On density of twin prime numbers

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Abstract: Like prime numbers, twin prime numbers are randomly distributed on the number axis. There is no deterministic equation that can be accurately expressed, so we can only use the uncertain probability distribution for research. Based on the latest research results, this paper uses "probability statistics" and "prime number pair distribution theorem" to conduct a preliminary study on the distribution interval of twin prime numbers. The results show that twin prime numbers and the interval occupied by prime numbers gradually increase as the integer value increases.

Keywords: twin prime number; distribution density; prime number pair distribution theorem

1. Introduction

Prime numbers are randomly distributed on the number axis, and there is no deterministic equation that can be accurately represented. The twin prime numbers are randomly distributed on the number axis like the prime numbers, so the uncertain probability distribution can only be used for research. Since 1849, Alphons de Polignac put forward the prime twin conjecture, the distribution of prime twins has been a subject of attention. Based on the latest research results, this paper uses "probability statistics" [1] and "prime number pair distribution theorem" [2] to conduct a preliminary study on the distribution interval of twin prime numbers.

2. Method

According to the prime number pair distribution theorem: let k, x, and y be natural numbers, and k<x<y, the number of prime number pairs greater than an integer (2x+2k) and less than an integer (2y+2k) is approximately:

$$y\cdot\left(\frac{2y+2k}{\ln(2y+2k)}-\frac{2y}{\ln(2y)}\right)\cdot\left(\frac{2y}{\ln(2y)}-\frac{2y-2k}{\ln(2y-2k)}\right)-x\cdot\left(\frac{2x+2k}{\ln(2x+2k)}-\frac{2x}{\ln(2x)}\right)\cdot\left(\frac{2x}{\ln(2x)}-\frac{2x-2k}{\ln(2x-2k)}\right)$$

When k=1, it is a twin prime number. If for any integer m, x, m<x, the probability function that exists approximately one twin prime number in the interval from 2m+2 to 2m+2x+2 is:

$$(m+x)\cdot\left(\frac{2(m+x)+2}{\ln(2(m+2)+2)}-\frac{2(m+x)}{\ln(2(m+x))}\right)\cdot\left(\frac{2(m+x)}{\ln(2(m+x))}-\frac{2(m+x)-2}{\ln(2(m+x)-2)}\right)$$
$$-m\cdot\left(\frac{2m+2}{\ln(2m+2)}-\frac{2m}{\ln(2m)}\right)\cdot\left(\frac{2m}{\ln(2m)}-\frac{2m-2}{\ln(2m-2)}\right)$$

When any integer m has different values, find the corresponding x value, then 2x value is the interval occupied by twin prime numbers.

In the same way, the probability function that exists approximately one prime number in the integer 2m+2 to 2m+2x+2 interval is:

$$\frac{2(m+x)+2}{\ln(2(m+2)+2)}-\frac{2m+2}{\ln(2m+2)}=1$$

Find out the corresponding x value, then the 2x value is the interval occupied by prime numbers.

3. Experimental Results

Calculation of the approximate interval occupied by twin prime numbers and prime numbers in the neighboring area greater than integer 2m+2 is shown in Table 1.

Table 1. Approximate intervals in different integer regions occupied by twin prime numbers and prime numbers.

Integer value （2m+2）	**Approximate interval occupied by twin prime numbers when the integer value greater than 2m+2 area (2x)**	**Approximate interval occupied by prime numbers when integer value greater than 2m+2 area (2x)**
10^5	94	12
10^25	1776	58
10^125	42000	288
10^625	1038400	1440
10^3125	25900000	7190
10^15625	647160000	35800

Note: Floating point 100,000 digits.

It can be seen from Table 1 that with the increase of the integer value 2m+2, the interval occupied by the twin prime numbers and the prime numbers in the adjacent area larger than the integer value 2m+2 also gradually increases, and the interval occupied by the twin prime numbers is much larger than the interval occupied by the prime numbers.

4. Conclusion

The interval occupied by twin prime numbers and prime numbers gradually increases as the integer value increases, and the interval occupied by twin prime numbers is much larger than the interval occupied by prime numbers.

References

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