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Operator Evolution Equations of Angular Motion Law

X. D. Dongfang

Wutong Mountain National Forest Park, Shenzhen, China

Quantum mechanics based on Planck hypothesis and statistical interpretation of wave function has achieved great success in describing the discrete law of micro motion. However, the idea of quantum mechanics has not been successfully used to describe the discrete law of macro motion, and the causality implied in the Planck hypothesis and the application scope of the basic principles of quantum mechanics have not been clarified. In this paper, we first introduce the angular motion law and its application, which seems to be of no special significance as a supplement to the perfect classical mechanics, but plays an irreplaceable role in testing whether the core mathematical procedure of quantum mechanics of operator evolution wave equation satisfies the unitary principle. Then, the operator evolution wave equations corresponding to the angular motion law are discussed, and the necessity of generalized optimization of differential equations are illustrated by the form of ordinary differential equations. Finally, the real wave equation which is superior to the Schrödinger equation in physical meaning but not necessarily the ultimate answer is briefly introduced. The implicit conclusion is that Hamiltonian can not be the only inevitable choice of constructing wave equation in quantum mechanics, and there is no causal relationship between operator evolution wave equation and quantized energy in bound state system, which indicates that whether the essence of quantum mechanics can be completely revealed is the key to unify macro and micro quantized theory.

Keywords: Centrifugal force equation; Angular motion law; Operator principle; Wave equations of multiwave functions; Optimal differential equation; Existence and uniqueness of solution.

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1 Introduction

From Planck's hypothesis[1,2] to Einstein's photon theory[3], to Bohr's hydrogen atom theory[4], to Schrödinger's equation[5] and Dirac's equation[6], quantum mechanics[7-11] has achieved great success in explaining the laws of blackbody radiation[12], photoelectric effect[13] and hydrogen atom spectrum[14], and has also constantly made new breakthroughs in the practice and applications[15-19] . Many problems of quantum mechanics have been clearly concluded.

However, the essence of quantum mechanics has not been clarified. This fundamental factor has led to the neglect of some fundamental problems which play a decisive role in quantum mechanics. For example, can or how to use basic principles to prove Planck's quantized energy hypothesis[20]? How to prove Bohr's[21] or Sommerfeld's[22,23] quantization conditions? How to derive Schrödinger wave equation[24,25] according to the basic principle? What is the scope of application of the basic principles of quantum mechanics such as the representation of mechanical quantities by operators? Can the fine structure constants[26] in the atomic system change? Is Bowen's statistical interpretation[27] the ultimate physical meaning of wave function? Is the concept of orbit compatible with the physical meaning of probabilistic wave? Is the quantum mechanics operation method of constructing wave equation by expressing mechanical quantities in the laws of mechanics as operators suitable for describing the discrete law of macroscopic motion? We try to find out the answers to these questions, clarify the scope of application of quantum mechanics principles, so as to systematically describe macro-and micro-discrete laws in the same logical framework. Some basic principles and mechanical laws neglected by standard theory will play an important role.

Here we introduce the universally applicable angular motion law and the simplified method of solving Kepler orbital equation by using the angular motion law. We further introduce the multi-wave function wave equation system of micro-motion corresponding to the angular motion law, and thus revealing the rich and colorful new mathematical problems and new physical problems hidden in quantum mechanics. The solution of all these problems is the premise to solve the basic difficulties of quantum mechanics, including revealing the essence of quantum mechanics, deciding the establishment of the basic principles of com quantum and the development

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https://orcid.org/0000-0002-3644-5170

direction of com quantum theory, and will also promote the further development of the analytical theory of differential equations.

2 Dongfang angular motion law

It is one of the important basic principles of quantum mechanics that the mechanical quantities of classical mechanical laws are expressed by operators and used to action on wave functions. New laws of mechanics or different expressions of existing laws of mechanics may have different effects on the construction of the wave equation. So, in addition to the equation describing the relationship between momentum and energy, are there any other classical mechanical equations that can be used to establish wave equations? What about the solutions of wave equations established by new mechanical equations? In order to find the answers to these questions, a universal law of angular motion and its operator evolution wave equations are proposed.

Assuming that the object moves in a curve, the mass of the object is$m$, the size of the position vector is$\mathbf{r}$, the direction angle is θ, the radial velocity of the motion is $\mathbf{v}_r = v_r\mathbf{e}_r$, and the transverse velocity is $\mathbf{v}_\theta = v_\theta\mathbf{e}_\theta$, then the radial force $\mathbf{F}_r$ and the transverse force F_θ of the object satisfy the equations respectively,

$$\begin{aligned} F_r &= \frac{m}{r}\left(v_\theta \frac{dv_r}{d\theta} - v_\theta^2\right) \\ F_\theta &= \frac{mv_\theta}{r}\left(\frac{dv_\theta}{d\theta} + v_r\right) \end{aligned} \tag{1}$$

Among them, the central force F_r is positive when it is repulsive force and negative when it is attractive; the transverse force F_θ is positive when it is dynamic force and negative when it is resistance force. Equation (1) is called the Dongfang angular motion law.

The centripetal force F_n of a body moving along a curve is usually expressed by the rate v and the radius ρ of curvature, $F_n = mv^2\rho^{-1} = p^2\left(m\rho\right)^{-1}$. The angular motion law is the generalized centripetal force equation. The common feature of the two methods is that they contain momentum squares and have no relation to energy, so they can not be used directly to construct Hamilton operators. This provides a basis for the comparison demonstration of the applicable scope of the basic operation methods of quantum mechanics.

Now let's prove the Dongfang angular motion law. The physical quantities are represented by complex numbers in exponential form[28]. The polar coordinate system or complex plane xoy is established, as shown in Figure 1. To avoid confusion with the imaginary number unit i of quantum mechanics, $j = \sqrt{-1}$ is used here to represent the imaginary number of units. The complex exponential form of orbital equation $\mathbf{r} = \mathbf{r}\left(t\right)$ is $\mathbf{r} = r\left(t\right)e^{j\theta(t)}$, where θ is the polar angle and t is the time. The dynamic radial and transverse unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ satisfy the right hand rule. The complex exponential form of the velocity vector is obtained by calculating the first derivative of $\mathbf{r}$ with respect to time t,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left(v_r + jv_\theta\right)e^{j\theta} \tag{2}$$

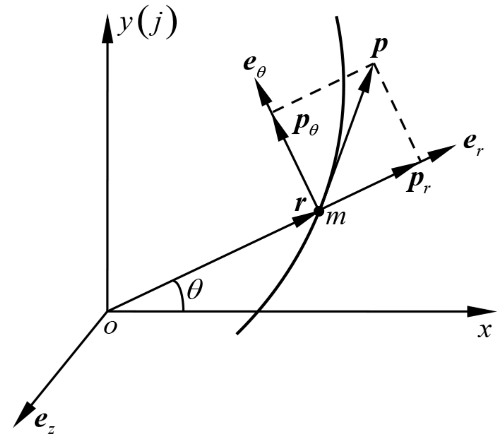

Figure 1 The momentum, radial momentum and transverse momentum of a particle in curvilinear motion on the plane of polar coordinates

The real part and the imaginary part in parentheses correspond to the radial velocity $v_r = dr/dt$ and the transverse velocity $v_\theta = rd\theta/dt$, respectively. The derivative of transverse velocity with respect to time t is transformed into a derivative with respect to angle θ. One has $dt = rd\theta/v_\theta$, so $dv_r/dt = \left(v_\theta/r\right)dv_r/d\theta$ and$dv_\theta/dt = v_\theta dv_\theta/\left(rd\theta\right)$. From this, the derivative of velocity (2) with respect to time t is calculated, and the following complex exponential form of the acceleration vector is obtained,

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{1}{r}\left[\left(v_\theta\frac{dv_r}{d\theta} - v_\theta^2\right) + j\left(v_\theta\frac{dv_\theta}{d\theta} + v_r v_\theta\right)\right]e^{j\theta} \tag{3}$$

The real part and the imaginary part in curly brackets are opposite to the radial acceleration a_r and the transverse acceleration a_θ, respectively. The transverse velocity formula $rd\theta/dt = v_\theta$ is used in the above expression.

The complex exponential representation of the central force is $\mathbf{F} = \left(F_r + jF_\theta\right)e^{j\theta}$. Where F_r takes positive value to represent repulsive force and the direction is the same as that of $\mathbf{e}_r$; F_r takes negative value to represent gravitation and the direction is opposite to that of $\mathbf{e}_r$; F_θ takes positive value to represent power and the direction is the same as that of $\mathbf{e}_\theta$; F_θ takes negative value to represent resistance and the direction is opposite to that of $\mathbf{e}_\theta$. In the complex plane, the complex exponential representation of Newton's law of motion[29] is as follows,

$$\left(F_r + jF_\theta\right)e^{j\theta} = \frac{m}{r}\left[\left(v_\theta\frac{dv_r}{d\theta} - v_\theta^2\right) + j\left(v_\theta\frac{dv_\theta}{d\theta} + v_r v_\theta\right)\right]e^{j\theta} \tag{4}$$

Hence, from the definition of complex number equality, the equation of the angular motion law (1) is obtained.

The angular motion law is a generalized form of centripetal force equation describing curve motion. The characteristic of the angular motion law is that the derivative of time can be transformed into the derivative of angle. Linear motion has no angular motion, so the angular motion law is not applicable to linear motion.

3 Angular motion law in central force field

From classical mechanics to quantum mechanics, the problem of a central force is one of the standard subjects. Coulomb's law and the law of gravitation are both central forces, which control the law of atomic spectrum and the law of celestial motion respectively, and maintain the dynamic stability of the micro-world and the macro-world. The transverse force in the central force field is zero. According to the second formula of equation (1), we obtain the relation $dv_\theta/d\theta+v_r=0$. So $v_r=-dv_\theta/d\theta$, $v_r^2=v_rv_r=-v_r\left(dv_\theta/d\theta\right)$. From $v^2=v_r^2+v_\theta^2$, we get the formula $v_\theta^2=v^2+v_rdv_\theta/d\theta$, which is substituted for the first formula of (1) to obtain several expressions of the angular motion law of the central force field,

$$\begin{aligned} F_r &= \frac{m}{r}\left(-v_\theta\frac{d^2v_\theta}{d\theta^2}-v_\theta^2\right)\\ F_r &= \frac{m}{r}\left(v_\theta\frac{dv_r}{d\theta}-v_r\frac{dv_\theta}{d\theta}-v_r^2-v_\theta^2\right)\\ F_r &= \frac{m}{r}\left[\left(\frac{dv_\theta}{d\theta}\right)^2-v_\theta\frac{d^2v_\theta}{d\theta^2}-v_r^2-v_\theta^2\right] \end{aligned} \tag{5}$$

The radial velocity of the circular motion is $v_r=0$, and the centripetal force is directed to the center of the circle, so F_r is negative. The centripetal force equation $F_r=mv_\theta^2/r$ is obtained by substituting this into the first formula (1) or (5) formula. It can be seen that the centripetal force equation is a special case of the angular motion law. Newton infers that the interaction force between planets and the sun follows the law of inverse square ratio by using the centripetal force equation of circular motion. By using the angular motion law or Newton's law of motion, it can be proved that the interaction force governing the motion of a conic curve has and only has the law of the inverse square ratio. The proof process is omitted here.

The Dongfang angular motion law itself is not a breakthrough discovery. Perhaps because the mathematical process of proving the two-dimensional angular motion law is too ordinary, we will even think that it should not be proposed as a new law or principle. However, if we try to write and prove the three-dimensional angular motion law and apply it to quantum mechanics, we will find that its proposal is not only necessary, but also of far-reaching significance.

According to Newton's law of motion and vector operation rules, we can derive the three-dimensional form of the angular motion law which contains many but not completely independent equations. The three-dimensional form of the angular motion law in spherical coordinate system of a central force field is a system of equations containing multiple equations,

$$\begin{aligned} &v_\theta\frac{dv_\theta}{d\theta}+v_\theta v_r-v_\varphi^2\cot\theta=0\\ &\frac{1}{\sin\theta}\frac{dv_\varphi}{d\varphi}+v_r+v_\theta\cot\theta=0,\quad \frac{dv_\varphi}{d\varphi}+v_r\sin\theta+v_\theta\cos\theta=0\\ &v_\varphi\frac{dv_\theta}{d\varphi}-v_\theta\frac{dv_\varphi}{d\varphi}+v_rv_\varphi\frac{d\theta}{d\varphi}-\left(v_\varphi^2+v_\theta^2\right)\cos\theta-v_rv_\theta\sin\theta=0\\ &F_r=\frac{m}{r}\left(v_\theta\frac{dv_r}{d\theta}-v_\theta^2-v_\varphi^2\right),\quad F_r=\frac{m}{r}\left(\frac{v_\varphi}{\sin\theta}\frac{dv_r}{d\varphi}-v_\theta^2-v_\varphi^2\right)\\ &F_r=\frac{m}{r}\left(v_\theta\frac{dv_r}{d\theta}-v_r\frac{dv_\theta}{d\theta}-v_r^2-v_\theta^2\right)+m\left(v_r\cos\theta-v_\theta\sin\theta\right)v_\varphi\frac{d\varphi}{d\theta}\\ &F_r=\frac{m}{r\sin\theta}\left[v_\varphi\frac{dv_r}{d\varphi}-v_r\frac{dv_\varphi}{d\varphi}-\left(v_r^2+v_\varphi^2\right)\sin\theta-v_rv_\theta\cos\theta-v_\varphi v_\theta\frac{d\theta}{d\varphi}\right] \end{aligned} \tag{6}$$

where F_r is positive for repulsion force and negative for attraction force. The momentum expression of three-dimensional angular motion law is omitted here. There are many equations here, and the form of Newton's law of motion in the spherical coordinate system can also be incorporated into the three-dimensional angular motion law.

It is very simple to solve Kepler orbital equation of motion of an object under the action of the inverse square ratio law $F_r=K/r^2$ with the angular motion law. Here F_r denotes the central force, K is a constant, $K>0$ for repulsion, $K<0$ for gravitation, and the transverse force is $F_\theta=0$. According to the first formula of (5), there is

$$F_r=-\frac{m}{r}\left(v_\theta\frac{d^2v_\theta}{d\theta^2}+v_\theta^2\right)=-\frac{mrv_\theta}{r^2}\left(\frac{d^2v_\theta}{d\theta^2}+v_\theta\right) \tag{7}$$

Where m denotes mass. $mrv_\theta=L$ is an angular momentum constant, from which an expression of transverse velocity $v_\theta=L/mr$ is obtained. Then the second order homogeneous linear differential equation with constant coefficients is obtained by using the central force

$F_r = K/r^2$, and its simplified form is

$$\frac{d^2}{d\theta^2}\left(\frac{L}{mr}\right)+\left(\frac{L}{mr}\right)=-\frac{K}{L}$$

According to the general and special solutions of second order homogeneous linear differential equation with constant coefficients, the conic solution $L/mr = A\cos(\theta+\alpha)-K/L$ is obtained. Among them, A and α are undetermined coefficients, which are determined by the orientation of the selected coordinate axis and the initial value conditions. Take $\alpha = 0$, and remember that $r = r_0$ when $\theta = 0$, one obtains $A = L/mr_0 + K/L$. The above Kepler orbital equation in polar coordinates expressed by angular momentum is transformed into the following form,

$$r=\frac{L^2/mK}{\left(1+L^2/mr_0K\right)\cos\theta-1} \tag{8}$$

Dongfang's angular motion law can also be used to solve the common physical problems related to satellite motion. Of course, in mechanics, because the development of classical mechanics has been very perfect, the angular motion law as a supplement to classical mechanics seems to have no special significance, although its physical meaning is more clear than the radial force formula of curvilinear motion because it describes radial force and transverse force at the same time, and it is more convenient to solve some complex problems of curvilinear motion. However, the angular motion law plays an irreplaceable role in testing the basic theoretical program of the operator evolution wave equation of quantum mechanics and thus the logical basis of quantum mechanics. This is because of the existence of a new law of mechanics expressed by momentum. If the mathematical art of wave equation evolution through mechanical law operator is an inevitable physical logic, then Hamiltonian is no longer the only choice for quantum mechanics to construct wave equation.

4 Operator evolution wave equations of angular motion law

The unitary principle is an important basic principle generally applicable to the logical self consistency test of natural science theory and social science theory[30, 31]. We use the unitary principle to prove that the assumption of constant speed of light can not be universally established, and find out the morbid equation of quantum numbers implied in quantum mechanics. As long as a theory implies discordant logical contradictions, it can be tested by the unitary principle. According to the unitary principle, the exact solution of the operator evolution wave equation of angular motion law must be consistent with the exact solution of the operator evolution wave equation of Hamiltonian. Otherwise, the operator theory, which is the key mathematical art of quantum mechanics, will be severely challenged because it violates the unitary principle.

Now let's discuss the operator evolution wave equation of angular motion law. The momentum $\mathbf{p}$ is decomposed into radial momentum $p_r = mv_r$ and transverse momentum $p_\theta = mv_\theta$. The momentum expressions of equation (1) and (5) of the angular motion law are as follows.

$$\begin{aligned}
&p_r+\frac{dp_\theta}{d\theta}=0\\
&F_r=\frac{1}{mr}\left(p_\theta\frac{dp_r}{d\theta}-p_\theta^2\right)\\
&F_r=-\frac{1}{mr}\left(p_\theta\frac{d^2p_\theta}{d\theta^2}+p_\theta^2\right)\\
&F_r=\frac{1}{mr}\left(p_\theta\frac{dp_r}{d\theta}-p_r\frac{dp_\theta}{d\theta}-p^2\right)
\end{aligned} \tag{9}$$

Where the third formula is derived by substituting the first formula into the second formula and eliminating p_r. The latter three formulas of this equation system are equivalent. It is one of the basic principles of quantum mechanics to replace mechanical quantities in mechanical laws by operators to construct the wave equation. Three-dimensional space usually constructs the rectangular coordinate form of the wave equation first, and then transforms it into spherical coordinate form. The mechanical quantity of the angular motion law is replaced by the operator to construct wave equation, which will encounter the problem of expression of the radial momentum operator which has been controversial. Historically, when solving Dirac equation of hydrogen atoms in spherical coordinates, the radial momentum operator was defined as $\hat{p}_r = -i\hbar\left(\partial/\partial r + 1/r\right)$. The result of the debate is to agree with this expression. However, the deduction of theoretical physics should follow the rules of the mathematical operation. Different mathematical forms of operators in different coordinates can be converted to each other, when they move from spherical coordinates to rectangular coordinates. The expression of radial momentum operators mentioned above will be mathematically difficult. The correct form of translating momentum operators in rectangular coordinates directly into radial and transverse momentum operators in plane polar coordinates is as follows,

$$\hat{p}_r=-i\hbar\frac{\partial}{\partial r},\quad \hat{p}_\theta=-\frac{i\hbar}{r}\frac{\partial}{\partial\theta} \tag{10}$$

where h is Planck constant and $\hbar = (2\pi)^{-1}h$. The above radial momentum operator and transverse momentum operator (10) are necessary mathematical inferences. If Dirac equation's rectangular coordinate system is directly transformed into a polar coordinate system, for the hydrogen atom, the expected solution of the standard theory can still be obtained by solving the equation with boundary conditions[32, 33]. To achieve this goal, it is not necessary to redefine the independent conservative quantity $\hbar\hat{\kappa}$ in relativistic quantum mechanics. Constructing the expected solution must not destroy any mathematics rule. The redefinition of the radial momentum operator $\hbar = (2\pi)^{-1} h$ violates the basic mathematical operation rules and is unnecessary.

There is a more complex relationship between radial momentum and angular momentum in the momentum representation of the angular motion law. When mechanical quantities are replaced by operators, the problem of composite operators arises in the equivalence relations $p_\theta\left(dp_r/d\theta\right) = \left(dp_r/d\theta\right)p_\theta$, $p_\theta\left(d^2p_\theta/d\theta^2\right) = \left(d^2p_\theta/d\theta^2\right)p_\theta$ of mechanical quantity contained in equation (9). Because of the lack of corresponding mathematical basis, the standard answers to these questions are still unknown. According to the stochastic expression of the system of equations (9), the forms of compound operators in the system of equations are written by using the formula (10), respectively. How to calculate one-step $\hat{p}_\theta\left(d\hat{p}_r/d\theta\right)$, $\hat{p}_\theta\left(d^2\hat{p}_\theta/d\theta^2\right)$ and $\hat{p}_r\left(d\hat{p}_\theta/d\theta\right)$? Is $\hat{p}_\theta\left(d\hat{p}_r/d\theta\right)$ and $\left(d\hat{p}_r/d\theta\right)\hat{p}_\theta$ commutative, $\hat{p}_\theta\left(d^2\hat{p}_\theta/d\theta^2\right)$ and $\left(d^2\hat{p}_\theta/d\theta^2\right)\hat{p}_\theta$ commutative, $\hat{p}_r\left(d\hat{p}_\theta/d\theta\right)$ and $\left(d\hat{p}_\theta/d\theta\right)\hat{p}_r$ commutative? If only a random expression of the system of equations (9) is taken, the forms of the related composite operators in the system of equations written by formula (10) are as follows,

$$\begin{aligned}
&\hat{p}_\theta^2 = \left(-\frac{i\hbar}{r}\frac{\partial}{\partial\theta}\right)\left(-\frac{i\hbar}{r}\frac{\partial}{\partial\theta}\right) = -\hbar^2\left(\frac{1}{r}\frac{\partial}{\partial\theta}\right)\left(\frac{1}{r}\frac{\partial}{\partial\theta}\right)\\
&\hat{p}_\theta\frac{d\hat{p}_r}{d\theta} = -\frac{i\hbar}{r}\frac{\partial}{\partial\theta}\left[\frac{d}{d\theta}\left(-i\hbar\frac{\partial}{\partial r}\right)\right] = -\frac{\hbar^2}{r}\frac{\partial}{\partial\theta}\left[\frac{d}{d\theta}\left(\frac{\partial}{\partial r}\right)\right]\\
&\hat{p}_\theta\frac{d^2\hat{p}_\theta}{d\theta^2} = -\frac{i\hbar}{r}\frac{\partial}{\partial\theta}\frac{d^2}{d\theta^2}\left(-\frac{i\hbar}{r}\frac{\partial}{\partial\theta}\right) = -\frac{\hbar^2}{r}\frac{\partial}{\partial\theta}\frac{d^2}{d\theta^2}\left(\frac{1}{r}\frac{\partial}{\partial\theta}\right)\\
&\hat{p}_\theta\frac{d\hat{p}_r}{d\theta} - \hat{p}_r\frac{d\hat{p}_\theta}{d\theta} = \hbar^2\left(\frac{\partial}{\partial r}\right)\frac{d}{d\theta}\left(\frac{1}{r}\frac{\partial}{\partial\theta}\right) - \frac{\hbar^2}{r}\frac{\partial}{\partial\theta}\frac{d}{d\theta}\left(\frac{\partial}{\partial r}\right)
\end{aligned}$$

Since the basic method of quantum mechanics expressed by operators is generally applicable, the radial and transverse momentum in formula (9) should be replaced by (10) radial and transverse operators and used to action on wave functions to describe the motion of the same particle, which requires at least four wave functions ψ_1, ψ_2, ψ_3 and ψ_4. The corresponding four wave equations constitute a wave equation system of real number,

$$\begin{aligned}
&\frac{\partial\psi_1}{\partial r} + \frac{d}{d\theta}\left(\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}\right) = 0\\
&\frac{\hbar^2}{mr^2}\left\{\frac{\partial}{\partial\theta}\left(\frac{1}{r}\frac{\partial\psi_2}{\partial\theta}\right) - \frac{\partial}{\partial\theta}\left[\frac{d}{d\theta}\left(\frac{\partial\psi_2}{\partial r}\right)\right]\right\} - F_r\psi_2 = 0\\
&\frac{\hbar^2}{mr^2}\left\{\frac{\partial}{\partial\theta}\left[\frac{d^2}{d\theta^2}\left(\frac{1}{r}\frac{\partial\psi_3}{\partial\theta}\right)\right] + \frac{\partial}{\partial\theta}\left[\left(\frac{1}{r}\frac{\partial\psi_3}{\partial\theta}\right)\right]\right\} - F_r\psi_3 = 0\\
&\frac{\hbar^2}{mr}\left\{\nabla^2\psi_4 + \frac{\partial}{\partial r}\left[\frac{d}{d\theta}\left(\frac{1}{r}\frac{\partial\psi_4}{\partial\theta}\right)\right] - \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{d}{d\theta}\left(\frac{\partial\psi_4}{\partial r}\right)\right]\right\} - F_r\psi_4 = 0
\end{aligned} \tag{11}$$

These equations are all linear equations, but unlike Schrödinger equation and Dirac equation, the equation does not contain time, and the central force replaces the energy parameter.

At present, it is difficult to find the exact solution of the wave equation system (11). This is not only because the exact solution method of each equation is not clear, but also because there is no answer to the question whether the wave functions in the equation system are consistent or not and how to explain the physical meaning of each wave function. Since the last two equations of the angular motion law equation (9) are derived from the first two equations, the last three equations of wave equation system (11) should be equivalent, but the forms of the last three wave equations are quite different. The more difficult problem is whether the operators are commutative or not because of the different writing order of mechanics quantities. In terms of mathematical form only, the third derivative of the last three equations of wave equation system (11) seems to be out of line with common sense. Whether the wave equation system means that the scope of application of momentum operators is actually very limited, so the momentum in the laws of mechanics can not always be replaced by operators to construct a wave equation. It is very interesting and important to study the answers to the questions.

The wave equation system (11) is derived by applying the operator operation principle of quantum mechanics to the angular motion law. The angular motion law describes the motion of curves, so the wave equation system (11) is applicable to the motion of curves. Schrödinger equation and Dirac equation do not seem to be confined to the angular motion conditions, but actually conceal more complex logic difficulties. For example, Schrdinger equation of linear harmonic oscillator has expected quantized energy solution, whereas Klein-Gordon equation[34, 35] or Dirac equation of relativistic linear harmonic oscillator has no quantized expected solution; the orbital differential equation of curvilinear motion under the action of the law of inverse square ratio has the exact solution of a conic curve, and the orbital differential equation of linear motion under the action of the law of inverse square ratio is a non-linear differential equation and can not give the exact solution of the orbital equation. These difficult mathematical problems of theoretical physics are more enlightening.

5 Ordinary differential form of wave equation

Wave equation system (11) contains composite differential operation, which reveals an important problem that has been neglected for a long time, that is, the classical concept of orbital equation $r = r(\theta)$ is not abandoned because of the principle of quantum mechanics. To determine the spatial orientation of particles, r and θ represent independent coordinate parameters. To study the law of motion of objects governed by interaction forces, the orbital equation $r = r(\theta)$ is a constraint equation, and r and θ are not independent parameters. However, in the Schrdinger equation, Dirac equation and even other mathematical and physical equations, r and θ have been treated as independent parameters and achieved great success. Therefore, a wave equation expressed by a partial differential equation may actually be an ordinary differential equation. Then, is the exact solution of such an ordinary differential equation the same as that of the original partial differential equation? This is a new mathematical and physical problem worthy of further study.

Considering two-dimensional hydrogen atom, the Coulomb force between electrons and protons is$F_r = -e^2\left(4\pi\varepsilon_0 r^2\right)^{-1} = -\alpha\hbar c r^{-2}$. Among them, r is the distance between electron and proton, e is the quantity of elementary charge, ε_0 is the dielectric constant in vacuum, α is the fine structure constant, $\hbar = h(2\pi)^{-1}$ and h is the Planck constant, c is the speed of light in vacuum. The polar coordinate form of the orbital equation of an electron moving around a proton is,

$$r = \frac{\delta}{1-\varsigma\cos\theta} \tag{12}$$

where polar angle θ represents the direction of the position vector. In order to avoid confusion caused by using the same symbols for different physical quantities, δ is used to represent the focus parameter and ς is used to represent the eccentricity. The Coulomb force between the electron and the proton is substituted for the wave equation group (11), and the wave equation group of the hydrogen atom is obtained,

$$\begin{aligned}
&\frac{\partial\psi_1}{\partial r} + \frac{d}{d\theta}\left(\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}\right) = 0\\
&\frac{\hbar^2}{m}\left\{\frac{\partial}{\partial\theta}\left(\frac{1}{r}\frac{\partial\psi_2}{\partial\theta}\right) - \frac{\partial}{\partial\theta}\left[\frac{d}{d\theta}\left(\frac{\partial\psi_2}{\partial r}\right)\right]\right\} - \frac{e^2\psi_2}{4\pi\varepsilon_0} = 0\\
&\frac{\hbar^2}{m}\left\{\frac{\partial}{\partial\theta}\left[\frac{d^2}{d\theta^2}\left(\frac{1}{r}\frac{\partial\psi_3}{\partial\theta}\right)\right] + \frac{\partial}{\partial\theta}\left[\left(\frac{1}{r}\frac{\partial\psi_3}{\partial\theta}\right)\right]\right\} + \frac{e^2\psi_3}{4\pi\varepsilon_0} = 0\\
&\frac{\hbar^2}{mr}\left\{\nabla^2\psi_4 + \frac{\partial}{\partial r}\left[\frac{d}{d\theta}\left(\frac{1}{r}\frac{\partial\psi_4}{\partial\theta}\right)\right] - \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{d}{d\theta}\left(\frac{\partial\psi_4}{\partial r}\right)\right]\right\} + \frac{e^2\psi_4}{4\pi\varepsilon_0 r^2} = 0
\end{aligned} \tag{13}$$

This system of equations has no energy parameters, so it is impossible to obtain quantized energy solutions like Schrödinger equation or the Dirac equation directly. However, since r and θ satisfy the constraint equation (10) and are no longer independent coordinate parameters, the energy parameters are actually expressed indirectly by the focal parameters δ and eccentricity ς of the elliptic equation. This is only a qualitative mathematical conclusion, but the quantitative problem is much more complicated. The last three equations of the wave equation system (13) should be equivalent. Although the equation system (13) is a system of linear differential equations, it is still difficult to find the exact solutions of the equation system so as to obtain scientific conclusions. Are their exact solutions consistent? Are there any indirect quantized exact solutions representing energy? Whether there is an indirect expression of the exact solution of energy quantization? Does the exact solution of quantization conform to the prediction of quantum mechanics? These problems puzzle us.

The first equation in the wave equation system (11) or (13) is the simplest. It has no controversial question whether the operator is commutative or not. It can be transformed into an ordinary differential equation by the constraint equation, i.e. orbital equation (12). Because r is a function of θ, the partial derivative of the equation is rewritten to ordinary differential. From the orbital equation (12), we get that,

$$\begin{aligned}
&\frac{dr}{d\theta} = \frac{d}{d\theta}\left(\frac{\delta}{1-\varsigma\cos\theta}\right) = -\frac{\varsigma\delta\sin\theta}{(1-\varsigma\cos\theta)^2}\\
&\frac{\partial\psi_1}{\partial r} = \frac{d\psi_1}{d\theta}\frac{d\theta}{dr} = -\frac{(1-\varsigma\cos\theta)^2}{\varsigma\delta\sin\theta}\frac{d\psi_1}{\partial\theta}\\
&\frac{d}{d\theta}\left(\frac{1}{r}\frac{\partial\psi_1}{\partial\theta}\right) = \frac{1-\varsigma\cos\theta}{\delta}\frac{d^2\psi_1}{d\theta^2} + \frac{\varsigma\sin\theta}{\delta}\frac{d\psi_1}{\partial\theta}
\end{aligned}$$

By substituting these relations into the first equation of the wave equation system (11) or (13), the second order ordinary differential equation with respect to the parameter θ is obtained,

$$\frac{d^2\psi_1}{d\theta^2} - \frac{(1-\varsigma\cos\theta)^2 - \varsigma^2\sin^2\theta}{\varsigma\sin\theta(1-\varsigma\cos\theta)}\frac{d\psi_1}{d\theta} = 0 \tag{14}$$

Let $\chi = \cos\theta$, then $\frac{d\psi_1}{d\theta} = -\sin\theta\frac{d\psi_1}{d\chi}$, $\frac{d^2\psi_1}{d\theta^2} =$

$\left(1-\chi^2\right)\frac{d^2\psi_1}{d\chi^2}-\chi\frac{d\psi_1}{d\chi}$, so equation (5.3) is reduced to

$$\frac{d\psi_1}{d\theta}=-\sin\theta\frac{d\psi_1}{d\chi}$$
$$\frac{d^2\psi_1}{d\theta^2}=\left(1-\chi^2\right)\frac{d^2\psi_1}{d\chi^2}-\chi\frac{d\psi_1}{d\chi}$$

the equation (14) is transformed into

$$\frac{d^2\psi_1}{d\chi^2}-\frac{3\varsigma^2\chi^2-3\varsigma\chi+1-\varsigma^2}{\varsigma^2\chi^3-\varsigma\chi^2-\varsigma^2\chi+\varsigma}\frac{d\psi_1}{d\chi}=0 \tag{15}$$

By using the orbital equation (12), the derivative of wave function to θ can also be transformed into the derivative to r, and then the first equation of wave equation system (11) can be simplified to the ordinary differential equation of the derivative to parameter r,

$$\frac{d^2\psi_1}{dr^2}-\frac{\left(1-\varsigma^2\right)r^2-2\delta r+2\delta^2}{\left(1-\varsigma^2\right)r^3-2\delta r^2+\delta^2 r}\frac{d\psi_1}{dr}=0 \tag{16}$$

By using the above methods, the last three partial differential wave equations of the wave equation system (13) can be transformed into ordinary differential equations. The existence and uniqueness of the exact solutions of these equations must be studied first. Only by finding the exact solution of the equation and comparing the solutions of the wave equation corresponding to the equivalent mechanical equation of different forms, can we know the true physical meaning of the wave function, and then clarify the scope of application of the basic principles of quantum mechanics in which mechanical quantities are replaced by operators. In order to study the exact solutions of equations (15) and (16), we need to develop the analytical theory of differential equations. The relevant mathematical basis is the optimization theorem of differential equations[36]. We need to extend the optimization theorem of differential equations to the generalized optimization theorem to study the existence and uniqueness of exact solutions of equations (15) and (16). Of course, finding the exact solution of equation (15) or (16) by any other method will also lead to important new physical conclusions including but not limited to the equation itself.

6 Comments and conclusions

In this paper, the law of angular motion is proposed, and the operator evolution wave equations of the law of angular motion of the central force field are written by using the quantum mechanics calculation principle of constructing the wave equation by replacing mechanical quantities with operators. The simplest equations of the wave equations of the hydrogen atom are simplified into two kinds of ordinary differential equations. At the same time, the necessity of generalized optimization of the differential equations is pointed out.

However, does the exact solution of each wave equation exist? Or the exact solutions exist, but are they consistent? These problems challenge the basic and most important computational rule of operator evolution of the wave equation in quantum mechanics. For any theory, it is only a necessary condition that the principle[37, 38], method and conclusion of the theory conform to the unitary principle. If it does not conform to the unitary principle, there must be major defects or even mistakes. The law of conservation of momentum, the law of conservation of energy, the law of conservation of angular momentum and the angular motion law are all the inevitable inferences of Newton's law of motion. Applying the angular motion law to quantum mechanics, the obtained wave equation set of the central force field implies the existence of many different wave functions. How to explain the physical meaning of these different wave functions is obviously an urgent topic. The statistical interpretation of wave functions may not be the only physical meaning of wave functions. Whether it belongs to functions that contain different physical meanings such as orbital density can not be affirmed or negated at present. From the point of view of statistical law, the concept of the electron cloud is very applicable, but the probability of a probability wave does not mean the negation of the concept of micro particle trajectory. The problem that quantized energy destroys the law of conservation of energy implied in the morbid equation of quantum numbers[31] and the exact solutions of the operator evolution equations of angular motion law all show that quantum mechanics can not pass the logical test of the unitary principle, so it is actually a very imprecise theory. But the idea of quantum mechanics is meaningful, and quantum mechanics needs to be revised systematically. We use the unitary principle to test more important physical theoretical problems, and find that we need to correct or prove all the basic physical hypotheses which lack the logical basis to solve the difficult theoretical physical problems fundamentally.

In quantum mechanics, it is considered that the energy and angular momentum of the interactive system are observable and can be expressed by a linear Hermitian operator. From the point of view of the com quantum phenomena of the macroscopic system, velocity is truly directly observable compared with angular momentum and energy. From the so-called plane wave function $\psi\left(\mathbf{r},t\right)=A\exp\left[\left(i/\hbar\right)\left(m\mathbf{v}\cdot\mathbf{r}-Et\right)\right]$, we can write the velocity operator $\hat{v}=-\left(i\hbar/m\right)\nabla$ or the velocity square operator$\hat{v}^2=-\left(\hbar^2/m^2\right)\nabla^2$. Therefore, a strict proof is needed to make a choice between the velocity operator and the momentum operator. Essentially, the steady wave equation means that the mathematical expression of the wave function independent of time is $\partial\psi/\partial t=0$. But this accurate mathematical expression will lead to time-dependent Schrödinger equation $i\hbar\partial\psi/\partial t=-\left(\hbar^2/2m\right)\nabla^2\psi \quad +U\left(r\right)\psi$ can not be transformed into the steady equation. The problem means different physical ideas, that is, the establishment of wave equations may not be limited to Hamiltonian operators. For example, considering the energy level transition of a quantum system, the absorbed or radiated energy of the system must follow the law of conservation of energy. Therefore, referring to the Maxwell equations, we can find a more reasonable reason than the Hamiltonian operator evolution method, but it may not be the true portrayal of the natural law, and write the following real number wave equation,

$$\nabla^2 \psi + \frac{4\pi^2}{\sigma c^2}\left(E - U\right)\psi - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = 0 \qquad (17)$$

Among them, m denotes the mass of particle, E and U denote energy and potential energy respectively, and σ is a constant for com quantum theory. The real wave equation is one of the corollaries of com quantum theory, which shows that the virtual number is not an indispensable element of quantum mechanics. There is no doubt about how to explain the concept of steady state in real wave equation. Because $\partial\psi/\partial t = 0$ corresponds to the steady-state Schrödinger wave equation, and the solution of this steady-state equation is just that have been given by quantum mechanics. Perhaps readers can write other forms of real wave equations and find the intrinsic relationship between different real wave equations.

It is generally believed that quantum mechanics is one of the perfect and precise scientific theories in mathematics. However, problems such as the scope of application of the basic principles of quantum mechanics and the nature of quantum mechanics have not been solved. The application of an operator operating method in quantum mechanics to the new laws of mechanics such as the angular motion law brings new mathematical and physical difficulties to be solved. Its scientific conclusion is of great significance to the design idea of the assumption that the quantum theory does not depend on the assumption that it is impossible to prove or is finally proved to be very limited. The assumption that accords with the natural law must have exact causality and can be proved and become a theorem. Otherwise, it will not be regarded as the basic principle in the com quantum theory. Even if some hypothetical form logic deduction can solve some difficult problems locally. The wave function of quantum mechanics is interpreted as a probability function. Why can't it be a new physical quantity that has not yet been discovered? What are the wave functions and wave equations of the com quantum theory, which can be used to describe both macroscopic and microscopic discrete laws? Let's find the answers to the questions together.

One of the basic principles of quantum mechanics, the mechanical quantity is replaced by an operator and acts on the wave function to construct the wave equation. Because of Dongfang's angular motion law, it exposes serious logical defects. It brings too undiscovered quantum mechanics problems that need to be solved urgently. The impact on quantum mechanics will be far-reaching and will promote the development of theoretical physics.

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