On entanglement swapping and teleportation with local hidden variables

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Abstract – A model with local hidden variables is presented, which describes phenomena such as entanglement swapping and teleportation and also reproduces the quantum mechanical expectation values for the measurement of entangled photons. It refutes Bell's theorem and at the same time expands our physical understanding of entangled states since it can also explain the phenomena mentioned above.

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1. Introduction – Entanglement swapping is a protocol that allows particles that have not previously been in contact to become entangled with each other. This entanglement can be done, for example, by means of a Bell state measurement [1]. Quite a few physicists are convinced that this process is not local. Ultimately, this belief is based on the assumption of the validity of Bell's theorem [2]. It states that quantum mechanics cannot be local, since it cannot be described by local realistic models with hidden variables. For a detailed description of the literature and arguments around Bell's theorem see [3]. Recently Bell's theorem has been refuted by a local contextual model with hidden variables [4] which correctly predicts the quantum mechanical expectation values with polarization entangled particles. In this model it was assumed that upon the generation of an entangled photon pair in the source each of its two partner particles gets the same value of the parameter λ. According to this model, measured values depend on the polarization of a particle and the value of the parameter λ. With the same rules acting upon the generated particles in both wings, the measurement results are correlated without nonlocal effects. Because Bell based his argument for possible models in his paper [5] solely on the prediction of expectation values the model [4] is still a valid refutation of Bell's theorem.

However, phenomena such as entanglement swapping and teleportation [6-9] cannot be explained by assuming a common value of a hidden parameter for the members of an entangled pair, as suggested by Bell [5]. When photons which have not previously interacted become entangled by swapping they cannot have a common parameter with a statistical distribution.

With a model that not only predicts the quantum mechanical correlations with entangled photons, but also explains teleportation and entanglement swapping, the understanding of the physical relationships also grows. To circumvent the difficulties mentioned above, we introduce here a model in which the indistinguishability of the entangled photons explains the physical states, as in [4], but in which the photon pairs do not share the value of a statistical parameter. Instead of a common statistical parameter for both partners in a pair, a weaker coupling is introduced. The value range of the statistical parameter also had to be changed for this. Then the question arises how the photons on side B get information about the polarizer position on side A without communication. This information results on the one hand from the mixing ratio of the horizontally and vertically polarized photons from the constituent initial states (see model assumption MA2), which contribute to a selection, and on the other hand from an additional connection between the two sides (see model assumption MA3), which, however, occurs to be random in the case of entanglement swapping and teleportation and therefore leads to four different end states.

The reader is assumed to be familiar with the Bell states Ψ+, Φ-, Φ+ *and* Φ- the definition of which can be found in the literature [1]. It will be shown for each Bell state what a selection of photons by a polarizer on one side means for the state on the other side. The resulting states are listed in table 1. Expectation values for correlation measurements on entangled photons and the states for entanglement swapping and teleportation are then derived from this.

2. A new model for polarization entangled photons with weakly coupled hidden variables

2.1 Model overview

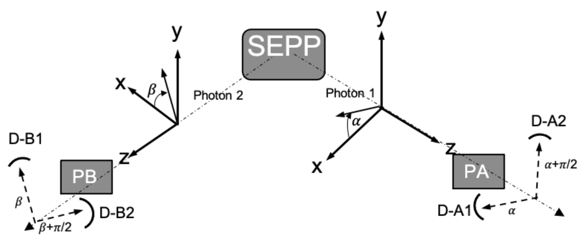

Figure 1: The SEPP (source of entangled photon pairs) emits entangled photons propagating towards the adjustable polarizers PA and PB and detectors DA-1 and DA-2 on wing A and DB-1 and DB-2 on wing B. A coincidence measuring device (not seen in the picture) encounters matching events. The

polarization angles are defined in the x–y-plane, which is perpendicular to the propagation direction of the photons. The coordinate systems are left-handed with the z-axis in propagation direction for each wing, with the x-axis in horizontal and the y-axis in vertical direction.

With polarization measurements, photons can choose one of two perpendicular polarizer exits. A hidden variable model has to describe which of these two possible exits a photon will take. Four model assumptions are introduced, an overview of which is given below with a description afterwards in italic letters:

MA1 introduces the statistical parameter λ which controls the polarizer exit that a photon will take.

MA2 describes the polarization of a selection of photons from an entangled pair.

MA3 describes how photons from the entangled pairs are coupled together.

MA4 states that photons carry the complete set of the hidden variable after a measurement.

2.2 Model assumptions

Model assumption MA1: *The statistical parameter λ which is equally distributed between -1/2 and +1/2 controls which of the two polarizer exits the photon will take.*

With polarizer setting α and photon polarization φ, we define $\delta = \alpha - \varphi$ as the difference between the polarizer setting and the polarization of the photon.

The function $A(\delta,\lambda)$ indicates which polarizer exit the photon will take. $A(\delta,\lambda)$ can have values +1 and -1.

For $0 \le \delta < \pi/2$, we define

$$A(\delta,\lambda) = +1 \quad \text{for} \quad -1/2 \le \lambda \le \cos^2(\delta)-1/2, \tag{1}$$

meaning the photon takes polarizer exit α and

$$A(\delta,\lambda) = -1 \quad \text{for} \quad \cos^2(\delta)-1/2 < \lambda \le +1/2, \tag{2}$$

meaning the photon takes polarizer exit $\alpha + \pi/2$.

MA1 is valid for single photons and for each wing of entangled photons as well..

The case $\pi/2 \le \delta < \pi$, is covered referring to the other exit of the polarizer. Then equation (2) applies and the value range of λ for positive results is $\cos^2(\delta)-1/2 < \lambda \le +1/2$. $\delta < 0$ is covered by reversing the polarizer direction by 180°. Thus, $-\pi \le \delta < -\pi/2$ is equivalent to $0 \le \delta < \pi/2$ and $-\pi/2 \le \delta < 0$ is equivalent to $\pi/2 \le \delta < \pi$.

Thus $A(\delta,\lambda) = +1$

1. for $0 \le \delta < \pi/2$ and $-1/2 \le \lambda \le \cos^2(\delta)-1/2$,
2. for $-\pi/2 \le \delta < 0$ and $\cos^2(\delta)-1/2 < \lambda \le +1/2$,
3. for $\pi/2 \le \delta < \pi$ and $\cos^2(\delta)-1/2 < \lambda \le +1/2$,
4. for $-\pi \le \delta < -\pi/2$, and $-1/2 \le \lambda \le \cos^2(\delta)-1/2$ and

$A(\delta,\lambda) = -1$ otherwise. (3)

Figure 2 shows the geometric relationships on which the model is based.

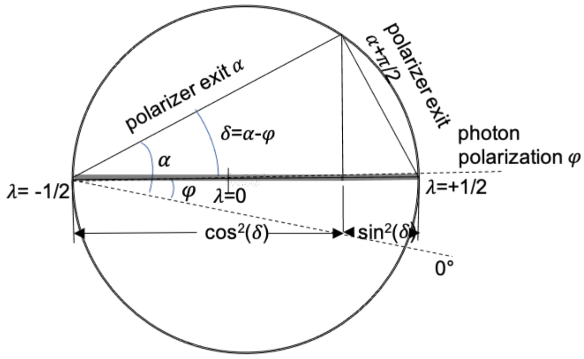

Figure 2: Distribution of polarized photons onto polarizer exits. The polarizer is set to the angle $\alpha/\alpha+\pi/2$ and the photon has a polarization φ. Thus, the difference is $\delta = \alpha - \varphi$. A parameter λ is evenly distributed over the generated photons in the value range $-1/2 \le \lambda \le +1/2$. Photons with $\lambda \le \cos^2(\delta)-1/2$ take the polarizer exit α, while photons with $\lambda > \cos^2(\delta)-1/2$ take the exit $\alpha + \pi/2$.

Model assumption MA2: *If the fractions of horizontally and vertically polarized photons from the entangled state which contribute to a stream of photons selected by a polarizer are $\cos^2(\alpha)$ and $\sin^2(\alpha)$ respectively then the common polarization is α or $-\alpha$ because of the indistinguishability of the photons. For a selecting polarizer on the same wing which is set to α the common polarization is α.*

A selection comprises all photons which take the same polarizer exit. Photons with polarization α and $\alpha+\pi/2$ come in equal shares, due to symmetry. MA2 accounts for the fact that the polarization of photons from the entangled state is undefined due to their indistinguishability but changed and re-defined by entanglement. Thus, the photons of a selection cannot be distinguished by their polarization. This argument was brought up already in [4] but only for photon pairs with common hidden variables. MA2 is a contextual assumption, as the polarization of a selection coincides with the setting of a polarizer. However, it is a local realistic assumption, as it assigns a real value to the physical quantity polarization.

Model assumption MA3 *Photons of a selected photon pair have always the same sign of the parameter λ for the Bell states Ψ- or Φ+ and the opposite sign for Ψ+and Φ-. Each Bell state is a mixture of indistinguishable constituent photon pairs in equal shares whose components have the same polarization 0° or 90° for Φ+ and Φ- and an offset of $\pi/2$ for Ψ+ and Ψ-. The constituent photon pairs make up the initial state.*

MA3 is the means of weakly coupling the photons of an entangled pair. It ensures that the polarization of partner photons are either both positive or both negative for Ψ- and Φ+ and of opposite sign for Ψ+and Φ- but not otherwise.
MA2 and MA3 have the consequence that the selection by a polarizer in position α on one side corresponds to a selection with polarization $\alpha+\pi/2$ or $-\alpha-\pi/2$ on the other side. (for Ψ+ or Ψ-) This can be seen from the following consideration: A polarizer PA set to α selects a fraction of $\cos^2(\alpha)$ of horizontally polarized photons 1 and a fraction of $\sin^2(\alpha)$ of vertically polarized photons 1 (see figure 2). This means a selection of partner photons 2 as well with perpendicular polarization yielding a selected fraction of $\cos^2(\alpha) = \sin^2(\alpha+ \pi/2)$ of vertically polarized photons 2 and a selected fraction of $\sin^2(\alpha) = \cos^2(\alpha+ \pi/2)$ of horizontally polarized photons 2. By MA2 the common polarization is $\alpha+\pi/2$ or $-\alpha-\pi/2$ due to the indistinguishability of the photons 2. For Φ+ and Φ- we have the same polarization for the photons of the constituent photon pairs and therefore a common polarization of α or $-\alpha$.

Which of the sign is correct follows from MA3. The same sign of the parameter λ is obtained if the value range of λ is the same for both partners of a pair. And this is the case for Ψ- or Φ+ with the same sign of $\delta = \alpha - \varphi$ in model assumption MA1. So we have for the common polarisation of the partner photon $\alpha+\pi/2$ for Ψ- and α for Φ+. The opposite sign of the parameter λ is obtained if the value range of λ is different for both partners of a pair. And this is the case for Ψ+ and Φ- with the opposite sign of $\delta = \alpha - \varphi$ in model assumption MA1. So we have for the common polarisation of the partner photon $-\alpha-\pi/2$ for Ψ+ and $-\alpha$ for Φ- .

One could object that according to MA1 the value ranges of λ overlap for positive δ and negative δ if $|\delta| < \pi/4$ so that the photon cannot decide which polarization to take over and the system has to collect several photons until a decision is possible. (see also figure (2)). This is true for the horizontally polarized constituent photon. The vertically polarized photon in this case has a $|\delta| > \pi/4$ where no overlap between the two alternatives occur so that there can be no ambiguity.

For all four Bell states it follows correspondingly:

Bell state	A	B
Ψ-:	α	$\alpha + \pi/2$
Φ+	α	α
Ψ+	α	$-\alpha - \pi/2$
Φ-	α	$-\alpha$

Table 1: polarization of partner photons 2 at wing B for different Bell states for a selection of photons 1 with a polarizer set to α at wing A.

Model assumption MA4: *Photons having left a polarizer exit α have polarization α with λ evenly distributed in the range $-1/2 \leq \lambda \leq +1/2$.*

MA5 emphasizes the fact that photons do carry the complete set of hidden variables after leaving a polarizer.

2.3 Predicting measurement results for single photons Using equation (3), a photon with polarization φ is found behind the exit α of a polarizer with probability

$$P_\delta = \int_{-1/2}^{\cos^2(\delta)-1/2} d\lambda = \cos^2(\delta) \qquad (4)$$

where $\delta = \alpha - \varphi$ with $0 \leq \delta < \pi/2$. For the case $\pi/2 \leq \delta < \pi$, we refer to the other exit of the polarizer and have, with $\vartheta^* = \delta - \pi/2$

$$P_\delta = \int_{\cos^2(\delta*)-1/2}^{1/2} d\lambda = 1-\cos^2(\vartheta^*) = \cos^2(\vartheta), \text{ as well.} \qquad (5)$$

With $\delta = \alpha - \varphi$, we obtain the same P_δ for a photon in state $\cos(\varphi)*|H> + \sin(\varphi)*|V>$ by projection onto $\cos(\alpha)*<H| + \sin(\alpha)*<V|$, according to QM (i.e., Born's rule).

2.3 Calculating expectation values with photons in singlet state We have seen above that all selected photons 1 from the singlet state which take PA exit α have polarization α while their partner photons 2 have polarization $\alpha+\pi/2$. The thus selected photons 2 are the same as would have been selected by a polarizer PB set to $\alpha+\pi/2$. Therefore λ is evenly distributed in the value range $-1/2 \leq \lambda \leq +1/2$. So we can conclude that if photon 1 passes PA at α we have matching events if partner photon 2 with polarization $\alpha + \pi/2$ exits polarizer PB at β. The probability that photons 2 with polarization $\alpha + \pi/2$ would pass PB at β can be obtained by equations (4) and (5), using $\delta = \beta - \alpha - \pi/2$ yielding

$$P_\delta = \cos^2(\delta) = \cos^2(\beta - \alpha - \pi/2) = \sin^2(\beta - \alpha), \qquad (6)$$

where δ is the angle between the PB polarizer setting β and the polarization $\alpha + \pi/2$ of photons 2 selected by PA.

The expectation value for a joint measurement with photon 1 detected behind detector PA at α and partner photon 2 detected behind detector PB at β is as obtained from [4]

$$E(\alpha,\beta) = \Sigma A*B*P_{A,\alpha} *P_{B,\beta|A,\alpha} \text{ for } A,B=+1,-1 \qquad (7)$$

where $P_{A,\alpha}$ is the unconditional probability to detect (A = 1) or not detect (A = -1) photon 1 at α and where $P_{B,\beta|A,\alpha}$ is the conditional probability for photon 2 to have the outcome B (+1 for passing or -1 for not passing) at PB set to β if photon 1 has outcome A (+1 for passing or -1 for not passing) at PA set to α. With P_δ from equation (6), we get, in particular, that:

$$P_{+1,\beta|+1,\alpha} = P_{-1,\beta|-1,\alpha} = P_\delta \text{ and}$$
$$P_{+1,\beta|-1,\alpha} = P_{-1,\beta|+1,\alpha} = 1 - P_\delta. \qquad (8)$$

The constituent photons of the singlet state with polarization 0° and 90° contribute to the probability $P_{1,\alpha}$ to find a photon 1 at α, according to equation (4,5), with fractions ½$\cos^2(\alpha)$ and ½$\cos^2(\alpha-\pi/2)$, respectively, taking into account that the photons of 0° and 90° contribute in equal shares to the total stream of photons on either wing. Thus,

$$P_{1,\alpha} = ½(\cos^2(\alpha) + \cos^2(\alpha-\pi/2)) = ½ = P_{-1,\alpha}. \quad (9)$$

With the above definitions, we get, from equations (6)–(9):

$$E(\alpha,\beta) = ½(1^* P_\delta -1^*(1- P_\delta) -1^*(1- P_\delta) +1^* P_\delta) =$$
$$= P_\delta - (1- P_\delta) = \sin^2(\alpha -\beta) - \cos^2(\alpha -\beta) = -\cos(2(\alpha -\beta)), \quad (10)$$

in accordance with QM. As the expectation value $E(\alpha,\beta)$ in equation (10) exactly matches the predictions of quantum physics, it also violates Bell's inequality.

2.4 Extending the model to spin ½ particles - The model also applies to spin ½ particles, by simply replacing every angle with its half in equations (1) and (2) and Figure 2, as well as in all subsequent derivations, yielding

$$E(\alpha,\beta) = -\cos(\alpha -\beta), \quad (11)$$

in accordance with QM.

Perpendicular polarizer exits correspond to opposite instrument exits. The coordinate systems is the same for both sides.

Figure 4 shows the geometric relationships on which the model is based for spin ½ particles.

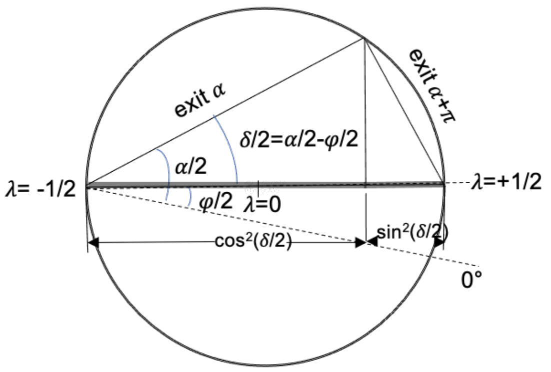

Figure 3: Distribution of spin ½ particles onto instrument exits. The instrument is set to the angle α/α+180°. The generated particle has a spin direction φ. The difference is $\delta = \alpha - \varphi$. The parameter λ is evenly distributed over the generated particles in the value range of $-1/2 \le \lambda \le +1/2$. Particles with $\lambda \le \cos^2(\delta/2)-1/2$ are assigned to the instrument exit α, while particles with $\lambda > \cos^2(\delta/2)-1/2$ take the exit α+180°.

2.5 Applying the model to teleportation Teleportation utilizes a protocol where an unknown state β is transferred to another wing B of a singlet state by means of a Bell state measurement between the unknown β and wing A of the singlet state [9].

With MA3 and table 1 we obtain the polarizations at wing A and B. AB are always in state Ψ-. βA is obtained by the Bell state measurement. So we get the results of table 2:

Bell state βA	A	B
Ψ-	$\beta + \pi/2$	$\beta\ (+\ \pi)$
Φ+	β	$\beta + \pi/2$
Ψ+	$-\beta-\pi/2$	$-\beta$
Φ-	$-\beta$	$-\beta +\pi/2$

Table 2: polarization of the photons of wings A and B for different Bell states obtained between the unknown β and wing A.

The results at B can be converted to state β by simply rotating or mirroring. This result is in accordance with the quantum mechanical calculations [9]. Note that $\beta + \pi$ and β are the same polarization.

2.6 Applying the model to entanglement swapping Entanglement swapping utilizes a protocol where two wings of different systems, each in singlet state, become entangled by means of a Bell state measurement of the two remaining wings. Let AB and CD be the two initial systems in singlet state. Then we define the outer pair AD and the inner pair BC. With a Bell state measurement between B and C we want to entangle A and D. Again there are four resulting Bell states possible. How do these results of the inner pair BC relate to the state of the outer pair AD? This is found out by applying table 1 to the pairs of channels. AB and CD are always in state Ψ-. BC is obtained by the Bell state measurement. So we get the results of table 3:

Bell state BC	A	B	C	D
Ψ-:	α	$\alpha + \pi/2$	$\alpha\ (+\ \pi)$	$\alpha + \pi/2$
Φ+	α	$\alpha + \pi/2$	$\alpha + \pi/2$	$\alpha\ (+\ \pi)$
Ψ+	α	$\alpha + \pi/2$	$-\alpha\ (-\ \pi)$	$-\alpha - \pi/2$
Φ-	α	$\alpha + \pi/2$	$-\alpha - \pi/2$	$-\alpha$

Table 3: polarization of the photons of wings B,C and A,D for different Bell states obtained between B and C by applying table 1 with an assumed selection of photons with a polarizer set to α at wing A.

Thus we see the Bell state of the outer pair AD is equal to the measured Bell state of the inner state BC in accordance with QM [1]. Note that $\alpha + \pi$ and α are the same polarization.

3. Results, discussion and conclusions - A local contextual model with weakly coupled hidden variables was presented. It correctly reproduces the QM predictions of expectation values for polarization measurements with entangled photons and for spin measurements with spin-1/2 particles in the singlet state and thus refutes again Bell's theorem which was already refuted by the model in [4]. The principle of the model presented here is that when photons are selected on one side of the system by a polarizer, their partner photons on the other side are also selected. The mixing ratio of horizontally and vertically polarized photons from the initial state is either the same for both sides or inverse depending on the Bell state. Because of their indistinguishability, the selected photons acquire a common polarization that depends on the fractions of the components of the mixture.

New is that we can also explain entanglement swapping and teleportation with weakly coupled hidden variables, because the Bell state measurement required for this reveals a specific relationship between the polarization of the two measured partner photons. The behaviour of entangled particles can be explained locally. There is no need for "spooky action at a distance" neither with the measurement of correlations, nor with teleportation and entanglement swapping.

Even if the coupling between the partners of an entangled pair is only weak, since it only exists via the sign of the statistical parameter λ, the model cannot do without a statistical parameter that controls which of two possible polarizer outputs a photon takes. If no spooky action at a distance is involved, this cannot be purely accidental in the case of entangled photons, which then also applies to single photons. Thus, the conclusions in [4] that quantum particles do not carry simultaneous information about mutually exclusive outcomes remain valid because the existence of a model with hidden variables indicates the opposite.

Conflicts of Interests/ Competing interests

- No funds, grants, or other support was received.
- The author has no financial or non-financial interests to disclose.

Data Availability statement

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

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