2 has the consequence that the selection by a polarizer in position α on one side corresponds to a selection with polarization $\alpha+\pi/2$ or $-\alpha-\pi/2$ on the other side. (for $\Psi+$ or $\Psi-$) This can be seen from the following consideration: According to equations (7,8) a polarizer PA set to α selects a fraction of $\cos^2(\alpha)$ of horizontally polarized photons 1 and a fraction of $\sin^2(\alpha)$ of vertically polarized photons 1. This means that partner photons 2 are also selected, but with perpendicular polarization, resulting in a selected fraction of $\cos^2(\alpha) = \sin^2(\alpha+\pi/2)$ of vertically polarized photons 2 and a selected fraction of $\sin^2(\alpha) = \cos^2(\alpha+\pi/2)$ of horizontally polarized photons 2. Due to MA2 the polarization of the selected photons 2 is $\alpha+\pi/2$ or $-\alpha-\pi/2$.

From equations (4.1) and (6) we get for $\Psi+$ the polarization $-\alpha-\pi/2$ of the partner photon 2 with the same normalized sign as that of the polarization α. For $\Psi-$ we get the polarization $\alpha+\pi/2$ of the partner photon 2 with an opposite normalized sign of the polarization α in accordance with equation (4.2).

For $\Phi+$ and $\Phi-$ we obtain that the selection by a polarizer in position α on one side corresponds to a selection with polarization α or $-\alpha$ on the other side. Again a polarizer PA set to α selects a fraction of $\cos^2(\alpha)$ of horizontally polarized photons 1 and a fraction of $\sin^2(\alpha)$ of vertically polarized photons 1. This means that partner photons 2 are also selected, but in this case with the same polarization, resulting in a selected fraction of $\cos^2(\alpha)$ of horizontally polarized photons 2 and a selected fraction of $\sin^2(\alpha)$ of vertically polarized photons 2. Due to MA2 the polarization of the selected photons 2 is α or $-\alpha$.

According to equation (4.1) we get the polarization of the partner photons 2 of α for $\Phi+$ as $\text{sign}'(\alpha)_1 = \text{sign}'(\alpha)_2$ and of $-\alpha$ for $\Phi-$ as $\text{sign}'(\alpha)_1 = -\text{sign}'(-\alpha)_2$ in accordance with equation (4.2). The results are summarized in Table 1.

Bell state	A	B
Ψ-:	α	α + π/2
Φ+	α	α
Ψ+	α	−α −π/2
Φ-	α	−α

Table 1: polarization of partner photons 2 at wing B for different Bell states for a selection of photons 1 with a polarizer set to α at wing A.

Calculating expectation values for photons in singlet state

We have seen above that all selected photons 1 from the singlet state which take PA exit α have polarization α while their partner photons 2 have polarization α+π/2. Seen from the other side we can conclude that if photon 2 leaves polarizer PB at β we have matching events if those photons 2 with polarization β would leave PB with an assumed setting of α+π/2. Note, that λ is evenly distributed in the value range $0 \le \lambda \le 1$ for the photons 2 with polarization β. This can be seen by looking at the initial states and applying equations (3.1)- (3.4) to the horizontally polarized photons and to the vertically polarized photons. For example, the horizontally polarized photons with $0 \le \lambda \le \cos^2(\delta)$ and the vertically polarized photons with $\cos^2(\delta) < \lambda \le 1$ contribute to a selection with polarization β for $0 \le \delta < \pi/2$.

Thus, the probability that photons 2 with polarization β would pass PB at α + π/2 can be obtained by equations (7) and (8), using δ = α+π/2 - β thus yielding

$$P_\delta = \cos^2(\delta) = \cos^2(\alpha+\pi/2 - \beta) = \sin^2(\alpha-\beta), \tag{9}$$

where δ is the angle between the PB polarizer setting β and the polarization α + π/2 of photons 2 selected by PA.

The expectation value for a joint measurement with photon 1 detected behind detector PA at α and partner photon 2 detected behind detector PB at β is as obtained from [4]

$$E(\alpha,\beta) = \Sigma A^*B^*P_{A,\alpha}{}^*P_{B,\beta|A,\alpha} \quad \text{for } A,B=+1,-1. \tag{10}$$

Here $P_{A,\alpha}$ is the unconditional probability to detect (A = 1) or not detect (A = -1) photon 1 at α and $P_{B,\beta|A,\alpha}$ is the conditional probability for photon 2 to have the outcome B (+1 for passing or -1 for not passing) at PB set to β if photon 1 has outcome A (+1 for passing or -1 for not passing) at PA set to α. With P_δ from equation (9), we get, in particular, that:

$$P_{+1,\beta|+1,\alpha} = P_{-1,\beta|-1,\alpha} = P_\delta \text{ and}$$
$$P_{+1,\beta|-1,\alpha} = P_{-1,\beta|+1,\alpha} = 1 - P_\delta. \tag{11}$$

The constituent photons of the singlet state with polarization 0° and 90° contribute to the probability $P_{1,\alpha}$ to find a photon 1 at α according to equation (7,8), with fractions ½cos²(α) and ½cos²(α-π/2) respectively, taking into account that the photons of 0° and 90° contribute in equal shares to the total stream of photons on either wing. Thus,

$$P_{1,\alpha} = ½(\cos^2(\alpha) + \cos^2(\alpha-\pi/2)) = ½ = P_{-1,\alpha}. \tag{12}$$

With the above definitions, we get, from equations (9)–(12):

$$E(\alpha,\beta) = ½(1^*\, P_\delta -1^*(1- P_\delta) -1^*(1- P_\delta) +1^*\, P_\delta) =$$
$$= P_\delta - (1- P_\delta) = \sin^2(\alpha-\beta) - \cos^2(\alpha-\beta) = -\cos(2(\alpha-\beta)), \tag{13}$$

in accordance with QM. As the expectation value E(α,β) in equation (13) exactly matches the predictions of quantum physics, it also violates Bell's inequality.

Applying the model to entanglement swapping

Entanglement swapping uses a protocol in which two wings of different systems, each in singlet state, become entangled by a Bell state measurement of the two remaining wings[1,2,8]. Let AB and CD be the two initial systems in singlet state. Then we define the outer pair AD and the inner pair BC. With a Bell state measurement between B and C we want to entangle A and D. However, this coupling is random in the case of entanglement swapping.Therefore four resulting Bell states are possible. How are these results of the inner pair BC related to the state of the outer pair AD? This is found out by applying table 1 to the pairs of channels. AB and CD are always in state Ψ-. BC is obtained by the Bell state measurement. So we get the results of Table 2. Comparing with Table 1 we see the Bell state of the outer pair AD is equal to the measured Bell state of the inner pair BC according to QM [8]. Note that $\alpha + \pi$ and α are the same polarization.

Bell state BC	A	B	C	D
Ψ-:	α	$\alpha + \pi/2$	$\alpha\ (+\pi)$	$\alpha + \pi/2$
Φ+	α	$\alpha + \pi/2$	$\alpha + \pi/2$	$\alpha\ (+\pi)$
Ψ+	α	$\alpha + \pi/2$	$-\alpha\ (-\pi)$	$-\alpha - \pi/2$
Φ-	α	$\alpha + \pi/2$	$-\alpha - \pi/2$	$-\alpha$

Table 2: polarization of the photons of wings B,C and A,D for different Bell states obtained between B and C by applying table 1 with an assumed selection of photons by a polarizer set to α at wing A.

Applying the model to teleportation

Teleportation uses a protocol in which an unknown state β is transferred to another wing B of a singlet state by a Bell state measurement between the unknown β and wing A of the singlet state [9]. Using MA3 and Table 1 we obtain the polarizations at wing A and B. AB are always in the Ψ- state. The polarization of βA is obtained by measuring the Bell state. Thus, we obtain the results of Table 3. The results at B can be converted to state β by simply rotation or mirroring. This result is in accordance with the quantum mechanical calculations [9]. Note that $\beta + \pi$ and β are the same polarization.

Bell state βA	A	B
Ψ-	$\beta + \pi/2$	$\beta\ (+\pi)$
Φ+	β	$\beta + \pi/2$
Ψ+	$-\beta - \pi/2$	$-\beta$
Φ-	$-\beta$	$-\beta + \pi/2$

Table 3: polarization of the photons of wings A and B for different Bell states obtained between the unknown β and wing A.

Results, discussion and conclusions

The model presented here is based on the selection of the photons by a polarizer (on one side of a photon pair in a Bell state). Due to their indistinguishability, the selected photons have a common polarization which depends on the mixing ratio of the constituent horizontally or vertically polarized components. This ratio is the same or inverse on both sides depending on the Bell state. Accordingly, there is a fixed connection between the polarization of a selection on one side and the corresponding polarization of the partner photons on the other side. These polarizations exist before any measurement.

Bell states were conceived in the model as mixtures of indistinguishable pairs of photons. Since Bell states are entangled they are as such not separable. This means that the individual photons of an entangled photon pair have no physical reality. The physical reason for this is the fact of indistinguishability. Only when photon pairs from a Bell state are selected by a polarizer, a Bell state splits into two systems of photon pairs with mutually perpendicular polarizations. A selection corresponds to a projection in quantum mechanics. By projecting the rotationally symmetric singlet state onto a direction on one side, the state on the other side is also fixed. In the model, this corresponds to the effect of a selection. The problem with quantum physics up to now was that the change of state on the opposite side was considered as a non-local interaction. This was suggested by Bell's theorem but refuted in [4] and by the model presented. Since a selection on one side implies a corresponding selection on the other side, there is no action associated with a selection.

The question, already discussed in [4], whether a photon can be in different physical states at the same time, must also be answered negative from the point of view of this manuscript. The model presented here is also based on the fact that the physical state of the photon is fixed before a measurement. Otherwise, correlation of the measurement results could be explained only by spooky action at a distance. However, after the refutation of Bell's theorem, there is no evidence for this.

References

1] Pan, J. W., Bouwmeester, D., Weinfurter, H., & Zeilinger, A. (1998). Experimental entanglement swapping: entangling photons that never interacted. Physical review letters, 80(18), 3891.

[2] Price, H., Wharton, K., Entanglement Swapping and Action at a Distance. Found Phys 51, 105 (2021)

[3] Wayne, M., Genovese, M. and Shimony, A., "Bell's Theorem". *The Stanford Encyclopedia of Philosophy* (Fall 2021 Edition)

[4] Muchowski, E., On a contextual model refuting Bell's theorem. EPL 2021 134 (10004)

[5] Bell, J.S., On The Einstein Podolsky Rosen Paradox. Physics (Long Island City N.Y.), 1964, 1, 195

[6] Kwek, LC., Cao, L., Luo, W. et al., Chip-based quantum key distribution. AAPPS Bull. 31, 15 (2021). https://doi.org/10.1007/s43673-021-00017-0

[7] Azuma, K., Bäuml S., Coopmans, T. et al., Tools for quantum network design. AVS Quantum Sci. 3, 014101

[8] Marshall, K. and Weedbrook, C., Continuous-Variable Entanglement Swapping. Entropy 2015, 17(5), 3152-3159 (2021);

[9] Bennett, Charles H., et al. "Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels." Physical review letters 70.13 (1993): 1895.

[10] Einstein A., Podolsky B., Rosen N., Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? Phys. Rev., 1935, 47, 777