Gravity, Quantum Mechanics and MOND

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Abstract

Most attempts to unite Quantum Mechanics (QM) and Gravity are focussed at the extreme energies of the Standard Model (SM). This paper, in complete contrast, approaches the problem from the opposite end of the energy spectrum. It starts by building the fundamental particles from infinite superpositions that fit the SM, apart from infinitesimal differences, with possibly profound consequences. All fundamental particles have at least an infinitesimal mass always inverse to the horizon radius. Cosmic wavelength ($k_{\text{min}}$) gravitons vastly outnumber all other particles and the invariant action they require comes from the horizon. Its surface area is always proportional to the density of matter and cosmic mass is always proportional to the horizon radius. When mass is distributed evenly as dust, gravitons have uniform spatial density. To maintain action invariance, the metric changes around mass concentrations in agreement with Einstein’s equations, apart from an infinitesimal difference effective only at cosmic radii. In large regions of space this difference makes the values of the Einstein tensor components in the Freidman equations average zero. Space is always flat, and Quantum Mechanics (QM) controls the expansion of space regardless of Omega, with or without inflation. The scale factors in the radiation era, and the start of the matter era, are similar to Lambda-CDM cosmology. Milrom’s acceleration $a_0$ has an inverse length that is always proportional to the maximum ($k_{\text{min}}$) graviton wavelength. This may facilitate early supermassive black hole generation. Just as the three SM coupling constants change at high energies, this paper proposes that the graviton coupling constant increases inversely with local acceleration when the gravitational gradient is less that $a_0$, but with an exponential cutoff close to halo radii. This behaves like an added mass, making low mass galaxies more MOND-like, and massive ones less so. Even though galaxies are not gravitationally bound, as the scale factor increases these vast MOND regions increase their volume, which adds more of this phantom-like mass and accelerates the expansion, just like dark energy. An approximate model of this (with just one free parameter related to the fraction of phantom mass) gives a deceleration parameter $q_0 = -1.16$, and a transition redshift of approximately $z_t = 0.37$, but it can be adjusted to fit deceleration/redshift surveys.
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1 Introduction

The current formulation of the Standard Model (SM) of particle physics was finalised in the mid-1970s. However, although extremely successful in providing testable experimental predictions and currently the best description we have of the subatomic world, the theory still leaves a significant number of phenomena unexplained. In the last forty years or so there have been a number of theories seeking to move physics beyond the SM, including supersymmetry and string theory. However, none of the particles predicted by supersymmetry have yet been found, despite a decade of work at CERN’s Large Hadron Collider (LHC), and string theory, widely considered the most likely path for including gravity in the SM, is not yet supported by any direct empirical evidence. Further, dark matter has yet to be directly detected, and dark energy remains elusive. In contrast to these disappointments however, the ATLAS and CMS experiments at CERN’s LHC announced in 2012 that they had each observed a new particle in the mass region around 126 GeV; a particle consistent with the Higgs boson predicted by the SM.

String theory has been strongly criticised over its inability to make testable predictions [1-6]. However, along with the multiverse theory, it has generated intense and important debate over the scientific standing of non-testable theories in physics. In 2009 Dawid, a theoretical physicist turned philosopher, noted substantial conflict between supporters and critics of string theory in assessing its status and success [7]. Dawid argued that this disagreement could best be understood in terms of a paradigmatic rift between the two sides over their understandings of theory assessment. Critics on the one hand believed that “it is a core principle that scientific theories must face continuous empirical testing [emphasis added] to avoid going astray” (p988). In contrast, supporters of string theory placed importance on theoretical criteria for theory assessment. In an interview several years later Dawid [8] suggested this emergence of non-empirical theory assessment, or post-empirical science, represented a Kuhnian paradigm shift in physics and that it would become increasingly important due to the difficulties associated with experimentally testing new theories. In Nature, Ellis and Silk in 2014 [9] made an appeal to “Defend the integrity of physics.” They expressed concern that when faced with the difficulties of applying fundamental theories to the observed universe, some researchers had begun explicitly advocating a change to how theories should be assessed, viz., if deemed sufficiently elegant and explanatory, experimental testing was unnecessary. Ellis and Silk disagreed, insisting that empirical testability is a necessary condition for a theory to be considered scientific, and concurred with Hossenfelder [10] that the concept of post-empirical science was an oxymoron.

Another important issue relating to the testability of theories in physics has been highlighted recently by the astrophysicist David Merritt [11]. In regard to the lambda cold dark matter model (ΛCDM), which contains Einstein’s theory of gravity, Merritt notes that dark matter, dark energy and inflation were all added to the theory in response to observations that would falsify it, i.e. they are ad hoc, or auxiliary hypotheses. Further, he argues that they are conventionalist hypotheses in that they add no empirical content and hence are unfalsifiable in the sense defined by the philosopher Karl Popper. Popper had set specific criteria for preserving falsifiability (or testability) when such “conventionalist stratagems” are employed,
i.e., the modified theory had to make some new, testable predictions, and at least some of the new predictions should be verified. Further, Popper’s student Imre Lakatos, tested and refined these criteria to distinguish between “progressive” and “degenerating” research programs. A progressive research program is one in which “its theoretical growth anticipates its empirical growth, that is, as long as it keeps predicting novel facts with some success.” The \( \Lambda CDM \), according to Merritt, fails to meet such requirements as the auxiliary hypotheses (dark matter, dark energy and inflation) have yet to be confirmed, and the \( \Lambda CDM \) is notably lacking in successful predictions. Steinhardt [12] one of the founders of inflationary cosmology, now also views that theory untestable and has become one of its sharpest critics. The failure to progress significantly beyond the SM during the past four decades, the increasing prominence of highly theoretical, mathematically elegant but difficult to test or untestable theories, and threats to undermine testability as a sine qua non for a theory to be considered scientific, all appear responsible for a succession of popular books expressing concern at the current state of physics [1-5]. In her recently published *Lost in Math: How Beauty Leads Physics Astray*, Hossenfelder [3] contends that the search for beauty has led physicists astray, giving wonderful mathematics but bad science; belief that the best theories are beautiful, natural and elegant has resulted in theories that are untestable. Lamenting the lack of a major breakthrough in the foundations of physics during the last forty years, she advocates physicists need to rethink their methods. In reviewing her book Wilczec [13] contends that Hossenfelder presents an overly pessimistic view, but concedes that “the malaise expressed...is not baseless and is widely shared among physicists” (p57).

In view of these concerns over the current state of physics we offer an alternative approach, but one which still uses very simple basic principles of quantum mechanics (QM) and special relativity (SR). Apart from infinitesimal differences it is consistent with the SM. It suggests that QM accelerates the horizon velocity in the matter era in a manner that is testable. This is so regardless of the value of \( \Omega \). It relates Milgrom’s MOND with an increasing graviton coupling constant at low accelerations in a covariant manner that cuts off exponentially and also relates with accelerating expansion.

We contend our theory is both simple and capable of making testable predictions; at the cosmological level, if not the quantum level. It is, however, radical in its proposals and implications. Consequently, it will require a significant shift in thinking, not only in regard to the fundamental particles, but also the evolution of the cosmos. Such a shift, however, may facilitate progress beyond the SM and/or the \( \Lambda CDM \).

Because these proposed ideas are so radical, we start with some preliminary explanatory notes. Part 1 of the paper follows and includes the forming of fundamental particles from infinite superpositions (section 2), their properties (section 3), and high energy superposition cutoffs (section 4). Part 2 looks at the cosmological consequences of these infinite superpositions. We end with a discussion about the overall implications of this paper, particularly the possibility of massive virtual gravitons forming galaxy halos consistent with the counter intuitive behaviour of QM, and the slightly different way of looking at the warping of spacetime which leads to a QM expansion model as an alternative to the \( \Lambda CDM \).
1.1 List of Some Abbreviations, Acronyms and Symbols Used in the Text

$\Lambda CDM$ The Lambda Cold Dark Matter Model of Cosmology.
AU The astronomical unit and defined as 149,597,870,700 metres.
CMB Cosmic Microwave Background.
EM Electromagnetic.
FLRW Friedmann-Lemaitre-Robertson-Walker metrics.
GR General Relativity.
ICM Intracluster Medium.
MOND Modified Newtonian Dynamics.
SM Standard Model.
SR Special Relativity.
QCD Quantum Chromodynamics
QED Quantum Electrodynamics
QM Quantum Mechanics.

$N$, $n$ & $s$. Integers $n = 3, 4, 5, 6 & 7$ are used in $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18)Y(\theta, \phi)$ virtual primary $(l = 3)$ wavefunctions at wavenumber $k$. The probability is $\left[ \frac{sN \cdot dk}{k} \right]$, where $s$ is spin, and $N=1$ for all massive $s=1/2$ fermions, as well as $s=1$ and $s=2$ massive bosons. For all $s=1$ and $s=2$ infinitesimal mass bosons $N = 2$.

$\chi_C$ is the primary to secondary coupling ratio $= \alpha_s^{-1}$ at the Planck energy superposition cutoff.
$k_{\text{min}}$ is the wavenumber of the maximum cosmic wavelength.
$R_{\text{OH}}$ is the observable horizon radius.
$Y = k_{\text{min}} R_{\text{OH}} \approx 0.223$ in radians for all cosmic time.
$\rho_{Gk_{\text{min}}}$ is the normal three dimensional density of $k_{\text{min}}$ gravitons
$K_{Gk_{\text{min}}}$ is the $k_{\text{min}}$ graviton invariant as in $\rho_{Gk_{\text{min}}} = K_{Gk_{\text{min}}} dk_{\text{min}}$ where $K_{Gk_{\text{min}}} \approx 0.0527 \alpha$.  
$K_{Qk_{\text{min}}}$ is the invariant $k_{\text{min}}$ action required by $k_{\text{min}}$ gravitons, where the four dimensional quanta density $\rho^{4D}_{Qk_{\text{min}}} = K_{Qk_{\text{min}}} \approx 0.067 \alpha$. 
$\alpha_G \approx 1/24$ is the graviton coupling constant between Planck masses but increases at low accelerations where $(\alpha'_G)^{-1} \propto \nabla \Phi$ when $\nabla \Phi \ll 2k_{\text{min}}$. 
$\alpha$ with no subscript is the usual electromagnetic coupling constant.
$M_U$ is the mass of the observable universe both phantom plus real.
$\rho_U$ is the average density of both phantom plus real mass in the universe.
$T^\mu_\nu$ is the infinitesimally modified Einstein tensor where $T^\mu_\nu = T^\mu_\nu \, \text{(Local)} - T^\mu_\nu \, \text{(Cosmos)}$.
$T^\mu_\nu \, \text{(Cosmos)}$ is the Einstein tensor averaged over the whole universe.
$\Omega = 1$ in the $\Lambda CDM$ at critical density for flatness.
1.2 Summary Flow Chart and Preliminary Explanatory Notes

This paper starts with the assumption that all “fundamental particles” are built from combinations of “virtual preons”. There are three preons, coloured red, green and blue, and their antiparticles. All preons are spin zero and electrically charged.

Different groups of 8 preons (with no weak charge) couple to the electromagnetic and 8 colour ground state fields, forming \( l = 3 \) spatially dependant wave functions. Infinite superpositions of these wave functions form all the spin \( \frac{1}{2} \) & spin 1 standard model particles, as well as spin 2 gravitons. The frequencies of these wavefunctions start at \( k_\text{min} \approx (\text{Horizon radius})^{-1} \) up to Planck scale maximum.

Cosmic wavelength zero point densities are very limited. Because spin zero preons are born with zero momentum and infinite wavelength they can couple to Doppler shifted Planck mode ground states from the vastly distant receding horizon.

High frequency coupling is to local ground state fields where the available densities are plentiful.

Low frequency coupling controls the average universe density at \( \rho_v \approx \frac{4.456 \pi \alpha_G}{\text{Horizon Area}} \) in Planck units, where \( \alpha_G \) is the graviton coupling between Planck masses, and the total cosmic mass \( M_u \approx 4.66 \rho_{oh} \).

The action required by maximum wavelength or \( k_\text{min} \) gravitons inside the horizon at a fixed cosmic time is invariant. When mass is distributed evenly as a dust there is a uniform density of \( k_\text{min} \) gravitons throughout a horizon radius sphere and space is flat everywhere. If any of this mass is moved to a central location it increases the density of \( k_\text{min} \) gravitons surrounding it and space has to expand locally to restore this density in agreement with Einstein’s equations, but with infinitesimal differences effective at cosmic radii:

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \left[ T_{\mu\nu} (\text{Local}) - T_{\mu\nu} (\text{Cosmos}) \right] = \frac{8\pi G}{c^4} T'_{\mu\nu}.
\]

In large regions of space the average values of \( T'_{\mu\nu} \approx 0 = G_{\mu\nu} \). Space is flat and the Freidman equation components average zero. QM controls the expansion of space regardless of \( \Omega \) with or without inflation. Writing the scale factor as \( a(t) \propto t^p \); in the radiation era \( p = 1/2 \), the horizon velocity \( V = 2 \); the matter era starts at \( p = 2/3 \) and \( V = 3 \), all as in current cosmology when \( \Omega = 1 \).

When \( \nabla \Phi \approx 2k_\text{min} \), the graviton coupling constant \( \alpha_G^{-1} \propto \nabla \Phi \) (increasing inversely with local acceleration) but with an exponential cutoff that is close to halo radii. This behaves like added mass giving galaxies their observed behaviour. This added mass also behaves like dark energy and accelerates expansion as the scale factor increases. A simplified mathematical model gives a deceleration parameter of \( q_0 \approx -1.16 \) and a transition redshift \( z_t \approx 0.37 \).
1.2.1 **General relativity as an initial guide**

GR informs us that all forms of mass, energy and pressure are sources of the gravitational field. Thus to create gravitational fields, all spin ½ leptons & quarks, spin 1 gluons, photons, W⁺ & Z⁺ particles etc. emit virtual gravitons, except possibly gravitons themselves (section 6.2.6), as gravitational energy is not part of the Einstein tensor.

The starting point of this paper assumes there is a common thread uniting these fundamental particles making this possible. Equations are developed that unite the amplitudes of the colour and electromagnetic coupling constants with that of gravity. The precision required by quantum mechanics for half integral and integral angular momentum allows gravity to be included, despite the vast disparity in magnitude between gravity and the other two. This combination of colour, electromagnetic and gravitational amplitudes in the same equation is possible because of a radically different approach taken in this paper: an approach using infinite superpositions of positive and negative integral \( h \) angular momentum virtual wavefunctions for spin \( \frac{1}{2} \), spin 1 and spin 2 particles. The result is almost identical to the SM, with infinitesimal but important differences. The total angular momentum can be summed over all wavenumbers \( k \); from \( k = 0 \) to some cutoff value \( k_{\text{cutoff}} \). We will assume (as with many unification theories) that the cutoff for these infinite superpositions is somewhere near Planck scale. Firstly, imagine a universe where the gravitational constant \( G \to 0 \). As \( G \to 0 \), the Planck length \( L_p \to 0 \), the Planck energy \( E_p \to \infty \) and \( k_{\text{cutoff}} \to \infty \) also. If we sum the angular momentum of these infinite superpositions when \( G \to 0 \) (i.e. from \( k = 0 \) to \( k_{\text{cutoff}} \to \infty \)) we get precisely half integral or integral \( h \) for the fundamental spin \( \frac{1}{2} \), spin 1 & spin 2 particles in appropriate \( m \) states. If we now put \( G > 0 \) the infinitesimal effect of including gravity can be balanced by an equal but opposite effect due to the non-infinite cutoff value in \( k \). A near Planck scale superposition cutoff requires gravity to be included to get precisely half integral or integral \( h \). (Section 4.2)

These infinite superpositions have another very relevant property relating to the fact that all experiments indicate that fundamental particles such as electrons can behave as point particles. Each wavefunction with wavenumber \( k \), which we label as \( \psi_k \), has a maximum radial probability at \( r \approx 1/k \) and they all look the same (Figure 1.1.1). Every wavefunction \( \psi_k \) of these infinite superpositions, interacts only with virtual photons (for example) of the same \( k \); if superpositions representing say an electron are probed with such photons (that interact only with wavefunction \( \psi_k \)) the resolution possible is of the same order as the dimensions of \( \psi_k \), both have \( r \approx 1/k \). The higher the energy of the probing particle the smaller the \( \psi_k \) it interacts with; the resolution of an observing photon can never be fine enough to see any \( \psi_k \) dimensions. Even if this energy approaches the Planck value, with a matching \( \psi_k \) radius near the Planck length it is still not possible to resolve it. This behaviour is consistent with the quantum mechanical properties of point particles.
1.2.2 Primary and secondary interactions

Supposing that superpositions can in fact build the fundamental spin ½, spin 1, and spin 2 particles, then what builds the superpositions? Answering that question requires dividing all interactions into two categories: primary and secondary.

Secondary interactions are those we are familiar with, and are covered by the SM; but with the addition of gravity, which is not included in the SM. They take place between the fundamental spin ½, spin 1 and spin 2 particles formed from infinite superpositions. They are the quantum electrodynamics/quantum chromodynamics (QED)/(QCD) etc, interactions of all real world experiments.

Primary interactions we conjecture on the other hand, are those that build virtual infinite superpositions. The base states of virtual infinite superpositions only last for time \( \Delta T \leq h/2\Delta E \), and the primary interactions that build them are completely hidden to the real world of experiments. Infinite superpositions cannot be decomposed into their base states, in the same way as base states of fundamental particles can be observed. The quantum world is always hidden until observation, even if we know base state probabilities. But virtual infinite superpositions are always hidden, and only fundamental particles can be observed.

Primary interactions are extremely simple. They are only one way; zero-point fields act on the particle, but the particle cannot act on, or influence, zero point fields. (Its invariance is guaranteed by Heisenberg’s uncertainty principle.) In contrast, secondary interactions involve all the excited modes above the ground state and are two way. These excited field modes both act on the particle which in turn acts back on the field. Quantum field theory (QFT) is all about these complicated two way interactions. Lagrangians are ideal for these two way interactions, predicting symmetries and conservations. However, Lagrangians are less relevant in primary interactions: the natural invariance of the ground state carries through into symmetries and conservation laws. In view of this, our proposals depart from the current practice of basing new theories on Lagrangians. In this regard, while acknowledging their enormous predictive power, Penrose [6] expresses unease with this modern trend, arguing against relying too strongly on Lagrangians in searches for improved fundamental theories (p 491). History tells us progress can be inhibited by assuming that what has worked so well up to now must always be so. Newton reigned supreme for almost two centuries until superseded by Einstein.
The first half of this paper is about these primary interactions, and the superpositions they build representing the fundamental spin $\frac{1}{2}$, spin 1 and spin 2 particles. Primary interactions are between spin zero particles borrowed from a Higgs type scalar field, and the zero-point vector fields. In the 1970’s models were proposed with preons as common building blocks of leptons and quarks [14-17]. In contrast with the virtual particles in this paper, some of these earlier models used real spin $\frac{1}{2}$ building blocks. However, real substructure has difficulties with large masses if compressed into the small volumes required to approach point particle behaviour. It was probably because of this high mass/small volume problem that these earlier preon proposals fell out of favour. On the other hand our proposed virtual substructure borrows energy from zero point fields where the mass contribution at high $k$ values can be cancelled (section 3.2.1). As in earlier models this paper also calls the common building blocks preons, but here the preons are both virtual and spin zero. They also now build all spin $\frac{1}{2}$ leptons and quarks, spin 1 gluons, photons, W & Z particles, plus spin 2 gravitons, in contrast to only the leptons and quarks in the earlier models. (See Table 2.2.) As these preons have zero spin they possess no weak charge. Primary interactions (section 2.2.1) can take place only with the zero point colour, electromagnetic and gravitational fields. The three primary coupling constants for each of these three zero-point fields are different from, but related to, secondary coupling constants.

The behaviour of primary coupling is also entirely different from secondary coupling. Secondary coupling strengths vary (or run) with wavenumber $k$ (the electromagnetic increasing with $k$ and colour decreasing with $k$). In contrast, we conjecture primary coupling strengths (or constants) do not run. In this paper virtual preons are continually born with mass out of a Higgs type scalar field, existing only for time $\Delta t \leq \hbar / 2E$. At their birth, they interact while still bare with zero point vector fields; at this instant of birth $t = 0$. The primary coupling constants consequently are fixed for all $k$; there is no time for charge cancelling or reinforcing, which in secondary interactions forms around the bare charge progressively after its birth. The equations work only if this is true, and they also work only if the primary colour coupling constant is one. (Sections 2.2.2). The ratio between the primary and secondary colour coupling constants labelled $\chi_C$ is thus (if primary colour coupling is one) the inverse of the secondary (or usual $\alpha_s^{-1}$ of QCD) colour coupling constant at the superposition cutoff at Planck Energy. (Sections 3.3 & 4.2.2) To enable the primary coupling to colour, electromagnetic and gravitational zero point fields, preons need colour, electric charge and mass. There are three preons, red, green & blue with positive electric charge, and their three antiparticles. Their mass borrowed from some type of scalar Higg’s field, or the time component of zero-point fields must always be non-zero. This is discussed further in section 1.2.3. As there are eight gluon fields, superpositions are built with eight virtual preons for each virtual wavefunction $\psi_i$. The nett sum of these eight electric charges is $0, \pm 2, \pm 4, \pm 6$, and never $> \pm 6$. This leads to the usual $0, \pm 1/3, \pm 2/3, \pm 1$ electric charge seen in the real world. Various combinations of these eight preons in appropriate superpositions can build leptons and quarks, colour changing and neutral gluons, neutral photons, neutral massive $Z^0$ photons and the charged massive $W^\pm$ photons. (Table 2.2.1)
1.2.3 Photons, gluons and gravitons with infinitesimal mass ($\approx 10^{-34}\text{eV}$).

Einstein taught us that regardless of how fast a particle with mass moves, a ray of light always passes it at the same velocity $c$. The SM builds on this principle with one group of particles travelling at less than $c$, and another group at $c$: massive and massless, with a clear division between them. In the SM the neutrino family was included in the massless group.

However, towards the end of last century evidence slowly emerged that this was not true, and the family of three neutrinos must have masses somewhere in the electron volt range. There is no explanation for this in the SM.

Due to their very low mass, and normal emitted energies, neutrinos invariably travel at virtually the velocity of light $c$. Photons also have always been included in the massless group traveling precisely at velocity $c$, except in the case of the massive $W$ & $Z$. Massless virtual photons have an infinite range, which has always been seen as an absolute requirement of the electromagnetic field. On the other hand, this paper requires some rest frame (even if this frame can move at virtually $c$) in which to build all the fundamental particles. Table 6.2.1 suggests photons, gluons and gravitons have $\approx 10^{-34}\text{eV}$ mass with a range of approximately the inverse of the causally connected horizon radius, and velocities sufficiently close to that of light their helicity remains essentially fixed. This allows some form of Higgs mechanism to increase this infinitesimal mass to the various values in the massive set. (These infinitesimal masses are also in line with some recent proposals [18,19] where gravitons have a mass of < $10^{-33}\text{eV}$ to explain accelerating expansion.)

The virtual wavefunction we use is $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta, \phi)$, an $l = 3$ wavefunction. This virtual $l = 3$ property is normally hidden. In the same way as scattering experiments on spin 0 pions show spin 0 properties, and not the properties of the two cancelling spin ½ component particles, this $l = 3$ property of the virtual components of superpositions is not visible in the real world. Scattering experiments can exhibit only the spin properties of the resulting particle. The individual angular momentum vectors $|L| = 2\sqrt{3}h$ of the infinite superposition all sum to a resulting: $|L_{total}| = (\sqrt{3}/2)h$, $\sqrt{2}h$ or $\sqrt{6}h$ for spin ½, spin 1 or spin 2 respectively, in a similar way to two spin ½ particles forming spin 0 or spin 1 states. We also use the fact that the angle to the $z$ axis of the angular momentum vector for $s = 1/2, m = \pm 1/2$ is identical to $l = 3, m = \pm 2$.

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta, \phi)$ has eigenvalues $P_{nk}^2 = n^2 h^2 k^2$ with $|P_{nk}| = nhk$, suggesting it borrows $n$ parallel $\hbar k$ quanta from zero point vector fields provided $n$ is integral. We can see this by letting $k \rightarrow \infty$ allowing energy $E \rightarrow nh\omega$ by absorbing $n$ quanta $\hbar\omega$ from the zero point vector fields (section 2.3.2). As spin 3 needs at least three spin 1 particles to create it, the lowest integral number $n$ can be is 3. The virtual $l = 3$ property can however be used to derive the magnetic moment of a charged spin ½, $m = \pm 1/2$ state as a function of $n$. Section 3.5 shows $g = 2$ Dirac electrons need an average (over integral $n$ states) of $\bar{n} \approx 6.0135$.

Three member superpositions $\psi_k = \sum c_n \psi_{nk}$ with $n = 5, 6, & 7$ achieve this, creating Dirac spin ½ states. We also find that $n = 6$ is the dominant member and each superposition $\psi_k$
needs at least three members to make all the equations consistent for Dirac particles. Secondary interactions at any wavenumber $k$ can occur with eigenvalues $|P| = nhk$ by $\pm hk$ where this can be only a temporary rearrangement of the triplets of values of $n$. This is true, whether the interaction is with leptons, quarks, photons, gluons, W & Z particles, or gravitons. (Section 3.3)

1.2.4 Superposition wavefunctions require only squared vector potentials

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18)Y'(\theta, \phi)$ requires an invariant in all coordinates spherically symmetric squared vector potential to create it: $Q^2 A^2 = n^4 h^2 k^4 r^2 / 81$. There are no linear potential terms in contrast with secondary interactions. The primary interaction operator is $\hat{P}^2 = -\hbar^2 \nabla^2 + Q^2 A^2$, with no linear potential terms included and $Q$ simply represents a collective symbol for all the effective charges concerned. As an example, the dominant $n = 6$ wavefunction of a spin $\frac{1}{2}$ Dirac $\psi_k$ requires a squared vector potential of $Q^2 A^2 = n^4 h^2 k^4 r^2 / 81 = 16h^2 k^4 r^2$ (section 2.3.1). Primary coupling between the eight virtual preons and the colour, electromagnetic and gravitational zero-point fields produces a vector potential squared value for all infinite superpositions which can be expressed as:

$$Q^2 A^2 = \frac{8 + 8\sqrt{\alpha_{\text{EM}}^c} + im_0 \sqrt{G_p I (2\hbar c)}}{3\pi (sN)(1 + \varepsilon)} \left( \frac{\hbar^2 k^4 r^2}{k} \right) \frac{(sN)(1 + \varepsilon)dk}{k}$$

(Where the length of the complex vector is simply squared here.) The significance of the cancelling top and bottom factors $(sN)$ is explained in section 2.1.2. Also the cancelling $(1 + \varepsilon)$ factors are due to gravity and explained in section 4.2. The primary colour coupling amplitude is conjectured to be 1 to each of the eight preons, and $\sqrt{\alpha_{\text{EM}}^c}$ the primary electromagnetic coupling. This equation applies regardless of the individual preon colour or electric charge signs, whether positive or negative (section 2.2.3). The primary gravitational coupling is to the particle mass $m_0$. The primary gravitational constant is $G_p$ divided by $\hbar c$ to put it in the same form as the other two coupling constants. The magnitude of the total angular momentum vector of the infinite superposition is $|L_{\text{total}}| = \sqrt{s(s+1)}$. This $Q^2 A^2$ without the gravity term generates superpositions with probability $(N \cdot s) dk / k$, where $s$ is the superposition spin, $N = 1$ for massive spin $\frac{1}{2}$ fermion & massive boson superpositions, but $N = 2$ for infinitesimal mass boson superpositions (Table 4.3.1 section 6 and its subsections cover this more fully). Section 4.2 includes gravity raising the superposition probability to $(1 + \varepsilon)(N \cdot s) dk / k$ where the infinitesimal $\varepsilon$ (not to be confused with infinitesimal mass) is $\varepsilon \approx 2m_0 / \text{Spin} \approx 7 \times 10^{-45}$ for electrons, and $\varepsilon \approx 10^{-34}$ for a $Z^0$ in Planck units $\hbar = c = G = 1$. The $\psi_k$ superpositions require at least three integral $n$ members. The following three member superpositions fit the SM best (see Table 4.3.1).

Spin $\frac{1}{2}$ massive $N = 1$ fermion superpositions

$$\psi_k = \sum_{n=5,6,7} c_n \psi_{nk}$$

Spin 1 massive $N = 1$ boson superpositions

$$\psi_k = \sum_{n=4,5,6} c_n \psi_{nk}$$
Spins 1 & 2 infinitesimal mass $N = 2$ boson superpositions $\psi_k = \sum_{n=3,4,5} c_n\psi_{nk}$. 

Below are infinite superpositions $|\psi_{\infty,m}\rangle$ for only spins $\frac{1}{2}$ & 1. The symbol $\infty$ refers to the infinite sum, $s$ the spin of the resulting real particle, $m$ its angular momentum state, and $ss$ a spherically symmetric state. Section 3.1.3 explains this format. Also, square cutoffs in wavenumber $k$ are used here for simplicity. Infinitesimal mass superpositions are introduced in section 6.2. (Complex number factors are not included here for clarity.)

$$\psi_{\infty,m} = \sum_{n=3,4,5} c_n\left(\sum_{k=0}^{k\text{(cutoff)}} \frac{1+\epsilon}{2k} \frac{k c_n}{\gamma_{nk}} + \beta_{nk} |\psi_{nk,4n}\rangle\right).$$ (1.1.1)

In these infinite superpositions the probability that the wavefunction is spherically symmetric is always $\gamma_{nk}^2 = 1 - \beta_{nk}^2$ and the probability that it is an $m$ state is $\beta_{nk}^2$, where $\beta_{nk}$ is the magnitude of the velocity of the centre of momentum frame (see Figure 3.1.1), which is where the primary interactions that generate each $\psi_{nk}$ take place. This is similar to the superposition of time and spatially polarized virtual photons in QED. For example, spin $\frac{1}{2}$ has probabilities of $\gamma_{nk}^2 = 1 - \beta_{nk}^2$ spherically symmetric $\psi_{nk}$ wavefunctions, and $\beta_{nk}^2 \times (\psi_{nk}, m = \pm 2)$ wavefunctions. Each $\psi_k$ is normalized to one but the infinite superpositions $\psi_{\infty,m}$ are not normalized, diverging logarithmically with $k$; the same logarithmic divergence that applies to virtual photon emission. (Real wavefunctions must be normalized to one as they refer to finding a real particle somewhere, but this need not apply here.) Section 3.1 finds that $m = \pm 2$ virtual wavefunctions have $\beta_{nk}^2$ probability of leaving an $m = -2$ debt. Integrating over all $k$ produces a total angular momentum for a spin $\frac{1}{2}$ state of $\hbar/2$. (The procedures for spin 1 & spin 2 particles are covered in section 3.2.2.)

The first half of this paper is about the primary interactions between spin zero preons and spin one quanta that build the fundamental particles. The SM is about the secondary interactions between them. (The weak force is only between spin $\frac{1}{2}$ particles and thus a secondary interaction. It cannot be involved in primary interactions.) Apart from infinitesimal effects, such as infinitesimal masses, the properties of fundamental particles covered in this paper should be consistent with their SM counterparts. All $N = 1$ & $N = 2$ superpositions as in Table 4.3.1 are conjectured to cutoff at Planck energy $E_p$. If this is so, both colour and electromagnetic interaction energies must cutoff at $E_p/\langle n \rangle \approx 2.03 \times 10^{18}$ GeV., or $\approx 1/6$ of the Planck energy. (The expectation value $\langle n \rangle$ is $\approx 6.0135$ for spin $\frac{1}{2}$ leptons and quarks Eq. (3.5.16)). The electromagnetic and colour coupling constants at this cutoff are consistent with SM predictions assuming three families of fermions and one Higgs field. (See Figure 4.1.1 & Figure 4.1.2).
Part 1

Fundamental Particles as Infinite Superpositions

2 Building Infinite Virtual Superpositions

2.1 The Possibility of Infinite Superpositions

2.1.1 Early ideas

After World War II there was still much confusion about QED. In 1947 at the Long Island Conference the results of the Lamb shift experiment were announced [21]. This conference was perhaps the starting point for the development of modern QED: perhaps the pinnacle of accurate theory supported by experiment. QED is also about what we have called secondary interactions. (See 1.2.2.) Part 1 of this paper is about the much simpler primary interactions and we start it with an oversimplified semi-classical way of explaining the Lamb shift. We are going to imagine that the Lamb shift involves primary interactions when, in fact, it doesn’t. It is a real world secondary interaction experiment, and therefore our illustration is not the correct QED way of handling this phenomenon. Picturing it as a primary interaction however, with zero point fields, may help illustrate the possibility of connections between fundamental particles and infinite virtual superpositions. Hopefully this is in a similar manner to the way Bohr’s original simple semi-classical explanation of quantized atomic energy levels played such a large part in the eventual development of full three dimensional wavefunction solutions of atoms, and quantum mechanics.

The density of transverse modes of waves at frequency $\omega$ is $\omega^2 d\omega / \pi^2 c^3$ and the zero point energy for each of these modes is $\hbar \omega / 2$. The electrostatic and magnetic energy densities in electromagnetic waves are equal, thus for electromagnetic zero point fields:

$$\frac{\varepsilon_0 E^2}{2} + \frac{\varepsilon_0 c^2 B^2}{2} = \frac{\hbar \omega}{2} \left[ \frac{\omega^2 d\omega}{\pi^2 c^3} \right] \text{ or } \varepsilon_0 E^2 = \varepsilon_0 c^2 B^2 = \frac{\hbar \omega^3 d\omega}{2\pi^2 c^5 \omega}.$$  

For a fundamental charge $e$ using $\alpha = e^2 / 4\pi e_0 \hbar c$, and provided $\beta << 1$, this gives an average force squared of

$$F^2 = e^2 E^2 = \frac{2\alpha \hbar \omega^3}{c^2} \frac{d\omega}{\omega}$$  \hspace{1cm} (2.1.1)

Thinking semi-classically, for an electron of rest mass $m$ this can generate simple harmonic motion of amplitude $r$, where $F^2 = m^2 \omega^2 r^2$ (if $\beta << 1$). Solving for $r^2$ (where $r^2$ is superimposed on the normal quantum mechanical electron orbit, $\lambda_c = \hbar / mc$ is the Compton wavelength, and $k = \omega / c$):

$$r^2 = \frac{\hbar^2}{m^2 c^2} \frac{2\alpha d\omega}{c^2} = \left[ \frac{\lambda_c^2}{\pi} \right] \left[ \frac{2\alpha}{\pi} \frac{d\omega}{k} \right]$$

Integrating $r^2$ (as directions are random): $r^2_{Total} = \frac{\lambda_c^2}{\pi} \frac{2\alpha}{k_{\min}} \int_{k_{\min}}^{k_{max}} \frac{dk}{k} = \frac{\lambda_c^2}{\pi} 2\alpha \log(k_{max} / k_{min})$. 

15
The minimum and maximum values for \( k \) can be chosen to fit atomic orbits, and a root mean square value for \( r \) can be found. Combining this with the small probability that the electron will be found in the nucleus, this small root mean square deviation shifts the average potential by approximately the Lamb shift. This can also be thought of as simple harmonic motion of amplitude \( \approx \frac{\hbar}{c} \), occurring with probability \( (2\alpha / \pi)dk/k \). It can also be interpreted as the electron recoiling by \( \approx \frac{\hbar}{c} \), (provided \( \beta_{\text{Recoil}} \ll 1 \)) in random directions due to virtual photon emission with a probability of \( (2\alpha / \pi)dk/k \).

### 2.1.2 Dividing probabilities into the product of two component parts

This probability \( (2\alpha / \pi)dk/k \) can be thought of as the product of two terms \( A \& B \), where \( A \) includes the electromagnetic coupling constant \( \alpha \), \( B \) includes \( dk/k \), and \( AB=(2\alpha / \pi)dk/k \). This suggests that this same behaviour is possible if we have an appropriate superposition of virtual wavefunctions occurring with probability \( B \), which emits virtual photons with probability \( A \) (by changing eigenvalues \( |p_{nk}|=n\hbar k \) by \( n=\pm 1 \)). For example, if a virtual superposition occurs with probability \( B=(N\cdot s)dk/k \), and has a virtual photon emission probability for each member of these superpositions of \( A=(N\cdot s)^{-1}(2\alpha / \pi) \), then the overall virtual photon emission probability remains as above at \( AB=(2\alpha / \pi)dk/k \).

In section 1.2.4 we said that these wavefunctions are built with squared vector potentials. If superpositions of them are to represent real particles they must be able to exist anywhere. This is possible only if they are generated by invariant fields. The only fields uniform in space-time are the zero point fields and looking at the electromagnetic field first we can use section 2.1.1 above. Consider a vector \( r \) from some central origin \( O \) and a magnetic field vector \( B \) through origin \( O \), then the vector potential at point \( r \) is \( A=(B\times r)/2 \) and the vector potential squared is \( A^2=(B^2r^2\sin^2\theta)/4 \) where the angle between vectors \( B \& r \) is \( \theta \).

As \( \sin^2\theta \) averages \( 2/3 \) over a sphere: \( A^2 = B^2r^2/6 \) \hspace{1cm} (2.1.2)

This requires the source of these fields to be spherically symmetric, where \( B^2 \) here is the magnetic field squared at any point due to the invariant cubic intensity of zero point electromagnetic fields, also as in section 2.1.1. This is only true at higher frequencies, and we will find later that at cosmic wavelengths we need a similarly invariant spherically symmetric source redshifted from the receding spherical horizon. Putting Eqs. (2.1.1) and (2.1.2) together the vector potential squared is

\[
\frac{e^2A^2}{6} = \frac{e^2B^2r^2}{6} = \frac{\alpha\hbar^2\omega^4r^2}{3\pi c^4} \frac{d\omega}{\omega} = \frac{\alpha\hbar^2k^4r^2}{3\pi} \frac{dk}{k}\]

\hspace{1cm} (2.1.3)
As in section 2.1.2 we can divide this into two parts, noting the inclusion of spin \( s \) and integer \( N \) in the numerator and denominator:

\[
\frac{e^2 A^2}{\alpha} = \left[ \frac{\alpha}{3\pi sN} \hbar^2 k^4 r^2 \right] \left[ \frac{sN \cdot dk}{k} \right]
\]  

(2.1.4)

But here a vector potential squared term \( \left[ \frac{\alpha}{3\pi sN} \hbar^2 k^4 r^2 \right] \) occurs with probability \( \left[ \frac{sN \cdot dk}{k} \right] \).

Another way of looking at this is that a wavefunction \( \psi_k \) that is generated by a vector potential squared term \( \left[ \frac{\alpha}{3\pi sN} \hbar^2 k^4 r^2 \right] \) can occur with \( \left[ \frac{sN \cdot dk}{k} \right] \) probability.

This is similar reasoning to that used in the semi-classical Lamb shift explanation of section 2.1.1. In the first bracketed term of Eq. (2.1.4), \( \alpha \) is the electromagnetic coupling constant, but the same logic applies for the eight gluon and gravitational zero point vector fields where we will sum appropriate amplitudes of these and square this total as our effective coupling constant in Eq. (2.1.4). But first we need to look at groups of spin zero preons that could build these wavefunctions. What mixtures of colours and electrical charges end up with the appropriate final colour and electrical charge for each of the fundamental particles or at least the ones we know of?

### 2.2 Spin Zero Virtual Preons from a Higgs Type Scalar Field

#### 2.2.1 Groups of eight preons that form superpositions

In this paper preons have zero spin and can have no weak charge. The only fields they can interact with (via primary interactions that build superpositions as in section 1.2.2) are colour, electromagnetic and gravity. In the simplest world there would be just one type of preon that comes in three colours, always positively charged say, with their three anti colours all negatively charged. We will indeed find that this seems to work. Looking at Table 2.2.1 we see that a minimum of 6 preons is required to get the correct charge ratios of 3:2:1 between electrons, and up and down quarks. To get vector potential squared values that make all our equations work however, we need to couple to all eight gluon fields requiring a total of eight preons. Table 2.2.1 has all the basic properties required to build infinite superpositions for the fundamental particles. We need to remember when looking at this table that from section 1.2.2 the effective secondary charge is much less than the primary charge and we have no idea yet of the effective value of the primary preon electric charge. Particles only are addressed in the groups of preons in Table 2.2.1. The first point to notice, however, is that both the electron and the \( W^- \) are predominantly antipreons, yet they are both defined as particles. Have we got something wrong? When we look at relativistic masses in section 3.2.1 we get the usual plus and minus solutions and Feynman showed us how to interpret the negative solutions as antiparticles.
Table 2.2.1 Groups of eight virtual preons forming the fundamental particles. The electric charges we measure in the real world are one sixth of the group electric charges in this table. The Higgs boson is discussed in section 11.3.5. If it is a superposition it would be in the neutral group at the top.

<table>
<thead>
<tr>
<th>Fundamental Particles</th>
<th>Preon colour</th>
<th>Preon electric charge</th>
<th>Group colour</th>
<th>Group electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin ½ Neutrino family</td>
<td>Any colour + its Anticolour</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spin 1</td>
<td>Red</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Photons, $Z_0$ &amp; Neutral gluons Spin 1 gravitons</td>
<td>Antired</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Green</td>
<td>1</td>
<td>Colourless</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Antigreen</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Blue</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antiblue</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spin ½ Electron family</td>
<td>Any colour + its Anticolour</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antired</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antired</td>
<td>-1</td>
<td>Colourless</td>
<td>-6</td>
</tr>
<tr>
<td>Spin 1 $W^-$</td>
<td>Antigreen</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antiblue</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antiblue</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spin ½ Blue up quark Family</td>
<td>Red</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antired</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Green</td>
<td>1</td>
<td>Blue</td>
<td>+4</td>
</tr>
<tr>
<td></td>
<td>Antigreen</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Green</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Blue</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Blue</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Red</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spin ½ Red down Quark family</td>
<td>Green</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antigreen</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Red</td>
<td>1</td>
<td>Red</td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td>Antired</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Green</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antigreen</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antiblue</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antigreen</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spin 1 Red to Green Gluons</td>
<td>Red</td>
<td>1</td>
<td>Red plus</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Antigreen</td>
<td>-1</td>
<td>Antigreen</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Red</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antired</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Green</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antigreen</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Blue</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Antiblue</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If this also applies in anti preons then because they are zero spin, and the weak force discriminates between particles and antiparticles by their helicity, this discrimination can apply only in secondary interactions. The preon antipreon content of the groups in Table 2.2.1 does not necessarily tell us whether they produce particles or antiparticles. We will discuss this further in section 3.2.1; also, as of now, there is still no good understanding of the predominance of matter over antimatter in our universe. In Table 2.2.1 only one example of colour is given for quarks and gluons. Different colours can be obtained by simply changing appropriate preon colours. Various combinations of eight preons in this table are borrowed from a scalar field for time $\Delta T \leq h/2\Delta E$, this process continually repeating in time. Conservation of charge normally allows only opposite sign pairs of electric charges to appear out of the vacuum. Let us imagine that these virtual preons are building an electron, for example, whose electric charge exists continually unless it meets a positron and is annihilated. This charged electron is thus due to a continuous appearance out of and back into the vacuum of virtual charged preons in a steady state process existing for the life of the superposition, and not conflicting with conservation of charge. If the electron itself does not conflict, then neither do the borrowed preons that build it.

2.2.2 Primary coupling constants behave differently and are constant

QED informs us that the bare (electric) charge of an electron, for example, increases logarithmically inversely with radius from its centre. Polarizations of the vacuum (of virtual charged pairs) progressively shield the bare charge from a radius of approximately one Compton radius $\lambda_c$ inwards towards the centre. When an electron (for example) is created in some interaction the full bare charge is exposed for an infinitesimal time.

Instantaneously after its creation, shielding due to polarization of the vacuum builds progressively outward from the centre of its creation at the velocity of light. For radii $\geq \lambda_c$ we measure the usual fundamental charge $e$. There are similar but more complicated processes that occur to the colour charge. Camouflage is the dominant one where the colour charge grows with radius as the emitted gluons themselves have colour charge. At the instant of their birth the preons are bare and at this time, $t = 0$ say, all the zero point vector fields can act on these bare colour and electric charges as there is simply no time for shielding and other effects to build. The primary coupling constants that we use must consequently be the same for all values of $k$, in complete contrast to those for secondary interactions. We don’t know what this primary electromagnetic coupling constant is, so we will just call it $\alpha_{EMP}$. Also, we will find that to get any sense out of our equations the primary colour coupling has to be very close to 1. A coupling of one is a natural number and simply reflects certainty of coupling. Provided the secondary colour coupling can be in line with the SM, and there does not seem to be any other good reason to pick a number less than 1, we will make the (apparently arbitrary) assumption that the bare primary colour coupling is exactly 1. (In section 4.1.1 we will find that this seems to be consistent with the SM.)
2.2.3  Primary interactions also behave differently

Let us define a frame in which the central origin of the wavefunctions \( \psi_k \) of our infinite superposition is at rest. The laboratory or rest frame we will refer to as the LF. The preons that build each \( \psi_k \) are born from a Higg’s type scalar field with zero momentum in this frame. This has very relevant consequences as their wavelength is infinite in this rest frame at time \( t = 0 \), and after they become wavefunction \( \psi_k \) their wavelength is of the order \( 1/k \) for times \( 0 < t < \hbar/2E \). This implies that there could possibly be significant differences in the way amplitudes are handled between primary and secondary interactions.

Let us consider secondary interactions first with an electron and positron, for example, located approximately distance \( r \) apart. For photon wavelengths \( << r \) both the electron and the positron each emit virtual photons with probabilities proportional to \( \alpha \), but for wavelengths \( >> r \) their amplitudes cancel. Returning to primary interactions, zero momentum preons must always have an infinite wavelength which is greater than the wavelengths (or \( 1/k \) values) of the zero point quanta they interact with, for all \( k \neq 0 \). This implies that we cannot simply add or subtract amplitudes algebraically as the charged preons can be always further apart than the wavelength of the interacting quanta (except when \( k = 0 \), but we will see there is always a minimum \( k \) value, i.e. \( k_{\text{min}} > 0 \) in sections 5 & 6). In fact, if algebraic addition of amplitudes did apply in primary interactions, infinite superpositions for colourless and electrically neutral neutrinos would be impossible. So how can infinitely far apart preons of differing charge generate wavefunctions of all dimensions down to Planck scale? This can happen only if the amplitudes of all eight preons are somehow linked over infinite space, all at the same time \( t = 0 \) contributing to generating the wavefunction \( \psi_k \). This non-local behaviour is not new.

All experiments confirm that what Einstein struggled to come to terms with is, in fact, true; he called it “spooky action at a distance”. While these experiments are currently limited in the distance over which they demonstrate entanglement, there is now wide acceptance that it can reach across the universe. In the same manner wavefunctions covering all space can instantly collapse. We want to suggest that this same non-locality applies in primary interactions; our eight virtual preons all unite instantaneously at time \( t = 0 \) across infinite space in generating each \( \psi_k \). Also, the vector potential squared equations that they generate must always be the same for all the preon combinations in Table 2.2.1. This can happen only if the amplitudes of all eight are added, regardless of charge sign for primary interactions. This applies to both colour and electric charge.

The opposite is true for the secondary interactions. At time \( t = 0 \) all eight preons instantaneously collapse into some sort of virtual composite particle that for times \( 0 < t < \hbar/2E \) obeys wavefunction \( \psi_k \). The dimensions of \( \psi_k \) are of the same order as the wavelength of the interacting quanta, and the usual algebraic total electric charge and nett colour charge must now apply as in the group charges in Table 2.2.1. All of this may seem contrary to current thinking which has gradually been built up over several centuries of secondary interaction experiments; however, it may not be so out of place when viewed in the context of the counter intuitive results of entanglement experiments. The key point to bear in mind is that the predictions of this paper must agree or at least be able to fit the SM, or
secondary interaction experiments; as we may never be able to look into virtual primary interactions, but only observe their effects.

Amplitudes to interact are complex numbers which we can draw as a vector. This applies to both colour and electric coupling, where these two vectors can be at the same complex angle or at different angles. The simplest case is if they are in line and we will assume this is true for both colour and electromagnetic primary interactions which are both spin 1. This seems to work and when we later include gravity, a spin 2 interaction, we find that the spin 2 vector only works if it is at right angles to the two in line spin 1 vectors. Let us start in a zero gravity world by simply adding the eight preon colour vectors of amplitude one and the eight primary electromagnetic vectors of amplitude $\sqrt{\alpha_{\text{EMP}}}$ together, as all this only works if they are all in line.

The total colour plus electromagnetic primary amplitude is $8 + 8\sqrt{\alpha_{\text{EMP}}}$ (2.2.1)

This equation is always true regardless of signs as in section 2.2.3

The colour plus electromagnetic primary coupling constant is $\left(8 + 8\sqrt{\alpha_{\text{EMP}}}\right)^2$ (2.2.2)

Inserting this into Eq. (2.1.4) we get

$$Q^2A^2 = \left[\frac{8 + 8\sqrt{\alpha_{\text{EMP}}}}{3\pi sN}\right]^2 h^2 k^4 r^2 \left[\frac{sN \cdot dk}{k}\right]$$

Again we interpret this just as we did in section 2.1.2 and Eq. (2.1.4) as a vector potential squared term

$$Q^2A^2 = \left[\frac{8 + 8\sqrt{\alpha_{\text{EMP}}}}{3\pi sN}\right]^2 h^2 k^4 r^2 \text{ occurring with probability } = \frac{sN \cdot dk}{k}$$

Where $Q$ is a symbol representing the entire eight colour and eight electric amplitudes combined, with $s$ the spin and $N = 1$ for massive superpositions, but $N = 2$ for infinitesimal mass superpositions. (Table 4.3.1, section 6 and its subsections cover this more fully.)

2.3 Virtual Wavefunctions that form Infinite Superpositions

2.3.1 Infinite families of similar virtual wavefunctions

Consider the family of wave functions where ignoring time:

$$\psi_{nk} = U(nrk)Y(\theta \phi)$$

$$U(nrk) = C_{nk} r^l \exp(-n^2 k^2 r^2 / 18)$$

(2.3.1)
$U(nrk)$ is the radial part of $\psi_{nk}, Y(\theta\phi)$ the angular part, $C_{nk}$ a normalizing constant, and we will find that $l$ is the usual angular momentum quantum number. There is an infinite family of $\psi_{nk}$, one for each value $k$ where $0 < k < \infty$ in a zero gravity world.

Now put $R(nrk) = rU(nrk) = C_{nk} r^{l+1} \exp(-n^2 k^2 r^2 / 18)$ \hfill (2.3.2)

As we are dealing with zero spin preons we use Klein-Gordon equations [22]. The Klein-Gordon equation is based on the relativistic equation $p^2 = E^2 / c^2 - m_0^2 c^2$ and in a spherically symmetric squared vector potential the time independent Klein Gordon Equation is

$$\hat{P}^2 \psi = -\hbar^2 \nabla^2 \psi + Q^2 A^2 \psi = \left[ \frac{E^2}{c^2} - m_0^2 c^2 \right] \psi$$ \hfill (2.3.3)

Using

$$\frac{\nabla^2 \psi}{\psi} = \frac{1}{R} \frac{\partial^2 R}{\partial r^2} - \frac{l(l+1)}{r^2}$$

we get the time independent radial Klein Gordon equation

$$\frac{\hbar^2}{R} \frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + Q^2 A^2 = \left[ \frac{E^2}{c^2} - m_0^2 c^2 \right]$$ \hfill (2.3.4)

For each $\psi_{nk}$ the energy is $E_{nk}$ a function of $n\& k$, and we will label the rest mass as $m_{nk}$ a function of spin $s, n\& k$, but also a function of the particle rest mass $m_0$. Using different colours to more clearly compare the next two equations this becomes

$$\frac{\hbar^2}{R} \frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + Q^2 A^2 = \left[ \frac{E_{nk}^2}{c^2} - m_{nk}^2 c^4 \right]$$ \hfill (2.3.5)

Differentiating $R(nrk) = rU(nrk) = C_{nk} r^{l+1} \exp(-n^2 k^2 r^2 / 18)$ twice with respect to $r$, multiplying by $\hbar^2$ and dividing by $R$

$$\frac{\hbar^2}{R} \frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + \frac{n^2 \hbar^2 k^4 r^2}{81} - \frac{(2l+3)n^2 \hbar^2 k^2}{9}$$ \hfill (2.3.6)

Comparing Eqs. (2.3.5) & (2.3.6) we see that $l$ is the usual angular momentum quantum number and the vector potential squared required to generate these wavefunctions is

$$Q^2 A^2 = \frac{n^4 \hbar^2 k^4 r^2}{81} = \left[ \frac{n}{3} \right] \hbar^2 k^4 r^2$$ \hfill (2.3.7)
The momentum squared is \[ p^2_{nk} = \frac{E^2_{nk}}{c^2} - m^2_{0\text{out}} c^2 = \frac{(2l + 3)n^2 \hbar^2 k^2}{9} \] (2.3.8)

For \( l = 3 \) wavefunctions this becomes \[ p^2_{nk} = n^2 \hbar^2 k^2 \quad \& \quad |p_{nk}| = n\hbar k \] (2.3.9)

2.3.2 Eigenvalues of these virtual wavefunctions and parallel momentum vectors

From Eqs. (2.3.8) & (2.3.9) as \( k \to \infty \), the energy squared \( E^2_{nk} \to p^2_{nk} c^2 = n^2 \hbar^2 \omega^2 \) and thus if \( l = 3 \) when \( k \to \infty \) energy \( E_{nk} \to n\hbar \omega \) (considering only the positive solution).

(2.3.10)

This suggests that \( n \) must be integral. If it is integral when \( k \to \infty \), we will conjecture that it must be integral for all values of \( k \). This is a virtual or “off shell” process, where energy can depart from \( E^2 = m^2_{0\text{out}} c^4 + \mathbf{p}^2 c^2 \) for time \( \Delta t \approx \hbar / 2\Delta E \). We can also perhaps think of Eq. (2.3.9) as integral \( n \) parallel momentum vector \( |p| = \hbar k \) quanta, transferring total momentum \( |p_{nk}| = n\hbar k \) and energy \( E \leq n\hbar \omega \) from the zero point fields to generate the virtual wavefunction \( \psi_{nk} \). Using different colours for both operator and wavefunction, we can say that provided \( Q^2 A^2 = (n / 3)^4 \hbar^2 k^4 r^2 \) as in Eq. (2.3.7) the operator \( \hat{P}^2 = (-n^2 \nabla^2 + Q^2 A^2) \) applied to the virtual wavefunction \( \psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18)Y(\theta \phi) \) produces \( \hat{P}^2 |\psi_{nk}\rangle = (-n^2 \nabla^2 + Q^2 A^2) |\psi_{nk}\rangle = n^2 \hbar^2 k^2 |\psi_{nk}\rangle \), where \( n \) is integral, but \( k \) is continuous as for free particles. Thus, we conjecture that:

\[ \psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18)Y(\theta \phi) \] are eigenfunctions with eigenvalues \( p^2_{nk} = n^2 \hbar^2 k^2 \) with continuous \( k \) but integral \( n \).

Also, there are no scalar potentials involved, only squared vector potentials, so this is a magnetic or vector type interaction. Particles in classical magnetic fields have a constant magnitude of linear momentum which is consistent with the squared momentum eigenvalues of Eq. (2.3.11). This also implies that each \( \psi_{nk} \) is formed from quanta of wave number \( k \) only and that secondary interactions with \( \psi_{nk} \) emit or absorb \( |\hbar k| \) virtual quanta if \( n \) changes by \( \pm 1 \). The wavefunction \( \psi_{nk} \) is virtual and in this sense both the energy \( E_{nk} \) and rest mass \( m_{0\text{out}} \) in Eq. (2.3.8) are also virtual quantities borrowed from zero point vector fields and its time component or a scalar Higgs type field. We use these virtual quantities to calculate the amplitude that the wavefunction \( \psi_{nk} \) is in an \( m \) state of angular momentum in section 3.1, and in section 3.2 to calculate the total angular momentum and rest mass. As in section 3.2 above, we can think of \( |p_{nk}| = n\hbar k \) as \( n \) parallel momentum vectors \( |p| = \hbar k \). As spin 3 (or \( l = 3 \)) needs at least three spin 1 quanta to build it, \( n \) must be at least 3. When \( n = 3 \) we can think of this as three of the eight preons each absorbing quanta \( |\hbar k| \) at time \( t = 0 \). We will find that a spin \( \frac{1}{2} \) state has a dominant \( n = 6 \) eigenfunction where six of the eight preons each absorb quanta \( |\hbar k| \). It needs at least two smaller side eigenfunctions \( n = 5 \) & \( n = 7 \) with either five or seven respectively, of the eight preons each absorbing quanta \( |\hbar k| \) respectively at \( t = 0 \). (Figure 3.1.4 illustrates the three \( n \) modes of a positron superposition.)
From Eq. (2.3.7) \( Q^2 A^2 = \left( \frac{n}{3} \right)^4 \hbar^2 k^4 r^2 = 16 \hbar^2 k^4 r^2 \) for this dominant \( n = 6 \) mode.

Thus using Eq. (2.2.4) \( Q^2 A^2 = \frac{\left[ 8 + 8\sqrt{\alpha_{EMP}} \right]^2}{3\pi sN} \hbar^2 k^4 r^2 = 16 \hbar^2 k^4 r^2 \) for an \( n = 6 \) mode.

Now \( s = 1/2 \) & \( N = 1 \) for spin \( 1/2 \) fermions and \( \frac{2 \left[ 8 + 8\sqrt{\alpha_{EMP}} \right]^2}{3\pi} = 16 \) if we have only an \( n = 6 \) mode. Thus \( 8 + 8\sqrt{\alpha_{EMP}} = \sqrt{24\pi} \) and \( \alpha_{EMP}^{-1} \approx 137.1 \), but this is true for an \( n = 6 \) eigenfunction only, and we have a superposition where the amplitudes of the smaller side eigenfunctions \( n = 5 \) & \( n = 7 \) determine the ratio between the primary to secondary (colour and electromagnetic) coupling amplitudes or the value of \( \alpha_{s}^{-1} \@ \text{cutoff} \) (Section 3.3). The \( Q^2 A^2 \) required to produce this superposition with amplitudes \( c_n \) is, using Eq. (2.3.7)

\[
Q^2 A^2 = \sum_{n=5,6,7} c_n \neq c_n \frac{n^4 \hbar^2 k^4 r^2}{81} \tag{2.3.12}
\]

Repeating the same procedure as above for three member superpositions using Eq. (2.3.12) we find the strength of \( \alpha_{EMP} \) required increases considerably (see section 4.1 & Table 4.1.1). As the secondary electromagnetic coupling \( \alpha_{EMS}^{-1} \@ \text{cutoff} \) must be constant for all spin \( 1/2 \) leptons and quarks, the amplitudes of the smaller side eigenfunctions \( n = 5 \) & \( n = 7 \) that determine this must also be constant for all the fermions, implying that Eq. (2.3.12) must be the same for all fermions. The same arguments apply to the other groups of fundamental particles but we return to this in sections 3.3 where we see that the same also applies with graviton emission.

### 3 Properties of Infinite Superpositions

#### 3.1 The Amplitude that Wavefunction \( \psi_{nk} \) is Spherically Symmetric

##### 3.1.1 Four vector transformations

The rules of quantum mechanics tell us that if we carry out any measurement on a real spherically symmetric \( l = 3 \) wavefunction it will immediately fall into one of the seven possible states \( l = 3, m = 0, \pm 1, \pm 2, \pm 3 \) [23]. But \( \psi_{nk} \) is a virtual \( l = 3 \) wave function so we cannot measure its angular momentum. During its brief existence it must always remain in some virtual superposition of the above seven possible states and we can describe only the amplitudes of these. So, is there any way to calculate these amplitudes, as they must relate to the amplitudes of the angular momentum states of the spin 1 quanta it absorbs from the zero point vector fields?
First consider the 4 vector wavefunction of a spin 1 particle and start with a time polarized state which has equal probability of polarization directions. It is thus spherically symmetric, which we will label as \( ss \). Using 4 vector \( (t, x, y, z) \) notation

\[
\text{In frame A, a time polarized or } ss \text{ spin 1 state is } (1,0,0,0).
\]

Let frame B move along the \( z \) axis at velocity \( \beta = v/c \) in the \( z \) direction.

\[
\text{In frame B the polarization state transforms to } (\gamma,0,0,\gamma \beta).
\]

But this is \( \gamma^2 \) time polarized \(| ss \rangle \) states minus \( \gamma^2 \beta^2 \times z \) polarized or \(| m = 0 \rangle \) states

\[
\text{In frame B the probabilities are } \gamma^2|ss\rangle - \gamma^2 \beta^2|m = 0\rangle \text{ states.}
\]

Now \( \gamma^2 - \gamma^2 \beta^2 = \gamma^2(1 - \beta^2) = 1 \) is an invariant probability in all frames and in removing \( \gamma^2 \beta^2 \times m = 0 \) states from \( \gamma^2 \times ss \) states, the new ratio of spherical symmetry is \( \gamma^2 - \gamma^2 \beta^2) / \gamma^2 = 1 - \beta^2 \). Thus, a spherically symmetric state is transformed from probability 1 in frame A, to \( 1 - \beta^2 \) in frame B. Also removing \( m = 0 \) states from spherically symmetric states leaves a surplus of \( m = \pm 1 \) states, as spherically symmetric states are equal superpositions of \(| m = -1 \rangle, |m = 0 \rangle, \& |m = +1 \rangle \) states.

Thus in Frame B the probabilities are \((1 - \beta^2)|ss\rangle + \beta^2|m = \pm 1\rangle \) states. \( (3.1.1) \)

We can describe this as a virtual superposition of \( \frac{1}{\gamma}|ss\rangle + \beta|m = \pm 1\rangle \) states. \( (3.1.2) \)

As \( \beta^2 \to 1 \) we have transverse polarized states, the same as real photons. Now transverse polarized spin 1 states can be either left \((m = -1)\), or right \((m = +1)\) circular polarization, or equal superpositions of \((1/\sqrt{2})|L\rangle + (1/\sqrt{2})|R\rangle \) as in \( x \& y \) polarization. If we think of individual spin zero preons absorbing these spin 1 quanta at \( t = 0 \) they must also have this same \( \beta^2 \) probability of transversely polarized spin 1 states. If they then merge into some composite \( l = 3 \) particle (as in Figure 3.1.4) for time \( 0 < t < h/2E \), the probability of it being in some particular state \((l = 3, m = 0), (l = 3, m = \pm 1), (l = 3, m = \pm 2) \) or \((l = 3, m = \pm 3)\), must be the same \( \beta^2 \). We initially write the amplitudes in these three equations in terms of \( \beta_{nk} \& \gamma_{nk} \) as this is the most convenient way to express them. Velocity operators are momentum operators over relativistic masses. Our eigenvalues are \( p_{nk}^2 = n^2 h^2 k^2 \) for each \( n \& k \), and this allows the velocity operators to give constant \( \beta_{nk}^2 \). Later in Eqs. (3.1.11) and (3.1.12) we write \( \beta_{nk} \& \gamma_{nk} \) in momentum terms. Even though the mass in these operators is virtual, we can still use it to calculate \( |\beta_{nk}| \). For each \( k \) and integral \( n \) there will be a constant \( |\beta_{nk}| \) and \( \gamma_{nk} = (1 - \beta_{nk}^2)^{-1/2} \). As we will see, \( \beta_{nk} \) can be thought of as the magnitude of the velocity of an imaginary centre of momentum frame in which these interactions take place. We will also draw our Feynman diagrams of these interactions in terms of \( \beta_{nk} \& \gamma_{nk} \) for convenience, even though this is unconventional. To proceed from here we define two frames as follows:

1) The Laboratory Frame (LF) or Fixed Frame as in section 2.2.3
The infinite superposition has rest mass $m_0$ and zero nett momentum in this frame. Each $\psi_{nk}$ is centred here with magnitude of momentum $|p_{nk}| = nhk$. Even though we have no idea of the direction of this momentum vector we will define it as the $z$ direction. The eight preons are born in this frame with zero momentum and can thus be considered here as being at rest or with zero velocity and infinite wavelength at their birth. The Feynman diagram of the interaction in this frame that builds $\psi_{nk}$ is illustrated in Figure 3.1.3.

2) The Centre of Momentum Frame (CMF)
This (imaginary) frame is the centre of momentum of the interaction that builds $\psi_{nk}$. The CMF moves at velocity $\beta_{nk}$ relative to the laboratory frame in the $z$ direction or parallel to the unknown momentum vector direction $p_{nk}$. In this CMF the momenta and velocities of the preons at birth and after the interaction are equal and opposite. This is illustrated in Figure 3.1.2 again in terms of $m_0, \beta_{nk}, \gamma_{nk}$. In the LF the velocity of the preons at birth is zero, in the CMF this is $-\beta_{nk}$ and after the interaction $+\beta_{nk}$, where both $-\beta_{nk}$ and $+\beta_{nk}$ are in the unknown $z$ direction. In the LF the particle velocity $\beta_{nk}(\text{particle}) = \beta_{nkp}$ is the simple relativistic addition of the two equal velocities $\beta_{nk}$ as in Figure 3.1.1.

$$\beta_{nk}(\text{Particle}) = \beta_{nkp} = \frac{2\beta_{nk}}{1 + \beta_{nk}^2}$$

$$\beta_{nk}$$

Laboratory Frame

Centre of Momentum Frame

Virtual Particle

Figure 3.1.1 Velocities in unknown but the same directions in different frames.

3.1.2 Feynman diagrams of primary interactions
Let us start with

$$\beta_{nk}(\text{Particle}) = \beta_{nkp} = \frac{2\beta_{nk}}{1 + \beta_{nk}^2} \quad \text{and} \quad \gamma_{nkp} = (1 - \beta_{nkp}^2)^{-1/2} = \gamma_{nk}^2 (1 + \beta_{nk}^2)$$

If the particle rest mass is $m_0$ let each preon have a virtual rest mass $m_0/ (8\gamma_{nk}\sqrt{2s})$.

The eight preons are effectively a virtual particle of rest mass $m_{0nk} = \frac{m_0}{\gamma_{nk}\sqrt{2s}}$.

The particle momentum in the LF is zero at birth. After the interaction using these equations

$$|p_{nk}| = nhk = m_{0nk}\beta_{nkp}\gamma_{nkp}c = \left[ \frac{m_0}{\gamma_{nk}\sqrt{2s}} \right] \left[ \frac{2\beta_{nk}}{1 + \beta_{nk}^2} \right] \left[ \gamma_{nk}^2 (1 + \beta_{nk}^2) \right] c$$
The particle momentum after the interaction in the LF \( |p_{nk}| = n\hbar k = \frac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}} \) (3.1.5)

Using Eq. (3.1.4), in the LF the particle energy at birth is

\[ m_{0\text{nk}}c^2 = \frac{m_0c^2}{\gamma_{nk}\sqrt{2s}} \] (3.1.6)

In the LF the particle energy after the interaction is by using Eq. (3.1.3)

\[ m_{0\text{nk}}\gamma_{nk}c^2 = \frac{m_0c^2}{\gamma_{nk}\sqrt{2s}} (1 + \beta_{nk}^2)c^2 = \frac{m_0\gamma_{nk}c^2}{\sqrt{2s}} (1 + \beta_{nk}^2)c^2 \] (3.1.7)

In the CMF the momentum at birth is using Eq. (3.1.4)

\[ -m_{0\text{nk}}\gamma_{nk}\beta_{nk} = \frac{-m_0\beta_{nk}}{\sqrt{2s}} \] (3.1.8)

In the CMF the momentum after the interaction is equal but in the opposite direction

\[ \frac{+m_0\beta_{nk}}{\sqrt{2s}} \] (3.1.9)

In the CMF the energy at birth, and after the interaction is

\[ m_{0\text{nk}}\gamma_{nk}c^2 = \frac{m_0c^2}{\sqrt{2s}} \] (3.1.10)

These values are all summarized in Figure 3.1.2 and Figure 3.1.3 but with \( c=1 \).

From Eq. (3.1.5) \( |p_{nk}| = n\hbar k = \frac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}} \) and \( \beta_{nk}\gamma_{nk} = \frac{n\hbar k\sqrt{2s}}{2m_0c} = \frac{\lambda_c nk\sqrt{2s}}{2} \)

(\( \lambda_c = \frac{\hbar}{m_0c} \) is the Compton wavelength). We can now express \( \beta_{nk} \) & \( \gamma_{nk} \) in momentum terms:

Let \( K_{nk} = \beta_{nk}\gamma_{nk} = \frac{n\hbar k\sqrt{2s}}{2m_0c} = \frac{\lambda_c nk\sqrt{2s}}{2} \) (3.1.11)

In terms of \( K_{nk} \): \( \beta_{nk}^2 = \frac{K_{nk}^2}{1 + K_{nk}^2} \) and \( \gamma_{nk}^2 = 1 + K_{nk}^2 \) (3.1.12)

Each infinite superposition has fixed \( \lambda_c \). Each wavefunction \( \psi_{nk} \) of this infinite superposition has fixed \( n \& s \), thus \( K_{nk} \propto k \).

For example, we can put \( \frac{dK_{nk}}{K_{nk}} = \frac{dk}{k} \) (3.1.13)

These simple expressions and what follows are not possible if \( m_{0\text{nk}} \neq m_0 / \gamma_{nk}\sqrt{2s} \), and when we include gravity we find \( m_{0\text{nk}} = m_0 / (\gamma_{nk}\sqrt{2s}) \) is essential (section 4.2).
The interaction in the Feynman diagrams above is with spin 1 quanta. The Feynman transition amplitude of this interaction shows that the polarization states of these exchanged quanta is determined by the sum of the components of the initial, plus final 4 momentum $(p_i + p_j)^\mu$. Ignoring all other common factors this says that the space polarized component is the sum of the momentum terms $(p_i + p_j)^\mu$. We have defined our momentum as in an unknown $z$ direction:

\[
\text{Eight preons at birth: } (\frac{m_0}{\sqrt{2s}}, 0, 0, 0, \frac{+m_0}{\sqrt{2s}} \beta_{nk})
\]

\[
q^\mu = (0, 0, 0, 2 \frac{m_0}{\sqrt{2s}} \beta_{nk})
\]

\[
\text{After merging: } (\frac{m_0}{\sqrt{2s}}, 0, 0, 0, \frac{+m_0}{\sqrt{2s}} \gamma_{nk} \beta_{nk})
\]

\[
q'^\mu = (0, 0, 0, 2 \frac{m_0}{\sqrt{2s}} \gamma_{nk} \beta_{nk})
\]

\[
\text{Eight preons at birth: } (\frac{m_0}{\sqrt{2s}}, 0, 0, 0, 0)
\]

The ratio of $z$ polarization to time polarization amplitudes is

\[
\frac{(p_i + p_j)^z}{(p_i + p_j)^0}
\]

In the CMF $(p_i + p_j)^z = 0$, thus an interaction in the CMF exchanges only time polarized, or spherically symmetric $l = 1$ states. In the LF the ratio of $z$ (or $m = 0$) polarization, to time polarization in the LF is $\beta_{nk}^2$,

\[
\frac{(p_i + p_j)^z}{(p_i + p_j)^0} = \frac{2m_0 \gamma_{nk} \beta_{nk}}{2m_0 \gamma_{nk}} = \beta_{nk}
\]

From section 3.1.1 these are probabilities of $\gamma_{nk}^2 |ss\rangle - \gamma_{nk}^2 \beta_{nk}^2 |m = 0\rangle$ states, or as $l = 1$ here $(1 - \beta_{nk}^2) |ss\rangle + \beta_{nk}^2 |m = \pm 1\rangle$ states.
In the LF this is a virtual superposition of \( \left( \frac{1}{\gamma_{nk}} |ss\rangle + \beta_{nk} |m = \pm 1\rangle \right) \) states. (3.1.16)

From section 3.1.1 as these quanta from the scalar and vector zero point fields build each \( \psi_{nk} \) this implies that:

In the LF \( \psi_{nk} \) has virtual superposition amplitudes \( \frac{1}{\gamma_{nk}} |ss\rangle + \beta_{nk} |m \rangle \) states. (3.1.17)

From section 3.1.1 appropriate \( l = 1, m = \pm 1 \) superpositions can build any \( l = 3, m \) state.

Figure 3.1.4 is an example of such a \( \psi_{nk} \) for \( n = 5,6,7 \) \( l = 3, m = +2 \) states.

### 3.1.3 Different ways to express superpositions

We have expressed all superpositions here in terms of spherically symmetric and \( m \) states for convenience and simplicity. We could have expressed them in the form:

\[
\frac{1}{\gamma_{nk}} \sqrt{\frac{1}{7}} \left[ |m = -3\rangle + |m = -2\rangle + |m = -1\rangle + |m = 0\rangle + |m = +1\rangle + |m = +2\rangle + |m = +3\rangle \right] + \beta_{nk} |m = +2\rangle
\]

This is equivalent to (as above we ignore complex number amplitude factors for clarity)

\[
\psi_{nk} = \frac{1}{\gamma_{nk}} |ss\rangle + \beta_{nk} |m = +2\rangle
\]

where we have put \( m = +2 \) in this example.

Because all these wavefunctions are virtual they cannot be measured in the normal way that collapses them into any of these eigenstates, it is more convenient to use the method adopted here which is similar to QED virtual photon superpositions.
Figure 3.1.4  Eight preons forming $m = +2$ states as part of a positron superposition. If a zero spin and zero momentum preon absorbs a quantum its momentum becomes $p = \hbar k$ and its angular momentum becomes either $m = \pm 1$, or an $m = 0$ equal superposition of $m = \pm 1$, states. When it does not absorb a quantum it remains at both spin zero and momentum $p = 0$. There is no significance in which preons have absorbed quanta.
3.2 Mass and Total Angular Momentum of Infinite Superpositions

3.2.1 Total mass of massive infinite superpositions

We will consider first the total mass of an infinite superposition, and to help illustrate, consider only one integral \( n \) eigenfunction \( \psi_{nk} \) at a time; temporarily assuming that the amplitude \( c_n \) of each \( \psi_{nk} \) has magnitude \( |c_n| = 1 \). Each time \( \psi_{nk} \) is born it borrows mass from a scalar Higgs field (or a zero point field time component) and momentum from a zero point field spatial component. The mass that it borrows is exactly cancelled by an equal debt in the Higgs scalar field (or the zero point field time component) so this sums to zero for all \( k \). (This is a different way of looking at what generates mass; however, the end result is identical.) But what about the momenta borrowed from the spatial component of zero point fields, do these momenta also leave momentum debts in the vacuum? At any fixed value of \( k \) the momentum is a constant of the motion in a squared vector potential \( A^2 \). We can think of this as in any particular direction there is some probability of momentum \( p_{nk} = n\hbar k \) due to this \( A^2 \) field. When interacting with the magnetic or the spatial component of any electromagnetic field the velocity squared factor \( \beta_{nk}^2 \) determines the rate of quanta absorbed. Our wavefunctions \( \psi_{nk} \) are generated from a vector potential squared term \( A^2 \) derived in section 2.1.2 which in turn came from a \( B^2 \) type term as in section 2.1.1. As discussed in section 2.3.2 the eigenvalues \( p_{nk}^2 = n^2\hbar^2 k^2 \) confirm the constant momentum squared feature of magnetic, or space mode interactions. Also in section 2.1.1 the scalar virtual photon emission probability is directly related to the force squared term \( F^2 = \epsilon^2 E^2 \). Magnetic type coupling probabilities are related to a magnetic type force squared term \( F^2 = \beta^2 \epsilon^2 B^2 / c^2 = \beta^2 \epsilon^2 E^2 \), where from section 3.1.2 and Eqs. (3.1.14) & (3.1.15) the ratio of this scalar to magnetic coupling is \( \beta_{nk}^2 \). Thus when \( k < \infty \) and the exchanged energy \( E_x \neq \pm \omega \), \( \beta_{nk}^2 n \) quanta \( |nk| \) are absorbed from the vacuum and

\[
\text{we can expect a momentum debt of } p_{nk}(\text{debt}) = -\beta_{nk}^2 n\hbar k
\]  

(3.2.1)

We could sum \( \sum p_{nk}^2 \) & \( \sum p_{nk}(\text{debt}) \) but both vectors \( p_{nk} \) and \( p_{nk}(\text{debt}) \) are antiparallel in the same unknown direction. We can pair them together giving a nett momentum per pair of:

\[
p_{nk}(\text{nett}) = p_{nk} + p_{nk}(\text{debt}) = (1 - \beta_{nk}^2) n\hbar k = n\hbar k / \gamma_{nk}^2 \text{ at wavenumber } k.
\]  

(3.2.2)

We have said above that the mass of each virtual particle is cancelled by an equal and opposite debt in the Higgs scalar field so we can now use the relativistic energy expression

\[
E_n^2 = \sum_{k=0}^{k=\infty} p_{nk}(\text{nett})^2 c^2 \text{ times the probability of each pair at each wavenumber } k.
\]

We will initially look at only \( N = 1 \) massive infinite superpositions in Eq. (2.2.4). Thus, using probability \( sN \cdot dk / k = s \cdot dk / k \), also Eqs. (3.1.11), (3.1.12), (3.1.13), & (3.2.2)
\[ E_{n}^{2} = c^{2} \int_{k=0}^{k=\infty} p_{nk}^{\text{(net)}} \frac{s \cdot dk}{k} = c^{2} \int_{0}^{\infty} n^{2}h^{2}k^{2} s \cdot dk \frac{1}{\gamma_{nk}^{4}} = 4m_{0}^{2}c^{4} \int_{0}^{\infty} \frac{K_{nk}^{2}}{(1 + K_{nk}^{2})^{2}} \frac{dK_{nk}}{2K_{nk}} \]

\[ E_{n}^{2} = m_{0}^{2}c^{4} \left[ \frac{-1}{1 + K_{nk}^{2}} \right]_{0}^{\infty} = m_{0}^{2}c^{4} \text{ or } E_{n} = \pm m_{0}c^{2} \]  

(3.2.3)

This energy is due to summing momenta squared and it must be real, with a mass \( \pm m_{0} \) for infinite superpositions of Eigenfunctions \( \psi_{nk} \). These superpositions can form all the non-infinite mass fundamental particles. The equations do not work if the mass \( m_{0} \) is zero. (We will look at infinitesimal masses in section 6.2.) Negative mass solutions in Eq. (3.2.3) must be handled in the usual Feynman manner, and treated as antiparticles with positive energy going backwards in time. If they are spin \( \frac{1}{2} \) this also determines how they interact with the weak force.

### 3.2.2 Angular momentum of massive infinite superpositions

We will use the same procedure for the total angular momentum of \( N = 1 \) type infinite superpositions with non-infinite mass in Eq. (2.2.4).

Wavefunctions \( \psi_{nk} = C_{m}r^{3} \exp(-n^{2}k^{2}r^{2}/18)Y(\theta, \varphi) \) have angular momentum squared eigenvalues \( L_{z} = mh \). We will treat both angular momentum and angular momentum debts as real just as we did for linear momentum. Even though \( m \) state wavefunctions are part of superpositions they still have probabilities, just as the linear momenta squared above, and it seemed to work. Using exactly the same arguments as in section 3.2.1, if \( \psi_{nk} \) is in a state of angular momentum \( L_{zk} = mh \), then it must leave an angular momentum debt in the vacuum of \( L_{zk}^{\text{(debt)}} = -\beta_{nk}^{2}mh \) (or as in section 3.2.1) \( L_{zk}^{\text{(nett)}} = L_{zk} - L_{zk}^{\text{(debt)}} \).

\[ L_{zk}^{\text{(nett)}} = (1 - \beta_{nk}^{2})mh = (1 - \beta_{nk}^{2})L_{zk} = \frac{L_{zk}}{\gamma_{nk}^{2}} \text{ (if } L_{zk} \text{ is in state } mh) \]

(3.2.4)

But from Eq. (3.1.17) the probability that \( L_{zk} \) is in an \( m \) state is also \( \beta_{nk}^{2} \) so that including this extra \( \beta_{nk}^{2} \) probability term: \( L_{zk}^{\text{(nett)}} = mh \frac{\beta_{zk}^{2}}{\gamma_{nk}^{2}} \) at wavenumber \( k \).

\[ \text{(3.2.5)} \]

For an \( N = 1 \) type infinite superposition \( L_{z}^{\text{(Total)}} = \int_{k=0}^{k=\infty} L_{zk}^{\text{(nett)}} \frac{s \cdot dk}{k} = smh \int_{0}^{\infty} \frac{\beta_{zk}^{2} \frac{dK_{nk}}{dk}}{2k} \]

Using Eqs. (3.1.11) to (3.1.13) \( L_{z}^{\text{(Total)}} = smh \int_{0}^{\infty} \frac{K_{nk}^{2}}{(1 + K_{nk}^{2})^{2}} \frac{dK_{nk}}{K_{nk}} = smh \left[ \frac{-1}{1 + K_{nk}^{2}} \right]_{0}^{\infty} \)

\[ L_{z}^{\text{(Total)}} = \frac{smh}{2} \text{ or } m' = \frac{s}{2} \]

(3.2.6)
Where \( m' \) is the angular momentum state of the infinite superposition and \( m \) the state of \( \psi_{nk} \). Thus for spin \( \frac{1}{2} \) particles with \( s = \frac{1}{2} \) in Eq. (3.2.6) \( m' = m/4 \) but \( m' \) can be only \( \pm \frac{1}{2} \), implying the \( m \) state of \( \psi_{nk} \) that generates spin \( \frac{1}{2} \) must be \( m = \pm 2 \). An \( N=1 \) massive spin 1 particle has \( s = 1 \) with \( m' = m/2 \). (\( N=2 \) is covered in section 6.2.) This is summarized in the following three member infinite superpositions ignoring complex number factors.

 Massive (\( N=1 \)) Spin \( \frac{1}{2} \), \( |\psi_{x,1/2,\pm 1/2}\rangle = \sum_{n=5,6,7} c_n \int_{k=0}^{k=e} \left[ \frac{\psi_{nk,1/2}}{\gamma_{nk}} + \beta_{nk} \psi_{nk,\mp 1/2} \right] \sqrt{\frac{1}{2k}} \right) \tag{3.2.7} \]

 Massive (\( N=1 \)) Spin 1, \( |\psi_{x,1,0}\rangle = \sum_{n=4,5,6} c_n \int_{k=0}^{k=e} \left[ \frac{\psi_{nk,1}}{\gamma_{nk}} + \beta_{nk} \psi_{nk,2m} \right] \sqrt{\frac{1}{k}} \right) \tag{3.2.8} \]

The spin vectors of each \( \psi_{nk} \) with \( |L| = 2\sqrt{3}h \), and their spin vector debts in the zero point vector fields, have to be aligned such that the sum in each case is the correct value: \( |L| = \sqrt{3}/2 \), \( |L| = \sqrt{2}h \) or \( |L| = \sqrt{6}h \) for spins \( \frac{1}{2} \), 1 & 2 respectively.

Spherically symmetric massive \( N = 1 \) spin 1 states are a superposition of three states

\[
\frac{1}{\sqrt{3}} \left[ |m' = -1 \rangle + |m' = 0 \rangle + |m' = 1 \rangle \right], \quad \text{and using Eq. (3.2.8) can be formed as follows}
\]

\[
\begin{align*}
\frac{1}{\sqrt{3}} |\psi_{x,1,m'=-1}\rangle &= \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=e} \left[ \frac{\psi_{nk,1}}{\gamma_{nk}} + \beta_{nk} \psi_{nk,m'=-2} \right] \sqrt{\frac{1}{k}} \right) \tag{3.2.9} \\
\frac{1}{\sqrt{3}} |\psi_{x,1,m'=0}\rangle &= \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=e} \left[ \frac{\psi_{nk,1}}{\gamma_{nk}} + \beta_{nk} \psi_{nk,m'=0} \right] \sqrt{\frac{1}{k}} \right) \\
\frac{1}{\sqrt{3}} |\psi_{x,1,m'=1}\rangle &= \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=e} \left[ \frac{\psi_{nk,1}}{\gamma_{nk}} + \beta_{nk} \psi_{nk,m'=1} \right] \sqrt{\frac{1}{k}} \right)
\end{align*}
\]

\[
3.2.3 \quad \text{Mass and angular momentum of multiple integer \( n \) superpositions}
\]

In sections 3.2.1 & 3.2.2 for simplicity we looked at single integer \( n \) superpositions \( \psi_{nk} \). For superpositions \( \psi_k = \sum_n c_n \psi_{nk} \), we replace \( K_{nk}^2 \) with \( \langle K_k \rangle^2 \). Equation (3.2.9) appears to suggest

\[
|p_k|^2 = \sum_n c_n^* c_n n^2 h^2 k^2 = \langle n^2 \rangle h^2 k^2 \quad \text{and} \quad \langle |p_k| \rangle = \hbar k \sqrt{\langle n^2 \rangle}. \quad \text{In section (3.5.1) we discuss why}
\]

\[
\langle |p_k| \rangle \neq \hbar k \sqrt{\langle n^2 \rangle} \quad \text{but} \quad \langle |p_k| \rangle = \hbar k \sum_n c_n^* c_n \cdot n = \hbar k \langle n \rangle. \quad \text{Thus using Eq. (3.1.11)}
\]

\[
\langle K_k \rangle = \frac{\hbar c k \sqrt{2s}}{2} \langle n \rangle \quad \text{&} \quad \langle K_k \rangle^2 = \frac{\hbar c^2 k^2 s}{2} \langle n^2 \rangle \quad \text{but} \quad \langle K_k \rangle^2 \neq \frac{\hbar c^2 k^2 s}{2} \langle n^2 \rangle \tag{3.2.10}
\]

Replacing \( K_{nk}^2 \) with \( \langle K_k \rangle^2 = \frac{\hbar c^2 k^2 s}{2} \langle n^2 \rangle / 2 \) in the key equations (3.2.3) & (3.2.6) does not change the final results. The laws of quantum mechanics tell us the total angular momentum is

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precisely integral $h$ or half integral $h/2$. Looking at the above integrals used to derive total angular momentum we see that $N$ must be 1 (we discuss $N=2$ in section 6.2) and $s$ must be exactly $\frac{1}{2}$ or one for spin $\frac{1}{2}$ & spin 1 massive particles respectively in our probability formula Eq. (2.2.4). Also, these integrals are infinite sums of positive and negative integral $h$ that are virtual and cannot be observed. If an infinite superposition for an electron is in a spin up state and flips to spin down in a magnetic field, a real $m = \pm 1$ photon is emitted carrying away the change in angular momentum. This is the only real effect observed from this infinity of $(l=3, m=+2)$ virtual wavefunctions all flipping to $(l=3, m=-2)$ states, plus an infinite flipping of the virtual zero point vector debts. Also, Eqs. (3.2.3) and (3.2.6) are true only if our high energy cutoff is at infinity and the low frequency cutoff is at zero. We look at high energy Planck scale cutoffs in section 4.2 and in section 6.1 low energy cutoffs near the radius of the causally connected horizon.

### 3.3 Ratios between Primary and Secondary Coupling

#### 3.3.1 Initial simplifying assumptions

This section is based on a special case thought experiment that tries to illustrate, hopefully in a simple way, how superpositions interact with one another; in the same way as virtual photons, for example, interact with electrons. It is unfortunately long and not very rigorous, but it illustrates how, in all interactions between fundamental particles represented as infinite superpositions, the actual interaction is between only the same $k$ single wavenumber superpositions of each particle. We will later conjecture that an interacting virtual particle is a single wavenumber $k$ superposition only, and not a full infinite superposition. Only real particles whose properties we can measure are full infinite superpositions. The full properties do not exist until measurement, just as in so many other examples in quantum mechanics. This will be clearer as we proceed. It is also important to remember here, that because primary coupling constants are to bare charges (section 2.2.2), and thus fixed for all $k$, while secondary coupling constants run with $k$, the coupling ratios can be defined only at the cutoff value of $k$ applying to the bare charge (sections 4.1.1 & 4.2.2). From Table 2.2.1 there are six fundamental primary charges for electrons and positrons. But electrons and positrons are defined as fundamental charges. In other words, what we define as a fundamental electric charge is in reality six primary charges. Of course, we can never in reality measure six as their effect is reduced by the ratio between primary and secondary coupling. Because electromagnetic and colour coupling are both via spin one bosons their coupling ratios are fundamentally the same, but because of the above they are related as $6^2 = 36:1$.

\[
\frac{1}{x_{\text{Colour}}} = \frac{36}{x_{\text{EM}}} 
\]

(3.3.1)

We define the colour and electromagnetic ratios as follows (leaving gravity till section 6.2.6)
The secondary coupling constants $\alpha_{\text{EMS}}$ & $\alpha_{3S}$ are the bare charge values, both at the fermion interaction cutoff near the Planck length Eq. (4.2.10). Also we assumed in section 2.2.2 that $\alpha_{3P} = 1$; thus from Eq. (3.3.2)

$$\chi_{\text{C}} = \alpha_{3S}^{-1} = \alpha_{3}^{-1} \@ \text{cutoff} \approx 2.029 \times 10^{18} \text{GeV}$$

In other words, provided $\alpha_{3P} = 1$, the ratio $\chi_{\text{C}}$ (or $\chi_{\text{Colour}}$) is also the inverse of the colour coupling constant $\alpha_{3}$ at the high energy interaction cutoff near the Planck length. In this respect $\chi_{\text{C}}$ or $\chi_{\text{Colour}}$ is the fundamental ratio we will use mainly from here on. From the above paragraphs, to find the coupling ratios we need secondary interactions that are between bare charges. But this implies extremely close spacing where the effects of spin dominate. If the spacing is sufficiently large the effects of spin can be ignored but then we are not looking at bare charges. However, we can ignore the effects of shielding due to virtual charged pairs by imagining, as a simple thought experiment, an interaction between bare charges even at such large spacing. We can also simplify things further by considering only scalar or coulomb type elastic interactions at this large spacing. We are also going to temporarily ignore Eq. (3.3.2) and imagine that we have only one primary electric and or one colour charge. Consider two superpositions and (due to the above simplifying assumptions) imagine them as spin zero charges. QED considers the interaction between them as a single covariant combination of two separate and opposite direction non-covariant interactions (a) plus (b) as in the Feynman diagram of Figure 3.3.1 below. The Feynman transition amplitude is invariant in all frames [22], so let us consider a special simple case in a CM frame where we have identical particles on a head-on (elastic) collision path with spatial momenta:

$$p_a = -p'_a = -p_b = +p'_b$$

From Eq. (3.3.4) the initial and final spatial momenta are reversed with mirror images of each other at each vertex. Of course, when we know momenta accurately we have no idea where the particles are when this takes place, so in reality there is no head-on collision. We are also going to assume in what follows that the vertices of this interaction are on opposite sides of the interacting boson superposition. While we have no idea where this boson superposition is centred, what we do know in this simple special scalar case is that the transferred four momentum squared is simply the transferred three momentum squared, and ignoring the minus sign for $q^2$ (due to $i^2$) in what we are doing here for simplicity we can say:

$$q^2 = (p_a - p'_a)^2 = (p_b - p'_b)^2 = 4p_a^2 = 4p_b^2$$
Figure 3.3.1 Feynman diagram of virtual photon exchange between two spin zero particles of charge $e$.

The Feynman diagram is drawn with a vertical photon line representing the superposition of two opposite direction and non-covariant processes (a) plus (b).

The exchanged 4 momentum is:

$$q = p_a - p'_a = p'_b - p_b.$$

Figure 3.3.2 All eigenfunctions $\psi_{nk}$ in the groups of three overlap at a fixed wavenumber $k$.

If we look at Figure 3.3.2 we see that at any fixed value of $k$, all modes $\psi_{nk}$ in the groups of three overlapping superpositions for the various spins $\frac{1}{2}$, 1 & 2 occupy similar sized regions of space. The directions of their linear momenta are unknown but let us imagine some particular vector $\hbar k$ that is parallel to the above vectors $p_a = p_b$. As we are considering only scalar interactions, all these modes must be spherically symmetric or time polarized. Equation (3.1.16) says spherical symmetry is $\propto 1/\gamma_{nk}$ and Eqs. (3.1.11) and (3.1.12) tell us $\gamma_{nk} \rightarrow 1$ as $\beta_{nk} \rightarrow 0$. But we are considering bare charges at large spacings where the exchanged virtual photons have small momenta and are time polarized as in Eq. (3.1.15). At a fixed value of $k$ they thus have momenta $\pm \hbar k$. Also, as they overlap each other, we can imagine units of $\pm \hbar k$ quanta somehow transferring between these superpositions so that the values of $n$ in each mode can change temporarily by $\pm 1$ for times $\Delta T = \hbar / \Delta E$. The directions of these
3.3.2 Restrictions on possible eigenvalue changes

Before we look at changing these eigenvalues by $n = \pm 1$ we need to consider what restrictions there are on these changes.

From Eq. (2.3.12) superposition $\psi_\downarrow$ requires $Q^2 A^2 = \sum_n c_n^* c_n n^4 h^2 k^4 r^2$ and Eq. (2.2.4) informs us the available $Q^2 A^2 = \frac{[8 + 8\sqrt{\sigma_{EMP}}]^2}{3\pi SN} - h^2 k^4 r^2$ occurs with probability $= \frac{sN \cdot dk}{k}$.

For very brief periods the required value of $Q^2 A^2$ can fluctuate, such as during these changes of momentum, but if its average value changes over the entire process then Eq. (2.2.4) says that the probability $sN \cdot dk / k$ changes also, and we have shown in section 3.2.1 that this is disallowed. For example, in a spin $\frac{1}{2}$ superposition $\psi_{s6}, \psi_{s7}, \psi_{s8}$, (see Table 4.3.1) the average values of $|c_s|, |c_6|, |c_7|$ must each remain constant. This can only happen if $n$ remains within its pre-existing boundaries of $(5 \leq n \leq 7)$. For example, if $\psi_7$ adds $+h\bm{k}$ (we will ignore the subscript $k$ in $\psi_{nk}$ from here assuming that it will be understood) it can create $\psi_8$, but $|c_8|$ must average zero, which it can do only if it fluctuates either side of zero, and $|c_n|$ cannot be negative. Similarly $|c_4|$ must average zero, thus $\psi_4$ & $\psi_5$ are forbidden fermion superposition states. Keeping the average values of $|c_n|$ constant is also equivalent to a constant internal average particle energy (we have shown in section 3.2.1 that rest mass is a function of $\sum c_n^* c_n \mathbf{p}_{nk}^2$). By changing these eigenvalues by $n = \pm 1$ there are only four possibilities: $\psi_5$ & $\psi_7$ can both reduce by $-h\bm{k}$ quanta; $\psi_6$ & $\psi_4$ can both increase by $+h\bm{k}$ quanta. If $\psi_6$ becomes $\psi_7$, $|c_4|$ also increases and $|c_6|$ decreases, but then $\psi_7$ has to drop back becoming $\psi_6$, with $|c_6|$ decreasing back down and $|c_4|$ increasing back up in exact balance. If we view this as one overall process the average values of both $|c_4|$ and $|c_6|$ remain constant but fluctuate continuously. We can use exactly the same argument if $\psi_5$ increases which has to be followed by $\psi_6$ dropping, where if we view this as one process again, the average values of both $|c_5|$ and $|c_6|$ remain constant. This is similar to a particle not being able to absorb a photon in a covariant manner, it has to re-emit within time $\Delta T \approx h / E$. Just as transversely polarized photons are the equal left and right superposition of circular polarizations $|L\rangle / \sqrt{2} + |R\rangle / \sqrt{2}$, we can perhaps express Eq. (2.3.9) $\mathbf{p}_{nk}^2 = n^2 h^2 k^2$ as equivalent to:

$$\mathbf{p} = \pm n h \bm{k}$$

is the equal superposition $\mathbf{p} = \{+n h \bm{k}\} / \sqrt{2} + \{-n h \bm{k}\} / \sqrt{2}$. (3.3.6)

This superposition is in opposite directions of the vector $\bm{k}$, implying equal 50% probabilities of momentum vectors for any pair of opposite directions. (It is a virtual superposition so neither of these two components can be observed.) Thus if $n$ changes by $+1$ say, there are equal 50% probabilities of the momentum transfers $\mathbf{p} = +h \bm{k}$ and $\mathbf{p} = -h \bm{k}$. And the same is true if $n$ changes by $-1$. Spin 1 bosons transfer momentum $\Delta \mathbf{p} = \pm h \bm{k}$, which means that two 50% probability transfers are required, such as $\psi_{s6} \rightarrow \psi_{s6}$ combined with a $\psi_{s6} \rightarrow \psi_{s5}$, provided the momentum directions add appropriately as in the Figure 3.3.3 top diagram. But if
\( \psi_{5k} \rightarrow \psi_{6k} \) and \( \psi_{6k} \rightarrow \psi_{5k} \), with \( p = \pm n\hbar k \) keeping the same sign during this process, there is no net 3 momentum transfer as in the lower half of Figure 3.3.3. The probability of these two processes is identical, and we will use this same probability for spin 2 graviton probability densities when looking at gravity which Einstein showed us it is not a force, as particles simply follow geodesics in the warped spacetime surrounding any mass.

For all the two way transitions at both vertices, similar to those discussed above, the following is true:

Probability of all transitions similar to \( \psi_5 \leftrightarrow \psi_6 \) is equal in either direction. (3.3.7)

As we are looking at virtual interactions between fermions and bosons we will use subscripts \( a \) for spin \( \frac{1}{2} \) and \( b \) for spins 1 and 2 superpositions in what follows.

**Figure 3.3.3** Covariant interaction (as in Eq. (3.3.4) and Figure 3.3.1) between fermion (subscript \( a \)) and boson (subscript \( b \) and in boxes) eigenfunctions, with spin 1 photons in the top diagram, and spin 2 gravitons in the bottom diagram. Different colours are used to help identify the transitions at each of the four spacetime corners. This is one process, but a superposition of two diagonal components splitting the 3 momentum \( \hbar k \) equally. Momentum is transferred in the spin 1 case only, but real spin 2 gravitons however, as in gravitational waves from rotating binary pairs for example, do carry energy and momentum,
We can think of the interactions in both the top and bottom of Figure 3.3 as a spacetime rectangle. Starting with the top left corner, the key factors are the superposition component/member amplitudes \( c_{6a} \) & \( c_{6b} \), then proceeding clockwise (the order is irrelevant) \( c_{5a} \) & \( c_{5b} \), \( c_{6a} \) & \( c_{6b} \), and finally \( c_{5a} \) & \( c_{5b} \). As this is part of one process, we can rearrange all terms and multiply them to get \( (c_{6b} \ast c_{6b})(c_{5b} \ast c_{5b})(c_{6a} \ast c_{6a})(c_{5a} \ast c_{5a}) \).

Putting \( P_{4b} = c_{4b} \ast c_{4b}, P_{5a} = c_{5a} \ast c_{5a} \) etc.

\[
(c_{6b} \ast c_{6b})(c_{5b} \ast c_{5b})(c_{6a} \ast c_{6a})(c_{5a} \ast c_{5a}) = P_{4b} \ast P_{5b} \ast P_{6a} \ast P_{5a} \tag{3.3.8}
\]

However, our superposition members (\( \psi_{nk} \) shortened to \( \psi_n \)) are all Eigenfunctions with Eigenvalues \( p_{nk}^2 = n^2 \hbar^2 k^2 \) having equal probabilities of momentum vectors \( k \) pointing in opposite directions, as in Eq.(3.3.7) and the following paragraph. Thus, we can interchange the red and orange boson \( \psi_{4b} \) & \( \psi_{5b} \) and also the blue and green fermion \( \psi_{5a} \) & \( \psi_{6a} \) in Figure 3.3 with no change in exchanged momentum. These four possibilities increase the amplitude factor for this group by four, so that (if all other factors are one) Eq. (3.3.8) becomes:

\[
2^2 (c_{6b} \ast c_{6b})(c_{5b} \ast c_{5b})(c_{6a} \ast c_{6a})(c_{5a} \ast c_{5a}) = 4 P_{4b} \ast P_{5b} \ast P_{6a} \ast P_{5a} \tag{3.3.9}
\]

But there are four different groups of four Eigenfunctions A, B, C & D as in Figure 3.3.4 below, and we have only been considering group C above.

**Figure 3.3.4 Interaction between the four Eigenfunction groups A, B, C and D**

Using Eq. (3.3.9), if all other factors are one the amplitudes for the groups in Figure 3.3.4 are:

\[
\begin{align*}
A &= 4(c_{6b} \ast c_{6b})(c_{5b} \ast c_{5b})(c_{6a} \ast c_{6a})(c_{5a} \ast c_{5a}) = 4 P_{4b} \ast P_{5b} \ast P_{6a} \ast P_{5a} \tag{3.3.10} \\
B &= 4(c_{5b} \ast c_{5b})(c_{6b} \ast c_{6b})(c_{6a} \ast c_{6a})(c_{5a} \ast c_{5a}) = 4 P_{3b} \ast P_{4b} \ast P_{6a} \ast P_{5a} \\
C &= 4(c_{6b} \ast c_{6b})(c_{5b} \ast c_{5b})(c_{6a} \ast c_{6a})(c_{5a} \ast c_{5a}) = 4 P_{4b} \ast P_{5b} \ast P_{6a} \ast P_{5a} \\
D &= 4(c_{6b} \ast c_{6b})(c_{5b} \ast c_{5b})(c_{6a} \ast c_{6a})(c_{5a} \ast c_{5a}) = 4 P_{4b} \ast P_{5b} \ast P_{6a} \ast P_{5a}
\end{align*}
\]

These amplitudes are all numbers as \( P_{4b} = c_{4b} \ast c_{4b}, P_{5a} = c_{5a} \ast c_{5a} \) etc. are just probabilities. But we can perhaps imagine these numbers as in the complex plane. From section 2.2.2 and Figure 3.1.4, however, the three eigenfunctions forming each of the interacting particles are born simultaneously. It would thus seem reasonable to assume that the amplitudes of each group of three eigenfunctions have the same complex phase angle. So whether they are in the complex plane or not, provided they are all at the same angle we can get the overall
probability of this virtual exchange by simply adding the four amplitudes $A, B, C & D$ from Eq. (3.3.10) and squaring the total to get:

$$
\text{Overall interaction probability if all other factors are one } = (A + B + C + D)^2
$$

$$
= 16 \left[ P_{ab} P_{ba} P_{da} P_{ad} + P_{ab} P_{aba} P_{dab} + P_{ab} P_{bda} P_{dab} + P_{ab} P_{bda} P_{dab} \right]^2
$$

(3.3.11)

Using different colours again for common terms in each of the equations following and then using $c_{3b}^* c_{3b} + c_{4b}^* c_{4b} + c_{3b}^* c_{3b} = c_{5a}^* c_{5a} + c_{6a}^* c_{6a} + c_{6a}^* c_{6a} = 1$ the interaction probability is

$$
(A + B + C + D)^2 = 2^4 \left[ c_{4b}^* c_{4b} (1 - c_{4b}^* c_{4b}) \right]^2 \left[ c_{6a}^* c_{6a} (1 - c_{6a}^* c_{6a}) \right]^2
$$

(3.3.12)

We have assumed to here that all other amplitude factors are one. However at each vertex there are both fermion and boson superposition probabilities from Eq. (2.2.4). Writing the superposition probability at each vertex as

$$
\frac{2 s_{i2} n_1 c_{6a}^* c_{6a} (1 - c_{6a}^* c_{6a})}{k^2} \left[ 2 s_{2} n_2 c_{4b}^* c_{4b} (1 - c_{4b}^* c_{4b}) \right]^2
$$

$$
\frac{2 s_{i2} n_2 c_{4b}^* c_{4b} (1 - c_{4b}^* c_{4b})}{k^2} \left[ 2 s_{2} n_2 c_{6a}^* c_{6a} (1 - c_{6a}^* c_{6a}) \right]^2
$$

$$
= \frac{2 s_{i2} n_1 c_{6a}^* c_{6a} (1 - c_{6a}^* c_{6a})}{(k)^4} \left[ 2 s_{2} n_2 c_{4b}^* c_{4b} (1 - c_{4b}^* c_{4b}) \right]^2
$$

The momentum per transfer is a total of $\pm h k$ and using Eqs. (3.3.5), (3.3.6) &

$$
(\pm h k)^4 = q^4
$$

then putting $h = 1$ the interaction probability is

$$
\left[ \frac{2 s_{i2} n_1 c_{6a}^* c_{6a} (1 - c_{6a}^* c_{6a})}{k^4} \right] \left[ 2 s_{2} n_2 c_{4b}^* c_{4b} (1 - c_{4b}^* c_{4b}) \right]^2
$$

(3.3.13)

This is the scalar interaction probability between two spin $\frac{1}{2}$ fermions exchanging infinitesimal rest mass spin 1 bosons at very large spacings, where the fermions are effectively spin zero, imagining them as bare charges and all other factors being one. When exchanging spin 2 infinitesimal rest mass time polarized gravitons (as in the bottom half of Figure 3.3 with no 3 momentum) we can simply keep using wavenumber $k$ in the denominator for the interaction probability between fermions and gravitons. If all other amplitude factors are one this interaction probability becomes (using subscript c for spin 2 and $N = 2 = N_2$ for clarity):

$$
\left[ \frac{2 s_{i2} n_1 c_{6a}^* c_{6a} (1 - c_{6a}^* c_{6a})}{k^4} \right] \left[ 2 s_{2} n_2 c_{4b}^* c_{4b} (1 - c_{4b}^* c_{4b}) \right]^2
$$

gravitons & fermions.
And if, for example, two spin 1 photons exchange spin 2 gravitons (all infinitesimal rest mass with \( N = 2 = N_s \)) the interaction probability if all other amplitude factors are one becomes

\[
\left[ 2s_1 N c_{4b} * c_{4b} (1 - c_{4b} * c_{4b}) \right]^2 \left[ 2s_2 N c_{4e} * c_{4e} (1 - c_{4e} * c_{4e}) \right]^2 \quad \text{for } N = 2 \text{ photons.}
\]  

(3.3.15)

If two massive \( N = 1 \) photons (as in Figure 3.3.2) exchange spin 2 gravitons the interaction probability if all other factors are one becomes

\[
\left[ 2s_1 N c_{3b} * c_{3b} (1 - c_{3b} * c_{3b}) \right]^2 \left[ 2s_2 N c_{3e} * c_{3e} (1 - c_{3e} * c_{3e}) \right]^2 \quad \text{for } N = 1 \text{ photons.}
\]  

(3.3.16)

According to GR (section 0) the emission of gravitons is identical for both mass and energy. Keeping all other factors (such as mass/energy) in Eqs. (3.3.14), (3.3.15) and (3.3.16) constant, the graviton interaction probabilities must be the same in each. We can thus put them equal to each other and cancel out all the common red terms on the RH sides above:

\[
2s_1 N c_{3b} * c_{3b} (1 - c_{3b} * c_{3b}) = 2s_1 N c_{3b} * c_{3b} (1 - c_{3b} * c_{3b}) = 2s_1 N c_{3b} * c_{3b} (1 - c_{3b} * c_{3b})
\]

or

\[
4 c_{4b} * c_{4b} (1 - c_{4b} * c_{4b}) = 2 c_{3b} * c_{3b} (1 - c_{3b} * c_{3b}) = c_{6a} * c_{6a} (1 - c_{6a} * c_{6a})
\]

(3.3.17)

\( N = 2 \) Spin 1

\( N = 1 \) Spin 1

\( N = 1 \) Spin 1/2

In this special case as in Eq. (3.3.4) we have shown that the time polarized interaction probabilities are the same whether 3 momentum is exchanged or not, and this equation for the above ratios is identical for both virtual spin 2 graviton and virtual spin 1 photon exchanges. Ignoring complex numbers for simplicity, we can use either 4 momentum \( q \) or wavenumber \( k \) interchangeably. Now assume that all other factors (other than coupling constants) are one, and remember that we are simplifying with a thought experiment by looking at spin \( \frac{1}{2} \) superpositions sufficiently far apart so we can treat them as approximately spherically symmetric or effectively spin zero, even if they are supposed to be bare charges with spin. Under these same scalar exchange conditions QED says that with electrons, for example:

\[
\text{The probability of scalar or coulomb exchange in Eq. (3.3.13)} \quad \left( \frac{4 \alpha^2}{q^4} \right).
\]

(This probability is for one momentum \( k \) direction only, but the mode density of these is \( k^2dk / \pi^2 \). We can perhaps think of \( \frac{4 \alpha^2}{k^4} \cdot \frac{k^2dk}{\pi^2} = \left( \frac{2 \alpha}{\pi k} \right)^2dk \) as an imaginary emission probability \( \frac{2 \alpha}{\pi k} \), multiplied by an imaginary absorption probability \( \frac{2 \alpha}{\pi k} \) in all possible directions. The rest of this paper is mainly about virtual particles which cannot be experimentally detected. However, as we will see, imaginary probability densities can have real world consequences. This is similar to our postulated infinite virtual superpositions being undetectable, but the particles they generate can certainly be experimented on in the real world. This paper uses these imaginary probabilities throughout, as it allows a very simple approximate way to look at gravity using only very long wavelength time polarized gravitons. We demonstrate how it works in the next section on electromagnetic energy between charges.)
Let us now temporarily ignore the fact that gluons have limited range and imagine our thought experiment applying to colour charges exchanging gluons. The \( \alpha \) of Eq. (3.3.18) becomes the usual colour coupling \( \alpha_c \). To get the fundamental coupling ratio labelled as \( \chi_c = \alpha_c^{-1} \) @ \( k_{\text{cutoff}} \) we substitute the \( \alpha \) of Eq. (3.3.18) with \( \alpha = \chi_c^{-1} \) as we assumed \( \alpha_c \), (Primary) 1.

Substituting \( 2s_1 = 1 \), \( 2s_1 = 2 \), \( N_1 = 1 \) & \( N_2 = 2 \) and equating Eqs. (3.3.13) & (3.3.18)

\[
\begin{align*}
\left[ c_{ab}^a * c_{ba}^b (1 - c_{ba}^a * c_{ba}^b) \right]^2 & \left[ 4c_{ab}^a * c_{ba}^b (1 - c_{ba}^a * c_{ba}^b) \right]^2 = 4(\chi_c^{-1})^2 \\
\left[ c_{6a}^a * c_{6a}^a (1 - c_{ba}^a * c_{ba}^b) \right] \left[ 4c_{ab}^a * c_{ba}^b (1 - c_{ba}^a * c_{ba}^b) \right] & = 2\chi_c^{-1}
\end{align*}
\]

(3.3.19)

But from Eq. (3.3.17) the blue and green terms are equal (also the magenta terms) and we can solve for the fundamental coupling ratio by combining Eqs. (3.3.17) & (3.3.19).

\[
\begin{align*}
N = 2 & \quad \text{Spin 1} & N = 1 & \quad \text{Spin 1} & N = 1 & \quad \text{Spin 1/2} \\
\text{Photons or Gluons} & \quad \text{Massive Photons} & \quad \text{Fermions} \\
4c_{ab}^a * c_{ab}^b (1 - c_{ba}^a * c_{ba}^b) & = 2c_{5b}^a * c_{5b}^a (1 - c_{5b}^a * c_{5b}^b) = c_{6a}^a * c_{6a}^a (1 - c_{6a}^a * c_{6a}^b) & = \sqrt{2} / \chi_c
\end{align*}
\]

(3.3.20)

The coupling ratio is fundamentally the same for colour and electromagnetism apart from the six primary electric charges of Eq. (3.3.1) because of the way electric charge is defined. Equations (3.3.17), (3.3.19) & (3.3.20) tell us that for any interactions between two superpositions, the inverse coupling ratio always involves the product of the central superposition member probability by the probability of the other two members combined \( \times N \times \text{spin} \) of the first superposition, times the equivalent product for the other superposition.

In section 4 we introduce gravity and solve these ratios. Despite all the simplifications and lack of rigour, the above equations are surprisingly consistent with the SM, provided there are only three families of fermions. Even though we used gravity to derive Eq. (3.3.17) we leave discussing the gravity coupling ratio till section 6.2.6.

### 3.4 Electrostatic Energy between two Infinite Superpositions

#### 3.4.1 Using an approximate but simple quantum mechanical approach

In section 3.3 we showed that fermion superpositions can exchange boson superpositions in the same way as electrons can exchange virtual photons for example. Providing the superposition amplitudes are appropriate, the coupling constants can be just as in QED, though we will look further at this in section 4.1.1. So, it might seem that evaluating electrostatic energy between superpositions is unnecessary. However, when we look at gravity, we find that the spacetime warping around mass concentrations could possibly be related to cosmic wavelength virtual graviton probability densities. Virtual particle exchange probabilities, in QED/QCD etc, use perturbation theory to calculate particle scattering cross-
sections, and electron $g$ factor corrections with incredible precision. Both space and time polarizations are involved. However, as we later focus on virtual cosmic wavelength graviton probability densities at large spacings, we will use a simple but only approximate (but true at large spacings) quantum mechanical method based on only time polarized photon probability densities to find the scalar potentials between two charges (or infinite superpositions). This same method also allows a simple solution to the magnetic energy between superpositions (again at large spacings) in section 3.5, where we modify relevant equations in a simple manner. In section 5 we will use some of these same equations when looking at why borrowing energy and mass from zero point fields requires the universe to expand after the Big Bang and distort spacetime around mass concentrations. We assume spherically symmetric $l = 3$ superpositions emit virtual scalar (time polarized) photons in this section and $l = 3, m = \pm 2$ superpositions emit virtual $m = \pm 1$ photons in section 3.5. As section 3.3 has shown that we can achieve the same electromagnetic coupling constant $\alpha$ we can use the scalar photon emission probability $(2\alpha/\pi)(dk/k)$ covered in section 2.1.1 and the section in italics after Eq.(3.3.18). From section 3.3 we can also see that the effective average emission point has to be the centre of superpositions. For a virtual photon $\Delta E \cdot \Delta T \leq h/2$, and the range over which it can be found is roughly $r \approx \Delta T \approx 1/2\Delta E \approx 1/2k$ when $h = c = 1$. The radial probability of finding the centre of the spin 1 superposition representing the interacting virtual photon decays exponentially with radius as $e^{-2kr}$. The normalized wavefunction $\psi$ for such a virtual scalar photon of wave number $k$ emitted at $r = 0$ is:

$$\psi = \sqrt{\frac{2k}{4\pi}} e^{-kr} e^{i(kr-\omega t)} = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr} e^{ikr}}{r} \text{ @ time } t = 0.$$  

![Radial probability of finding the virtual photon superposition centre of the same $k$ value.](image)

**Figure 3.4.1** Radial probabilities of $\psi_{6k}$ and the exponential decay with radius of a virtual photon of the same $k$ value $R*R \propto 2ke^{-2kr}$. These curves look the same for all $k$, applying equally to virtual photons, gravitons and to large $k$ value gluons etc.

Wavefunction $\psi$ is spherically symmetric as scalar photons are time polarized. Figure 3.4.1 plots the radial probabilities of the exponentially decaying virtual photon and the dominant $n = 6$ mode of its relating superposition $\psi_{6k}$. The effective range of a wavenumber $k$ virtual
photon is of a similar order to the radial probability dimensions of $\psi_{6k}$. For simplicity, in what follows we locate two superpositions (which we refer to as sources) in cavities that are small in relation to the distance between them. The accuracy of our results depends on how far apart they are in relation to the cavity size. Consider two spherically symmetric sources distance $2C$ apart emitting virtual scalar photons as in Figure 3.4.2 where point $P$ is $r_1$ from source one, and $r_2$ from source 2. Let $\psi_1$ be the amplitude from source one, and $\psi_2$ be the amplitude from source two and for simplicity and clarity let $t = 0$.

Thus

$$\psi_1 = \sqrt{\frac{2k}{4\pi r_1}} e^{-kr_1+ikr_1}$$
$$\psi_2 = \sqrt{\frac{2k}{4\pi r_2}} e^{-kr_2+ikr_2}$$

Consider $(\psi_1 + \psi_2) \ast (\psi_1 + \psi_2) = \psi_1 \ast \psi_1 + \psi_1 \ast \psi_2 + \psi_2 \ast \psi_1 + \psi_2 \ast \psi_2$

Now $\psi_1 \ast \psi_1$ & $\psi_2 \ast \psi_2$ are just the normal probability densities around sources one and two as though they are infinitely far apart but the work done per pair of superpositions $k$ on bringing two sources closer together is in the interaction term: $\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1$.

$$\psi_1 \ast \psi_2 = \frac{2k}{4\pi r_2} e^{-kr_1} e^{-ikr_1} e^{+ikr_2} = \frac{2k}{4\pi r_2} e^{-k(r_1+r_2)} e^{-ik(r_1-r_2)}$$
$$\psi_2 \ast \psi_1 = \frac{2k}{4\pi r_1} e^{-kr_2} e^{-ikr_2} e^{+ikr_1} = \frac{2k}{4\pi r_1} e^{-k(r_1+r_2)} e^{-ik(r_1-r_2)}$$
$$\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1 = \frac{2k}{4\pi r_2} e^{-k(r_1+r_2)} \left[ e^{ik(r_1-r_2)} + e^{-ik(r_1-r_2)} \right]$$
$$= \frac{4k}{4\pi r_2} e^{-k(r_1+r_2)} \cos (r_1 - r_2)$$

Now put $(A = r_1 + r_2, B = r_1 - r_2)$ & $\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1 = \frac{4k}{4\pi r_2} e^{-Ak} \cos (kB)$

Real work is done when bringing superpositions together and we can treat these interacting virtual photons as having real energy $\hbar \omega = \hbar kc$. Using virtual photon emission probability $(2\alpha / \pi)(dk / k)$ from section 2.1.1

Energy per virtual photon × Probability = $\hbar kc \left[ \text{Probability} \frac{2\alpha}{\pi} \frac{dk}{k} \right] = \frac{2\alpha hc}{\pi} \frac{dk}{k}$

Including Eq. (3.4.3) the interaction energy @ $k$ is thus $\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1 \left[ \frac{2\alpha hc}{\pi} \frac{dk}{k} \right]$ and using Eq. (3.4.2) the interaction energy @ $k$ is $\frac{4k}{4\pi r_2} e^{-Ak} \cos (kB)$.

The total interaction energy density due to $\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1$ for all $k$ is

$$\frac{2\alpha hc}{\pi} \frac{4}{4\pi r_2} \int_0^\infty ke^{-Ak} \cos (kB) dk$$
\[
\int_0^\infty ke^{-Ak} \cos(Bk)dk = \frac{A^2 - B^2}{(A^2 + B^2)^2}
\]  
(3.4.5)

Where  
\[A^2 = (r_1 + r_2)^2 = r_1^2 + 2r_1r_2 + r_2^2 \quad \& \quad B^2 = (r_1 - r_2)^2 = r_1^2 - 2r_1r_2 + r_2^2\]

Thus  
\[A^2 = (r_1 + r_2)^2 = r_1^2 + 2r_1r_2 + r_2^2 \quad \& \quad A^2 + B^2 = 2(r_1^2 + r_2^2)\]  
(3.4.6)

\[= 2(r^2 + C^2) \quad \text{as} \quad \cos(180 - \theta) = -\cos \theta\]

and  
\[A^2 + B^2 = 4(r^2 + C^2)\]  
(3.4.7)

![Diagram](image)

**Figure 3.4.2** Distances to a point from two sources as a function of angle \(\theta\) and radius \(r\).

Putting Eqs. (3.4.4), (3.4.5), (3.4.6) \& (3.4.7) together  
\[
\frac{A^2 - B^2}{(A^2 + B^2)^2} = \frac{4r_1r_2}{16(r^2 + C^2)^2}
\]

\[
\int_0^\infty ke^{-Ak} \cos(Bk)dk = \frac{r_1r_2}{4(r^2 + C^2)^2}
\]

\[
\frac{2\alphahc}{\pi} \frac{4}{4\pi r_1r_2} \int_0^\infty ke^{-Ak} \cos(Bk)dk = \frac{2\alphahc}{\pi} \frac{4}{4\pi r_1r_2} \frac{r_1r_2}{4(r^2 + C^2)^2}
\]

\[
\frac{2\alphahc}{\pi} \frac{4}{4\pi r_1r_2} \int_0^\infty ke^{-Ak} \cos(Bk)dk = \frac{2\alphahc}{\pi} \frac{1}{4\pi} \frac{1}{(r^2 + C^2)^2}
\]  
(3.4.8)

This is the total interaction energy density of time polarized virtual photons at point \(P\) due to \(\psi_1 \ast \psi_2 \ast \psi_1\) for all \(k\) and there are no directional vectors to take into account. We will use similar equations for the vector potential \((m = \pm 1)\) photons for magnetic energies but will then need directional vectors. Equation (3.4.8) is the energy due to the interaction of amplitudes at any radius \(r\) from the centre of the pair. It is independent of \(\theta\), and to get the total energy of interaction we multiply by \(4\pi r^2 dr\) for layer \(dr\) and integrate from \(r = 0 \rightarrow \infty\).  

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The total interaction energy is
\[ \frac{2\alpha \hbar c}{\pi} \int_0^\infty \int_0^\infty (\psi_1^* \psi_2 + \psi_2^* \psi_1) dk \, 4\pi r^2 dr \]

Using Eq. (3.4.8)
\[ = \frac{2\alpha \hbar c}{\pi} \frac{1}{4\pi} \int_0^\infty 4\pi r^2 dr \frac{r^2}{(r^2 + C^2)^2} \]

Thus
\[ \frac{2\alpha \hbar c}{\pi} \int_0^\infty (\psi_1^* \psi_2 + \psi_2^* \psi_1) dk dv = \frac{2\alpha \hbar c}{\pi} \int_0^\infty \frac{r^2 dr}{(r^2 + C^2)^2} \]
\[ \int_0^\infty \frac{r^2 dr}{(r^2 + C^2)^2} = \frac{1}{2C} \]

The interaction or potential energy is
\[ \frac{\alpha \hbar c}{2C} = \frac{\alpha \hbar c}{R} \] (3.4.9)

If \( R = 2C \) is the distance between the centres of our assemblies, this is the classical potential.

The procedure used here, with small changes, simplifies the derivation of the magnetic moment; we reuse some equations, but in a slightly modified form taking polarization vectors into account. We also reuse some of these simple but approximate derivations when looking at gravity in Section 5.

### 3.5 Magnetic Energy between two Spin Aligned Infinite Superpositions

In this section we are going to consider two infinite superpositions that form Dirac spin ½ states. We will look at the magnetic energy between them when they are both in a spin up state, say along some \( z \) axis as in Figure 3.5. 1. We are not looking at the magnetic energy here when they are both coupled in a spin 0 or spin 1 state. That is, both Dirac spin ½ states have their \( \sqrt{3\hbar/2} \) spin vectors randomly oriented around the \( z \) axis with \( \hbar/2 \) components aligned along this \( z \) axis. Also, in this section we will be dealing with transversely polarized virtual photons and must take account of polarization vectors. In section 3.2.2 and Eq. (3.2.7) spin ½ states are generated only from \( l = 3, m = 2 \) states and as transversely polarized photons are superpositions of \( m = \pm 1 \) photons they can only be emitted from these \( l = 3, m = 2 \) states; the remaining states are spherically symmetric and cannot emit transversely polarized photons. We don’t yet know the value of amplitudes \( |c_n| \) so we will derive the magnetic energy in terms of these. We will then equate this energy to the Dirac values assuming a value of \( g = 2 \) before QED corrections; this allows us to evaluate in section 4.3 the amplitudes \( |c_n| \) in terms of the ratio \( \chi_{EM} \) between primary and secondary electromagnetic coupling. We can then evaluate in section 4.1 the primary electromagnetic coupling constant \( \alpha_{EMP} \) in terms of the ratio \( \chi_{EM} \). (Section 3.5 uses the same format as Chapter 18, “The Feynman Lectures on Physics” Volume 3, Quantum Mechanics[24]).
Figure 3.5. 1 Two spin aligned superpositions.

An \(l = 3, m = 2\) state can emit a right hand circularly (R.H.C.) polarized \((m = +1)\) photon in the \(+z\) direction. Let the amplitude for this be temporarily \(\left| R \right\rangle\).

An \(l = 3, m = -2\) state can emit a left hand circularly (L.H.C.) polarized \((m = -1)\) photon in the \(+z\) direction. Let the amplitude for this also be temporarily \(\left| L \right\rangle\).

First rotate the \(z\) axis about the \(y\) axis by angle \(\theta\) (call this operation \(\text{SR}\)) then use 

\[
\frac{1}{\sqrt{2}}\left[ \langle R' \left| S \right| R \rangle + \langle L' \left| S \right| R \rangle \right]
\]

and multiply on the right by operation \(\text{SR}\).

The amplitude to emit a transversely polarized photon in the \(x'\) direction is thus

\[
\langle x' \mid S \mid R \rangle = \frac{1}{\sqrt{2}}\left[ \langle R' \left| S \right| R \rangle + \langle L' \left| S \right| R \rangle \right]
\]

Where \(\langle R' \left| S \right| R \rangle = \langle 3, +2' \mid S \mid 3, +2 \rangle = (1/4) \left[ 2 + 2 \cos \theta - 4 \sin^2 \theta + 3 \sin^2 \theta \cos \theta \right]\) is the amplitude an \(l = 3, m = 2\) state remains in an \(l = 3, m = 2\) state after rotation by angle \(\theta\).

Also \(\langle L' \left| S \right| R \rangle = -\langle 3, -2' \mid S \mid 3, +2 \rangle = (1/4) \left[ 2 - 2 \cos \theta - 4 \sin^2 \theta - 3 \sin^2 \theta \cos \theta \right]\) is minus the amplitude that an \(l = 3, m = 2\) state is in an \(l = 3, m = -2\) state after rotation by \(\theta\).

Putting this together

\[
\langle x' \mid S \mid R \rangle = \frac{1 - 2 \sin^2 \theta}{\sqrt{2}} = \frac{\cos 2\theta}{\sqrt{2}}
\]

An \(l = 3, m = 2\) state can also emit an \((m = +1)\) photon in the \(-z\) direction but it will now be left hand circularly polarized. Let this amplitude be temporarily: \(\left| L \right\rangle\).

Similarly an \(l = 3, m = -2\) state can emit an \((m = -1)\) photon in the \(-z\) direction which is right hand circularly polarized. Let this amplitude be temporarily: \(\left| R \right\rangle\).

We can go through the same procedure as above to get

\[
\langle x' \mid S \mid L \rangle = \frac{\cos 2\theta}{\sqrt{2}}
\]

This amplitude Eq. (3.5.2) is for a photon emitted in the opposite direction to amplitude Eq. (3.5.1) but \(\cos 2\theta = \cos 2(180 + \theta)\) and we can simply add these two amplitudes. Let us assume, however, that an \(l = 3, m = 2\) state has equal amplitudes to emit in the \(+z\) & \(-z\) directions of \(\left| R \right\rangle/\sqrt{2}\) and \(\left| L \right\rangle/\sqrt{2}\).
With these amplitudes; \[
\frac{1}{\sqrt{2}} \left[ \langle x' | S | R \rangle + \langle x' | S | L \rangle \right] = \frac{\cos 2\theta}{2} + \frac{\cos 2\theta}{2} = \cos 2\theta \quad (3.5.3)
\]

Equation (3.5.3) is the angular component of the amplitude for a transverse \( x' \) polarization in the new \( z' \) direction where \( x \rightarrow x' & z \rightarrow z' = \theta \). When \( \theta = 0 \) or 180 the on-axis amplitude for transverse polarization is one as expected ignoring other factors. Using the same normalization factors (we check the validity of this in section 3.5.2) we can still use the amplitudes and phasing of our original time mode photons Eqs. (3.4.1) but instead of including polarization vectors we will for simplicity just use the cosine of the angle \( \gamma - \delta \) between them (as in Figure 3.5.2) as a multiplying factor. Including the angular factor Eq. (3.5.3) in our earlier scalar amplitudes Eqs. (3.4.1) we have for our new wavefunctions:

\[
\psi_1 = \cos 2\delta \sqrt{\frac{2k}{4\pi r_1}} e^{-ikr_1} & \psi_2 = \cos 2\gamma \sqrt{\frac{2k}{4\pi r_2}} e^{-ikr_2} \quad (3.5.4)
\]

The transverse polarized photons from sources (1) & (2) have polarization vectors \( |x_1\rangle \) and \( |x_2\rangle \) at angle to each other \( \gamma - \delta \), (Figure 3.5.2) and the complex product becomes:

\[
(\psi_1 + \psi_2) \ast (\psi_1 + \psi_2) = \psi_1 \ast \psi_1 + (\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1)(\cos(\gamma - \delta)) + \psi_2 \ast \psi_2
\]

Where the interaction term is now: \( (\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1)\cos(\gamma - \delta) \) and as in the scalar case (section 3.4.1) but now using Eqs. (3.5.4)

\[
\psi_1 \ast \psi_2 \cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-ik(r_1 + r_2)} e^{-ik(r_1 - r_2)} \cos(\gamma - \delta)
\]

\[
\psi_2 \ast \psi_1 \cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-ik(r_1 + r_2)} e^{ik(r_1 - r_2)} \cos(\gamma - \delta)
\]

\[
(\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1) \cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos(kB) \cos(\gamma - \delta) \quad (3.5.5)
\]

(Where as in section 3.4.1, Eq. (3.4.2) \( A = r_1 + r_2 \& B = r_1 - r_2 \).)

**Figure 3.5.2** Two sources 2C apart, both with \( \beta_{mk}^2 \times (m = +2) \) states along the joining line, \( \delta \& \gamma \) are the respective angles to \( P \), \( r_1 \& \theta \) are the respective distances to point \( P \).
3.5.1 Amplitudes of transversely polarized virtual emitted photons

In the laboratory frame $\psi_{nk}$ has amplitude $\beta_{nk}$ to be in an $m = \pm 2$ state (section 3.1). For a multiple integer $n$ superposition $\psi_k = \sum c_n \psi_{nk}$. At each fixed wavenumber $k$, we cannot distinguish which integer $n$ a virtual photon comes from, so we must add amplitudes from each individual integer $n$ superposition. To keep integrals simple we will assume that $\beta_{nk} \ll 1$ or that spacing $2C$ is very large, and our interacting $k$ values are very small. (We can make a comparison with the Dirac values at any large spacing, so accuracy need not be affected.) Thus if $\beta_{nk} \ll 1$ and $\gamma_{nk} \approx 1$, we can approximate Eq. (3.1.11) as

$$K_{nk} = \beta_{nk} \gamma_{nk} \approx \beta_{nk} \approx \frac{n \hbar k \sqrt{2s}}{2m_e c} = \frac{|p_{nk}| \sqrt{2s}}{2m_e c} \approx \frac{\lambda_s nk \sqrt{2s}}{2} \approx \frac{\lambda_s nk}{2} \text{ for spin } \frac{1}{2} \text{ fermions.}$$

Adding amplitudes for multiple integer $n$ superpositions $\langle \beta_k \rangle \approx \frac{\lambda_s \langle n \rangle_k}{2}$ (3.5.6)

(When deriving Eq. (3.2.10) we said $\langle |p_k| \rangle = \hbar k \langle n \rangle$ and not $\langle |p_k| \rangle = \hbar k \sqrt{\left\langle n^2 \right\rangle}$. How do we justify this? When $\beta_{nk} \ll 1$ as above $\beta_{nk} \approx n \hbar k = |p_{nk}|$ So adding amplitudes $\beta_{nk}$ to get $\langle \beta_k \rangle$ is equivalent to adding $p_{nk}$ to get $\langle p_k \rangle$ and not adding $p_{nk}^2 = n^2 \hbar^2 k^2$ to get $\langle |p_k| \rangle = \hbar k \sqrt{\left\langle n^2 \right\rangle}$. If this is true when $\beta_{nk} \ll 1$ it must be true for $0 \leq \beta_{nk} \leq 1$.)

3.5.2 Checking our normalization factors

Let us pause and check the reasonableness of all this and our normalization factors. From Eqs. (3.4.1) for scalar photons $\left[ \psi^* \psi = \frac{2k e^{-2kr}}{4\pi r^2} \right] \times \text{(emission probability } \frac{2\alpha dk}{\pi k} \text{)}$ gives a

Scalar $\psi_k$ emission probability density $\psi^* \psi \left[ \frac{2\alpha dk}{\pi k} \right] = \frac{2k e^{-2kr}}{4\pi r^2} \left[ \frac{2\alpha dk}{\pi k} \right]$.  

The transversely polarized probability density, using Eqs. (3.5.4) & (3.5.7) plus $\langle \beta_k \rangle^2$ is

$$\langle \beta_{nk} \rangle^2 \psi^* \psi \frac{2\alpha dk}{\pi k} = \langle \beta_{nk} \rangle^2 \left[ \cos^2 2\delta \frac{2k e^{-2kr}}{4\pi r^2} \right] \frac{2\alpha dk}{\pi k}$$

(Where $2\delta = 2\gamma$ & $r_i = r_z$.) If we now consider the on-axis $\delta = 0$ case the transverse polarized on axis emission probability density at $k$ is:

$$\langle \beta_k \rangle^2 \left[ \frac{2k e^{-2kr}}{4\pi r^2} \right] \frac{2\alpha dk}{\pi k} = \langle \beta_k \rangle^2 \psi^* \psi \frac{2\alpha dk}{\pi k}$$

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Just as in QED the factor $\left(\beta_k\right)^2$ is the factor we need for this on-axis emission probability density ratio between transverse and scalar polarization. This justifies using the same normalization constant $(2k/4\pi)^{1/2}$ for both the scalar and magnetic wavefunctions. Using the same virtual photon emission probability and energy $\hbar c$ as in Eq. (3.4.3) for both the scalar and transverse polarization cases:

$$\text{Energy per transverse photon } \times \text{Probability} = \hbar c \times \left[ \text{Probability} \frac{2\alpha}{\pi k} \right] = \frac{2\alpha \hbar c}{\pi} dk \quad (3.5.7)$$

Multiplying Eq. (3.5.5) by Eq. (3.5.6) squared, and Eq. (3.5.7) we get the transverse interaction energy at wavenumber $k$:

$$\left\langle \beta_k \right\rangle^2 (\psi_1^* \psi_2 + \psi_2^* \psi_1) \cos(\gamma - \delta) \left[ \frac{2\alpha \hbar c}{\pi} \right]$$

$$= \left[ \frac{\hbar c^2 \langle n \rangle^2 k^2}{4} \right] \cos 2\delta \cos 2\gamma - \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos(kB) \cos(\gamma - \delta) \left[ \frac{2\alpha \hbar c}{\pi} \right]$$

Rearranging this:

$$\left\langle \beta_k \right\rangle^2 (\psi_1^* \psi_2 + \psi_2^* \psi_1) \cos(\gamma - \delta) \left[ \frac{2\alpha \hbar c}{\pi} \right]$$

$$= \frac{2\langle n \rangle^2 \hbar c \alpha \hbar c}{\pi} \cos 2\delta \cos 2\gamma \cos(\gamma - \delta) \left[ \frac{k^3 e^{-Ak} \cos(kB)dk}{4\pi r_1 r_2} \right] \quad (3.5.8)$$

As in the scalar case we integrate over $k$ first but now with a $k^3$ term due to the inclusion of the $\left\langle \beta_k \right\rangle^2$ factor which is approximately proportional to $k^2$ from Eq. (3.5.6).

Using $A = r_1 + r_2$ & $B = r_1 - r_2$ and Eqs. (3.4.6) & (3.5.6)

$$\int_0^\infty \left[ k^3 e^{-Ak} \cos(kB)dk \right] = \frac{3}{8} \left[ \frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right]$$

And thus:

$$\int_0^\infty \left\langle \beta_k \right\rangle^2 (\psi_1^* \psi_2 + \psi_2^* \psi_1) \cos(\gamma - \delta) \frac{2\alpha \hbar c}{\pi} dk$$

$$= \frac{2\langle n \rangle^2 \hbar c \alpha \hbar c}{\pi} \cos 2\delta \cos 2\gamma \cos(\gamma - \delta) \frac{2r_1^2 r_2^2}{4\pi r_1 r_2} \times \frac{3}{8} \left[ \frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] \quad (3.5.9)$$

Equation (3.5.9) is the magnetic interaction energy density at point $P$ for all wave numbers $k$.

Figure 3.5.2 is a plane of symmetry that can be rotated through angle $2\pi$ around the axis of symmetry (the joining line along the axis of the two spin aligned sources). To evaluate the total magnetic energy density over all space we multiply by $4\pi r^2 \sin \theta d\theta dr$.

We thus integrate Eq. (3.5.9) $\times$ $4\pi r^2 \sin \theta d\theta dr$ to get
\[
\frac{3(n)^2 \hbar^2 c^2 \alpha h c}{4\pi} \int_0^\infty \int_0^\infty \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{r_1^2 r_2^2} \left[ \frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] r^2 \sin \theta d\theta dr
\]

(3.5.10)

Now
\[
\int_0^\infty \int_0^\infty \cos 2\delta \cos 2\gamma \cos(\gamma - \delta) \frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} r^2 \sin \theta d\theta dr
\]

can be reduced to the single integral:
\[
\int_0^\infty \frac{1}{8C^3} \left[ \frac{1}{2p + 3} - \frac{10}{2p + 1} \right] \frac{2p - 1!}{(p - 1)!(p + 1)!} \frac{\pi}{2} = \frac{1}{8C^3} \frac{(160 - 51\pi)}{6}, \frac{\pi}{2}
\]

(Putting \( R = 2C \))

(3.5.11)

This infinite series is approximately
\[
\approx - \frac{1}{R^3 54(1.0045062...)}
\]

(3.5.12)

Putting Eq. (3.5.12) into Eq. (3.5.9) the total magnetic interaction energy over all frequencies and all space for two spin aligned infinite superpositions is:
\[
U' \approx \frac{3(n)^2 \hbar^2 c^2 \alpha h c}{4\pi} \left[ -\frac{1}{R^3 54(1.0045062...)} \right]
\]

(3.5.13)

We will call this \( U \) (superpositions) \approx \left[ \frac{(n)^2 \hbar^2 c^2 \alpha h c}{72R^3 (1.0045062...)} \right]

We can equate this magnetic energy to the classical value assuming the Dirac value of \( g = 2 \) for spin \( \frac{1}{2} \) (No QED corrections have been applied so it must be \( g = 2 \)). For the arrangement of spins as in Figure 3.5.1 the Dirac magnetic energy between two spin \( \frac{1}{2} \) states is

\[
U(\text{Dirac}) = \left[ \frac{2\mu^2}{4\pi\varepsilon_0 c^2 R^3} \right]
\]

(3.5.14)

Using the Dirac magnetic moment \( \mu = \frac{e\hbar}{2m_0} = \frac{e\hbar c}{2m_0 c} = \frac{e\hbar c}{2} \) the Dirac magnetic energy is

\[
U(\text{Dirac}) = \left[ \frac{\hbar^2 c^2 \alpha h c}{2R^3} \right]
\]

The approximation used in deriving Eq. (3.5.6) \( \gamma^2 \beta^2 \approx \beta^2 \) for \( \beta^2 << 1 \) is true only when \( R >> \hbar c \). This error in \( \beta^2 \) is of the order of \( \hbar^2 c / R^2 \) and rapidly tends to zero with
increasing \( R \). There is no upper limit on the value of distance \( R \) we can choose. Thus, comparing our estimate of the magnetic energy with Dirac's value when \( R \gg \hbar c \).

\[
U(\text{Dirac}) = U(\text{Superpositions}) \text{ or } \left[ \frac{\hbar c^2 \alpha \hbar c}{2 R^3} \right] \approx - \left[ \frac{\langle n \rangle^2 \hbar c^2 \alpha \hbar c}{72 R^3 (1.0045062\ldots)} \right] \tag{3.5.15}
\]

All symbols cancel except \( \langle n \rangle \) leaving: \( \langle n \rangle^2 \approx 36(1.0045062\ldots) \)

The expectation value \( \langle n \rangle \) in our superposition is slightly more than \( n = 6 \) our dominant mode. This is why we have used a three member superposition centred on this dominant \( n = 6 \) mode. The two side modes \( n = 5 \) and \( n = 7 \) are smaller so that:

\[
\langle n \rangle = \sum_{n=5,6,7} (c_n^* c_n) n \approx \sqrt{36(1.0045062\ldots)} \approx 6.01350345 \tag{3.5.16}
\]

This is for Dirac spin \( \frac{1}{2} \) particles. This mean value of \( n \) creates a \( g = 2 \) fermion which QED corrections (which are secondary interactions) increase slightly to the experimental value. In section 4.1 we solve the primary electromagnetic coupling constant in terms of ratio \( \mathcal{Z}_{EM} \) using Eq. (3.5.16). It is important to remember this magnetic energy derivation applies to two infinite assemblies (or particles) localized in small cavities in relation to their distance \( R \) apart. They must be both on the \( z \) axis with spins aligned (or anti aligned) along this \( z \) axis as in Figure 3.5. 1 & Figure 3.5. 2. Also, the agreement with Dirac and in what follows is possible if superposition \( \Psi_k \) interacts only with virtual photons of the same wavenumber \( k \).
4 High Energy Superposition Cutoffs

4.1 Electromagnetic Coupling to Spin ½ Infinite Superpositions

Equation (3.5.16) is the key requirement for spin ½ superpositions to behave as Dirac fermions, allowing us to solve $\alpha_{\text{EMP}}^{-1}$ as a function of coupling ratio $\chi$ using Eq. (3.5.16).

$$\langle n \rangle = \sum_{n=5,6,7} (c_n^* c_n)n \approx \sqrt{36(1.0045062...)} \approx 6.01350345$$

Thus $5c_5^* c_5 + 6c_6^* c_6 + 7c_7^* c_7 = 6.01350345$ but $6c_5^* c_5 + 6c_6^* c_6 + 6c_7^* c_7 = 6$

and $c_7^* c_7 - c_5^* c_5 = 0.01350345$

As $c_7^* c_7 + c_5^* c_5 = 1 - c_6^* c_6$ we can now solve for $c_7^* c_7$ & $c_5^* c_5$ in terms of $c_6^* c_6$

$$c_7^* c_7 \approx 0.50675172 - \frac{c_6^* c_6}{2} \quad \text{&} \quad c_5^* c_5 \approx 0.49324827 - \frac{c_6^* c_6}{2} \quad (4.1.1)$$

From Eq. (2.3.12) the $Q^2 A^2$ required to produce this superposition with amplitudes $c_n$ is

$$Q^2 A^2 = \sum_{n=5,6,7} c_n^* c_n \frac{n^4\hbar^2 k^4 r^2}{81}$$

and using Eq.(4.1.1)

$$\sum_{n=5,6,7} c_n^* c_n n^4 = 625c_5^* c_5 + 1296c_6^* c_6 + 2401c_7^* c_7 \approx 1524.991 - 217c_6^* c_6$$

Thus $Q^2 A^2 = \sum_{n=5,6,7} c_n^* c_n \frac{n^4\hbar^2 k^4 r^2}{81} \approx [18.82705 - 2.67901c_6^* c_6]\hbar^2 k^4 r^2$ is the required vector potential squared to produce this spin ½ superposition. From Eq. (2.2.4) with $s = \frac{1}{2}$ & $N=1$

for massive fermions $Q^2 A^2 = \frac{2\left[8 + 8\sqrt{\alpha_{\text{EMP}}} \right]^2}{3\pi}$

$\hbar^2 k^4 r^2$ is the available $Q^2 A^2$.

Equateing required and available: $2\left[8 + 8\sqrt{\alpha_{\text{EMP}}} \right] \approx 3\pi [18.82705 - 2.67901c_6^* c_6]$

$$\left[1 + \sqrt{\alpha_{\text{EMP}}} \right] \approx [1.386256 - 0.197258c_6^* c_6]$$

$$\alpha_{\text{EMP}} \approx \left[1.386256 - 0.197258c_6^* c_6 - 1 \right]^2 \quad (4.1.2)$$

From Eqs. (3.3.1) & (3.3.20), $c_6^* c_6 (1-c_6^* c_6) = \sqrt{2/\chi_C} = 6\sqrt{2/\chi_{EM}}$ and we can solve for $\alpha_{\text{EMP}}$ as a function of either $\chi_{EM}$ or $\chi_C$. We then use Eq. (3.3.20) again to get $\alpha_{\text{EMS}}^{-1} @ k_{\text{cutoff}}$.

Now both $\chi_{EM}$ and $\chi_C$ are fundamentally the same ratio differing only by 36:1, because electron superpositions have six primary charges whereas we define them as one fundamental charge (section 3.3.1) and quarks have only one colour charge (Table 2.2.1) Because $\chi_C = \alpha_3^{-1}$ at the cutoff near $L_p$ it is more convenient to work with. From Eq. (3.3.20)

$$c_6^* c_6 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4/\chi_C}$$

and there are two solutions for each $\chi_C$. 

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One has $c_6^*c_6$ dominant with two smaller $c_5^*c_5$ & $c_7^*c_7$ side modes, the other is the reverse with $c_6^*c_6$ the minor player and two larger $c_5^*c_5$ & $c_7^*c_7$ side modes. As the values for $\alpha_{\text{EMP}}$ with $c_6^*c_6$ dominant fit the SM very closely, we include only these. (This only applies to spin $\frac{1}{2}$ fermions, and the spins 1 & 2 boson superpositions in Table 4.3.1 are the opposite, with minor centre modes.) Table 4.1.1 shows possible coupling ratios $\chi_c$ in the range $\chi_c = 50 \rightarrow 51$. The yellow row corresponds to the cutoff energy in Eq. (4.2.10) and Figure 4.1.2. Table 4.1.1 shows these dominant $c_6^*c_6$ mode results for $\chi_c = \alpha_3^{-1}$ at various possible cutoffs in the range $\chi_c = 50 \rightarrow 51$, as this range fits the SM.

<table>
<thead>
<tr>
<th>Coupling Ratio $\chi_c$</th>
<th>$c_6^*c_6$</th>
<th>$\alpha_{\text{EMP}}^{-1}$</th>
<th>$\alpha_{\text{EM}}^{-1}\text{Secondary}@k_{\text{cutoff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.00</td>
<td>$\approx 0.723607$</td>
<td>$\approx 75.4414$</td>
<td>$\approx 104.7798$</td>
</tr>
<tr>
<td>50.20</td>
<td>$\approx 0.724497$</td>
<td>$\approx 75.5447$</td>
<td>$\approx 105.3429$</td>
</tr>
<tr>
<td>50.40</td>
<td>$\approx 0.725378$</td>
<td>$\approx 75.6472$</td>
<td>$\approx 105.9060$</td>
</tr>
<tr>
<td>50.4053</td>
<td>$\approx 0.725401$</td>
<td>$\approx 75.6499$</td>
<td>$\approx 105.9210$</td>
</tr>
<tr>
<td>50.60</td>
<td>$\approx 0.726250$</td>
<td>$\approx 75.7488$</td>
<td>$\approx 106.4692$</td>
</tr>
<tr>
<td>50.80</td>
<td>$\approx 0.727115$</td>
<td>$\approx 75.8497$</td>
<td>$\approx 107.0324$</td>
</tr>
<tr>
<td>51.00</td>
<td>$\approx 0.727970$</td>
<td>$\approx 75.9499$</td>
<td>$\approx 107.5956$</td>
</tr>
</tbody>
</table>

Table 4.1.1. Possible coupling ratios $\chi_c$ versus $\alpha^{-1}$ (EM Secondary) in the range $\chi_c = 50 \rightarrow 51$. The yellow row corresponds to the interaction cutoff energy in Figure 4.1.2 & Eq. (4.2.10) as there can be only one solution for this cutoff.

### 4.1.1 Comparing this with the standard model

In the real world of SM secondary interactions the electromagnetic force splits into two components $\alpha_1$ & $\alpha_2$ at energies greater than the mass/energy of the $Z_b$ boson or $\approx 91.1876 \text{ GeV}$ [25]. However we want to compare these SM couplings with the values derived in Table 4.1.1 at the $\approx 2.0288 \text{ GeV}$ cutoff of Eq. (4.2.10). Assuming three families of fermions and one Higgs field the SM [26] predicts

$$\alpha_1^{-1} \approx 58.98 \pm 0.08 - \frac{4.1}{2\pi} \log_\epsilon \frac{Q}{91.1876}$$

$$\alpha_2^{-1} \approx 29.60 \pm 0.04 + \frac{19}{6 \times 2\pi} \log_\epsilon \frac{Q}{91.1876}$$

$$\alpha_3^{-1} \approx 8.47 \pm 0.22 + \frac{7}{2\pi} \log_\epsilon \frac{Q}{91.1876}$$

(4.1.3)

The weak force split obeys

$$\alpha_{\text{EM}}^{-1} = \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1}$$

(4.1.4)

Also $\alpha_1^{-1} = \frac{3}{5} \alpha_{\text{EM}}^{-1} \cos^2 \theta_w$ & $\alpha_2^{-1} = \alpha_{\text{EM}}^{-1} \sin^2 \theta_w$ where $\theta_w$ is the Weinberg angle.

Combining these equations $\alpha_{\text{EM}}^{-1} = \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1} \approx 127.90 \pm 0.173 - \frac{11}{3 \times 2\pi} \log_\epsilon \frac{Q}{91.1876}$

(4.1.5)
Figure 4.1.1 plots these four inverse coupling constants. Figure 4.1.2 plots the intersection of $\alpha_{EM}^{-1}$ predicted in Table 4.1.1 and the SM prediction for $\alpha_{EM}^{-1}$ in Eq. (4.1.5). It would initially seem in Figure 4.1.2 that there is an unusually large error band in the predicted results. However $\Delta \alpha_{EM}^{-1}/\Delta \chi^{-1} \approx 2.8$ is approximately constant in this table and the error band in the SM colour coupling $\alpha_3^{-1}$ of $\pm 0.22$ in Eqs (4.1.3) translates into the larger error band for $\alpha_{EM}^{-1}$ of $\pm 0.22 \times 2.8 \approx \pm 0.62$ in Figure 4.1.2.

$\alpha_{EM}^{-1} = \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1} \approx 105.934 \pm 0.173 \approx 2.029 \times 10^{18} \text{GeV}$.

$\alpha_3^{-1} \approx 50.405 \pm 0.22 \approx 2.029 \times 10^{18} \text{GeV}$.

Figure 4.1.2 is a close up of this region.

Figure 4.1.1 Standard Model based on three families of fermions and one Higgs field.

Possible values for $\alpha_{EM}^{-1}$ (Secondary) from Table 4.1.1. There can of course, only be one solution here.

$\frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1} = \alpha_{EM}^{-1}$

Figure 4.1.1 expanded

$Q$ in GeV. $\rightarrow$

Figure 4.1.2 A close up of the intersecting region of the SM that Eq. (4.1.5) and Table 4.1.1 predicts. This fermion interaction cutoff is perhaps more consistent with the SM than we might expect; as we have assumed, for simplicity, a square superposition cutoff at $k_{\text{cutoff}}$. An exponential cutoff of some type is much more likely, but it may have only small effect.
4.2 Introducing Gravity into our Equations

4.2.1 Simple square superposition cutoffs

In section 3.2 we looked at single integer \( n \) superpositions of \( \psi_{nk} \) initially for clarity, and later found multiple integer \( n \) superpositions gave the same results; we will do the same here. We also found in Eqs. (3.2.3) & (3.2.6) that the integrals for both angular momentum and rest masses are of similar form. They both ended up including the term

\[
\left[ \frac{-1}{1 + K_{nk}^2} \right]_0^\infty \text{ which if } K_{nk}^\text{cutoff} < \infty \text{ becomes } \left[ \frac{-1}{1 + K_{nk}^2} \right]_0^{K_{nk}^\text{cutoff}} \text{ and this is equal to } \]

\[
1 - \frac{1}{1 + K_{nk}^2} = \frac{K_{nk}^\text{cutoff}}{1 + K_{nk}^2} = \frac{1}{1 + 1/K_{nk}^2 (\text{cutoff})} = \frac{1}{1 + \varepsilon} \tag{4.2.1}
\]

where using Eq. (3.1.11) the infinitesimal \( \varepsilon = \frac{1}{K_{nk}^2 \text{cutoff}} = \frac{2m_{nk}^2c^2}{n^2\hbar^2(k_{\text{cutoff}})^2s} \tag{4.2.2} \)

For integral or half integral \( h \) angular momentum precision is required but Eq. (3.2.6) now gives us \( L_c^c (\text{Total}) = \frac{\sinh}{2} \left[ -\frac{1}{1 + K_{nk}^2} \right]_0^{K_{nk}^\text{cutoff}} = \frac{\sinh}{2} \frac{1}{1 + \varepsilon} \). So, can the effect of gravity increase our probabilities from \( sN \cdot \frac{dk}{k} \) to \( sN \cdot (1 + \varepsilon) \cdot \frac{dk}{k} \)?

We will initially address only massive infinite superpositions where \( N = 1 \) in Eq. (2.2.4).

The first question we need to address is what is the effective preon mass to be used when coupling to gravity? In Eq. (3.1.4) we said the preon rest mass is \( m_0 / (8\gamma_{nk} \sqrt{2s}) \) for each of the eight preons that build a spin \( \frac{1}{2} \) particle of rest mass \( m_0 \). Now gravitons couple to the total mass including the kinetic energy. At the start of the interaction each preon mass is \( m_0 / (8\gamma_{nk} \sqrt{2s}) \) and after the interaction (Figure 3.1.3) it is \( m_0\gamma_{nk}(1 + \beta_{\text{nk}}^2) / (8\sqrt{2s}) \). Let us think semi-classically again and see where it leads us. We have been using magnitudes of velocities as they are the most convenient way to express our equations, even if not the conventional language of quantum mechanics. The interaction with the zero point fields takes the momentum of each preon from zero to \( 2m_0\gamma_{nk}\beta_{\text{nk}}c / (8\sqrt{2s}) \) (Figure 3.1.3). While this happens as a quantum step change, let us imagine it as a virtually infinite acceleration from zero velocity to \( 2\beta_{\text{nk}}c^2 / (1 + \beta_{\text{nk}}^2) \), which is the relativistic velocity addition (see Figure 3.1.1) of two equal steps of \( \beta_{\text{nk}} \). At the half way point after one step the velocity is \( \beta_{\text{nk}} \) (the velocity of the CMF, the preon mass has increased to \( m_0 / (8\sqrt{2s}) \). We can imagine this as being like the central point of a quantum interaction.

We will conjecture this midway point preon mass \( m_0 / (8\sqrt{2s}) \) is the mass value that gravitons couple to and we will see that it is indeed the only value that fits all equations. Also, it does not make sense to choose either of the end point masses. We can also get reassurance from the properties of the Feynman transition amplitude which informs us in Eq. (3.1.15) that
\[
\frac{(p_i + p_f)^2}{(p_i + p_f)^0} = \frac{2m_0\gamma_{nk}p_{nk}}{2m_0\gamma_{nk}} = \beta_{nk} \quad \text{and the ratio of space to time polarization in the LF is } \beta_{nk}^2.
\]

This centre of momentum velocity gives us the key properties of the interaction. We will thus assume we have eight preons in each \(\psi_{nk}\) of effective gravitational mass \(m_0 / (8\sqrt{2} s)\) with effective total gravitational mass \(m_0 / \sqrt{2} s\). To put the gravitational constant in the same form as the other coupling constants we need to divide it by \(\hbar c\). The gravitational coupling amplitude is thus \(m_0\sqrt{G_p} / (2\hbar c)\) to the gravitational zero point field, where \(\sqrt{G_p}\) is the primary amplitude for a Planck mass to emit or absorb a graviton. Now this gravitational amplitude can be regarded as a complex vector just as colour and electromagnetism. We assumed for simplicity, as they are both spin 1 field particles, that colour and electromagnetism are parallel. Spin 2 gravity could be at a different complex angle to the other two. In fact, the equations only have the correct properties if gravity is at right angles to colour and electromagnetism. Putting \(G_{\text{primary}} = \chi'_G \cdot G_{\text{secondary}}\) we conjecture that the gravitational coupling amplitude is \(im_0\sqrt{G_p} / (2\hbar c) = im_0\sqrt{\chi'_G \cdot G_s} / (2\hbar c)\),

\[
= im_0\sqrt{\chi'_G \cdot G} / (2\hbar c)
\]

(4.2.3)

We have put the secondary gravitational coupling constant to a bare Planck mass \(G\), in Eq. (4.2.3) equal to the measured gravitational constant \(G\). We can only do this if \(\alpha_G = 1\) between Planck masses. (See Eq. (5.1.7) and the preceding paragraph.) We will later find that \(\alpha_G\) does not need to be one and is in fact less than one. Consequently we temporarily label the ratio between the primary and secondary gravitational constants as \(\chi'_G\), returning to it in section 6.2.6. So, modifying Eqs. (2.2.1) to (2.2.3) by adding Eq. (4.2.3)

\[
Q^2A^2 = \left[\frac{8 + 8\sqrt{\alpha_{EMP}}}{3\pi sN} \hbar^2 k^4 r^2\right][sN \cdot dk] \quad \text{where } \varepsilon' = \frac{m_0^2 \chi'_G \cdot G}{2\hbar^2 c(8 + 8\sqrt{\alpha_{EMP}})^2}
\]

Our previous wavefunctions \(\psi_k\) required \(Q^2A^2 = \left[\frac{8 + 8\sqrt{\alpha_{EMP}}}{3\pi sN} \hbar^2 k^4 r^2\right][sN \cdot dk]\) from Eq. (2.2.4).

Thus primary graviton interaction can increase the probability of our previous wavefunctions \(\psi_k\) by \(1 + \varepsilon'\) as required to obtain precision in our integrals for \(h/2 \& h\) if \(K_{nk} \text{cutoff} < \infty\).

Using Eq.(4.2.2) now put \(\varepsilon' = \frac{m_0^2 \chi'_G \cdot G}{2\hbar^2 c(8 + 8\sqrt{\alpha_{EMP}})^2} = \varepsilon = \frac{1}{K_{nk}^2 \text{cutoff}} = \frac{2m_0^2 c^2}{sN^2 h^2 (k_{\text{cutoff}})^2}\)

Thus

\[
\frac{\chi'_G \cdot G}{4\hbar^2 c(8 + 8\sqrt{\alpha_{EMP}})^2} \approx \frac{c^2}{n^2 h^2 (k_{\text{cutoff}})^2}
\]

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For \( N = 1 \) single integer \( n \) superpositions \( \psi_k = \sum_n c_n \psi_{nk} \), we can use the logic of section 3.5.1; replacing \( K_{nk}^2 \) with \( \langle K_k^2 \rangle \), and \( n^2 \) with \( \langle n \rangle^2 \) in Eq. (4.2.4) so that Eq. (4.2.5) becomes

\[
\chi'_G \approx \frac{256(1 + \sqrt{\alpha_{EMP}})^2}{n^2 (k_{\text{cutoff}})^2} \]  

(4.2.5)

If we now go back to Eqs. (2.3.9) & (2.3.10) as \( k \to \infty \) the energy squared \( E_{nk}^2 \to p_{nk}^2 c^2 = n^2 \hbar^2 c^2 \). Again, using the logic of section 3.5.1 for multiple integer \( n \) superpositions the expectation value for energy squared as \( k \to \infty \) is \( \langle E_k^2 \rangle \to \langle |p_k|^2 \rangle c^2 = \langle n \rangle^2 \hbar^2 k^2 c^2 \) thus

for multiple integer \( n \) superpositions as \( k \to \infty \), \( \langle E_k \rangle \to \langle |p_k|^2 \rangle c = \langle n \rangle \hbar k c \)  

(4.2.6)

### 4.2.2 All \( N = 1 \) superpositions cutoff at Planck energy but interactions at less

It is reasonable to assume that the cutoff superposition energy cannot exceed the Planck energy \( E_{\text{Planck}} \) (at least for square cutoffs) and that this is true for all \( N = 1 \) superpositions. (Section 6.2.1 discusses \( N = 2 \) superposition \( E_{\text{Planck}} \) cutoffs.) So, for simple square cutoffs:

\( N = 1 \) multiple integer \( n \) superpositions cutoff energy \( \langle E_{k(\text{cutoff})} \rangle = \langle n \rangle \hbar k_{\text{cutoff}} c = E_{\text{Planck}} \)  

(4.2.7)

This can be written as \( \langle n \rangle k_{\text{cutoff}} \hbar c = E_{\text{Planck}} = \frac{hc}{L_{\text{Planck}}} \)  

(4.2.8)

For \( N = 1 \) multiple integer \( n \) superpositions \( \langle n \rangle k_{\text{cutoff}} = \frac{1}{L_{\text{Planck}}} \) & \( \langle n \rangle k_{\text{cutoff}} L_p = 1 \)  

(4.2.8)

\( N = 1 \) multiple integer \( n \) superposition interaction cutoff energy \( \hbar k_{\text{cutoff}} = \frac{E_{\text{Planck}}}{\langle n \rangle} \)  

(4.2.9)

Using Eq. (4.2.9) with Planck energy of \( 1.22 \times 10^{19} \text{GeV} \) and \( \langle n \rangle \approx 6.0135 \) from Eq.(3.5.16) for simple square cutoffs (also see Figure 4.1.2).

Interactions between \( N = 1 \) fermions cutoff \( \approx 2.0288 \times 10^{18} \text{GeV} \).

From Table 4.3.1 we see that all other particles such as photons, gluons and gravitons etc. have \( \langle n \rangle < 6 \) and thus higher interaction cutoff energies than fermions i.e.
\( > 2.03 \times 10^{18} GeV \), but \( < E_p \). Putting \( 2.0288 \times 10^{18} GeV \) in the SM equations (4.1.3) and (4.1.4).

\[
\begin{align*}
\alpha_i^{-1} \text{ @ } 2.0288 \times 10^{18} GeV & \approx 34.4179 \pm 0.08 @ k(\text{cutoff}) \\
\alpha_2^{-1} & \approx 48.5707 \pm 0.04 \\
\alpha_3^{-1} & \approx 50.4053 \pm 0.22 \\
\alpha_{EM}^{-1} & = \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1} \approx 105.934 \pm 0.173
\end{align*}
\]  

Real world high energy secondary interactions only involve \( \alpha_i, \alpha_j, \& \alpha_3 \), but spin zero primary interactions do not involve the weak force. Table 4.1.1 can thus only predict \( \alpha_{EM}^{-1} \approx 105.921 \) at the cutoff compared to the SM combination of \( (5/3)\alpha_1^{-1} + \alpha_2^{-1} = \alpha_{EM}^{-1} \approx 105.934 \pm 0.173 \) of Eq. (4.2.10). (See Figure 4.1.1 & Figure 4.1.2). Also using Eqs. (3.3.3) and (4.2.10) we get the primary to secondary fundamental coupling ratio \( \chi_c \).

\[
\text{Coupling Ratio } \chi_c = \alpha_3^{-1} @ k_{\text{cutoff}} \approx 50.405 \pm 0.22 \text{ (ie. @ } 2.0288 \times 10^{18} GeV.\text{)}
\]  

The secondary coupling constants in Eq. (4.2.10) can perhaps be thought of as those to the bare colour and electromagnetic charges.

If we now put Eq. (4.2.8) into Eq. (4.2.5) we get \( \chi'_G \approx \frac{256(1 + \sqrt{\alpha_{EM}})^2}{(n)^2(k_{\text{cutoff}}L_p)^2} = 256(1 + \sqrt{\alpha_{EM}})^2 \)

From Eqs.(4.1.2) and Table 4.3.1 we find \( (1 + \sqrt{\alpha_{EM}}) \approx 1.115 \) and Eq.(4.2.5) becomes

\[
\chi'_G \approx 256(1.115)^2 \approx 318.3
\]  

From the paragraph following Eq. (4.2.3) we see that this equation temporarily assumes \( \alpha_G = 1 \). If it does equal one then \( \chi'_G \approx 318.3 \) is the ratio between the primary graviton coupling to bare preons, and the normal measured gravitational constant G. Approximate calculations using Eq. (5.3.20) in Eq. (11.1.18) suggest that \( \alpha_G \) is probably somewhere in the range \( \alpha_G \approx 1/24 \). (In section 6.2.6 we assume an approximate value of \( \alpha_G \approx 1/24 \) in Eq.(6.2.8) to get a primary to secondary graviton coupling ratio of \( \chi_G \approx 7632 \).) When \( \chi'_G \approx 318 \) the contribution from gravity cancels any deficit in primary interactions providing these superpositions cutoff at Planck energy, which we argue is true for all \( N = 1 \) superpositions. (This section and all its equations are derived by equating the contribution due to gravity and the deficit due to the Planck energy cutoff in primary interactions.). To enable Planck energy interactions \( N = 2 \) infinitesimal mass bosons must also cutoff at Planck energy just as \( N = 1 \) superpositions do, or as in Eq. (4.2.9). Sections 6.2 & 6.2.1 discusses these \( N = 2 \) superposition Planck energy cutoffs.
Figure 4.2.1 plots radial probabilities for all $n = 3, 4, 5, 6 & 7$ Planck Energy cutoff modes. They are identical as the radial probability $P_n \propto r^8 \times \text{Exp}(n^2 k^2 r^2 / 9)$, but from Eq. (4.2.6) $nk = 1$ in each Planck energy mode, so they all have radial probability $P_n \approx 8.74 \times 10^{-6} \times r^8 \times \text{Exp}(r^2 / 9)$.

![Planck region](image)

All Planck energy $n$ modes look identical

$\rightarrow$ Radius in Planck units.

Figure 4.2.1 Plot of radial probability of all $n = 3, 4, 5, 6 & 7$ Planck energy modes. Despite each mode having Planck energy, the probability in every case of being inside the Planck region is virtually zero at $\approx 8.9 \times 10^{-7}$.

### 4.3 Solving for Spin $\frac{1}{2}$, Spin 1 and Spin 2 Superpositions

Superpositions with $N = 2$ are covered in section 6.2. Equation (4.2.11) and Eq. (3.3.20) can be extended by keeping $N \cdot s$ constant as in Eq. (4.4.1) allowing us to solve various combinations of spins $\frac{1}{2}$, 1 or 2 and $N = 1$ or $N = 2$.

$$(N = 2) \times \text{(Spin 1)} \quad (N = 1) \times \text{(Spin 1)} \quad (N = 1) \times \text{(Spin $\frac{1}{2}$)}$$

or

$$(N = 1) \times \text{(Spin 2)} \quad \text{or (N = 2) \times (Spin $\frac{1}{2}$)}$$

$$4c_{4b} \times c_{4b} (1 - c_{4b} \times c_{4b}) = 2c_{5b} \times c_{5b} (1 - c_{5b} \times c_{5b}) = c_{6a} \times c_{6a} (1 - c_{6a} \times c_{6a})$$

$$= \sqrt{2 / \chi_c} \approx \sqrt{2 / 50.4053} \approx 0.199194$$

Starting with spin $\frac{1}{2}$ we can solve this to get $c_b \times c_b \approx 0.7254$ as the dominant value.

Putting $c_b \times c_b \approx 0.7254$ into Eq.(4.1.2) or alternatively using Table 4.1.1

$$\alpha_{EMP} \approx \left[ \sqrt{1.386256 - 0.197258 c_b \times c_b} - 1 \right] \approx 75.6499^{-1}$$

(4.4.2)

From Eq. (2.2.4) the available $Q^2 A^2 = \left[ \frac{8 + 8 \sqrt{\alpha_{EMP}}}{3 \pi sN} \right]^2 \hbar^4 k^4 r^2$ with probability $\frac{sN \cdot dk}{k}$ where we ignore the infinitesimal factor of $(1 + \varepsilon)$ due to gravitons. And from Eq. (2.3.12)
\[ Q^2 A^2 = \sum_n c_n^* c_n \frac{n^4 \hbar^2 k^4 r^2}{81} = \frac{\left(8 + 8 \sqrt{\alpha_{\text{EMP}}} \right)^2}{3\pi sN} \hbar^2 k^4 r^2 \]

\[ \sum c_n^* c_n \cdot n^4 \approx 1367.58 \text{ for (spin } 1/2 \times N = 1) \]
\[ \approx 683.79 \text{ for (spin } 1 \times N = 1) \text{ or (spin } 1/2 \times N = 2) \]  (4.4.3)
\[ \approx 341.9 \text{ for (spin } 1 \times N = 2) \text{ or (spin } 2 \times N = 1) \]
\[ \approx 170.95 \text{ for (spin } 2, N = 2) \text{ by extension.} \]

The same primary electromagnetic coupling \( \alpha_{\text{EMP}} \) builds all fundamental particles, allowing Eq.(4.4.3) to be true. Using Eqs, (4.4.1), (4.4.3) \& \( \sum c_n^* c_n = 1 \) we get Table 4.3.1. We define the coupling ratio for gravitons \( \chi_C \approx 6364 \) in Eq.(6.2.8) section 6.2.6, where we also solve infinitesimal mass graviton superpositions. In Table 4.3.1 three member superpositions fit the SM best. In section 4.1 we solved spin ½ superpositions with a dominant centre mode \( c_n^* c_n \) that fitted the SM. However when solving for spins 1 & 2 we must initially comply with Eq. (4.4.1) which defines interaction probabilities (see Eq. (3.3.20) and the following paragraph). We must also comply with Eq. (4.4.3) which determines centre or side mode dominance. In this table we have also included a massive \( N = 1 \), spin 2 graviton superposition as a dark matter possibility which we will look at in Section 11. There are other possibilities which we have not included.

<table>
<thead>
<tr>
<th>Mass Type</th>
<th>Spin</th>
<th>N</th>
<th>( c_3^* c_3 )</th>
<th>( c_4^* c_4 )</th>
<th>( c_5^* c_5 )</th>
<th>( c_6^* c_6 )</th>
<th>( c_7^* c_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinitesimal mass gravitons</td>
<td>2</td>
<td>2</td>
<td>0.8344</td>
<td>0.0003</td>
<td>0.1653</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Massive Spin 2 gravitons</td>
<td>2</td>
<td>1</td>
<td>0.4847</td>
<td>0.0526</td>
<td>0.4627</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Infinitesimal mass bosons</td>
<td>1</td>
<td>2</td>
<td>0.4847</td>
<td>0.0526</td>
<td>0.4627</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Massive bosons</td>
<td>1</td>
<td>1</td>
<td>0.0134</td>
<td>0.8878</td>
<td>0.0988</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Massive fermions</td>
<td>½</td>
<td>1</td>
<td>0.1305</td>
<td>0.7254</td>
<td>0.1441</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.3.1** Approximate probabilities for known and one possible superposition.

To this point this paper has attempted to demonstrate that infinite superpositions can behave as the SM fundamental particles. The methods used may be unconventional, but it is important to remember that primary interactions are very simple and very different in comparison to secondary interactions (see sections 2.2.2 & 2.2.3). These methods are also based on simple basic quantum mechanics and SR. There is also surprising consistency with the SM. If the principles behind the outcomes of these derivations are at least on the right track, and fundamental particles can be built by borrowing energy and mass from (space and time mode) zero point fields then, as we will see in what follows, this may have some significant and profound consequences.
Part 2
Consequences of Infinite Superpositions

5 Exploring Possible Connections with Gravity

5.1 Zero Point Energy Densities are Limited

If the fundamental particles can be built from energy borrowed from the spatial component of zero point fields and this energy source is limited, particularly at cosmic wavelengths, there must be implications for the maximum possible densities of these particles. In section 2.2.3 we discussed how the preons that build fundamental particles are born from a Higg’s type scalar field with zero momentum in the laboratory rest frame. Infinitesimal mass particles such as gravitons borrow their mass from the time component of the same zero point fields. In this frame they have infinite wavelength and can borrow from anywhere in the universe. This suggests there should be little effect on localized densities, but possibly on overall average densities in any universe. So, which fundamental particle is likely to be most abundant? Working in Planck, or natural units with $G = 1$, assume a graviton coupling constant between Planck masses of one. The total baryonic matter in the universe according to the $\Lambda$CDM is $\approx 4.5 \times 10^{51} K_g$ or $\approx 2 \times 10^{59}$ Planck masses. Including dark matter this is $\approx 10^{60}$ Planck masses. Their average distance apart is approximately the radius $R_{oh}$ of this region. As a crude illustration assume one graviton of this wavelength per pair of Planck masses. Thus there should be approximately $M^2 \approx 10^{120}$ virtual gravitons with wavelengths of the order of radius $R_{oh}$ within this same volume. (This number is in line with more accurate later calculations.) No other fundamental particle is likely to approach these values; for example, the number of virtual photons of this extreme wavelength is much smaller. (Virtual particles emerging from the vacuum are covered in section 6.2.3) If this density of virtual gravitons needs to borrow more energy from the zero point fields than what is available at these extreme wavelengths, does this somehow control the maximum possible density of a causally connected universe?

5.1.1 Virtual particles and infinite superpositions

Looking carefully at Section 3.3 we showed there that, for all interactions between fundamental particles represented as infinite superpositions, the actual interaction is only between a single wavenumber $k$ superposition of each particle. We are going to conjecture that a virtual particle of wavenumber $k$, for example, is just such a single wavenumber $k$ member. Only if we somehow interact with it do we observe the properties of the full infinite superposition representing that particle. They are virtual before this interaction, always only lasting for $\Delta T \leq h / 2\Delta E$, and the full properties do not exist until observed, as in the Copenhagen interpretation of quantum mechanics. Even though they are only a single wavenumber their three superposition modes as in Table 4.3.1 partially identify them. This combination and its probability as in Eq. (2.2.4) $\mathcal{N} \cdot s \cdot dk / k$ and the first paragraph after Eq. (3.3.12) defines the full particle properties. We will use this conjectured virtual property when
looking at the probability density of virtual gravitons of the maximum cutoff wavelength. Virtual gravitons are thus a superposition of the three modes \( n = 3, 4, 5 \) as in Table 4.3.1, but of a single wavenumber \( k \) only. Time polarized or spherically symmetric versions are a further equal \((1/\sqrt 5)\) superposition of \( m = \pm 2, \pm 1, 0 \) states of the above \( n = 3, 4, 5 \) mode superpositions. A spin 2 virtual graviton in an \( m = +2 \) state is simply a superposition of the three modes \( n = 3, 4, 5 \) as above, but all in an \( m = +2 \) state.

5.1.2 Virtual graviton density at wavenumber \( k \) in a causally connected universe

From this point on we will use Planck units \( \hbar = c = G = 1 \). General Relativity predicts nonlinear fields near black holes, but in the low average densities of typical universes we can approximate linearity. The majority of mass moves slowly relative to comoving coordinates so we can ignore momentum (i.e. \( \beta << 1 \)), provided we limit this analyses to comoving coordinates. Spin 2 gravitons transform as the stress tensor in contrast to the 4 current Lorentz transformations of spin 1, but, at low mass velocities the only significant term is the mass density \( r_{00} \). In comoving coordinates the vast majority of virtual gravitons will thus be time polarized or spherically symmetric which we will for simplicity call scalar. We will initially do all our calculations in these comoving coordinates where we should be able to simply apply the equations in sections 3.4 & 3.5 to spin 2 virtual graviton emissions, as they should apply equally to both spins 1 & 2 at low mass velocities. (This is not necessarily so near black holes.) We will assume spherically symmetric \( l = 3 \) wavefunctions emit both spin 1 & 2 scalar virtual bosons, and \( l = 3, m = \pm 2 \) states can emit both \( m = \pm 1 \) spin 1 bosons and \( m = \pm 2 \) spin 2 gravitons. Section 3.4 derived the electrostatic energy between infinite superpositions. In flat space we looked at the amplitude that each equivalent point charge emits a virtual photon, and then focused on the interaction terms between them. Thus we can use the same scalar wavefunctions Eq’s. (3.4.1) for virtual scalar gravitons as we did for virtual scalar photons. Using \( (\psi_1 + \psi_2) (\psi_1' + \psi_2') = \psi_1' \psi_1 + \psi_1' \psi_2 + \psi_2' \psi_1 + \psi_2' \psi_2 \) we showed in section 3.4.1 that the interaction term for virtual photons is

\[
\psi_1' \psi_2 + \psi_2' \psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-k(r_1 + r_2)} \cos k(r_1 - r_2)
\]

(5.1.1)

This equation is strictly true only in flat space but it is still approximately true if the curvature is small or when \( 2m/r <<<<1 \), which we will assume applies almost everywhere throughout the universe except in the infinitesimal fraction of space close to black holes. In both sections 3.4 & 3.5, for simplicity and clarity, we delayed using coupling constants and emission probabilities in the wavefunctions until necessary. We do the same here. There will also be some minimum wavenumber \( k \) which we call \( k_{\text{min}} \) where for all \( k < k_{\text{min}} \) there will be insufficient zero point energy available, and Eq. (5.1.1) cuts off exponentially. We will find that this maximum wavelength is where \( k_{\text{min}} \approx 1/R_{\text{DU}} \approx 1/R_{\text{ObservableUniverse}} \). In Section 6 we find gravitons have an infinitesimal rest mass \( m_{\text{0}} \) of the same order as this minimum wavenumber \( k_{\text{min}} \). At these extreme \( k \) values this rest mass must be included in the wavefunction exponential term. It is normally irrelevant for infinitesimal masses. Section 6.2 looks at \( N = 2 \) infinitesimal rest masses finding \( \{K_{\text{min}}\}^2 \approx 1 \).
Using Eq. (3.1.11) and \( \hbar = c = 1 \)

\[
\langle K_{\min} \rangle^2 = \frac{s \langle n \rangle^2 k_{\min}^2}{2m_0^2} \approx 1 \quad \text{so for spin 2 gravitons} \quad \frac{\langle n \rangle^2 k_{\min}^2}{m_0^2} \approx 1 \quad \text{or} \quad m_0 \approx \langle n \rangle k_{\min}
\]  
(5.1.2)

Table 4.3.1 tells us for \( N = 2 \) spin 2 gravitons \( \langle n \rangle \approx 3.33 \) so that \( m_0 \approx 3.33k_{\min} \)  
(5.1.3)

This virtual mass \( m_0 \) increases the \( \Delta E \) term in \( \Delta E \cdot \Delta T \leq \hbar / 2 \) for a virtual graviton from \( \Delta E = k \) to \( \Delta E = \sqrt{k^2 + m_0^2} \) when \( \hbar = c = 1 \), reducing the range \( r \approx \Delta T \approx \Delta E^{-1} \) over which it can be found. This range is controlled by the exponential decay term \( e^{-kr} \) in its wavefunction, becoming \( e^{-\sqrt{k^2 + m_0^2}} \) near \( k_{\min} \). So we can define a \( k' \) using Eq. (5.1.3)

\[
k' = \sqrt{k^2 + m_0^2} \approx \sqrt{k^2 + 11.09k_{\min}^2} \quad \text{and} \quad k_{\min}' \approx \sqrt{k_{\min}^2 + 11.09k_{\min}^2} \approx 3.477k_{\min}
\]  
(5.1.4)

Using the normalized virtual graviton wavefunction Eq. (3.4.1) we can say that:

A massless \( \psi = \sqrt{\frac{2k}{4\pi k}} e^{-k r + i k r} \) becomes with infinitesimal mass \( \sqrt{\frac{2k^2}{4\pi k} e^{-k' r + i k' r}} \)  
(5.1.5)

Thus the massless interaction term in Eq. (5.1.1) becomes with this infinitesimal mass \( m_0 \)

\[
\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1 = \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1 + r_2)} \cos k(r_1 - r_2)
\]  
(5.1.6)

Let point \( P \) in Figure 5.1.1 be anywhere in the interior region of a typical universe. Let the average density (or its equivalent transformed value) be \( \rho_U \) (subscript \( U \) for universe) Planck masses/energy density per unit volume. Consider two spherical shells initially in comoving coordinates around the central point \( P \) of radii \( r_1 \& r_2 \) and thicknesses \( dr_1 \& dr_2 \) with masses \( dm_1 = \rho_U dv_1 = 4\pi r_1^2 dr_1 \rho_U \) and \( dm_2 = \rho_U dv_2 = 4\pi r_2^2 dr_2 \rho_U \)

**Figure 5.1.1** Two spherical shells surrounding a central point.

In Planck units we know that the gravitational constant \( G = 1 \) applies between Planck masses, so we might expect the graviton coupling constant is \( \alpha_G = 1 \) between Planck masses also, but we don’t actually know this. (Its actual value has no effect on what we are going to do in section 5, but we will find in Eq. (11.1.18) that \( \alpha_G \approx 1/24 \).
Until we can calculate the actual value of $\alpha_g$ we will temporarily define it as
\[
\text{The secondary graviton coupling constant between Planck masses } = \alpha_g \quad (5.1.7)
\]
(We will see later that the graviton coupling constant increases at extremely low accelerations as in MOND, and the above definition is for higher accelerations only). Section 3.4.1 in Eq. (3.4.3) used scalar emission probability $(2\alpha / \pi)(dk/k)$ which becomes $(2\alpha_g / \pi)(dk/k)$ between Planck masses. Equation (6.2.7) suggests all superpositions cutoff exponentially @ $k_{\min}$. Looking at what we did in deriving Eq. (3.3.14) for graviton emission probability, we must include this cutoff twice i.e. $(1 - \text{Exp}[-0.61k^2 / k_{\min}^2])^2$, first for graviton superpositions, then for mass superpositions. (We are looking at emission probability here, not exchange probability, which requires four powers.) Now distant galaxies recede at light like and greater velocities, but all clocks in comoving coordinates tick at the same rate and quantum interactions are instantaneous over all space. Thus, as we integrate over radii $r_i$ & $r_j = 0 \to \infty$, we can still use the same equations as if the distant galaxies are not moving. (The vast majority of mass is moving relatively slowly in these comoving coordinate systems and we return to this important comoving coordinate property in section 5.3.1). Using this new coupling probability between Planck masses $(2\alpha / \pi)(dk/k)$ we can now integrate over both radii $r_i$ & $r_j$; but to avoid counting all pairs of masses $dm_i$ & $dm_j$ twice, we must divide the integral by two. The total probability density of virtual gravitons at any point $P$ in the universe at wavenumber $k$, is using Eq.(5.1.6)
\[
\rho_{G_k} = \frac{\rho^2_{\mu}}{2} \frac{\alpha_g}{(1 - e^{-0.61k^2/k_{\min}^2})^2} \frac{2}{\pi} \int_{0}^{\infty} [4\pi r_i^2 dr_i \cdot 4\pi r_j^2 dr_j \cdot \frac{4k'}{4\pi r_i r_j} e^{-k'(r_i + r_j)} \cos k(r_i - r_j)]
\]
\[
= 16\alpha_g (1 - e^{-0.61k^2/k_{\min}^2})^2 \rho^2_{\mu} k' \int_{0}^{\infty} [r_i r_j e^{-k'(r_i + r_j)} \cos k(r_i - r_j) \cdot dr_i \cdot dr_j]
\]
Expanding $\cos k(r_i - r_j) = \cos kr_i \cos kr_j + \sin kr_i \sin kr_j$ we can then use:
\[
\int_{0}^{\infty} e^{-k'(r_i + r_j)} \cos kr_i \cos kr_j dr_i dr_j = \frac{k'^4 - 2k'^2k^2 + k^4}{(k'^2 + k^2)^4}
\]
\[
\int_{0}^{\infty} e^{-k'(r_i + r_j)} \sin kr_i \sin kr_j dr_i dr_j = \frac{4k'^2k^2}{(k'^2 + k^2)^4}
\]
Adding both together $\rho_{G_k} = 16\alpha_g (1 - e^{-0.61k^2/k_{\min}^2})^2 \rho^2_{\mu} k' \int_{0}^{\infty} [k'^2 - k^2 + \frac{1}{(k'^2 + k^2)^4}]
\]
\[
= 16\alpha_g (1 - e^{-0.61k^2/k_{\min}^2})^2 \rho^2_{\mu} \frac{k'}{k} \frac{1}{(k'^2 + k^2)^4}
\]
\[
\text{From Eq.(5.1.4) } k' = \sqrt{k^2 + m^2_0} \approx \sqrt{k^2 + 11.09k_{\min}^2} \text{ and we can write Eq.(5.1.8) as}
\]
\[
\rho_{G_k} = 16\alpha_g \rho^2_{\mu} (1 - e^{-0.61k^2/k_{\min}^2})^2 \sqrt{k^2 + 11.09k_{\min}^2} \frac{dk}{(2k^2 + 11.09k_{\min}^2)^2 k} \]
\[
= 16\alpha_g \rho^2_{\mu} \sqrt{1 - e^{-0.61x^2}} \frac{1}{x(2x^2 + 11.09)^2} \text{ where } x = \frac{k}{k_{\min}}
\]
65
Wavelength $k$ probability density $\rho_{Gk} \approx \frac{0.0677a_G \rho_U^2}{k_{\text{min}}^4} \int \frac{[237 \left(1 - e^{-0.61^2}\right)^2 \sqrt{x^2 + 11.09}]}{x \left(2x^2 + 11.09\right)^2} dk$

Where the blue square bracket is approximately one when $k / k_{\text{min}} = x = 1$

Cutoff wavelength probability density $\rho_{Gk_{\text{min}}} \approx \frac{0.0677a_G \rho_U^2}{k_{\text{min}}^4} d k_{\text{min}}$ when $k / k_{\text{min}} = x = 1$

As we think $K_{G_{\text{min}}}$ will prove to be a spacetime invariant we will write this as follows:

Cutoff wavelength probability density $\rho_{Gk_{\text{min}}} = K_{Gk_{\text{min}}} d k_{\text{min}}$ where $K_{Gk_{\text{min}}} \approx \frac{0.0677a_G \rho_U^2}{k_{\text{min}}^4}$ (5.1.10)

5.2 Can we Relate all this to General Relativity?

The above assumes a homogeneous universe that is essentially flat on average. At any cosmic time $T$ it also assumes there is always some value $k_{\text{min}}$ where the borrowed energy density $E_{Gk_{\text{min}}} = E_{ZP_{\text{min}}}$, the available zero point energy density @ $k_{\text{min}}$. We have initially assumed comoving coordinates, but at peculiar velocities our spherical shells become ellipses and our equation $\rho_{Gk_{\text{min}}} = K_{Gk_{\text{min}}} d k_{\text{min}}$ should remain true at any peculiar velocity, also in all coordinates, as we hope to show later. So, what happens if we put a small mass concentration $+m_1$ at some point? The gravitons it emits must surely increase the local density of $k_{\text{min}}$ gravitons, upsetting the balance between borrowed energy and that available. However, GR informs us that near mass concentrations the metric changes, radial rulers shrink and local observers measure larger radial lengths. This expands locally measured volumes lowering their measurement of the background $\rho_{Gk_{\text{min}}}$. But clocks slowdown also, increasing the locally measured value of $k_{\text{min}}$. Let us look at whether we can relate these changes in rulers and clocks with the $\rho_{Gk_{\text{min}}} = K_{Gk_{\text{min}}} d k_{\text{min}}$ of Eq. (5.1.10).

5.2.1 General Relativity may not be accurate at large cosmic radii

In this section we will only consider the probability density of $k_{\text{min}}$ virtual gravitons emitted from a mass concentration $+m_1$ at a distance $r$ from a central observer at the point $P$ as in Figure 5.2.1. We will assume that space is approximately flat with errors $\propto 1 - (1 - 2m/r)^{1/2}$ or $\approx m/r$. We can again use Eq.(5.1.6) for this probability density but now use the radial distances as in Figure 5.2.1 and $k'_{\text{min}}$ & $k_{\text{min}}$ as we are only focussing on $k_{\text{min}}$.

$$\psi_1 \ast \psi_2 + \psi_2 \ast \psi_1 = \frac{4k_{\text{min}}'}{4 \pi r_1 r_2} e^{-k_{\text{min}}' (r_1 + r)} \cos[k_{\text{min}} (r_1 - r)]$$

(5.2.1)

We are assuming the background gravitons are time polarized as we are effectively looking at the scalar potential of a central mass relative to the rest of the universe, or a time polarized/scalar interaction with no directional effects due to spatial polarization.
We can consider simple spherical shells (again initially in comoving coordinates) of thickness $dr$ and radius $r$ around a central observer at the point $P$ which have mass $dm = 4\pi \rho_U r^2 dr$.

At each radius $r$ the coupling factor including cutoff is $(1 - e^{-0.61k/k_{\min}})^2 (2\alpha_G / \pi) (dk \ell / k)$ and when $k = k_{\min}$ is $(1 - e^{-0.61})^2 (2\alpha_G / \pi) (dk_{\min} / k_{\min})$ between Planck masses. Again, assuming instantaneous quantum coupling as if space is not expanding:

\[
\text{Coupling factor} = (1 - e^{-0.61})^2 \frac{2\alpha_G m_i}{\pi} \frac{dk_{\min}}{k_{\min}} = (1 - e^{-0.61})^2 \frac{2\alpha_G m_i}{\pi} \rho_U 4\pi r^2 dr \frac{dk_{\min}}{k_{\min}} \quad (5.2.2)
\]

Including this coupling factor in Eq. (5.2.1)

\[
(1 - e^{-0.61})^2 \left( \frac{2\alpha_G m_i}{\pi} \frac{dk_{\min}}{k_{\min}} \rho_U 4\pi r^2 dr \right) (\psi^*_1 \psi^*_2 + \psi^*_2 \psi^*_1)
\]

\[
= (1 - e^{-0.61})^2 \left( \frac{2\alpha_G m_i}{\pi} \frac{dk_{\min}}{k_{\min}} \rho_U 4\pi r^2 dr \right) \left( \frac{4k'_{\min} e^{-2k'_{\min}(r_1 - r)} \cos[k_{\min}(r_1 - r)]}{4\pi r_{1} r} \right)
\]

\[
= (1 - e^{-0.61})^2 \frac{8\rho_U}{r_1} k'_1 \frac{dk_{\min}}{k_{\min}} \int_{r = 0}^{\infty} e^{-k'_{\min}(r_1 - r)} \cos[k_{\min}(r_1 - r)] dr
\]

This is the virtual graviton density at point $P$ due to each spherical shell. (Ignoring the relatively small number of $k_{\min}$ gravitons emitted by mass $m$, itself $\psi^* m \psi m$, (Section 8)). Integrating over radius $r = 0 \to \infty$ the virtual graviton density at wavenumber $k_{\min}$ using Eqs.(5.1.4) and (5.2.3) is

\[
\Delta \rho_G = (1 - e^{-0.61})^2 \frac{m_i}{r_1} \frac{8\rho_U}{\pi} k'_1 \frac{dk_{\min}}{k_{\min}} \int_{r = 0}^{\infty} \left[ e^{-k'_{\min}(r_1 - r)} \cos[k_{\min}(r_1 - r)] \right] dr
\]

\[
\int_{r = 0}^{\infty} re^{-k'_{\min}(r_1 - r)} \cos[k_{\min}(r_1 - r)] dr = \int_{r = 0}^{\infty} re^{-k'_{\min}t} [\cos(k_{\min} r_1) \cos(k_{\min} r) + \sin(k_{\min} r_1) \sin(k_{\min} r)] dr
\]
Using Eq. (5.1.4) \( k'_{\text{min}} \approx 3.477 k_{\text{min}} \) this becomes

\[
\int_{r=0}^{r=\infty} e^{-k'_{\text{min}} r} \cos(k_{\text{min}} r_1) dr \approx \frac{1}{15.45k_{\text{min}}^2} e^{-3.44k_{\text{min}} r} \left[ \cos(k_{\text{min}} r_1) + 0.627 \sin(k_{\text{min}} r_1) \right]
\]

Inserting this into Eq. (5.2.4) and using \( k'_{\text{min}} \approx 3.477 k_{\text{min}} \) again

\[
\Delta \rho_G \approx (1 - e^{-0.61})^2 \frac{m_l}{r_1} \frac{8 \rho U}{k_{\text{min}}} \int_{r=0}^{r=\infty} e^{-3.44k_{\text{min}} r} \cos(k_{\text{min}} (r_1 - r)) dr
\]

\[
\Delta \rho_G \approx (1 - e^{-0.61})^2 \frac{m_l}{r_1} \frac{8 \rho U}{k_{\text{min}}} 3.477 k_{\text{min}} \frac{1}{15.45k_{\text{min}}^2} e^{-3.44k_{\text{min}} r} \left[ \cos(k_{\text{min}} r_1) + 0.627 \sin(k_{\text{min}} r_1) \right]
\]

\[
\Delta \rho_G \approx (1 - e^{-0.61})^2 \frac{m_l}{r_1} \frac{0.573 \rho U}{k_{\text{min}}^2} k_{\text{min}} e^{-3.44k_{\text{min}} r} \left[ \cos(k_{\text{min}} r_1) + 0.627 \sin(k_{\text{min}} r_1) \right]
\]

Equation (5.1.10) hypothesizes \( \rho_{G_{k_{\text{min}}}} = K_{G_{k_{\text{min}}}} k_{\text{min}} \). In a metric far from masses where \( g_{\mu \nu} = \eta_{\mu \nu} \), \( k_{\text{min}} \) has its lowest value. As we approach any mass \( k_{\text{min}} \) increases to \( k''_{\text{min}} \) where we use blue/green double primes when \( g_{\mu \nu} \neq \eta_{\mu \nu} \) due to metric changes. This avoids confusion with the \( k' \) & \( k''_{\text{min}} \) of Eq.(5.1.4). At a radius \( r \) from mass \( m \) the Schwarzschild metric is \((1 - 2m/r)^{1/2}\) for the time and radial terms. Radial rulers shrink and clocks slow, measured volumes and frequencies both increase locally as \( \approx 1 + \frac{m}{r} \).

Thus both \( \frac{V + \Delta V}{V} \approx 1 + \frac{m}{r} \) and also \( k''_{\text{min}} \approx 1 + \frac{m}{r} \) if \( r >> m \)

Then using \( \rho_{G_{k_{\text{min}}}} = K_{G_{k_{\text{min}}}} k_{\text{min}} \) & \( \rho''_{G_{k_{\text{min}}}} = K_{G_{k_{\text{min}}}} k''_{\text{min}} \)

\[
1 + \frac{m}{r} \approx \frac{V + \Delta V}{V} = 1 + \frac{\Delta V}{V} \approx k''_{\text{min}} \frac{d k''_{\text{min}}}{d k_{\text{min}}} = \frac{\rho''_{G_{k_{\text{min}}}}}{\rho_{G_{k_{\text{min}}}}} = \frac{\rho_{G_{k_{\text{min}}}}}{\rho_{G_{k_{\text{min}}}}}
\]

(5.2.6)

So in this metric the total number of \( k_{\text{min}} \) gravitons is the original \((g_{\mu \nu} = \eta_{\mu \nu})\ \rho_{G_{k_{\text{min}}}} \) of Eq. (5.1.10) plus the extra due to a local mass of Eq.(5.2.5), but we have to divide this number by the increased volume to get the new density \( \rho''_{G_{k_{\text{min}}}} \approx (1 + \frac{m}{r}) \rho_{G_{k_{\text{min}}}} \). Thus using Eq. (5.2.6)
The new \( \rho_{G_{\text{min}}}^{\text{min}} \) is given by:

\[
\rho_{G_{\text{min}}}^{\text{min}} + \Delta \rho_{G_{\text{min}}}^{\text{min}} = (1 + m/lr) \rho_{G_{\text{min}}}^{\text{min}} \approx (1 + 2m/lr) \rho_{G_{\text{min}}}^{\text{min}} \quad \text{(if } r \gg m) \]

(5.2.7)

We can now put Eqs. (5.1.9) and (5.2.5) into Eq. (5.2.7) and dropping the now unnecessary subscripts, the graviton coupling constant \( \alpha_G \) and exponential cutoff \( (1 - e^{-0.61r^2})^2 \) cancel:

\[
\begin{align*}
\frac{\Delta \rho_{G_{\text{min}}}^{\text{min}}}{\rho_{G_{\text{min}}}^{\text{min}}} &\approx \frac{(1 - e^{-0.61r^2})^2 \alpha_G^{\text{min}} m}{r} 
\times \left[ \frac{1}{2} \frac{k_{\text{min}}^2}{P_U^2} \frac{d}{dk_{\text{min}}} \left[ e^{-3.44 \rho_{\text{min}}^2} \left( \cos(k_{\text{min}}r_1) + 0.627 \sin(k_{\text{min}}r_1) \right) \right] \right] \\
&\approx \frac{(1 - e^{-0.61r^2})^2 \alpha_G^{\text{min}} 0.3247 \frac{P_U^2}{k_{\text{min}}^2}}{r} \frac{d}{dk_{\text{min}}} \left[ e^{-3.44 \rho_{\text{min}}^2} \left( \cos(k_{\text{min}}r_1) + 0.627 \sin(k_{\text{min}}r_1) \right) \right] \\
&\approx \frac{2m}{r}
\end{align*}
\]

(5.2.8)

(We will do all this more accurately in 5.2.2.) GR has been accurately proven inside the solar system and Figure 5.2.2 shows that \( e^{-3.44 \rho_{\text{min}}^2} \left( \cos(k_{\text{min}}r_1) + 0.627 \sin(k_{\text{min}}r_1) \right) \approx 2 \) if \( r << R_{\text{OH}} \), where we have used \( k_{\text{min}} \approx 0.223 R_{\text{OH}}^{-1} \) from Eq.(11.1.18) When this is true, Eq. (5.2.8) tells us that in any metric both \( \rho_U \) & \( k_{\text{min}} \) transform their values but \( k_{\text{min}}^2 / \rho_U \) must be invariant.

\[ \frac{r_1}{R_{\text{OH}}} \rightarrow \]

\[ \text{Figure 5.2.2 A plot of } e^{-3.44 \rho_{\text{min}}^2} \left[ \cos(k_{\text{min}}r_1) + 0.627 \sin(k_{\text{min}}r_1) \right] \text{ with } \gamma = k_{\text{min}} R_{\text{OH}} \approx 0.223. \]

We have plotted it out to four cosmic radii for illustration only. We can think of this number as a coefficient of the \( 2m/lr_1 \) term in the metric tensors \( g_{00} \) & \( g_{rr} \) that is irrelevant at solar system and galactic scales, but not at very large or multiple cosmic radii. We will ignore it in the following sections until Section 10.

To be consistent with GR, for small cosmic radii where it is proven, Eq.(5.2.8) suggests that: At all points inside the horizon, and at any cosmic time \( T \), the red highlighted part of Eq. (5.2.8) is \( \approx 2 \) when using Planck units. This is simply equivalent to putting \( G/c^2 = 1 = G = c \).

Thus we can say the average density of the universe \( \rho_U \approx (0.8823) k_{\text{min}}^2 \approx 0.8823 \frac{\gamma^2}{R_{\text{OH}}^2} \)

where the parameter \( \gamma = k_{\text{min}} R_{\text{OH}} \) is in radians, and if \( \alpha_G^{-1} \approx 24 \) then \( \gamma \approx 0.223 \). (5.2.9)
Putting Eq. (5.2.9) the average density $\rho_U$ into Eq. (5.1.10) gives $\rho_G \min & K_G \min$.

Cutoff wavelength graviton probability density $\rho_G \min \approx \frac{0.0677 \alpha_G \rho_U^2}{k_{\min}^4} dk_{\min}$

$$\rho_G \min \approx \frac{0.0677 \alpha_G (0.8823 k_{\min}^2)^2}{k_{\min}^4} \approx 0.0527 \alpha_G \rho_U^2 dk_{\min} = K_G \min rk_{\min} & K_G \min \approx 0.0527 \alpha_G \quad (5.2.10)$$

is "The $k_{\min}$ Graviton Invariant" with $dk_{\min}$ the infinitesimal band $k_{\min} \pm (dk_{\min} / 2)$

If our conjectures are true, this is the number density of maximum wavelength gravitons excluding possible effects of virtual particles emerging from the vacuum. In section 6.2.3 we argue these do not change the $K_G \min$ of Eq.(5.2.10). However $K_G \min$ does depend on the graviton coupling constant $\alpha_G$ between Planck masses, but $\alpha_G$ cancels out in Eq. (5.2.8). We will later find that both the average density $\rho_U$ and $Y$ depend on $\alpha_G$. See Eq. (5.3.19).

5.2.2 The Schwarzchild metric near large masses

At a radius $r$ from a mass $m$ (dropping the now unnecessary suffixes) the Schwarzchild metric is $(1 - 2m/r)^{1/2}$ for the time and radial terms which can be written as

$$\sqrt{g_{rr}} = \frac{1}{\sqrt{1 - 2m/r}} = \frac{1}{\sqrt{g_{\theta\theta}}} = \frac{1}{\sqrt{1 - \beta_M^2}} = \gamma_M \quad (5.2.11)$$

Velocity $\beta_M (c = 1)$ is that reached by a small mass falling from infinity and $\gamma_M^{\pm 1}$ is the metric change in clocks and rulers due to mass $m$. We use blue/green symbols with the subscript $M$ for metrics $g_{\mu\nu} \neq \eta_{\mu\nu}$, as we did for $k_{\min}^\mu$ above. The symbols $\gamma_M^{\pm 1}$ help clarity in what follows.

$$\beta_M^2 = \frac{2m}{r}$$

$$\gamma_M^2 = \frac{1}{1 - 2m/r} = g_{\mu\nu} = \frac{1}{g_{\theta\theta}}$$

Using these symbols $k_{\min}^\mu = \gamma_M k_{\min} \quad & dk_{\min}^\mu = \gamma_M dk_{\min} \quad & \rho_G \min = \gamma_M \rho_G \min \quad (5.2.12)$

When near large masses $\left[ e^{-3.444 m r} \left( \cos(k_{\min} r) + 0.627 \sin(k_{\min} r) \right) \right] \approx 1$ in Eq.(5.2.5) so what effect does non-flat space have on Eq.(5.2.5) for $\Delta \rho_G$? We derived it by looking at the graviton coupling between a local mass and the mass of the rest of the universe. We know from GR that the mass $m$ is the value when measured at infinity. In any metric, a local observer measures both the mass $m$ after falling to his location from infinity, and that of all the distant matter, as having increased by $\gamma_M$. The $\Delta \rho_G$ we derived in flat space transforms as $\gamma_M^2 \rho_G \min$, and $\frac{\Delta \rho_G \min}{\rho_G \min}$ becomes $\approx \gamma_M^2 \frac{2m}{r}$ in non flat space. We can thus expect this same local observer to measure the extra $k_{\min}$ virtual gravitons emitted by this mass to be $\Delta \rho_G \min \approx \frac{2m}{r} \rho_G \min$ in this changed metric.
Using $\beta^2_M = \frac{2m}{r}$ in $1 + \gamma_M^2 \beta^2_M = \gamma_M^2$ and multiplying by $\rho_{Gk \text{ min}}$, $\gamma_{M} \rho_{Gk \text{ min}} + \frac{2m}{r} \rho_{Gk \text{ min}} = \gamma_{M} \rho_{Gk \text{ min}}$. 

To get the new density we divide this by the increased volume which also transforms as $\gamma_M$. 

$$\frac{\rho_{Gk \text{ min}}}{\gamma_M} \gamma_M + \frac{2m}{r} \rho_{Gk \text{ min}} = \gamma_M \rho_{Gk \text{ min}} = \rho_{Gk \text{ min}}^*$$

(5.2.13)

If, for example, $\gamma_M = 2$, frequencies are doubled so $k_{\text{min}}^* = 2k_{\text{min}}$, the number density of gravitons ($\rho_{Gk \text{ min}}^* = 2\rho_{Gk \text{ min}}$) is doubled, but so is the measurement of a local small volume element, which is now $V = 2$. The above equations tell us that the original $\rho_{Gk \text{ min}}$ background gravitons which occupied one unit of volume is now only requires 1/2 a unit of volume (because of frequency doubling) and the remaining 3/2 units of volume is taken up by the gravitons due to the central mass. Figure 5.2.3 illustrates this. The metric adjusts itself so that $K_{Gk \text{ min}}$ (the cutoff wavelength graviton probability constant) is an invariant number, and this should be true in all metrics at any peculiar velocity also. What we have done in this section is only true if the increase in measured volume is equal to the increase in measured frequency. In the Schwarzschild metric this is equivalent to saying that $g_{rr} \cdot g_{rr} = 1$ or $|g| = 1$. We discuss angular momentum in section 7.

**Figure 5.2.3** An infinitesimal local volume in a metric where $\sqrt{g_{rr}} = \gamma_M = 2$.

Differentiating $\gamma_M = \frac{1}{\sqrt{1 - \frac{2m}{r}}}$ at a fixed radius $\frac{d\gamma_M}{\gamma_M} = \frac{dV}{V} = \gamma_M^2 \frac{dm}{r}$, or

$$\frac{\text{The change in Frequency}}{\text{Frequency before change}} = \frac{\text{The change in Volume}}{\text{Volume before change}} = \gamma_M^2 \frac{\text{The change in Mass}}{\text{Radius of Measurement}}$$

(5.2.14)

This is just another way of saying exactly what the above equations are telling us. The change in both frequency and volume is $\gamma_M^2 \frac{dm}{r}$, and caused by the extra gravitons emitted in this metric, when adding an infinitesimal mass that measured $dm$ at infinity.
5.3 A Different Expansion to the Lambda Cold Dark Matter Model

Section 5.1.1 describes virtual gravitons as superpositions of the three modes \( n = 3,4,5 \) at a single wavenumber \( k \), as in Table 4.3.1 from which we find \( \langle n \rangle = 3.33 \). Using Eqs. (3.1.11), (3.1.12) and (3.2.10) \( \langle \gamma_k^2 \rangle = 1 + \langle K_k^2 \rangle^2 \) and \( \langle \beta_k^2 \rangle = \frac{\langle K_k^2 \rangle^2}{1 + \langle K_k^2 \rangle^2} \). For \( N = 2 \) spin 2 gravitons

\[
\langle K_k \rangle = \frac{\langle n \rangle k}{m_0} = \frac{\langle n \rangle k_{\text{min}}}{k_{\text{min}}} \quad \text{and thus we can express these as}
\]

\[
\langle \gamma_k^2 \rangle = 1 + x^2 \quad \text{and} \quad \langle \beta_k^2 \rangle = \frac{x^2}{1 + x^2} \quad \text{where} \quad x = \frac{k}{k_{\text{min}}} \tag{5.3.1}
\]

Using Eq. (5.1.2) \( m_0 = \langle n \rangle k_{\text{min}} = 3.3k_{\text{min}} \) and from Eq. (3.1.4), we find spin 2 gravitons borrow from time polarized quanta a mass \( m_0 / (\gamma^2 2\gamma) = m_0 / (2\gamma) = 3.3k_{\text{min}} / (2\sqrt{1 + x^2}) \). Now the total energy squared of a superposition mode \( n \) is the rest mass squared that is borrowed plus the momentum squared \( \langle p^2 \rangle = \langle n^2 \rangle k^2 = 3.3^2 k^2 \) if \( n = 1 \). Thus for gravitons

\[
\Delta E_{k,\text{Superposition}} = \sqrt{\frac{m_0^2}{2\gamma^2}} + \langle n \rangle^2 k^2 = \langle n \rangle \sqrt{\frac{k_{\text{min}}^2}{4(1 + x^2)}} + k^2 = \frac{3.33k_{\text{min}}}{2} \sqrt{\frac{1 + 4x^2 + 4x^4}{1 + x^2}} \tag{5.3.2}
\]

Now the superposition lasts for time \( \Delta T \approx 1/2\Delta E \) from which we obtain

\[
\text{superposition at wavenumber } k \quad \text{lifetime} \quad \Delta T_{k,\text{Superposition}} \approx \frac{1}{3.33k_{\text{min}}} \sqrt{\frac{1 + x^2}{1 + 4x^2 + 4x^4}} \tag{5.3.3}
\]

Equation (3.2.1) says \( \langle p_k (\text{debt}) \rangle = -\langle \beta_k \rangle^2 \langle n \rangle \hbar k \) is the vacuum debt of spatially polarized quanta. But as in Eq. (2.2.4) even though we are considering only a single graviton emitted, its superposition occurs with probability \( N \cdot s \cdot dk / k \) so we need to multiply this spatial debt by \( N \cdot s = 4 \) for \( N = 2 \) infinitesimal mass spin 2 gravitons. (See first paragraph after Eq. (3.3.12).) So putting \( h = 1 \), multiplying by 4 and ignoring minus signs

Spatial vacuum debt per graviton at wavenumber \( k \) is \( \Delta E_{k,\text{Spatial}} = 3.3k \frac{4x^2}{1 + x^2} \tag{5.3.4} \)

Going through similar arguments for the time mode vacuum debt we can show that

\[
\text{Time vacuum debt per graviton at } k \quad \text{is} \quad \Delta E_{k,\text{Time}} = 4 \times \frac{3.3k_{\text{min}}}{2\sqrt{1 + x^2}} = \frac{6.6k_{\text{min}}}{\sqrt{1 + x^2}} \tag{5.3.5}
\]

The spatial, and time, action quanta for such graviton superpositions is the lifetime \( \Delta T \) of Eq. (5.3.3) times \( \Delta E_{k,\text{Spatial}} \) and \( \Delta E_{k,\text{Time}} \).

\[
\text{Spatial action quanta per graviton at } k \quad \text{is} \quad \Delta E_{k,\text{Spatial}} \Delta T_k = \frac{4x^3}{1 + x^2} \sqrt{\frac{1 + x^2}{1 + 4x^2 + 4x^4}} \tag{5.3.6}
\]

\[
\text{Time action quanta per graviton at } k \quad \text{is} \quad \Delta E_{k,\text{Time}} \Delta T_k = \sqrt{\frac{4}{1 + 4x^2 + 4x^4}} \tag{5.3.6}
\]

However to plot these near \( k = k_{\text{min}} \) we need the number density of gravitons at any wavenumber \( k \), so rewriting Eq. (5.1.9) using Eq. (5.2.9) for \( \rho_u^2 / k_{\text{min}}^4 \) and Eq. (5.2.10)
Graviton density at wavenumber $k$

$$\rho_{\text{G}} \approx 12.45 \alpha_G dk \left[ \frac{(1-e^{-0.61x^2})^2}{x(2x^2+11.09)^2} \right]$$

(5.3.7)

We can now multiply the quanta per graviton by this density to get quanta required densities

$$\rho_{\text{SpatialQuanta @ k}} \approx 12.45 \alpha_G dk \left[ \frac{(1-e^{-0.61x^2})^2}{x(2x^2+11.09)^2} \right]$$

$$\rho_{\text{TimeQuanta @ k}} \approx 12.45 \alpha_G dk \left[ \frac{(1-e^{-0.61x^2})^2}{4x^2} \right] \left[ \frac{1+4x^2+4x^4}{1+4x^2+4x^4} \right]$$

(5.3.8)

But the density of zero point modes available @ $k_{\min}$ is $k_{\min}^2 dk / \pi^2$. Even if $\alpha_G << 1$ this is too small by about $k_{\min}^2 \approx 1/R_{\text{GH}}^2$. However, the area of the causally connected horizon $4\pi R_{\text{GH}}^2$ suggests possible connections with holographic horizons and the AdS/CFT correspondence, but as we will find, in a very different way.

5.3.1 Receding horizons and redshifted Planck scale zero point modes

Malcadena [27] proposed anti-de Sitter or hyperbolic spacetime where Planck modes on a 2D horizon are infinitely (almost) redshifted at the origin by an (almost) infinite change in the metric. In contrast, we have assumed flat space on average to the horizon. In section 2.2.3 we defined a rest frame in which preons born with zero momentum and infinite wavelength build superpositions. These superpositions cutoff at $k \approx 1$ near Planck energy but there are higher frequency vibration modes $1 < k < \infty$ still available. If we have a spherical horizon with these $k > 1$ modes receding locally at the velocity of light, they can be absorbed by infinite wavelength preons (from that receding horizon) and redshifted inwards, just as if they were radially focussed. This is instantaneous redshifting, and not due to the expansion of space when looking back in time. It is simply SR applying locally and is different to the normal Lorentz invariant density of zero point fields. We will argue in what follows, that at the centre where the infinite superpositions are built, 1/6 of these less than or equal to Planck length modes, can be absorbed from that horizon with almost infinitely redshifted wavelengths of the horizon radius and greater. This potential possibility only exists because zero momentum preons have an infinite wavelength. If any source of radiation recedes at velocity $\beta = v/c$ the frequency/wavenumber reduces as $k_{\text{observer}} = k_{\text{source}} \left[ \gamma(1-\beta) \right]$ where $\gamma = (1-\beta^2)^{-1/2}$. In the extreme relativistic limit $\beta \rightarrow 1$ & we can put $1-\beta = \Delta \beta = \varepsilon$.

$$1-\beta = \Delta \beta = \varepsilon \text{ or } \beta = 1-\varepsilon \text{ or } \beta^2 \approx 1-2\varepsilon \text{ or } 1-\beta^2 \approx \gamma^2 \approx 2\varepsilon \text{ & } \gamma \approx 1/\sqrt{2\varepsilon}$$

(5.3.9)

Thus

$$k_{\text{observer}} / k_{\text{source}} = [\gamma(1-\beta)] \approx \frac{\varepsilon}{\sqrt{2\varepsilon}} \approx \sqrt{\frac{\varepsilon}{2}} \approx \frac{1}{2\gamma}$$

There is always some rest frame travelling at virtually light velocity that can almost infinitely redshift these $1 < k < \infty$ modes into extreme wavelengths. Again, this is simply SR applying locally. But in sections 5.1.2 & 5.2.1 we used the fact that clocks in comoving coordinates tick
at the same rate. So how do we use Eq.(5.3.9)? Space between comoving galaxies expands with cosmic or proper time \( t \) and is called the scale factor \( a(t) \). It is normally expressed as \( a(t) \propto t^p \) and we will start at time \( t = T_0 \) with time \( T \) now.

Thus \( \dot{a}(t) \propto pt^{p-1} \) and the Hubble parameter \( H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{p}{t} \) \( (5.3.10) \)

We have been assuming to here that space is flat on average and will use the properties that in flat space at the current time the coordinate, proper and comoving distances are all equal. Writing the present scale factor normalized to one so that \( a(T) = 1 \) implies \( a(t) = t^p / T^p \), we can get the causally connected horizon radius and the horizon velocity \( V \). Using Eq.(5.3.10)

The horizon radius \( R_{oh} = \int \frac{dt}{a(t)} = T^p \int _{t_0} ^T \frac{dt}{t^p} = \frac{T-T_0}{1-p} \approx \frac{T}{1-p} \) if \( T >> T_0 \) & \( p \) constant. \( (5.3.11) \)

In flat space horizon velocity \( V = \frac{dR_{oh}}{dT} = \frac{d}{dT} \left[ T^p \int _{t_0} ^T \frac{dt}{t^p} \right] \) then using \( d(u \cdot v) = u \cdot dv + v \cdot du \) for \( (T >> T_0) \):

\[
\frac{dR_{oh}}{dT} = \frac{T^p}{T^p} + \frac{R_{oh}}{T^p} (pT^{p-1}) = 1 + \frac{p}{T} R_{oh}.
\]

But \( \frac{p}{T} \) is the Hubble parameter at time \( T \), so that in flat space the horizon velocity \( V = 1 + H(T)R_{oh} \) regardless of how \( p \) behaves. \( (5.3.12) \)

The Hubble flow velocity of a comoving galaxy on the horizon is \( V' = H(T)R_{oh} \) and thus from this equation the horizon velocity is always \( V = 1 + V' \). In other words, the horizon is moving at light velocity relative to comoving coordinates instantaneously on the horizon as measured by a local comoving observer. Also clocks tick at the same rate in all comoving galaxies, but clocks moving at almost the horizon light velocity (relative to comoving coordinates instantaneously on the horizon) will virtually stop (1/\( \gamma \)→1/\( \infty \)→0 from Eq (5.3.9) as SR applies locally in this case). Thus, extreme modes on the receding horizon will obey Eqs.(5.3.9) as seen in all comoving coordinates. Now imagine a Planck scale thickness shell travelling at virtually light velocity (relative to comoving coordinates instantaneously on the horizon) and radially as seen by central observers. The total transverse area of this shell is:

\[
\text{Horizon surface area} = 4\pi R_{oh}^2.
\] \( (5.3.13) \)

Before redshifting consider zero point waves in this Planck scale shell with \( 1 < k' < \infty \) (temporarily using a single primed \( k' \) that is not the \( k' \) of Eq. (5.1.4) and \( k \) after redshifting) have a mode density of

\[
\frac{k'^2}{\pi^2} dk' = \frac{k^3}{\pi^2} \frac{dk'}{k'}.
\] \( (5.14.1) \)

Wavenumbers \( 1 < k' < \infty \) can be redshifted to \( 0 < k << 1 \) using Eqs.(5.3.9) \( k = k' \sqrt{\varepsilon / 2} \) provided \( \varepsilon = \Delta \beta \to 0 \), also as \( k = k' \sqrt{\varepsilon / 2} \) and \( dk = dk' \sqrt{\varepsilon / 2} \) then \( dk'/k' = dk/k \). As zero point energies are \( h \omega / 2 \) per vibration mode, which is equivalent to half an action quanta \( h \) per mode, we need to divide by 2. Remembering this is dif ferent to Lorentz invariant zero point densities Eq.(5.14.1) becomes

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\[ k' > 1 \text{ mode density before redshifting} \]

\[ \frac{k'^3}{\pi^2 k'} \]

If they were all redshifted mode density

\[ \frac{k^3}{\pi^2} \frac{dk}{k} = \frac{k^2}{6\pi^2} dk \]

But only \( \frac{1}{6} \) can be radially redshifted inwards so mode density

\[ \frac{k^2}{6\pi^2} \frac{dk}{k} \]  

But zero point energy per mode is \( \frac{\hbar \omega}{2} \) so quanta density

\[ \frac{k^2}{12\pi^2} \frac{dk}{k} \]

Multiplying Eq. (5.3.15) by the horizon surface area, then multiplying both numerator and denominator by \( k_{\min} \) and using Eq. (5.2.9) \( Y = k_{\min} R_{OH} \) we get

\[ \rho_{\text{Quanta @ }k} \approx 4\pi R_{OH}^2 \times \frac{k^2}{12\pi} \frac{dk}{dk} \approx \frac{k_{\min}^2 R_{OH}^2}{3\pi} \left[ \frac{k}{k_{\min}} \right]^2 \approx \frac{Y^2}{3\pi} \frac{dk}{x^2} \text{ where } x = \frac{k}{k_{\min}} \]

But these quanta are half time mode and half spatial so dividing by 2 again we get

\[ \rho_{\text{Quanta @ }k} = \frac{k_{\min}^2 R_{OH}^2}{6\pi} \frac{x^2}{x^2} \frac{dk}{dk} = \frac{Y^2}{6\pi} \frac{dx}{x^2} \]

Space and time mode quanta density available

Horizon surface area \( 4\pi R_{OH}^2 \)

Comoving observers instantaneously on the horizon see radial inwards modes redshifted from \( 1 < k' < \infty \) to \( 0 < k << 1 \) that can only be borrowed by preons born at centre of sphere with zero momentum and infinite wavelength. The whole surface area of the horizon is thus available for this supply, but only one sixth of these modes can be radially redshifted inwards. This amplifies the effective density available at the centre by \( 4\pi R_{OH}^2 / 6 \) and is only possible for preons born with zero momentum at the centre.

Figure 5.3.1 Preons forming a central mode \( k \) are born with zero momentum and infinite wavelength. They can borrow \( 1 < k' < \infty \) modes that are redshifted from the entire surface area of the horizon, massively amplifying the available central quanta density. This is quite different to the normal Lorentz invariant density of zero point fields and is only possible for infinite wavelength preons born in a continually expanding flat universe.

In the first half of this paper where we looked at building superpositions we assumed that mass was borrowed from a Higgs type field whereas energy was borrowed from zero point spatial modes. We will now conjecture that all infinitesimal mass superpositions borrow their mass from the time modes of zero point fields. Figure 5.3.2 plots both Eqs. (5.3.8) as a function of \( x = k / k_{\min} \). Assuming the mass borrowed comes from time modes, the parabola \( 0.067 \alpha_G x^2 \) fits closely, and in the case of spatial modes this same parabola leaves a surplus at all wavelengths. Both these plots look the same at all cosmic times \( T \) and in any metric only the value of \( k_{\min} \) changes. This is different to Lorentz invariant zero point densities and only works in flat expanding space with infinite wavelength preons born with zero momentum.
If the above ideas are on the right track these equations may well control the expansion of space. We will first use them to show that space expansion in the radiation dominated era is virtually identical to current cosmology models. Section 10 looks at the possibility of massive spin 2 virtual gravitons behaving as what is called dark matter, and also the possibility they control the acceleration of space expansion in the matter dominated era.

The parabola $0.067\alpha_G x^2$ of spatial mode quanta available from the horizon show a surplus at all frequencies if we let time mode requirements as below control this.

Spatial mode quanta required with $(1 - e^{-0.61x^2})^2$ cutoff

The parabola $0.067\alpha_G x^2$ of time mode quanta available from the horizon fits the $0 < k < k_{min}/2$ frequency requirements closely.

Time mode quanta required with $(1 - e^{-0.61x^2})^2$ cutoff

**Figure 5.3.2** Redshifted from horizon, and required time and space mode quanta.

Equating $0.067\alpha_G x^2$ with Eq. (5.3.17) $\frac{Y^2}{6\pi} x^2 dk \approx 0.067\alpha_G x^2 dk$, and when $x = \frac{k}{k_{min}} = 1$ we find

Time and space quanta available density at $k_{min}$ is $\frac{Y^2}{6\pi} dk_{min} \approx 0.067\alpha_G dk_{min} = K_{Q_{min}} dK_{min}$

Where $K_{Q_{min}} \approx 0.067\alpha_G$

and $Y^2 = k_{min}^2 R_{OH}^2 \approx 1.26\alpha_G$

We find in Eq.(11.1.18) that $Y \approx 0.223$ and $k_{min} \approx 0.223 R_{OH}$ at all cosmic times.

Using Eq. (5.2.9) $\rho_u \times R_{OH}^2 \approx 0.8823 Y^2$, in Planck units $\rho_u R_{OH}^2 \approx 1.114\alpha_G$

Mass inside the Horizon $= \frac{4\pi R^3}{3} \rho_u \approx \frac{4\pi R_{OH}^3}{3} \frac{1.114\alpha_G}{R_{OH}} \approx 4.66 R_{OH} \alpha_G$
5.3.2 A constant horizon velocity in flat on average radiation dominated space

In the introductory notes we discussed why we can simplify things greatly in this different way of looking at gravity in flat on average space. We have equal coordinate, proper and comoving distances at the current time. The FLRW equation becomes a very simple $ds^2 = -c^2 dt^2 + a(t)^2 dr^2$ in this flat space (as explained in section 10.1.1) where $a(t)$ is the scale factor at time $t$. (We sometimes simply use scale factor $a$ with density $\rho \propto a^{-3}$.) Also the observable horizon radius can be very simply expressed as $R_{oh} = \int V dt$ with $V = \frac{dR}{dt}$, and this is only true if space is flat in average.

The universe density Eq. (5.3.20) $\rho_U \approx \frac{1.114\alpha_G}{R_{oh}^2} = \frac{K}{R_{oh}^2} = \frac{1}{a^4}$ in radiation dominated space

$$a^4 \propto R^2 \rightarrow a \propto R^{1/2} \text{ where } R = R_{oh} \tag{5.3.22}$$

The Hubble parameter $H = \frac{\dot{a}}{a} = \frac{R^{-1/2} \frac{dR}{dt}}{2R^{1/2}} = \frac{V}{2R}$ as in flat space $\frac{dR}{dt} = V$ the horizon velocity.

The Hubble flow velocity at $R_{oh} (= R$ here) is $V' = H \cdot R = \frac{V}{2}$ \tag{5.3.23}

As in Eq.(5.3.12) the Hubble flow velocity at the horizon is $V' = H \cdot R = V - 1$ (a comoving observer instantaneously on the horizon sees it passing him at velocity $c$ as in local SR.)

Thus $V' = V - 1 = \frac{V}{2}$ or the Horizon velocity is $V = 2$ in the radiation era \tag{5.3.24}

Planck temperature $\approx 1.4 \times 10^{32} K @ t \approx 1.07 \alpha_G^{1/2}$ Planck time.

For many powers of ten until transition, temperature $T \approx 1.61 \times 10^{32} t^{-1/2} \alpha_G^{1/4}$ K

After transition, this gradient is initially set by $a(t) \propto t^{-2/3}$

Transition @ $t \approx 10^{55}$ Planck time.

$T \approx 3000K @$ recombination

$T \approx 2.75K$ now

Time in Planck units

Figure 5.3.3 A logarithmic temperature plot with $\alpha_G \approx 1/24$ from $t = 1$ Planck units until now.
A horizon velocity of \( V = 2 \) implies a horizon radius in flat space of \( R_{\text{OH}} = \int_0^t V dt = 2t \). From Eq. (5.3.22) \( a \propto R^{1/2} \propto t^{1/2} \) as in current cosmology where \( R = \int_0^T \frac{dt}{a(t)} = \int_0^T \frac{dt}{t^{1/2}} = 2T \).

The Stefan-Boltzmann law says \( \rho_{\text{thermal}} = \frac{\pi^2 k^4 T^4}{60 c^2 h^3} = \frac{\pi^2}{60} T^4 \approx 0.1645 T^4 \) in Planck units.

We can put \( R_{\text{OH}} = 2t \) into Eq. (5.3.20) to get the temperature in Planck units:

\[
\rho_{\text{thermal}} \approx \frac{1.11 \alpha_G}{R_{\text{OH}}^2} \approx \frac{1.11 \alpha_G}{(2t)^2} \approx 0.1645 t^4 \rightarrow T \approx 1.14 t^{-1/2} \alpha_G^{1/4} \text{ and converting to degrees } K
\]

Temperature \( T \approx 1.14 t^{-1/2} \alpha_G^{1/4} \times 1.417 \times 10^{32} K \approx 1.61 \times 10^{12} t^{-1/2} \alpha_G^{1/4} K \) (5.3.25)

Planck temperature \( T_{\text{Planck}} @ t \approx \sqrt{1.14 \alpha_G^{1/4}} \approx 1.07 \alpha_G^{1/2} \) Planck units

Figure 5.3.3 plots radiation temperature starting at \( T \approx 1.4 \times 10^{32} K \) at \( t \approx 1.07 \alpha_G^{1/2} \) Planck time, dropping to \( T \approx 3000 K \) at recombination. Equation (5.3.25) controlling this plot is based on Eq. (5.3.20), which this paper argues is true in all flat comoving coordinates and there is no need for a finely tuned critical density to achieve flat space as it is inherent in this model. Of course, we do not know exactly how long after \( t = 1 \) Planck time all these equations start to apply, but if it happened to be about the same time that inflation is thought to end \( (t \approx 10^{10} \text{ Planck time or } \approx 10^{-33} \text{ seconds}) \) the causally connected radius would be \( R_{\text{OH}} \approx 10^{10} \) Planck lengths or \( \approx 10^{-25} \text{ metres} \) with quantum temperature fluctuations (as a fraction of the average) that could be similar to what inflation predicts \( (\approx 10^{-5}) \), and as observed in the CMB. Nucleosynthesis is virtually identical to the \( \Lambda \text{CDM} \) model and there should be less need for inflation as all these equations also apply in regions initially out of causal contact.

5.4 Non-Comoving Coordinates and Spatial Polarization

To this point we have been working in comoving coordinates for simplicity. Velocities relative to comoving coordinates are called peculiar velocities, so, does all our previous work still apply in non-comoving coordinates with these peculiar velocities? In section 5.1.2 we calculated the density of \( k_{\text{min}} \) virtual gravitons in comoving coordinates where they are spherically symmetric or time polarized. So at peculiar velocities there can be spatially polarized probability densities of them. However we can apply here the same thinking that Poincare used over a century ago. At that time there were various models of the electron, the Abraham-Lorenz probably being the most well-known [28], [29]. All these models suffered the problem of electromagnetic mass in the field being 4/3 times the relativistic mass, where the extra 1/3 came from the spatially polarized component due to velocity. In 1906 Poincare showed that if the bursting forces due to charge were balanced by stresses (or forces) in the same rest frame as the particle, these would cancel the extra 1/3 figure, restoring covariance [30]. We can use the same principles here. In comoving coordinates we can think of our time polarized gravitons with their centres of momentum at rest. (See the scalar type interaction example as in section 3.3.2). In section 2.3 we looked at spherically symmetric wavefunctions that build these superpositions around such zero nett momentum centres. They had squared orbital momentums that would generate bursting pressures balanced by zero point forces in
the same frame as that centre, which for our time polarized gravitons is at rest in comoving coordinates. These can be thought of as equivalent to the Poincare stresses holding a charged particle together. So in any other frame moving relative to it at a peculiar velocity these zero point balancing forces cancel any extra momentums or energies due to spatially polarized components. Thus spatially polarized virtual gravitons due to peculiar velocities do not add to the zero point energies borrowed from the distant horizon. We can ignore them, and only consider time polarized $k_{\text{min}}$ gravitons in all other frames when equating zero point energies required to build virtual gravitons with that available to be borrowed from the receding horizon. In other words they do not change the metric.

We can think of a box of these $k_{\text{min}}$ gravitons fixed in comoving coordinates. It will have a 3 volume density (or 3 dimensions) $\rho_{3D_{k_{\text{min}}}} = \kappa_{k_{\text{min}}} dk_{\text{min}}$ as we have previously calculated where by 3 volume we mean $3V = d^3x = dx dy dz$. If we now move relative to it at peculiar velocity $\beta_p$ (where red symbols will be used from here for peculiar velocities, to distinguish from blue/green metric changes near mass concentrations) it will shrink in size as $\gamma_p^{-1} = (1 - \beta_p^2)^{1/2}$ so that its new 3 volume density $\rho^*_{3D_{k_{\text{min}}}} = \kappa_{k_{\text{min}}} dk_{\text{min}}^*$, where $dk_{\text{min}}^* / dk_{\text{min}} = \gamma_p$ is the local increase in wavenumber $k_{\text{min}}$. If we repeat our derivations of the background 3 volume density, and the extra emitted by local mass concentrations, we find they also both increase by $dk_{\text{min}}^* / dk_{\text{min}} = \gamma_p$ with no change in the ratio $\Delta \rho / \rho$, so all our logic is unchanged at any peculiar velocity. But all this, is the same as saying that at any peculiar velocity, and in any metric, the 4 volume density of $k_{\text{min}}$ gravitons is invariant at any cosmic time $T$, where 4 Volume is $4V = d^4x = dx dy dz dt$ for 4 dimensions. (It is important to note here that we are discussing above the number densities of $k_{\text{min}}$ gravitons which increase as $\gamma_p$ with peculiar velocity, as distinct from energy densities of $k_{\text{min}}$ gravitons which increase as $\gamma_p^2$ with peculiar velocity.)

### 5.4.1 Invariant 4 volume or 4D cosmic wavelength graviton densities

Define $\rho_{3D_{k_{\text{min}}}} = k_{\text{min}} \frac{\text{Gravitons}}{\text{3Volume}} = k_{\text{min}} \frac{\text{Gravitons}}{\Delta x \Delta y \Delta z}$ and as 4 volume $\Delta x \Delta y \Delta z \Delta t = \Delta x' \Delta y' \Delta z' \Delta t'$

$$\rho_{4D_{k_{\text{min}}}} = k_{\text{min}} \frac{\text{Gravitons}}{\text{4Volume}} = k_{\text{min}} \frac{\text{Gravitons}}{\Delta x \Delta y \Delta z \Delta t} = k_{\text{min}} \frac{\text{Gravitons}}{\Delta x' \Delta y' \Delta z' \Delta t'}$$ is an invariant.

In flat comoving coordinates only we will define 4 volume $k_{\text{min}}$ graviton density as

$$\text{4 Volume Density } \rho_{4D_{k_{\text{min}}}} = 3 \text{ Volume Density } \rho_{3D_{k_{\text{min}}}}$$

however $\rho_{4D_{k_{\text{min}}}}$ is invariant in all coordinates and in any metric. (5.4.1)

This is equivalent to dividing $k_{\text{min}}^*$, in any metric, at any peculiar velocity, by $\gamma_p \gamma'_M$ thus returning it to flat space comoving value $k_{\text{min}}$ at any cosmic time $T$. As both $\rho_{4D_{k_{\text{min}}}}$ & $\Delta \rho_{4D_{k_{\text{min}}}}$ are invariant, their ratio is also invariant in any coordinates, and at any peculiar velocity at any particular cosmic time. But the flat space comoving value of $k_{\text{min}}$ decreases with cosmic time.
5.4.2 Cosmic wavelength graviton and 4 volume or 4D action densities

In deriving Eq.(5.3.8) we said that each $k_{\text{min}}$ graviton always borrows a fixed amount of action, where $\text{Action} = \Delta E \cdot \Delta T$ per graviton is constant but $\Delta E \propto k_{\text{min}}$. So if four volume (4D) $k_{\text{min}}$ graviton density ($\rho_{4D_{Gk_{\text{min}}}}$) is invariant, the four volume action density required by $k_{\text{min}}$ gravitons must also be invariant.

Our hypothesis is that at any point in spacetime, gravity is determined by the 4 volume $k_{\text{min}}$ action density available from a receding horizon always being equal to the 4 volume $k_{\text{min}}$ action density required by gravitons; with both remaining invariant in any coordinates.

Using Eq. (5.3.18) and defining 4 volume action density @ $k_{\text{min}}$ as $\rho_{4D_{Gk_{\text{min}}}}$

$$\rho_{4D_{Gk_{\text{min}}}} \text{ action density available} \approx 0.067 \alpha_G = K_{Gk_{\text{min}}}. \quad (5.4.2)$$

This equation is true in any coordinates, and at any point in spacetime.

We will discuss the connection between all this and Einstein’s field equations in section 10.

6 Infinitesimal Mass Bosons

6.1 Cosmic Wavelength Superposition Cutoffs

In section 4.2 when we introduced gravity, for the lower limit in our integrals we assumed $k_{\text{min}} = 0$, and then in section 5 showed that there is a lower limit $k_{\text{min}} > 0$. It turns out that for massive $N=1$ superpositions the effect of this is negligible in comparison to the high frequency cutoff $k_{\text{cutoff}} < \infty$, which we showed gravity can address in section 4.2. For infinitesimal rest mass $N=2$ superpositions we cannot, however, ignore the effect of $k_{\text{min}} > 0$.

6.1.1 Quantifying the approximate effect of $k_{\text{min}} > 0$ on infinite superpositions

If we look again at section 4.2.1 we can repeat what we did there as follows. Initially to illustrate these effects we will consider only $N=1$ superpositions where we can say that when $K_{nk_{\text{cutoff}}} \to \infty$, and (for $N=1$ only) $K_{nk_{\text{min}}} \to 0$, and thus

$$\left[ \frac{-1}{1 + K_{nk_{\text{min}}}^2} \right]_{K_{nk_{\text{cutoff}}}} = \frac{1}{1 + K_{nk_{\text{min}}}^2} - \frac{1}{1 + K_{nk_{\text{cutoff}}}^2} \approx 1 - \frac{1}{K_{nk_{\text{cutoff}}}^2 + K_{nk_{\text{min}}}^2} \approx 1 - e^\pi \approx \frac{1}{1 + e^\pi} \quad (6.1.1)$$

Our earlier infinitesimal in Eq.(4.2.2) $\varepsilon = \frac{1}{K_{nk_{\text{cutoff}}}^2}$ becomes $\varepsilon^* = \frac{1}{K_{nk_{\text{cutoff}}}^2} + K_{nk_{\text{min}}}^2$.

Using section 4.2.1 and 4.2.2 equations, we can show

$$\frac{1}{K_{nk_{\text{cutoff}}}^2} \approx \frac{L_{\text{cutoff}}^2}{\lambda_C^2} \quad \text{and} \quad K_{nk_{\text{min}}}^2 \approx \frac{\lambda_C^2}{R_{GH}}.$$
For our purposes here we are ignoring small numerical factors such as \( \langle n \rangle^2 \) to show in Planck units where \( L_p = 1 \) that

\[
\varepsilon'' \approx \frac{1}{K_{nk_{\text{Cutoff}}}^2} + K_{nk_{\text{min}}}^2 = \varepsilon + \Delta \varepsilon \approx \frac{1}{K^2} + \frac{\kappa_C^2}{R_{OH}^2}
\]

The ratio of the extra contribution \( \Delta \varepsilon \) to \( \varepsilon \) is

\[
\frac{\Delta \varepsilon}{\varepsilon} \approx \left( \frac{\kappa_C^2}{R_{OH}^2} \right)^2
\]

(6.1.2)

In Planck units \( R_{OH}^2 \approx 10^{22} \) and \( \frac{\kappa_C^4}{R_{OH}} \) for electrons say is \( \approx 10^{22} \), so the effect is of order \( \Delta \varepsilon / \varepsilon \approx 10^{22} / 10^{-22} \approx 10^{-30} \) which we have been ignoring. We cannot ignore this, however, in the case of infinitesimal rest masses as we will see.

### 6.2 Infinitesimal Masses and \( N = 2 \) Superpositions

Looking again at angular momentum and rest masses discussed in section 3.2 the key factor in our final integrals is in Eq. (6.1.1). Using Eq. (3.1.12) we can rewrite Eq. (6.1.1) as

\[
\left[ -\frac{1}{1 + K_{nk}} \right]_{K_{nk_{\text{Cutoff}}}} = -\frac{1}{\gamma_{nk_{\text{min}}}^2} - \frac{1}{\gamma_{nk_{\text{Cutoff}}}^2}
\]

(6.2.1)

With massive \( N = 1 \) superpositions, as above, the difference between \( \gamma_{nk_{\text{min}}}^2 \) & 1 is vanishingly small, i.e. \( (\gamma_{nk_{\text{min}}}^2 - 1) \rightarrow 1/\infty \) and as in section 6.1.1 this first term is of much less significance than the \( \gamma_{nk_{\text{Cutoff}}}^2 \) term. Now define an approximate equality between \( N \) & \( \langle \gamma_{k_{\text{min}}}^2 \rangle \) using Eq. (3.1.12) as follows

\[
N \approx \left[ \langle \gamma_{k_{\text{min}}}^2 \rangle \right]^2 = 1 + \langle K_{k_{\text{min}}}^2 \rangle
\]

(6.2.2)

In section 3.2 we derived angular momentum and rest masses for only massive, or what we called \( N = 1 \), particles. To get integral angular momentum we had to assume in deriving Eq. (3.2.6) that the minimum value of \( K_{nk} \) or \( K_{nk_{\text{min}}} = 0 \). For massive \( N = 1 \) particles, such as electrons, the error in this assumption (as in section 6.1.1) is \( \approx 10^{-30} \) times smaller than \( \varepsilon \), which for an electron is already \( \varepsilon \approx 10^{-45} \) due to the high frequency cutoff @ \( \approx 10^{18.31} \) GeV. (We allowed for this \( \varepsilon \approx 10^{-45} \) when we included gravity in section 4.2.) From section 6.1.1 above we approximated \( K_{nk_{\text{min}}}^2 \) as \( \approx \frac{\kappa_C^2}{R_{OH}^2} \) for all massive particles. So, we can express Eq. (6.2.2) in terms of this approximation for fermions with non-infinitesimal mass

\[
N \approx \left[ \langle \gamma_{k_{\text{min}}}^2 \rangle \right]^2 = 1 + \frac{\kappa_C^2}{R_{OH}^2} \approx 0
\]

(6.2.3)

A \( 10^{-10} \) eV mass particle has \( \frac{\kappa_C^2}{R_{OH}^2} \approx 10^{-50} \)

For the massive particles it appears we can say that \( N = 1 \). However in section 11 we explore the possibility of galaxy halos as spin 2 virtual gravitons of \( \approx 10^{-29} \) eV mass where \( \langle \gamma_{k_{\text{min}}}^2 \rangle \approx 1 \approx 10^{-10} \). At this extreme low mass (but still \( 10^5 \) larger than infinitesimal masses (which we discuss further in section 11) Eq. (6.2.1) shows that we cannot get the correct angular momentum unless something else changes, perhaps by a small change in the actual high frequency cutoff details.
So if massive particles are a group with $N = 1$, then it would not seem unreasonable to imagine there could possibly be another group with $N = 2 = 1 + \langle K_{k_{\text{min}}} \rangle^2$ implying that $\langle K_{k_{\text{min}}} \rangle^2 = 1$. Repeating the derivation of Eq. (3.2.6) but with $N = 2 = 1 + \langle K_{k_{\text{min}}} \rangle^2$ and for clarity and simplicity let $K_{\text{cutoff}} \to \infty$.

$$L_c^\prime (\text{Total}) = s \cdot (N = 2) \cdot m \cdot h \int_{K_{\text{min}}}^{\infty} \frac{K_{n_k}^2}{(1 + K_{n_k}^2)^2} \frac{dK_{n_k}}{K_{n_k}} = s m h \left[ \frac{-1}{1 + K_{n_k}^2}, K_{\text{cutoff}} \right]$$

$$L_c^\prime (\text{Total}) = s m h \left[ \frac{1}{1 + K_{n_k}^2}, K_{\text{cutoff}} \right] = s m h \left[ \frac{1}{(N = 2) \cdot (N = 2)} \right] = \frac{s m h}{2} \text{ as previously.}$$

Provided we have doubled the probability of superpositions as in Eq. (2.1.4) from $s \cdot (N = 1) dk/lk$ to $s \cdot (N = 2) dk/lk$, the final angular momentum results in Eqs. (3.2.6) and (6.2.4) are identical. The same is true for rest mass calculations. For multiple integer $n$ infinite superpositions if $N = 2$ then the expectation value $\langle K_{k_{\text{min}}} \rangle^2 = 1$. We thus conjecture that all $N = 2$ infinite superpositions have $\langle K_{k_{\text{min}}} \rangle^2 = 1$. Using Table 4.3.1

$N = 2$ infinitesimal rest mass spin 1 superpositions have $\langle n \rangle \approx 3.98$

$N = 2$ infinitesimal rest mass spin 2 superpositions have $\langle n \rangle \approx 3.33$

Using Eqs. (3.1.11) and Eq. (5.2.9)

$$\langle K_{k_{\text{min}}} \rangle^2 = \left( \frac{n}{2} \right)^2 \frac{s}{2} \lambda_c^2 k_{n_k}^2 \approx 15.82 \frac{k_{n_k}^2}{2} \lambda_c^2 = 1 \text{ or } \lambda_c \approx 0.355 \frac{R_{\text{OH}}}{\gamma} \text{ for Spin 1}$$

$$\approx 11.09 \times 2 \frac{k_{n_k}^2}{2} \lambda_c^2 = 1 \text{ or } \lambda_c \approx 0.300 \frac{R_{\text{OH}}}{\gamma} \text{ for Spin 2}$$

From Eq. (11.1.18) $\gamma \approx 0.223$ and from Table 11.2.1 $R_{\text{OH}} \approx 40 \times 10^9 ly \approx 2.34 \times 10^{64} Lp$ and using these values the above equations provide the infinitesimal masses of $N = 2$ photons, gluons and gravitons as in Table 6.2.1 below.

<table>
<thead>
<tr>
<th>Spin</th>
<th>$\langle n \rangle$</th>
<th>Compton Wavelength $\lambda_c$</th>
<th>Infinitesimal Rest Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.98</td>
<td>$\approx 1.27 R_{\text{OH}}$</td>
<td>$\approx 2.5 \times 10^{-34} eV.$</td>
</tr>
<tr>
<td>2</td>
<td>3.33</td>
<td>$\approx 1.07 R_{\text{OH}}$</td>
<td>$\approx 2.9 \times 10^{-34} eV.$</td>
</tr>
</tbody>
</table>

**Table 6.2.1** Infinitesimal masses and Compton wavelengths of $N = 2$ photons, gluons & gravitons. They limit the range of virtual photons and gravitons to approximately the horizon. The graviton rest masses above are reasonably close to recent proposals for the accelerating expansion of the cosmos [18,19].

6.2.1 **Cutoff behaviours for $N = 1$ & $N = 2$ superpositions**

Equation (6.2.1) can be written for both $N = 1$ & $N = 2$ superpositions using the results of sections 4.2 & 6.2 as follows
\[
\begin{align*}
&\frac{-1}{1 + K_{nk}^2} \gamma_{nk,\text{cutoff}}^2 = \frac{1}{\gamma_{nk,\text{min}}^2} = \frac{1}{2(1 + \epsilon^\prime)} \quad \text{when } N = 2 \\
&\frac{-1}{1 + K_{nk}^2} \gamma_{nk,\text{cutoff}}^2 = \frac{1}{\gamma_{nk,\text{min}}^2} \approx 1 = \frac{1}{1 + \epsilon^\prime} \quad \text{when } N = 1
\end{align*}
\]

(We should be using expectation values, but for clarity we simply imply them.) We have shown in section 6.2 that \( \langle 1/\gamma^2_{\text{min}} \rangle = 1/2 \) when \( N = 2 \), but in reality it is Eq. (6.2.6) that must be true. In section 4.2 we showed that for \( N = 1 \) superpositions the primary coupling of gravity to preons infinitesimally increased the interaction probability by \( \epsilon^\prime \) to \( (1 + \epsilon^\prime) \) where from Eq. (4.2.4) \( \epsilon^\prime = \frac{m_0^2 x^\prime K_G}{2hC(8 + 8\sqrt{\alpha_{\text{EMP}}})^2} = \epsilon = \frac{1}{K_{nk,\text{cutoff}}^2} = \frac{2m_0^2 c^2}{sn^2 h^2 (k_{\text{cutoff}})^2} \).

In the \( N = 1 \) case this meant that any deficits due to a non-infinite cutoff were exactly balanced by the contribution from gravity, but in the \( N = 2 \) case this infinitesimal correction is out by a factor of two. However Eq. (6.2.6) says that exactness can be maintained in the \( N = 2 \) case by an infinitesimal change from \( \langle 1/\gamma^2_{\text{min}} \rangle = 1/2 \) to \( \langle 1/\gamma^2_{\text{min}} \rangle \approx 1/2 \). Thus both \( N = 1 \) & \( N = 2 \) superpositions can cutoff at Planck energy as in section 4.2.2. The low frequency cutoff for all superpositions must be at \( k_{\text{min}} \approx \gamma / R_{\text{OH}} \) if they are to affect gravity.

6.2.2 An exponential cutoff at cosmic wavelengths for infinite superpositions

We used a square cutoff above for \( k_{\text{min}} \) but an exponential cutoff is more likely, and a squared exponential cutoff fits best (Figure 5.3.2). Going over what we did in Eq.(6.2.4) and putting \( x = \frac{k}{k_{\text{min}}} \), then using \( \int_{x=0}^{x=\infty} xdx \left(1 + x^2\right)^{-3} = 0.25 \approx \int_{x=0}^{x=\infty} (1 - e^{-0.61x^2}) xdx \approx 0.249991 \)

An exponential cutoff \( (1 - e^{-0.61x^2}) @ k_{\text{min}} \) is \( \approx \) the same as a square cutoff @ \( k_{\text{min}} \) (6.2.7)

\[ \rightarrow x = \frac{k}{k_{\text{min}}} \]

**Figure 6.2.1** A \((-0.61x^2)\) exponential cutoff @ \( k_{\text{min}} \) for all infinite superpositions that gives the correct angular momentum for \( N = 2 \), spins 1 & 2 infinitesimal mass bosons. This fits the very low frequency time mode quanta required and quanta available nicely. A simple linear exponential does not work. (See Figure 5.3.2.)
6.2.3 Virtual particle pairs from the vacuum and spacetime curvature

For almost a century it has been a puzzle why spacetime appears to be flat on average and not massively curved by Planck scale zero point energy densities. In section 5.1.1 we conjectured that virtual particles are just single wavenumber \( k \) superposition members, whereas real particles are full infinite superpositions of all wavenumbers \( k \) from \( k_{\text{min}} \) to \( k_{\text{Planck}} \). We assumed this was true in all of section 5. If this is the actual difference between virtual and real particles, then only full infinite superpositions (representing that particle) have real properties that can be measured (such as measured mass/energy) rather than implied. If \( k_{\text{min}} \) virtual gravitons are such single members they can couple to \( k_{\text{min}} \) members of full infinite superpositions. On the other hand, virtual particles out of the vacuum, are mainly short lived high \( k \) single value members that will not couple to \( k_{\text{min}} \), if our conjectures are true. The density of \( k_{\text{min}} \) virtual pairs from the vacuum is virtually zero as it is based on the Lorentz invariant supply of local zero point fields, not from the receding horizon (see sections 6.2.4 & 6.2.5 below). But this is not the full story. The virtual particles that dress electrons and quarks for example add mass to the real particles. In fact, the majority of the proton and neutron mass is due to the virtual gluons interacting between quarks. If short lived virtual particles somehow contribute to the mass of full infinite superpositions, then these virtual particles indirectly contribute to the \( k_{\text{min}} \) virtual graviton coupling, which is based on the actual mass of the infinite superposition as in Eq. (3.2.3). The conservation of energy, or in reality 4 momentum, says that what we call “real matter or energy” can last for close to the age of the universe. It will have mass and by definition it can be weighed. It can move around, even close to the speed of light, but it is conserved. Gravitons that last this long we have called \( k_{\text{min}} \) gravitons and they can only couple to real, or long lasting energy/matter that can be weighed in whatever manner. The particle beams in accelerators have real energy which can be temporarily converted into virtual particles. The total energy or 4 momentum is always conserved but can fluctuate for time \( \Delta T \geq 1/2\Delta E \). The long term average is what counts. In this sense the mass of short lived virtual particles can contribute to \( k_{\text{min}} \) virtual graviton coupling, just as it does in the virtual particle dressing of real charged particles.

6.2.4 Redshifted zero point energy from the horizon behaves differently to local

As we said above local zero point energies are Lorentz invariant. At high frequencies there is no shortage locally to build the high frequency components of full infinite superpositions. But as we have shown this is not so as we approach cosmic wavelengths. If there were no redshifted supply from the horizon there would be only a few modes of the local supply of \( k_{\text{min}} \approx 1/R_{\text{OU}} \) quanta inside the horizon. Because preons are born with zero momentum and infinite wavelength they can, however, absorb different redshifted \( k_{\text{min}} \approx 1/R_{\text{OU}} \) quanta from the receding horizon as we have discussed. This \( k_{\text{min}} \) quanta redshifted supply behaves differently to normal Lorentz invariant zero point local fields. It behaves as

\[
K_{Qk\text{min}} = 0.067 \alpha_G \quad \text{"The Quanta required @ } k_{\text{min}} \text{Constant" of Eq. (5.3.18)}
\]

Where \( K_{Qk\text{min}} \approx 1.27 \times K_{Gk\text{min}} \quad \text{"The } k_{\text{min}} \text{Graviton Constant" of Eq. (5.2.10)} \)
This redshifted supply is only available to zero spin preons that are born with zero momentum, or infinite wavelength, in the rest frame in which infinite superpositions are built.

6.2.5 Revisiting the building of infinite superpositions

In section 2 we developed equations to determine the probability of each mode of a superposition using local zero point fields. In section 5 when we found the cosmic wavelength supply inadequate, we switched to a supply of long range quanta redshifted from the horizon. So how do we justify our use of the local zero point fields to determine mode probabilities and behaviours? As we noted above there is a plentiful supply of high frequency local zero point fields. This local supply is adequate for high densities of superpositions for all modes from the Planck energy \( k \approx 1 \) high energy mode cutoffs to somewhere around \( k \approx 10^{-17} \), or near the Higgs boson energy. The coupling to local zero point fields in this high frequency region determines the behaviour of all the SM particles. There is, however, a gradual transition to absorbing quanta from the redshifted horizon supply as the wavelength increases. Because the redshifted supply of \( k_{\text{min}} \) quanta behaves as the invariant \( k \) or \( Q_k G_k \) above, and entirely differently to Lorentz invariant local zero point fields, spacetime has to warp around masses concentrations and the universe has to expand.

6.2.6 The primary to secondary graviton coupling ratio \( \chi_G \)

In Eq. (4.2.12) we found \( \chi_G' \approx 318.3 \) as the ratio between the primary graviton coupling to a bare Planck mass and the normal measured gravitational constant \( G \). Equation (5.1.7) defined graviton coupling between Planck masses \( \alpha_G \). But Eq.(11.1.18) finds that \( \alpha_G \approx 1/24 \). The primary to secondary graviton coupling ratio (as for colour and electromagnetism in Eq. (3.3.2)) is

\[
\chi_G = \alpha_G^{-1} \chi_G' \approx 24 \times 318.3 \approx 7632 \quad (6.2.8)
\]

To solve graviton superpositions we can use Eq. (3.3.14), which is the gravitational interaction probability between fermions, and we can now put on the RHS the coupling ratio \( \chi_G = 1878 \) in the same way as we did for Eq. (3.3.19). This \( c_{4c} \times c_{4c} (1 - c_{4c} \times c_{4c}) \) we are going to calculate here is for spin 2 & \( N = 2 \). It is different to the double combination of \((N = 2) \times (\text{Spin 1})\) or \((N = 1) \times (\text{Spin 2})\) for \( 4c_{ab} \times c_{ab} (1 - c_{ab} \times c_{ab}) \) we derived in Eq. (4.4.1).

\[
\frac{[2s_{1/2} N_1 c_{ba} \times c_{ba} (1 - c_{6a} \times c_{6a})]^2}{q^4} \cdot \frac{2s_2 N_2 c_{4c} \times c_{4c} (1 - c_{4c} \times c_{4c})]^2}{q^4} = \frac{4(\chi_G^{-1})^2}{q^4}.
\]

For \( n_{1/2} = 1, N_1 = 1 \) & \( 2s_2 = 4, N_2 = 2 \) so \( \left[c_{ba} \times c_{ba} (1 - c_{6a} \times c_{6a})\right] \left[8c_{4c} \times c_{4c} (1 - c_{4c} \times c_{4c})\right] = \frac{2}{\chi_G} = \frac{2}{7632} \)

or \( c_{4c} \times c_{4c} (1 - c_{4c} \times c_{4c}) \approx \frac{1}{7632} \left[\frac{1}{8} c_{ba} \times c_{ba} (1 - c_{6a} \times c_{6a})\right] \)

But from Eq. (4.4.1) \( c_{6a} \times c_{6a} (1 - c_{6a} \times c_{6a}) = \sqrt{2 / \chi_C} \approx \sqrt{2 / 50.4053} \approx 0.199194 \)
So \( c_4 \cdot c_4 \sqrt{1-c_4 \cdot c_4} \approx \frac{1}{7632} \frac{1}{\{0.199194\}} \approx \frac{1}{6105} \approx 1.64 \times 10^{-4}. \)

Using Eq.(4.4.3), \( \sum c_n \cdot c_n \cdot n^4 = 1.64 \times 10^{-4} \) for spin 2, \( N=2 \) we get the infinitesimal mass graviton superposition values in Table 4.3.1.

6.2.7 Massive bosons and the Higg’s mechanism

In the SM the Higg’s mechanism adds mass to zero mass photons but here we say it adds mass to infinitesimal mass photons. But additionally, it also converts them from \( N = 2 \) to \( N = 1 \), and also from \( n = 3, 4, 5 \) to \( n = 4, 5, 6 \) superpositions.

7 Angular Momentum and the Kerr Metric

In the next two sections we revert to simple 3 volume densities for \( k_{min} \) gravitons. Also, what we do here is more to illustrate a possible way that the Kerr Metric can relate with the way we have been looking at gravity. But clearly our proposals must be able to include angular momentum. The ideas presented here may not survive and are intended only as initial tentative proposals. In the Schwarzschild metric the increase in measured volume is the same as the frequency increase, as \( \sqrt{g_{rr}} = r_{Sch} \) and \( g_{\theta \theta} \cdot g_{\phi \phi} = r^2 \sin^2 \theta \) is invariant if there is no angular momentum. With angular momentum both \( g_{\theta \theta} \) & \( g_{\phi \phi} \) change. The volume ratio of \( g_{\mu \nu} \neq \eta_{\mu \nu} \) space, to \( g_{\mu \nu} = \eta_{\mu \nu} \) flat space, in any metric at fixed \( r \) & \( \theta \) is

\[
\frac{V'}{V} = \sqrt{\frac{(g'_{\nu \nu} : g'_{\theta \theta} \cdot g'_{\phi \phi})(g_{\mu \nu} \neq \eta_{\mu \nu})}{(g_{\nu \nu} : g_{\theta \theta} \cdot g_{\phi \phi})(g_{\mu \nu} = \eta_{\mu \nu})}} = \sqrt{\frac{(g'_{\nu \nu} : g'_{\theta \theta} \cdot g'_{\phi \phi})(g_{\mu \nu} \neq \eta_{\mu \nu})}{r^4 \sin^2 \theta}} \tag{7.1.1}
\]

The Kerr metric [31] can be written in Boyer-Lindquist coordinates as

\[
\begin{align*}
g_{\theta \theta} &= r^2 + \alpha^2 \cos^2 \theta \\
g_{\phi \phi} &= (r^2 + \alpha^2 + \frac{r_s r}{g_{\theta \theta}} \alpha^2 \sin^2 \theta) \sin^2 \theta \\
g_{\theta \phi} &= -\frac{r_s r}{g_{\theta \theta}} \alpha \sin^2 \theta & \text{&} & g_{rr} = 1 - \frac{r_s r}{g_{\theta \theta}} \\
g_{rr} &= \frac{g_{\theta \theta}}{\Delta} \text{ where } \Delta = r^2 - r_s r + \alpha^2
\end{align*}
\]

Where \( \alpha = \frac{J}{mc} \) and the Schwarzschild radius \( r_s = \frac{2Gm}{c} = 2m \) in natural units with \( G = c = 1 \).

Everything is in units of length or \( (\text{length})^2 \), except \( g_{rr} \) & \( g_{rr} \) which are dimensionless. Because we want volume ratios as in Eq. (7.1.1) we can write the above version of the Kerr metric in a dimensionless form, leaving the length squared, and length terms \( (r^2, r^2 \sin^2 \theta \text{ & } r \sin \theta \text{ in } r^2 d\theta^2, r^2 \sin^2 \theta d\phi^2 \text{ & } r \sin \theta d\phi \text{ etc}) \) outside the metric tensor. This effectively gives us the denominator \( r^4 \sin^2 \theta \) we want in Eq. (7.1.1) as we will see. Also, the angular momentum parameter \( \alpha \) is a length dimension.
A dimensionless form of the Kerr metric

\[ g_{\phi\phi} = 1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\phi\phi}} \frac{\alpha^2}{r^2} \sin^2 \theta \]

where \( \Delta = 1 + \frac{\alpha^2}{r^2} - A \) and \( A = \frac{2m}{r} \) but

\[ g_{\phi\phi} = 1 + \frac{\alpha^2}{r^2} \cos^2 \theta \]

we will add an also dimensionless \( \frac{m^2}{r^2} \) term later. (See section 8.)

The space surrounding a rotating mass corotates with it. In such corotating reference frames we can write the Kerr metric, using these dimensionless coefficients, as

\[ ds^2 = -(g_{tt} + \frac{g_{\phi\phi}}{g_{\phi\phi}} dr^2 + g_{rr} dr^2 + g_{\phi\phi} r^2 d\theta^2 + g_{\phi\phi} r^2 \sin^2 \theta (d\phi + \frac{g_{\phi\phi}}{g_{\phi\phi}} dt)^2) \]  

(7.1.3)

The corotating time component becomes \( g'_{tt} = g_{tt} + \frac{g_{\phi\phi}^2}{g_{\phi\phi}} \) with positive proper time world lines for all particles outside the inner horizon provided they rotate at least \( \frac{\Omega}{\frac{\Delta}{g_{\phi\phi}}} \) inside the ergosphere. Thus using Eq. (7.1.2)

\[
g'_{tt} = \frac{g_{tt} + \frac{g_{\phi\phi}^2}{g_{\phi\phi}}}{g_{\phi\phi}} = \frac{1 - \frac{A}{g_{\phi\phi}} + \frac{A^2 \alpha^2}{g_{\phi\phi} r^2} \sin^2 \theta}{1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\phi\phi} r^2} \sin^2 \theta} = \frac{1 - \frac{A}{g_{\phi\phi}}}{1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\phi\phi} r^2} \sin^2 \theta}
\]

(7.1.4)

We have explicitly gone through this to show that if the parameter \( A = 2m/r \) is dimensionless there is potentially freedom to change it without altering this equation. (See section 8.1.5 as
this is similar to what happens in the Kerr-Newman metric, where instead of a dimensionless \( m^2/r^2 \) term, a dimensionless \( r_0^2/r^2 \), or equivalently a dimensionless \( Q^2/r^2 \), is included in term \( A \). (Note for example Table 8.1.2 and Table 8.1.3.) We will work in corotating frames, or alternatively use the measurements of a zero angular momentum observer (ZAMO). (Note also for example [32]). Space is swirlling around effectively at rest in these frames, simplifying our calculations and equations. (Section 9 puts this into a four vector form, invariant in all frames.) If a small mass, at rest at infinity in the same rest frame as the rotating black hole centre, falls inwards, it will have the same circumferential velocity as the corotating rest frames at all radii. It will be falling radially through these corotating frames. As in section 5.2.2 we call this radial velocity \( \beta_M \) whereas in the non-rotating case

\[
\frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M \quad \text{but now} \quad \frac{1}{\sqrt{1-\beta_M^2}} = \gamma_M = \frac{1}{\sqrt{g_{tt}}} \quad \text{the corotating inverse rate of clocks.}
\]

In corotating frames

\[
\gamma_M' = g_{rr} = \frac{\Delta}{g_{\phi\phi}} \quad \gamma_M = \frac{g_{\phi\phi}}{\Delta}
\]

(7.1.5)

Frequencies measured in corotating frames increase as \( \gamma_M \). We can also get the three volume element in this corotating frame by using the three spatial components \( dr, d\theta, d\phi \) with \( dt = 0 \) in Eq. (7.1.3) and then using Eq. (7.1.5) to get the corotating 3 volume element ratio

\[
V = \sqrt{(g_{rr} - g_{\phi\phi})^2} = \frac{\sqrt{g_{\phi\phi}}}{\Delta} g_{\phi\phi} g_{\phi\phi} g_{\phi\phi} = \frac{g_{\phi\phi}}{\Delta} g_{\phi\phi} = g_{\phi\phi} \gamma_M
\]

(7.1.6)

With angular momentum we no longer have the same increase in frequency as volume as in the Schwarzschild case. With no angular momentum we found that the probability density of time polarized \( k_{min} \) gravitons Eq. (5.2.13) \( \Delta \rho_{Gk_{min}} \approx \gamma_M^2 \beta_M^2 K_{Gk_{min}} dk_{min} = \gamma_M^2 \frac{2m}{r} K_{Gk_{min}} dk_{min} \).

(Again, temporarily including \( \gamma_M' \)). With rotation we will find a circularly polarized \( \cos^2 \theta \) type distribution of \( k_{min} \) gravitons around the axis which as we will see must behave differently to transversely polarized \( k_{min} \) gravitons in the equatorial plane (See section 5.4)

These circularly polarized gravitons add to the time polarized dimensionless number \( \frac{2m}{r} \) to get an as yet unknown number we simply label as \( X \) where \( X > \frac{2m}{r} \)

\[
\text{Undiluted } \Delta \rho_{Gk_{min}} \approx \gamma_M^2 XK_{Gk_{min}} dk_{min} \quad \text{with rotation (7.1.7)}
\]

\[
\rho_{Gk_{min}} \quad \text{(Undiluted Total)} = K_{Gk_{min}} dk_{min} + \gamma_M^2 XK_{Gk_{min}} dk_{min} = (1 + \gamma_M^2 X) K_{Gk_{min}} dk_{min}
\]

As in section 5.2.2 Eq.(5.2.13) but in a slightly different order we divide this undiluted total by the new volume \( V = g_{\phi\phi} \gamma_M \) in Eq. (7.1.6) to get the new \( k_{min} \) graviton density \( \rho_{Gk_{min}}^* \). If our conjectures are correct \( \rho_{Gk_{min}}^* = K_{Gk_{min}} dk_{min} \) is always true, and as our measurement of \( k_{min} \) increases to \( k''_{min} = \gamma_M k_{min} \) in the new metric, \( \rho_{Gk_{min}}^* = K_{Gk_{min}} \gamma_M dk_{min} \); rewriting as follows
\[ \rho_{G_k \text{min}}^* = \frac{(1 + \gamma_M^2 X) K_{G_k \text{min}} d_{k_{\text{min}}}}{\sqrt{g_{\theta\theta}}} = \frac{(1 + \gamma_M^2 X) K_{G_k \text{min}} d_{k_{\text{min}}}}{g_{\theta\theta} \gamma_M^2} = \gamma_{M} K_{G_k \text{min}} d_{k_{\text{min}}} = K_{G_k \text{min}} d_{k_{\text{min}}}^* \]

\[ (1 + \gamma_M^2 X) K_{G_k \text{min}} d_{k_{\text{min}}} = g_{\theta\theta} \gamma_M^2 K_{G_k \text{min}} d_{k_{\text{min}}} \]

\[ 1 + \gamma_M^2 X = g_{\theta\theta} \gamma_M^2 \]

\[ X = g_{\theta\theta} - \frac{1}{\gamma_M^2} = (1 + \frac{\alpha^2}{r^2} \cos^2 \theta) - \frac{1}{\gamma_M^2} \]

\[ X = (1 + \frac{\alpha^2}{r^2} \cos^2 \theta) - \frac{\Lambda}{g_{\theta\theta}} \text{ using Eq. (7.1.5)} \]

\[ X = 1 + \frac{\alpha^2}{r^2} \cos^2 \theta - \frac{1 + \frac{\alpha^2}{r^2} - A}{1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2 \theta} \text{ using Eqs.(7.1.2)} \]

We can write this as:

\[ X = \frac{\alpha^2}{r^2} \cos^2 \theta + \left[ \frac{A \left( \frac{1 + \alpha^2}{r^2} \sin^2 \theta \right)}{1 + \frac{\alpha^2}{r^2} + \frac{A}{g_{\theta\theta}} \frac{\alpha^2}{r^2} \sin^2 \theta} \right] \]

\[ X = \frac{\alpha^2}{r^2} \cos^2 \theta + \left[ \frac{A \left( \frac{1 + \alpha^2}{r^2} \sin^2 \theta \right)}{g_{\phi\phi}} \right] \text{ using Eqs.(7.1.2)} \]

This suggests both circularly polarized \( \cos^2 \theta \) and transversely polarized \( \sin^2 \theta \) extra \( k_{\text{min}} \) gravitons. However in section 5.4 we discussed how transversely polarized virtual gravitons due to peculiar velocities do not need to borrow additional zero point energy from the horizon. (Real transversely polarized gravitons in contrast do need to borrow additional zero point energy from the horizon thus changing the local metric as they pass by.) We are going to conjecture here that there is no extra expansion of space due to transversely polarized virtual gravitons because of rotational velocities. We will argue in section 7.1.2 that circular polarization is very different. Because of this we will write the \( X \) above as follows:
\[ X = \frac{\alpha^2}{r^2} \cos^2 \theta + \frac{Ag_{\phi\phi}}{g_{\phi\phi} g_{\theta\theta}} + A \frac{\alpha^2 \sin^2 \theta}{r^2} g_{\phi\phi} g_{\theta\theta} \]

\[ X = \frac{\alpha^2}{r^2} \cos^2 \theta + \frac{A(1 + \frac{\alpha^2}{r^2} \cos^2 \theta)}{g_{\phi\phi} g_{\theta\theta}} + A \frac{\alpha^2 \sin^2 \theta}{r^2} g_{\phi\phi} g_{\theta\theta} \]

Which we finally write as

\[ X = \frac{\alpha^2}{r^2} \cos^2 \theta + \frac{A(1 + \frac{\alpha^2}{r^2})}{g_{\phi\phi} g_{\theta\theta}} \]  

(7.1.8)

Putting \( A = \frac{2m}{r} \), the extra \( k_{\text{min}} \) virtual gravitons \( \gamma_M^2 X \) (due to a mass \( m \) rotating with angular parameter \( \alpha \) that has dimensions of length) are the following two polarization groups (The background \( k_{\text{min}} \) virtual gravitons have been normalized to one when \( \gamma_M = 1 \)) i.e.

Circularly polarized spin 2:

\[ \left[ \frac{\alpha^2}{r^2} \cos^2 \theta \right] \]

& Time polarized spin 2:

\[ \left[ \frac{2m}{r} \frac{(1 + \frac{\alpha^2}{r^2})}{g_{\phi\phi} g_{\theta\theta}} \right] \]

Using \( \frac{1}{\gamma_M^2} + X = g_{\theta\theta} = 1 + \frac{\alpha^2}{r^2} \cos^2 \theta \) we can say that

\[
\frac{1}{\gamma_M^2} + \frac{\alpha^2}{r^2} \cos^2 \theta + \frac{A(1 + \frac{\alpha^2}{r^2})}{g_{\phi\phi} g_{\theta\theta}} = 1 + \frac{\alpha^2}{r^2} \cos^2 \theta \quad \text{or} \quad \frac{1}{\gamma_M^2} + \frac{A(1 + \frac{\alpha^2}{r^2})}{g_{\phi\phi} g_{\theta\theta}} = 1
\]

where \( \frac{A(1 + \frac{\alpha^2}{r^2})}{g_{\phi\phi} g_{\theta\theta}} = \beta_M^2 \)

We have a 4 vector equation again as circular polarization cancels on both sides. The main thing to notice here is that the circularly polarized \( k_{\text{min}} \) gravitons are independent of the central mass, suggesting they are due to the effect of the rotation of space, or frame dragging, on the \( k_{\text{min}} \) graviton background. We will discuss this in section 7.1.2. The extra \( k_{\text{min}} \) gravitons due to the central mass have a \( (1 + \frac{\alpha^2}{r^2}) / (g_{\phi\phi} g_{\theta\theta}) \) factor, distorting them from spherical symmetry. Figure 7.1.1 illustrates this and there are some parallels with spinning charged spheres in electromagnetism. The electrostatic energy density surrounding a charged sphere however, reduces with radius as \( r^{-4} \), and magnetic energy as \( r^{-6} \), or two more powers of radius. With gravity however we have been looking at the probability density of minimum wavenumber \( k_{\text{min}} \) gravitons surrounding a mass. With no angular momentum there are only time polarized \( k_{\text{min}} \) gravitons, and their extra probability density drops as \( r^{-1} \), as so far we have only focussed on those \( k_{\text{min}} \) gravitons (the vast majority) that interact with the rest of the mass in the universe. If a charged sphere rotates, there is a radial magnetic field of circularly polarized \( m = \pm 1 \) photons varying in intensity as \( \cos^2 \theta \), and a transverse magnetic field of transversely polarized \( m = \pm 1 \) photons varying as \( \sin^2 \theta \).
Figure 7.1.1 Spinning mass $m$ with angular momentum length parameter $\alpha$ as viewed in a corotating frame. There are circularly polarized $k_{\text{min}}$ virtual gravitons due to the effect of frame dragging on the background time polarized $k_{\text{min}}$ gravitons. Any transversely polarized $k_{\text{min}}$ gravitons due to a rotating mass $m$ do not alter the metric. Time polarized extra $k_{\text{min}}$ gravitons due to mass $m$ are distorted from spherical symmetry as $(1 + (\alpha^2 / r^2)) / (g_{\phi\phi} g_{\theta\theta})$. For $r >> r_{Sw}$ we can ignore the effects of $g_{\phi\phi}, g_{\theta\theta}$. As they rapidly tend to one, with the metric written in dimensionless form as in Eqs.(7.1.2)

7.1.1 Stress tensor sources for spin 2 gravitons & 4 current sources for spin 1

Spin 1 particles behave like a 4 vector as they come from a 4 current source, transforming with velocity as in the SR transformations of Minkowski spacetime. Spin 2 gravitons in contrast come from mass/energy density sources. There are two factors in their transformations with velocity. One from the mass increase per source particle, and the second from the increase in particles per unit length due to length contraction. Thus spin 2 particles transform as a 4x4 rank 2 tensor, which Einstein connected with spacetime curvature. The rules of quantum mechanics tell us that spherically symmetric spin 2 particles should be equal $1/\sqrt{5}$ superpositions of $m = \pm 2, \pm 1, 0$ states. But the shape of gravitational waves behaves like transversely polarized $m = \pm 2$ particles, suggesting the $k_{\text{min}}$ gravitons surrounding mass concentrations may only consist of time polarized, plus $m = \pm 2$ polarized, spin 2 particles. Also, the distortion of space in a gravitational field has an elliptical shape consistent with $m = \pm 2$ states. It appears that $m = \pm 1$ are inconsistent with the gravitational field and thus may not be present.

When we looked at non-rotating spherical masses it appeared that, even close to black holes, the spherical symmetry of the Schwarzchild metric suggested similarly spherically symmetric, time polarized, extra $k_{\text{min}}$ gravitons down to the horizon; with space expanding only radially. Thus, before we considered angular momentum we could treat all $k_{\text{min}}$ gravitons as only time polarized. A stress tensor source with no angular momentum has spherically symmetric
spacetime curvature with time polarized \( k_{\text{min}} \) gravitons. But angular momentum in the source produces cylindrically symmetric spacetime curvature. We still have radially polarized \( k_{\text{min}} \) gravitons (in co-rotating coordinates) due to the central mass, but distorted from spherical symmetry as \( (1 + \alpha^2 / r^2) / \left( g_{\varphi\varphi} \right) \) which only affects the close in region, disappearing as \( \alpha \to 0 \). But there are also circularly polarized \( k_{\text{min}} \) gravitons only related to angular momentum. These circularly polarized \( k_{\text{min}} \) gravitons do not have the \( 2m/r \) factor and must be very different. As we will discuss below it appears that they are generated from the background time polarized \( k_{\text{min}} \) gravitons by the swirling velocity of corotating space.

### 7.1.2 Circularly polarized gravitons from corotating space

The circularly polarized gravitons do not have a \( 2m/r \) factor and appear to be related to the angular momentum parameter \( \alpha \) which has dimensions of length and is defined as \( \alpha = \frac{J}{mc} \).

Because angular momentum is the cross product of momentum by radius or \( m\mathbf{v} \times \mathbf{r} \), we can think of this length parameter as a vector of length \( \alpha \), pointing along the axis of spin, with components \( \alpha \cos \theta \) at any polar angle \( \theta \) to the spin axis. Space corotates around spinning masses with angular velocity \( \Omega = \frac{g_{\varphi\varphi}}{g_{\phi\phi}} \) which in the plane of the equator simplifies to

\[
\Omega = \frac{r_5 \alpha c}{r^3 + r \alpha^2 + r_5 \alpha^2} \approx \frac{r_5 \alpha c}{r^3} \quad \text{when} \quad r \gg r_5 \text{ & } \alpha.
\]

At large radii the (equatorial) corotating velocity \( \mathbf{V} = \Omega \times \mathbf{r} \approx \frac{r_5 \alpha c}{r^2} \).

Because \( r_5 \) & \( \alpha \) have dimensions of length this equation has dimensions of velocity, and if we divide it by \( c \) it is dimensionless. We will call it \( \beta_{\text{corotating}} = \beta_c \).

At large radii

\[
\beta_{\text{corotating}} = \beta_c = \frac{V}{c} = \frac{\Omega \times r}{c} \approx \frac{r_5 \alpha c}{r^2} \quad \text{a dimensionless number.}
\]

If we now think of \( \alpha = \frac{J}{mc} \) as \( \alpha = \frac{m \mathbf{v} \times \mathbf{r}}{mc} = \frac{\mathbf{v} \times \mathbf{r}}{c} \) we can consider a similar vector along the spin axis consisting of the cross product of the corotating velocity of space \( \frac{\mathbf{V}}{c} \approx \frac{r_5 \alpha c}{r^2} \) by the radius \( r \). The length along the spin axis of this cross product vector \( \frac{\mathbf{V} \times \mathbf{r}}{c} \) is simply \( \frac{r_5 \alpha}{r} \).

At the equator: Length of vector \( \frac{\mathbf{V} \times \mathbf{r}}{c} \) along the spin axis is \( \approx \frac{r_5 \alpha}{r} \) for \( r \gg r_5 \).

We need this vector length to be a dimensionless number representing the amplitude that a background time polarized \( k_{\text{min}} \) graviton generates a circularly polarized \( k_{\text{min}} \) graviton.
around the spin axis. If we divide Eq. (7.1.11) by the Schwarzschild radius \( r_S \), all rotating black holes with the same percentage of maximum spin look identical, and we get a dimensionless magnitude as required. We also divide by \( \frac{\alpha}{r^2} \) and explain why below.

Magnitude of normalized dimensionless vector
\[
\frac{V \times r}{r_c \sqrt{2}} \approx \frac{r_S \alpha}{r_S r_c \sqrt{2}} = \frac{\alpha}{r \sqrt{2}}
\]  

(7.1.12)

The whirling velocity of space (as seen by a distant observer) varies as \( \sin \theta \), and the magnitude of this dimensionless amplitude vector varies as \( \cos \theta \) around the axis. The average of \( \sin^2 \theta \) is \( \frac{2}{3} \) over a sphere and \( \cos^2 \theta \) is \( \frac{1}{3} \), a two to one ratio, which is why we introduced the above \( \sqrt{1/2} \) factor. We thus conjecture that the probability of these \textit{background} corotating time polarized \( k_{\text{min}} \) gravitons, generating circularly polarized \( k_{\text{min}} \) gravitons around the spin axis is

\[
\text{Probability of } \frac{\text{Extra circularly polarized } k_{\text{min}} \text{ gravitons}}{\text{Background time polarized } k_{\text{min}} \text{ gravitons}} \approx \frac{2 \alpha^2 \cos^2 \theta}{2r^2} = \frac{\alpha^2 \cos^2 \theta}{r^2}
\]  

(7.1.13)

For simple explanatory purposes, we approximated at large radii only. At small radii we must use Eq. (7.1.2) \( \Omega = \frac{g_{\phi \phi}}{g_{\phi \phi}} = \frac{r_S \alpha c}{r^3 g_{\phi \phi} g_{\phi \phi}} \). On the equator \( g_{\phi \phi} = 1 \), the co-rotation velocity \( V = \Omega \times r \sqrt{g_{\phi \phi}} \). The circumferential volume generating these circularly polarized gravitons also expands as \( \sqrt{g_{\phi \phi}} \). Rederiving Eqs. (7.1.10) and those following, the effective angular momentum term becomes \( \Omega \times r \times \sqrt{g_{\phi \phi}} \cdot \sqrt{g_{\phi \phi}} = \left[ \frac{r_S \alpha c}{r^3 g_{\phi \phi}} \right] \cdot r^2 g_{\phi \phi} = \frac{r_S \alpha c}{r} \) just as before.

Thus the swirling velocity of background \( k_{\text{min}} \) gravitons generates extra circularly polarized \( k_{\text{min}} \) gravitons as in Figure 7.1.1. Equation (7.1.13) should apply in all regions outside the central singularity, and certainly outside the inner horizon. While all this may appear to be somewhat contrived, these circularly polarized \( k_{\text{min}} \) gravitons are necessary if our hypothesis that the spatial density of \( k_{\text{min}} \) gravitons as in Eq. (5.2.10) \( \rho_{Gk_{\text{min}}} \approx K_{Gk_{\text{min}}} d k_{\text{min}} \) is true in all spacetime. If we reread section 5.1 we can conjecture that the emission of \( k_{\text{min}} \) gravitons from matter/energy, combined with the warping of spacetime these gravitons cause (as in the rest of section 5) is essential, if the way we built fundamental particles in the first part of this paper is true. When this matter has angular momentum it has to also emit circularly polarized \( k_{\text{min}} \) gravitons for \( \rho_{\text{Gk}_{\text{min}}} \approx K_{Gk_{\text{min}}} d k_{\text{min}} \) to stay true. While it appears to be different to spin one electromagnetism with its equatorial transverse polarization, it is important to remember that spin one transmits real forces by exchanging momentum. On the other hand gravitational forces, as Einstein taught us, are fictitious. There can be no exchanged 3 momentum with spin 2 gravitons (see section 3.3.2) as all particles simply follow geodesics in spacetime that is warped by mass plus its angular momentum. (Gravitational waves are real transversely polarized \( m = \pm 2 \) \( k_{\text{min}} \) gravitons as discussed in section 11.3.3)
7.1.3 Does our time polarized $k_{\text{min}}$ value in corotating coordinates make sense?

The inner horizon radius $R$ is defined when $\Delta = 1 + \frac{\alpha^2}{r^2} - A = 1 + \frac{\alpha^2 - 2m}{r} = 0$ where we initially define the dimensionless number $A = \frac{2m}{r}$. Using $A = \frac{2m}{r}$ the inner horizon is where $r^2 - 2mr + \alpha^2 = 0$. So horizon radius $R = r = m + \sqrt{m^2 - \alpha^2} = 2m$ when $\alpha = 0$, and at maximum spin $r = R = m$ when $\alpha = m$. But we will, for generality, revert to the dimensionless $A$ and look at what happens near the horizon for various spins.

Let $A_H$ be the value of $A$ at the horizon where $1 + \frac{\alpha^2}{R^2} - A_H = 0$ is always true

So $\frac{\alpha^2}{R^2} = A_H - 1$ and $A_H = 1 + \frac{\alpha^2}{R^2}$ (7.1.14)

At the horizon $g_{\theta\theta} g_{\phi\phi} = (1 + \frac{\alpha^2}{R^2} \cos^2 \theta)(1 + \frac{\alpha^2}{R^2} + A_H \frac{\alpha^2 \sin^2 \theta}{R^2} g_{\theta\theta}) = g_{\theta\theta}(A_H + \frac{\alpha^2 \sin^2 \theta}{R^2} g_{\theta\theta})$ = $g_{\theta\theta}A_H(1 + \frac{\alpha^2 \sin^2 \theta}{R^2} g_{\theta\theta}) = A_H(g_{\theta\theta} + \frac{\alpha^2}{R^2} \sin \theta)$

At the horizon $g_{\phi\phi} g_{\theta\theta} = A_H (1 + \frac{\alpha^2}{R^2} \cos^2 \theta + \frac{\alpha^2}{R^2} \sin^2 \theta) = A_H (1 + \frac{\alpha^2}{R^2}) = A_H^2$ (7.1.15)

and independant of angle $\theta$ near the horizon only. This is true regardless of spin from zero to maximum as the radius shrinks.

Near the horizon the black hole surface area is $4\pi R^2 \sqrt{g_{\theta\theta}g_{\phi\phi}} = 4\pi R^2 A_H$ (7.1.16)

We have shown in co-rotating frames the extra time polarized $k_{\text{min}}$ graviton density due to a central mass is $\Delta \rho_{\text{Gkmin}} = A \cdot \frac{1 + \frac{\alpha^2}{r^2}}{g_{\theta\theta}g_{\phi\phi}} K_{\text{Gkmin}}dk_{\text{min}}$ where $A = \frac{2m}{r}$ so far, and dimensionless.

Using Eqs. (7.1.14) & (7.1.15) above, near the horizon in a corotating frame, this becomes the extra time polarized $k_{\text{min}}$ graviton density near the horizon for all black holes

$$\Delta \rho_{\text{Gkmin}} = A_H \cdot \frac{1 + \frac{\alpha^2}{R^2}}{g_{\theta\theta}g_{\phi\phi}} K_{\text{Gkmin}}dk_{\text{min}} = A_{H}^2 K_{\text{Gkmin}}dk_{\text{min}} + K_{\text{Gkmin}}dk_{\text{min}}$$ (7.1.17)

Ignoring the factor $K_{\text{Gkmin}}dk_{\text{min}}$, the extra radially polarized $k_{\text{min}}$ graviton probability density is always one in Planck units regardless of spin and is spherically symmetric, but only near the horizon where the background density $K_{\text{Gkmin}}dk_{\text{min}} / \gamma_m^2 \rightarrow 0$ as $\beta_{\text{H}} \approx 1$ & $\gamma_m^2 \rightarrow \infty$, providing it is observed (somehow by a ZAMO) in a corotating reference frame.

It can also be shown that near the horizon of a black hole $\dot{\gamma}_m^2 \approx \frac{4R^2}{s^2}$ and $\gamma_m^2 \approx \frac{2R}{s}$ is always true regardless of the degree of spin, and the value of the dimensionless number $A_H$, where $R$ is the horizon radius and $s$ the proper distance from it, providing it is all measured in co-rotating coordinates. The region well above the horizon is not spherically symmetric.
until several Schwarzschild radii away where spherical symmetry is gradually retained in the non-rotating case. Further, near the horizon this density due to a central mass is so great that we can effectively ignore the background value, but the rotation of space generates circularly polarized gravitons, of probability density \( \frac{\alpha^2}{r^2} \cos^2 \theta \) from this background. The extra radially, plus circularly, polarized graviton probability density near the horizon ignoring the factor \( K_{Gk} dk_{Gk} \) is

\[
\text{Time polarized } 1 + \text{Circularly polarized } \frac{\alpha^2}{R^2} \cos^2 \theta = (1 + \frac{\alpha^2}{R^2} \cos^2 \theta) \quad (7.1.18)
\]

\( = g_{\theta\theta} \) as in our original derivation, but ignoring the background, which is infinitesimal near the horizon.

Because the Kerr metric is an exact solution for rotating black holes, we can say that if the extra gravitons due to a rotating mass are consistent with \( \chi \) as in Eq. (7.1.8), then it is also consistent with keeping the graviton constant \( K_{Gk} \) as in Eq. (5.2.10) invariant in the spacetime surrounding it. We come back to this, and potential changes to the dimensionless term \( A = 2m/r \) below. When we looked at non-rotating black holes in section 5.2 we used simple first principles to show that the warping of spacetime around them is consistent with an invariant graviton constant \( K_{Gk} \). With rotating black holes we turned the argument around and assumed this invariance to derive the extra probability densities of time, and circularly polarized gravitons, before the density dilution from the expansion of space around the rotating mass. Equations (7.1.17) and (7.1.18) can perhaps increase our confidence that our hypothesis is possibly correct. If it is correct on the horizon, and also far from a rotating black hole, we will conjecture that it is also correct in all regions between, even if it might not initially appear so. It is also important to remember that the Kerr metric is an exact solution for rotating black holes, not for rotating masses in general. We have only considered here the exact solution. We have not yet included the relatively small number of gravitons emitted by the mass itself \( (\psi_m^* \psi_m) \), which mainly has significant effect close to black holes.

## 8 Virtual Gravitons and Mass Interacting with Itself

### 8.1 The Effect of \( \psi_m^* \psi_m \) on \( k_{Gk} \) Densities

In section 5 we began by finding the average graviton probability density in a uniform universe. We then placed a mass concentration in it, and calculated the extra probability density of gravitons (before the dilution due to local space expansion) due to the amplitude of this mass multiplied by the amplitude of the rest of the mass in the universe. This ended up being proportional to \( 2m/r \) in Planck units. Ignoring the \( \psi_m^* \psi_m \) term we got

\[
\Delta \rho_{Gk} = (\psi_{\text{Universe}}^* \psi_m) + (\psi_m^* \psi_{\text{Universe}}) \propto 2m/r \quad \text{as in Eq.}(5.2.5).
\]
In this section we use simple 3 volume \( k_{\min} \) graviton probability densities to evaluate \( \psi_{\text{m}} \psi_{\text{m}} \).

We also use Eq.(5.1.5) and the coupling probability \( (1 - e^{-0.61k^2/k_{\min}^2})^2 \left[ \frac{2\alpha_G m^2}{\pi k} \right] \).

\[
\psi_{\text{m}} \psi_{\text{m}} = (1 - e^{-0.61k^2/k_{\min}^2})^2 \left[ \frac{2\alpha_G m^2}{\pi k} \right] \left[ \frac{2k' e^{-2k'r}}{4\pi r^2} \right] = (1 - e^{-0.61k^2/k_{\min}^2})^2 \alpha_G \frac{m^2}{r^2} \frac{k' e^{-2k'r}}{\pi^2} \frac{dk}{k}
\]

Also using Eq. (5.1.4) \( k' = \sqrt{k^2 + 11.09k_{\min}^2} \approx 3.477k_{\min} \) when \( k = k_{\min} \)

\[
\psi_{\text{m}} \psi_{\text{m}} = (1 - e^{-0.61k^2/k_{\min}^2})^2 \alpha_G \frac{m^2}{r^2} \frac{3.477k_{\min} e^{-2(3.477k_{\min}r)}}{\pi^2} \frac{dk_{\min}}{k_{\min}} = \alpha_G \frac{m^2}{r^2} \frac{0.725e^{-6.95k_{\min}r}}{\pi^2} \frac{dk_{\min}}{k_{\min}} \text{ when } k = k_{\min}
\]

The radial exponential decay term \( e^{-6.95k_{\min}r} \approx 1 \) as we are only interested in small radii \( r \) in relation to the observable radius of the universe \( R_{\text{OU}} \approx k_{\min}^{-1} \), just as the assumptions we made in section 5.2.1. In these regions we can approximate this equation with good accuracy as

\[
\psi_{\text{m}} \psi_{\text{m}} \approx \alpha_G \frac{m^2}{r^2} \frac{0.725}{\pi^2} \frac{dk_{\min}}{k_{\min}}
\]

\( \Delta \rho_{Gk_{\min}} \) due to self emission \( \psi_{\text{m}} \psi_{\text{m}} \approx \alpha_G \frac{m^2}{r^2} 0.0735dk_{\min} \)

\[
\approx 1.4 \frac{m^2}{r^2} 0.0527\alpha_G dk_{\min} \text{ when } k = k_{\min}
\]

Using Equ’s. (5.2.8)\&(5.2.10) \( \Delta \rho_{Gk_{\min}} \) due to \( \psi_{\text{m}} \psi_{\text{m}} \approx 1.4 \frac{m^2}{r^2} K_{Gk_{\min}}dk_{\min} \) \( \quad \text{(8.1.1)} \)

\[
\text{and } \frac{\Delta \rho_{Gk_{\min}}}{\rho_{Gk_{\min}}} \text{ total } \approx \frac{2m}{r} + 0.7 \frac{m^2}{r^2} \quad \text{(8.1.2)}
\]

### 8.1.1 What does this extra term mean for non-rotating black holes?

Equation (8.1.2) suggests a modified non rotating metric of

\[
g_{00}' = 1 - \frac{2m}{r} - 1.4 \frac{m^2}{r^2} = \frac{1}{g_{rr}'} \quad ; \quad \gamma_M^2 = \frac{1}{1 - 2m/r - 1.4m^2/r^2} \quad \text{(8.1.3)}
\]

\[
\beta_M^2 = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}
\]
where $\beta_m$ is the velocity reached by a small test mass falling in from infinity in the same rest frame. The modified nonrotating horizon radius occurs when $r^2 - 2mr - 1.4m^2 = 0$.

Modified nonrotating horizon radius $r \approx 2.55m$ \hspace{1cm} (8.1.4)

or $\approx 27.5$% larger than the Schwarzchild value.

Differentiating $\gamma_M = \frac{1}{\sqrt{1 - 2m/r - 1.4m^2/r^2}}$ at a fixed radius similarly to Eq. (5.2.14) \hspace{1cm}

$$\frac{d\gamma_M}{\gamma_M} = \frac{dV}{V} = \gamma_M^2 \frac{dm}{r} + \gamma_M^2 \frac{1.4mdm}{r^2}$$ \hspace{1cm} (8.1.5)

where $\gamma_M$ is the inverse of the local clock rate. If we similarly differentiate Eq.(8.1.2) at a fixed radius $d\left(\frac{\Delta \rho_{Gk_{\min}}}{\rho_{Gk_{\min}}}\right) \approx \frac{dm}{r} + 0.7 \frac{mdm}{r}$, where Eq. (8.1.2) is $\gamma_M^2$ times larger, just as in section 5.2.2, and for the same reasons.

The central mass $m$ is the original mass as measured at infinity (in the same rest frame). At a radius $r$ its total mass/energy has increased to $\gamma_M m$. We can think of the two terms in Eq. (8.1.5) as follows: At any radius $r$ the small mass $dm$ at infinity becomes $\gamma_M dm$ and the locally measured mass of the universe also increases as $\lambda_M$. Thus, both the first amplitude interaction term between the infinitesimal mass $dm$ and the rest of the universe, and the second interaction term with the central mass, both increase proportionally to $\gamma_M^2 dm$. As discussed in section 5.2.2 this requires changes in rulers and clocks in the metric to restore the effective local spatial 3D density of $k_{\min}$ gravitons to the allowed value $\rho_{Gk_{\min}} = K_{Gk_{\min}} dk_{\min}$ as in Eq. (5.2.10) where $k_{\min}$ is the new measured frequency in this local metric. Or equivalently, all of this simply keeps the four volume density of either $k_{\min}$ gravitons, or $k_{\min}$ action, invariant.

8.1.2 What does it mean for rotating black holes?

In section 7 when we looked at the Kerr metric, we used a dimensionless form of the metric in Eqs. (7.1.2). We also used a dimensionless parameter $A$ where we initially put $A = 2m/r$. We also showed that we could change $A$ without changing $g'_{t} = \Delta / g_{\phi\phi}$, the time component in the corotating frame, provided there is a modified $\Delta = 1 + \frac{\alpha^2}{r^2} - A$. So again temporarily ignoring potential conflicts with GR, let us change $A = \frac{2m}{r}$ to $A = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}$ and look at the consequences. Firstly, from Eqs. (8.1.3) we can see that $A = \beta_M^2$ where $\beta_M$ is
the radial inward velocity in a corotating rest frame of a small test mass falling from infinity (in the rest frame of the rotating black hole centre). The inner event horizon is the radius where \( g_{rr} \to \infty \) so using Eqs. (7.1.2) let \( g_{rr} = \frac{g_{\theta\theta}}{\Delta} \to \infty \) or \( \Delta = 1+\frac{\alpha^2}{r^2} - A = 0 \).

Thus \( 1+\frac{\alpha^2}{r^2} - \frac{2m}{r} - 1.4\frac{m^2}{r^2} = 0 \)

or \( r^2 + \alpha^2 - 2mr - 1.4m^2 = 0 \)

\[
 r = \frac{2m \pm \sqrt{4m^2 + 5.6m^2 - 4\alpha^2}}{2}
\]

Event horizon radius \[
 r = \frac{2m \pm \sqrt{9.6m^2 - 4\alpha^2}}{2} = m \pm \sqrt{2.4m^2 - \alpha^2}
\]

When \( \alpha = 0 \) \( r = m \pm \sqrt{2.4m^2} \approx m + 1.55m \approx 2.55m \) as in the non-rotating case.

Maximum spin is when \( \alpha^2 = 2.4m^2 \) or \( \alpha_{\text{max}} \approx 1.55m \) \( (8.1.6) \)

At this maximum spin \( r = m \) as in the usual Kerr metric.

The outer horizon occurs when \( g_{tt} = 1 - \frac{A}{g_{\theta\theta}} = 0 \) or \( g_{\theta\theta} - A = 0 \) and using Eqs. (7.1.2)

\[
 1 + \frac{\alpha^2}{r^2} \cos^2 \theta - A = 1 + \frac{\alpha^2}{r^2} \cos^2 \theta - \frac{2m}{r} - 1.4\frac{m^2}{r^2} = 0
\]

\[
 r^2 - 2mr - 1.4m^2 + \alpha^2 \cos^2 \theta = 0
\]

\[
 r = \frac{2m + \sqrt{4m^2 + 5.6m^2 - 4\alpha^2 \cos^2 \theta}}{2}
\]

\[
 r = \frac{2m + \sqrt{9.6m^2 - 4\alpha^2 \cos^2 \theta}}{2}
\]

Ergosphere radius \( r = m + \sqrt{2.4m^2 - \alpha^2} \) @ \( \theta = 0 \) & \( \pi \)

\[
 = m + \sqrt{2.4m^2} \approx 2.55m \ @ \theta = \frac{\pi}{2}
\]  \( (8.1.7) \)

Figure 8.1.1 illustrates these changes from the Kerr metric. The main effect from changing \( A \) is to allow an increase in maximum spin from \( \alpha = m \) to \( \alpha \approx 1.55m \), and \( \approx 27.5\% \) increase in the maximum ergosphere radius from \( r = 2m \) to \( 2.55m \). It appears to contradict GR which we discuss in sections 10 & 8.1.5, but provided the extra densities of time polarized and \( m = \pm 2 \) circular gravitons are as in Eq. (7.1.8) with \( A = \frac{2m}{r} + 1.4\frac{m^2}{r^2} \) then \( \rho_{GR \min} = K_{GR \min} dk_{\min} \) is still true in rotating space outside black holes.
Event horizon $r = m$ @ maximum spin is same as Kerr metric, but maximum spin has increased by $\approx 55\%$.

Ergosphere maximum radius $r \approx 2.55m$ is the same as a modified non-spinning black hole.

**Figure 8.1.1** Modified Kerr metric with the dimensionless parameter $A$ changed from $A = \frac{2m}{r}$ to $A = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}$.

Figure 8.1.2 below is a spinning black hole mass $m$ with angular momentum length parameter $\alpha$, but with the dimensionless parameter $A$ changed from $A = \frac{2m}{r}$ to $A = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}$. The determinant of the metric is independent of $A$. The denominator terms $g_{\phi\phi}$ & $g_{\theta\theta}$, in dimensionless form as in Eqs. (7.1.2) rapidly tend to one for radii $r >> r_{Sw}$, and can then be ignored. It shows the probability densities of time polarized and circularly polarized $m = \pm 2$, $k_{\text{min}}$ gravitons (as in Eq. (7.1.8) and the text and equations below) in this modified metric keep the $k_{\text{min}}$ graviton constant $K_{Gk_{\text{min}}}$ invariant outside the black hole. This is as observed in corotating coordinates.

**Figure 8.1.2** Virtual $k_{\text{min}}$ polarizations outside a spinning black hole with an $m^2 / r^2$ term.

8.1.3 **Determinant of the metric and the $k_{\text{min}}$ graviton constant $K_{Gk_{\text{min}}}$**

Working in dimensionless form as in Eqs. (7.1.2) using Eq. (7.1.4) $g'_{\phi\phi} = \frac{\Delta}{g_{\phi\phi}}$ and the steps used in its derivation, the determinant of the metric is:
\[|g_{\mu\nu}| = (g_{\mu r} g_{r\phi} - g_{\mu\phi}^2) g_{\phi\phi} g_{rr} = g'_{\mu r} g_{r\phi} g_{\phi\phi} g_{rr} = \frac{\Delta}{g_{\phi\phi}} g_{\phi\phi} g_{r\phi} g_{rr} g_{\phi\phi} = g_{\phi\phi}^2 = (1 + \frac{\alpha^2}{r^2} \cos^2 \theta)^2\]

(It is usually written as \(|g_{\mu\nu}| = (r^2 + \alpha^2 \cos^2 \theta)^2 \sin^2 \theta\) (see for example [33]) but we have been working in an equivalent dimensionless form of the metric tensor as in Eqs.(7.1.2). As 4 volumes are invariant in relativity and \(\rho_{Gk_{\min}} = K_{Gk_{\min}} dk_{\min}\) is true in corotating frames,

if \(|g_{\mu\nu}| = g_{\phi\phi}^2 = (1 + \frac{\alpha^2}{r^2} \cos^2 \theta)^2\) then \(\rho_{Gk_{\min}} = K_{Gk_{\min}} dk_{\min}\) is true in all frames, and is independent of the dimensionless parameter \(A\).

Despite what appears to be a conflict with GR, if the metric determinant Eq. (8.1.8) is \(g_{\phi\phi}^2\) then the \(k_{\min}\) graviton probability density is always \(\rho_{Gk_{\min}} = K_{Gk_{\min}} dk_{\min}\) in all frames outside the black hole, and this is also true if there is no rotation, regardless of the value of the dimensionless parameter \(A\). (See section 9).

8.1.4 The Reissner-Nordstrom metric and \(m^2 / r^2\) terms

Reissner [34],[35] solved the metric surrounding an electrically charged non-rotating mass not long after Schwarzschild had solved the metric around a static mass. He added the electromagnetic stress tensor surrounding a charge to the usual Einstein energy-momentum tensor in the region where the mass density term had previously been zero, as in the Schwarzschild case. As before we will put \(G = c = 1\) so we can work in Planck masses. The Schwarzschild radius \(r = 2Gm / c^2\) has length dimension and thus \(2Gm / rc^2\) becomes \(2m / r\), and both \(2m / r\) and \(m^2 / r^2\) are effectively dimensionless as, in these units, mass effectively has a length dimension.

Reissner similarly used the characteristic length \(r_Q\) where \(r_Q^2 = \frac{Q^2G}{4\pi\varepsilon_0 c^4}\)

Working in length units of charge with the Coulomb force constant \(\frac{1}{4\pi\varepsilon_0} = 1\) (8.1.9)

If \(G = c = 1\) & these units of charge \(r_Q^2 = \frac{Q^2}{r^2}\) are both dimensionless numbers.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Schwarzschild</th>
<th>Modified Schwarzschild</th>
<th>Reissner-Nordstrom</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g_{00} = g_{rr}^{-1})</td>
<td>(1 - \frac{2m}{r})</td>
<td>(1 - \frac{2m}{r} - 1.4 \frac{m^2}{r^2})</td>
<td>(1 - \frac{2m}{r} + \frac{Q^2}{r^2})</td>
</tr>
</tbody>
</table>

Table 8.1.1 Both parameters, mass \(m\) and charge \(Q\) effectively have dimensions of length.

Using our modified Schwarzschild metric from Eq. (8.1.3) Table 8.1.1 shows the similarities to the Reissner-Nordstrom metric for a charged mass, providing we measure charge parameter \(Q\) in a similar manner to measuring mass in Planck units. The signs are reversed, however. The relevance of this comparison is that the effect on the Reimannian curvature tensor of a \(-m^2 / r^2\) metric term, will be similar, but of opposite sign to a \(+Q^2 / r^2\) term.
8.1.5 The Kerr-Newman metric and $m^2/r^2$ terms

In 1965 Newman [36],[37] solved the charged version of the axisymmetric rotating black hole solved in 1962 by Kerr [31]. In section 7 and Eqs. (7.1.2) we introduced the dimensionless parameter $A = 2m/r$ and in section 8 modified this to get a dimensionless $A = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}$.

We showed in section 7 that provided A is dimensionless it does not change Eqs. (7.1.4). If we look carefully at the Kerr-Newman metric, we can see that it fits Eqs. (7.1.4) provided we put $A = \frac{2m}{r} - \frac{r_0^2}{r^2}$ which is equivalent to putting $A = \frac{2m}{r} - \frac{Q^2}{r^2}$ where $r_0^2 = \frac{Q^2G}{4\pi\varepsilon_0c^4}$ and again we measure charge $Q$ in length units as in Eqs.(8.1.9)

Thus, our modified Kerr metric where $A = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}$ is again similar to:

The Kerr-Newman metric where $A = \frac{2m}{r} - \frac{Q^2}{r^2}$ but with opposite signs.

These two metrics are the rotating versions of our modified Schwarchild metric and the Reissner-Nordstrom metrics. We can summarize this in the two tables below. The small changes in the Riemannian curvature tensor, due to this $m^2/r^2$ term, are of opposite sign for both our modified Kerr and Schwarchild metrics, when compared to the Kerr-Newman and Reissner-Nordstrom metrics, but of exactly the same form.

<table>
<thead>
<tr>
<th>Schwarchild</th>
<th>Modified Schwarchild</th>
<th>Reissner-Nordstrom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = \frac{2m}{r}$</td>
<td>$A = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}$</td>
<td>$A = \frac{2m}{r} - \frac{Q^2}{r^2}$</td>
</tr>
</tbody>
</table>

Table 8.1.2 The non-rotating metrics where the dimensionless parameter $A$ is as in Eq. (7.1.2) The modified Schwarchild and Reissner-Nordstrom metrics both have the same form of changes to the Riemannian curvature tensor but of opposite sign.

<table>
<thead>
<tr>
<th>Kerr</th>
<th>Modified Kerr</th>
<th>Kerr-Newman</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = \frac{2m}{r}$</td>
<td>$A = \frac{2m}{r} + 1.4 \frac{m^2}{r^2}$</td>
<td>$A = \frac{2m}{r} - \frac{Q^2}{r^2}$</td>
</tr>
</tbody>
</table>

Table 8.1.3 The rotating versions of the above. Again, the modified Kerr and Kerr-Newman metrics both have the same form of changes to the Riemannian tensor but of opposite sign.

8.1.6 Parallels with QED corrections to Poisson’s equation and Gauss’s law

Einstein based his remarkable equation on the “Equivalence Principle”, or the same physics in all free falling frames as in empty space; with covariant tensor equations that apply in all coordinates throughout all spacetime. He wanted it to be similar to Gauss’s law and Poisson’s equation $\nabla^2 \varphi = \rho$ ignoring constants, but in curved spacetime. This naturally led to inverse square force laws with inverse potentials where masses are concerned, so the inclusion of an $m^2/r^2$ potential term in the metric due to $\psi_\infty * \psi_\infty$ seems to mess it all up. But does it really? Quantum mechanics in the form of QED predicts that near the Compton wavelength, the normally simple inverse square force law starts to change, close in shielding makes
fundamental electric charge also appear to change, and QED takes over with incredible accuracy. Simple inverse square electric force laws had ruled with remarkable accuracy for over a century before QED arrived on the scene. In fact it was the announcement of the Lamb Shift at the Long Island conference in 1947 that started the big breakthroughs in QED. The second part of this paper has been trying to relate gravity with quantum mechanics. Are there parallels with an $m^2/r^2$ term mainly effective near black hole radii, and QED modifications to Gauss/ Poisson’s laws near the Compton radius? Section 10 develops all this further.

8.1.7 What is the effect of this term in the solar system?

The distance to Mars can be measured very precisely as we have instruments on the surface that can reflect radar from Earth at known locations. The astronomical unit is quoted as $149,597,870,700$ metres. The orbital period of Earth is known to extreme accuracy and one of the early definitions of the AU is based on “The radius of an unperturbed circular GR orbit of an infinitesimal mass with a constant angular velocity based on that known period.” So, let us do a crude first order approximation of what happens if we include this $1.4m^2/r^2$ term in the metric. Using low velocity (compared with light) Christoffel symbol approximations and circular orbits for simple comparisons the accelerations are:

$$\omega^2 r \approx \frac{1}{2} \frac{d}{dr} g_{00} \approx \frac{1}{2} \frac{d}{dr} (1 - \frac{2m}{r}) \approx \frac{m}{r^2}$$ in the usual Schwarzschild case if $g_{00} \approx 1$ and

$$\omega^2 r \approx \frac{1}{2} \frac{d}{dr} g'_{00} \approx \frac{1}{2} \frac{d}{dr} (1 - \frac{2m}{r} - \frac{1.4m^2}{r^2}) \approx \frac{m}{r^2} + \frac{1.4m^2}{r^3} \approx \frac{m}{r^2} (1 + \frac{1.4m}{r})$$ in the modified metric case. So $\omega^2 \approx \frac{m}{r^3}$ in the usual Schwarzschild case and $\omega'^2 \approx \frac{m}{r^3} (1 + \frac{1.4m}{r})$ in the modified metric case. In weak gravitational field accelerations we can replace mass $m$ with a new effective mass $m' = m(1 + \frac{1.4m}{r})$ but orbital periods and angular velocities $\omega$ cannot change as we know them very precisely. So, we will try the following modification to all planetary radii to account for the effect of the extra metric term.

$$\omega^2 = \frac{m}{r^3} \approx \frac{m'}{r'^3} = \frac{m}{(r + \Delta r)^3} (1 + \frac{1.4m}{r + \Delta r}) \approx \frac{m}{r'^3} (1 + \frac{1.4m}{r + \Delta r})$$

and if $\omega$ is unchanged

$$\frac{1 - \frac{3\Delta r}{r} + \frac{1.4m}{r + \Delta r}}{1 + \frac{3\Delta r}{r} + \frac{1.4m}{r + \Delta r}} \approx 1$$

and

$$\Delta r \approx \frac{1.4m}{3} \approx \frac{1.4R_{sw}}{6}$$

The Schwarzschild radius $R_{sw} = 2m$ and the extra distance to the Sun $\Delta r \approx \frac{1.4m}{3} \approx \frac{1.4R_{sw}}{6}$.

The Schwarzschild radius of the Sun is $R_{sw} \approx 3$ km so $\Delta r \approx \frac{1.4R_{sw}}{6} \approx 0.7$ km.

The change $\Delta r$ for all planets in our solar system is about 700 metres which would also be added to the AU figure above. Thus all interplanetary radial separations do not change. With this single change all orbital periods are identical (to what we currently observe) to a very high accuracy in this modified metric. The gravitational constant does not change.
What we have done here is a bit like dipoles with the electrostatic field dropping as $\propto 1/r^2$ and the resultant field as $\propto \Delta r/r^3$ where $\frac{1}{r^2} - \frac{1}{(r+\Delta r)^2} \approx \frac{2\Delta r}{r^4}$. However, in the non-spinning gravity case there is spherical symmetry but not in an electric dipole.

8.1.8 Can we measure this difference?

We used circular orbits for a simple crude calculation, but the same arguments apply in a slightly more complicated way for eccentric orbits; in a similar manner as Kepler’s original arguments with elliptical orbits that sweep out equal angular segment areas with time. The orbit of Mars in particular is highly eccentric and Earth much less so. If the eccentricity of both Earth and Mars orbits were known, to better than say a hundred metres or so between max and min, we should be able to check this difference by measuring the distance (also to around a 100 metre or so accuracy) between Mars and Earth at various points around their orbits. It would seem, however, that this would be pushing at the very border of current technology, as radar measurements to the Sun are inherently a little blurry due to surface variability. We need these to get a very precise value for the eccentricity of Earth’s orbit. Even if we can measure Earth-Mars with complete accuracy we have to add in errors due to lack of accuracy in Earth’s eccentricity. Also, when Mars is on the opposite side of the Sun to us, if the distance measuring signal grazes the Sun there will a Shapiro type delay that is equivalent to roughly a 15 km error that reduces logarithmically with the minimum radial distance of the signal from the Sun. Even if the beam passes through a half Earth-Sun radius the error is still a few km. All these effects introduce possible errors making it difficult to measure 700 metres difference in all planetary radii. Future accuracy improvements will no doubt change this.

![Figure 8.1.3](image)

Figure 8.1.3 Proposed changes to the radial distances of the planets from Sun due to the extra $m^2/r^2$ metric term and that do not change orbital periods. Scales are grossly exaggerated for clarity. We have also assumed circular orbits here for simplicity, and ignored errors due to the centre of mass of the solar system not being the centre of Sun. We have also assumed infinitesimal planet masses so we can simply ignore the effect they have on each other.
8.1.9 What about the Hulse-Taylor binary pulsar, can it show this change?
The timing of this pulsar is accurate to 14 significant figures and it would initially seem that
this accuracy would show up such differences. However, the semi-major orbit of this binary is
≈ 2×10^6 metres, with a decay rate of ≈ 3.5 metres per orbit, or a change ∆r ≈ 100 metres
over 30 years due to gravitational radiated energy where ∆r / r ≈ 2×10^{-7}. If we totally ignore
this change in radius, treating it as effectively zero, the accumulated time delay is parabolic;
or proportional to elapsed time squared. At the current time \( \frac{m}{r} \approx 5 \times 10^{-7} \) and approximating
the normal term over 30 years \( \frac{2m}{r} \) increases to \( \frac{2m}{(r - \Delta r)} \) a change of \( \frac{2m\Delta r}{r^2} \approx 2 \times 10^{-13} \).
If we however include the extra squared term in this approximation we now find that
\[
\frac{2m}{r} + 1.4 \frac{m^2}{r^2} \quad \text{increases to} \quad \frac{2m}{(r - \Delta r)} + \frac{1.4m^2}{(r - \Delta r)^2} \quad \text{a change of} \quad \frac{2m\Delta r}{r^2} (1 + \frac{0.7m}{r}).
\]
As \( \frac{m}{r} \approx 5 \times 10^{-7}, \quad \frac{0.7m}{r} \approx 3.5 \times 10^{-7} \) is the new increase. The usual \( \frac{2m\Delta r}{r^2} \approx 2 \times 10^{-13} \) increases
by ≈ 7×10^{-20}. This is much smaller than the current 14 significant figure accuracy. Even the
effect of a ∆r ≈ 100 metre change in the usual \( m/r \) term on the accumulated parabolic time
delay over 30 years is miniscule. The chances of measuring the extra effect of the \( m^2/r^2 \)
term is very small in the foreseeable future. The best chance of measuring any difference will
almost certainly turn out to be gravitational wave observations. We have ignored in the above
the fact that our extra \( m^2/r^2 \) implies an ≈ 0.7m/r ≈ 3.5×10^{-7} reduction in the calculated
mass compared to the value derived without it.

8.1.10 Can millimetre accuracy lunar distance measurements show this change?
The Swarzchild radius of Earth is 8.7 mm. so satellites occupy a region where \( m/r \approx 7	imes10^{-11} \).
Orbital dynamics are very accurately known and the mass of the Earth can be calculated from
GR using just the \( 2m/r \) term. If the \( 1.4m^2/r^2 \) is included an Earth mass value reduced by
10^{-10} gives the same satellite dynamics. The lunar orbital period is very accurately known so
using \( \omega^2 = \frac{m}{r^3} = \frac{m'}{r'^3} \approx \frac{m(1-10^{-10})}{r^3} \) = a constant, we find that the lunar radius is reduced by
\( \approx 380,000 \times 10^{-10} / 3 \) kilometres or \( \approx 13 \text{mm} \) which is a reduction in radius of the lunar centre
of mass not the surface. The laser reflectors placed on the lunar surface by Apollo astronauts
fifty years ago enable millimetre accuracy lunar distance measurements to the surface where
they are placed not the Lunar centre. Tidal friction on Earth causes the moon to move away
from Earth by about 38mm per year to conserve angular momentum. This amounts to about
two metres in fifty years. We can go through similar reasoning to that used in section 8.1.9 to
show that while the millimetre precision agrees with current GR, higher precision would be
necessary to show the effect of an \( m^2/r^2 \) term on this spiralling rate.
8.1.11 Gravitational wave observations of black hole mergers

Some of the mergers observed so far [38] have been suggesting relatively larger black hole masses than current astrophysics theory had expected. Section 0 shows that the extra $m^2/r^2$ term in the metric increases the maximum spin by $\approx 55\%$ (see Figure 8.1.1) for the same mass. This must surely increase the total merging energy and hence that in the resulting gravitational waves. However, computer simulations with these changed metrics would be required to model all this in detail, but the above suggests that the masses of the black holes before merging could well be less than what they have so far seemed. In other words, a pair of smaller black holes merging might create the gravitational waves current theory predicts from the mergers of two, up to maybe $55\%$ larger black holes. Spins had also been expected to be roughly perpendicular to their orbiting plane, but this does not tie up with their merging speeds. Is it possible that this unexpected behaviour results from something different in the metric as we get close to black hole horizons?

8.1.12 Do this extra term fit within known Solar system constraints on GR?

Probably the most important constraint in Will’s review [39] of the status of experimental tests of GR and the theoretical frameworks for measuring them is the Cassini time delay data. This has a fit with GR of $\approx 2.3 \times 10^{-5}$ for signals passing close to the solar horizon, where our extra $1.4m^2/r^2$ term is $\approx 3 \times 10^{-6}$. So, it should be within the Cassini Constraint and also within the light deflection constraint but future accuracy improvements may change this.

8.1.13 Are there higher power terms than a squared term?

In QED there is a series of terms in $\alpha, \alpha^2, \alpha^3$ and so on where the electromagnetic coupling constant $\alpha \approx 1/137$ is relatively small so they rapidly decrease in importance, but the final result agrees with all current experiments no matter how precise. So shouldn’t we be looking for higher power terms than $m^2/r^2$? The electromagnetic coupling constant is between fundamental charges whereas we have been using a gravitational coupling constant (which we labelled as $\alpha_G \approx 1/16$) between Planck masses for simplicity. Gravitons will be emitted by fundamental particles that have a mass many orders of magnitude less than a Planck mass. Electric charges emit a single photon with a probability $\propto \alpha$, two photons $\propto \alpha^2$ and so on, where $\alpha^2 \approx (1/137)^2$. Our $m^2/r^2$ term is connected to the probability of a single graviton being exchanged between two Planck masses or a single Planck mass emitting and absorbing the same graviton. As the probability of this between two fundamental particles is so much smaller, the probability of two gravitons being exchanged can be completely ignored as it is infinitesimal. We thus don’t need to consider any power greater than $m^2/r^2$.

Finally in this section, does this extra $m^2/r^2$ term alter what we said in Eq.(7.1.17) where we first used $A_n = \frac{2m}{R}$. Before we introduced the self-emission term $1.4m^2/r^2$ we found that the extra time polarized $k_{\min}$ graviton density near the horizon for all black holes is:
where $\alpha'$ is the increased spin parameter due to the extra $1.4m^2/R^2$ term; we have also reused Eqs. (7.1.14) and (7.1.15). Everything we did there is unaffected by this extra term.

9 Four Vectors and Four Volume (4D) Action Densities

9.1.1 Graviton densities represented as invariant 4 velocities

Four velocity vectors have the property that $U^0_0 - U^2_1 = 1$ is invariant under local Lorentz transformations; where $U^0_0$ is the time component of the four velocity, and $U^2_1$ the spatial component. We will, as previously, use the notation

$U^0_0 = \gamma^2_M$ and $U^2_1 = \gamma^2_M \beta^2_M$ where $\gamma^2_M = \frac{1}{1 - \beta^2_M}$

We can think of the spatial component $U^2_1$ as the four velocity $\gamma_m \beta_m$ of a free falling mass that came from rest at infinity, in the same coordinate frame as the black hole. The normal velocity always points radially inwards in corotating frames. We can also write

$U^0_0 - U^2_1 = 1$ as $1 + U^2_1 = U^0_0$ or $1 + \gamma^2_M \beta^2_M = \gamma^2_M$.

This was what we did for the Schwarzschild metric when we had temporarily multiplied both sides by $\gamma^2_M$ and normalized the background $k_{\text{min}}$ graviton three volume, or 3D, probability density to one, with $\gamma^2_M \beta^2_M$ the extra $k_{\text{min}}$ graviton density due to a central mass, and $\gamma^2_M$ the total; this equation only applies before we have expanded the volume and changed time in the new metric. Because this is a 4 vector relationship it is true in all coordinates. Multiplying both sides temporarily by $\gamma^2_M$ does not change its validity.

We can also add a term $\gamma^2_M X^2$ to both sides to get $1 + \gamma^2_M \beta^2_M + \gamma^2_M X^2 = \gamma^2_M + \gamma^2_M X^2$ and still maintain covariance as

$(\gamma^2_M + \gamma^2_M X^2) - (\gamma^2_M \beta^2_M + \gamma^2_M X^2) = 1$, and we can put $X^2 = \frac{\alpha^2}{r^2} \cos^2 \theta$

so that:

$1 + \gamma^2_M \beta^2_M + \gamma^2_M \frac{\alpha^2}{r^2} \cos^2 \theta = \gamma^2_M + \gamma^2_M \frac{\alpha^2}{r^2} \cos^2 \theta = g_{00} \gamma^2_M$.

We are not adding another 4 vector here; we are simply adding squared terms, which are equal on each side, so that Lorenz invariance is not affected. This is still an invariant equation in any coordinates. In the above the local metric clock rate is always $1/\gamma^2_M$.

The three volume probability density of circularly polarized $k_{\text{min}}$ gravitons due to rotation, before volume expansion and time changes in the new metric, always obeys $\gamma^2_M X^2 = \gamma^2_M \frac{\alpha^2}{r^2}$ and the remaining $k_{\text{min}}$ graviton three volume probability density is $\gamma^2_M \beta^2_M$.

This section on invariant 4 vectors, is equivalent to saying that four volume, or 4D, densities of $k_{\text{min}}$ gravitons are invariant.
### 9.1.2 Four volumes in changing metrics

Using our dimensionless form of the metric tensor, the nonrotating space metric determinant has magnitude $|\text{Det } g| = |g| = \left| g_{\mu\nu} g_{\rho\sigma} \right| = 1$, but we want the square root of this: $\sqrt{|g|} = 1$.

However in rotating space this becomes $\sqrt{|g|} = g_{\phi\phi} = 1 + \cos^2(\alpha^2 / r^2)$ which reverts to $\sqrt{|g|} = g_{\phi\phi} = 1$ when the angular momentum length parameter $\alpha = 0$. At a large radius from any mass concentration let us start with a unit four volume such that $\Delta^4 x = \Delta t \Delta x \Delta y \Delta z = 1$ when $g_{\mu\nu} = \eta_{\mu\nu}$, where for simplicity we use $x, y, z$ for the space components. As we approach the central mass in the new metric, this four volume becomes:

$$\Delta^4 x' = \sqrt{\text{Det } g} \Delta t' \Delta x' \Delta y' \Delta z' = g_{\phi\phi} = 1 + \frac{\alpha^2}{r^2} \cos^2 \theta = \Delta t' \Delta x' \Delta y' \Delta z'$$

When angular momentum is present the behaviour of a unit 4 volume as we approach from a large distance depends on the local value of $g_{\phi\phi}$.

**Curved spacetime 4 volume**

**Flat spacetime 4 volume**

We also know that clocks change as $\Delta t' = \sqrt{g} \Delta t = \frac{\Delta t}{\gamma_M}$ in curved spacetime so that

$$\frac{\Delta t' \Delta x' \Delta y' \Delta z'}{\Delta t \Delta x \Delta y \Delta z} = g_{\phi\phi} = 1$$

The expanded spatial volume in the new metric $\Delta^4 x' = \Delta x' \Delta y' \Delta z' = \gamma_M g_{\phi\phi} \Delta x \Delta y \Delta z = \gamma_M g_{\phi\phi} \Delta^4 x$.

Spatial volume in any metric expands as $V' = \frac{\Delta^4 x'}{V} \Delta x' = \frac{d^4 x'}{d^3 x} = \gamma_M g_{\phi\phi} (1 + \frac{\alpha^2}{r^2} \cos^2 \theta) \gamma_M$

Where, as above, we have defined $\gamma_M = g_{\phi\phi}^{-1/2}$ as the local metric clock rate.

The 4 volume density invariance of $k_{\text{min}}$ graviton still applies, as the extra circularly polarized gravitons due to angular momentum occupy this expanded volume.

### 10 Warping Spacetime Slightly Differently to Einstein

#### 10.1 An Infinitesimal change to General Relativity at Cosmic Scale

Let us review how we have tried to connect gravity with our infinite superpositions in the second half of this paper. We started out with the hypothesis that space is flat when there are no mass concentrations, assuming that uniform densities don’t curve spacetime. We introduced mass concentrations and spacetime had to curve around them so as to keep our spherically symmetric 4 volume $k_{\text{min}}$ action densities, required and available, invariant. If we think of the mass in the universe as a dust of density $\rho$, essentially at rest in comoving coordinates we can define a tensor $T_{\mu\nu}(\text{Cosmos})$. In comoving coordinates $T_{\mu\nu}(\text{Cosmos})$ has only one significant non zero term $T_{00}(\text{Cosmos}) = \rho$, a density of only a few atoms per cubic metre. In any other coordinates this same $T_{\mu\nu}(\text{Cosmos})$ tensor is transformed by the usual tensor transformations that apply in GR. If these coordinates move at peculiar velocity $\beta_p$ then $T_{00}^\prime(\text{Cosmos}) = \gamma_p^2 \rho_p = \gamma_p^2 T_{00}(\text{Cosmos})$. The same transformation happens in any metric coordinates.
but with $\gamma'_M \rho_U$. We argue that Eq’s. (5.3.18) & (5.4.2) are consistent with the infinitesimally modified Einstein field equations

\[
\text{Modified Einstein } G_{\mu \nu} = R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \frac{8\pi G}{c^4} \left[ T_{\mu \nu} \text{(Local)} - T_{\mu \nu} \text{(Cosmos)} \right]
\] (10.1.1)

Einstein had always wanted his theory of gravity in curved spacetime to be similar to the Gauss/Poisson equation $\nabla^2 \phi = \rho$ the electric charge density. But this charge density is in reality the local difference in the normally very accurately (on average) balanced positive and negative charge densities. We can express this difference as follows:

\[
\text{Poisson/Gauss } \nabla^2 \phi = [\rho(\text{positive}) - \rho(\text{negative})] = \rho(\text{nett charge})
\] (10.1.2)

If we write the electric charge equation this way we can see that our modified Einstein tensor equation above is perhaps not too different, and hopefully Einstein might have approved. Equation (10.1.1) has some parallels to, but is not the same as, the $\Lambda$ term he introduced to stabilize the cosmos. The red terms are zero if $T_{\mu \nu} \text{(Local)} = T_{\mu \nu} \text{(Cosmos)}$ and $\rho(\text{positive}) = \rho(\text{negative})$. Just as the average value over the whole universe of $\rho(\text{positive})$ is the same as that of $\rho(\text{negative})$, the same is also true for the average value of $T_{\mu \nu} \text{(Local)}$ which has to be $T_{\mu \nu} \text{(Cosmos)}$. This forces the universe to be flat on average. It also means that there is no nett gravitational attraction of matter over any large cosmic sphere. (We will look at what this means for the FLRW metric in the next section) Thus this infinitesimal modification is most relevant in the extreme case as $T_{\mu \nu} \text{(Local)}$ approaches $T_{\mu \nu} \text{(Cosmos)}$.

Far from mass concentrations $T_{\mu \nu} \leq T_{\mu \nu} \text{(Cosmos)}$. Spacetime curvature, in these remote voids, is in general somewhere between slightly negative and zero; but the causally connected universe is always flat on regardless of the value of $\Omega$. (See section 10.1) Equation (10.1.1) is also consistent at all cosmic times with Eq. (5.2.10) $\rho_{k_{\text{min}}} = K_{k_{\text{min}}} dk_{\text{min}}$. If there is no inflation, in flat comoving coordinates, at the Big Bang or shortly after, (using $\bar{Y} = k_{\text{min}} R_{\text{off}} \approx 1$) $k_{\text{min}}$ starts at some value less than one and is always proportional to the inverse of the causally connected horizon radius. It is also close to the inverse of cosmic time $T$. It is always at its minimum far from mass concentrations, but increases with the slower clock rates in the local metric around mass concentrations as in Figure 10.1.1 below. The effect of our modified Einstein equation at large scale can be seen with a very oversimplified (for illustrative purposes) example. Consider a very large spherical hole of radius $R$ in an otherwise homogenous background of the cosmos with average density $\rho_U$, where the matter that was inside this spherical hole has now collapsed into a central concentrated mass $M = \frac{4}{3} \pi R^3 \rho_U$. Providing everything is spherically symmetric we can use our modified Einstein Eq.(10.1.1) to get $T_{\text{on}} \text{(Local)} - T_{\text{on}} \text{(Background)} = 0 - \rho_U = -\rho_U$ with the hole having an effective negative density of $\rho_U$ and a negative effective mass inside any particular radius $r$ of $-m = \frac{4}{3} \pi r^3 \rho_U$ such that $\frac{m}{M} = \frac{r^3}{R^3}$. 

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At any cosmic time $T$ in any coordinates, and in any metric, in the infinitesimal band $dk_{\alpha_{G}}$, $\rho_{Gk_{\alpha_{G}}} = K_{Gk_{\alpha_{G}}}dk_{\alpha_{G}}$ is always true. $K_{Gk_{\alpha_{G}}}$ is an invariant number in all spacetime, but the value of $k_{\min}$ depends on both local metric clockrates and cosmic time $T$.

$k_{\min}$ is some value $< 1$ when these equations start to apply sometime after the Big Bang.

Figure 10.1.1 A plot showing the invariance of $T$ and how $k_{\min}$ is roughly inverse to cosmic time when measured in flat comoving coordinates.

Spherical hole in background of radius $R$

Hole mass concentrated at centre $M = \frac{4}{3}\pi R^3\rho_U$.

Background density of the universe $\rho_U$, only a few atoms per cubic metre.

Figure 10.1.2 An illustration of an oversimplified example of a spherical hole in the background density of the universe that has collapsed into a central mass.

Modified gravitation attraction $\uparrow$

General Relativity attraction

Figure 10.1.3 A plot of the ratio of the modified gravitational attraction over the usual GR attraction. It has dropped to 50% at about 80% of the hole radius.

In weak gravitational fields Newton and Einstein are approximately the same, with Newton predicting that provided both the negative hole mass and the concentrated central mass are spherically symmetric, at the outside radius $R$ they must cancel each other, with no gravitational force/acceleration at this radius. (Provided the mass of the background dust
outside radius \( R \) is spherically symmetric it has no effect on this.) Thus \( 1 - (r^3 / R^3) \) is the ratio of the modified gravitational attraction to normal GR as plotted above in Figure 10.1.3. A galaxy of say \( 10^{12} \) solar masses is roughly the same mass as a background sphere of \( \approx 4 \times 10^6 \) light years. At this radius the attraction has dropped to zero, but it is roughly halved at \( \approx 3.2 \times 10^6 \) light years, or about 80% of the hole radius. Our example is quite unrealistic as the transition will not be abrupt, but it does illustrate something that has been observed in recent surveys of the universe where the rate at which matter is falling into denser regions of the universe appears to be less than predicted by GR [40].

10.1.1 Friedmann-Lemaitre-Robertson-Walker Metrics and Friedmann’s Equations
The FLRW metric and Friedmann equations have been the bedrock of cosmology for nearly a century, a cosmology that in its current \( \Lambda \)CDM form has been recently showing some cracks. (See Reiss [41] and New Physics required due to Hubble tensions). It is an exact solution to the 00 component of GR so how can it possibly be questioned? If we relook at our infinitesimally modified Einstein Eq. (10.1.1) and re-express it as follows

\[
W = T'_{\mu \nu} = T_{\mu \nu} (\text{Local}) - T_{\mu \nu} (\text{Cosmos}) \tag{10.1.3}
\]

Modifying Einstein Infinitesimally \( G_{\mu \nu} \equiv R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \frac{8\pi G}{c^4} T'_{\mu \nu} \)

Over large homogenous and isotropic regions of space the average values of all 16 components of \( T_{\mu \nu} (\text{Local}) \) are the same as \( T_{\mu \nu} (\text{Cosmos}) \) and this is true for all large volumes inside the horizon.

All 16 components of both \( G_{\mu \nu} \) & \( T'_{\mu \nu} \) average zero in large regions \( \tag{10.1.4} \)

If the components of the Friedmann equations average zero it cannot control cosmic expansion and we argue that QM does. We will consequently continue to use the following metric.

FLRW flat space metric with no KE or gravitation effects: \( ds^2 = -c^2 dt^2 + a(t)^2 dr^2 \) \( \tag{10.1.5} \)

11 Galaxies MOND and Cosmic Acceleration

11.1 Graviton Coupling Constants and MOND
All three of the SM coupling constants \( \alpha_1, \alpha_2 & \alpha_3 \) change their values in the energy region greater than that of massive bosons (\( \approx 100 \text{GeV} \)). This paper, in complete contrast, hypothesises that gravity is directly related to the densities of \( k_{\text{min}} \) (or maximum wavelength) gravitons, as they vastly outnumber all other particles in the cosmos. The energies of these \( k_{\text{min}} \) gravitons is at the very opposite extreme of that relevant in the SM. We will argue that the graviton coupling constant increases at these extreme low energies, in a way that relates with MOND. This is a clear violation of the Strong Equivalence Principle, as the laws of physics applying to gravity would not be the same in all free falling frames. This was one of Einstein’s original GR postulates. These proposed changes to the laws of physics in free falling frames depend on the local gradient of the potential energy at that point in space time.
Section 6.2.2 and Figure 5.3.2 proposed that $k_{\text{min}}$ gravitons cut off exponentially at a maximum wavelength. We can also express Milrom’s acceleration as an inverse length.

$$a_0 \approx 1.2 \times 10^{-10} \text{ m/s}^2 = \frac{1}{R_{\text{Milgrom}}} \approx \frac{1}{4.55 \times 10^6 \text{lp}}$$

(11.1.1)

This length which is $\approx 80$ billion light years is about twice our predicted horizon radius and about half the maximum cosmic wavelength $\lambda_{\text{max}} = k_{\text{min}}^{-1}$. We will simply postulate that:

$$a_0^{-1} = R_{\text{Milgrom}} \approx 4.55 \times 10^6 \text{lp}$$

(11.1.2)

This is equivalent to saying that there is a particular potential energy gradient $\nabla \Phi_0 = a_0$ which we can think of as an inverse length near $\lambda_{\text{max}} / 2$, and we propose it is always true.

Define $\nabla \Phi_0 = a_0 = \frac{1}{R_{\text{Milgrom}}} \approx \frac{2}{\lambda_{\text{max}}} = 2k_{\text{min}}$ at all cosmic time.

(11.1.3)

We also find in Eq. (11.1.18) that $\Psi = k_{\text{min}} R_{\Omega H} \approx 0.223$ is a cosmic constant and thus:

$$\nabla \Phi_0 = a_0 \propto \frac{1}{R_{\Omega H}} \propto k_{\text{min}}$$

is always true.

(11.1.4)

If this is so, it must surely have facilitated early era galaxy and super massive black hole formation. MOND tells us that galaxies behave as $v^2 = m a_0$. If the central mass increases approximately linearly with time $(m \propto t)$, where time is approximately proportional to the horizon radius, and $a_0$ is inverse to the horizon radius $(a_0 \propto 1/t)$, the rotation velocities $v = (ma_0)^{1/4}$ can be roughly constant, even with initially extremely small central masses.

So let us look more carefully at how this might all be possible.

11.1.1 A possible covariant metric relating with MOND with an exponential cutoff

If our arguments that galaxy behaviour could be due to an increase in the graviton coupling constant at extremely low accelerations, it would seem reasonable to assume that there will be some cutoff radius where it no longer applies. In section 10.1 and Eq (10.1.1) we proposed an infinitesimal change to Einstein’s equations. We also used an unrealistic spherical bubble example in Figure 10.1.2 where at the boundary the central concentrated mass was simply the average cosmic background density by the volume of that bubble. There is no nett gravitational attraction at the bubble radius where the $\frac{\Delta \rho_{\text{gl,min}}}{\rho_{\text{gl,min}}} = 2 \frac{m}{r}$ of Eq. (5.2.8) and $\nabla \Phi = 0$ plus $g_{\text{eff}} = 1$, so what sort of radius is this sphere/bubble? Before we look at what it might be, we need to make some simplifying assumptions which are similar to the usual cosmology assumptions at vast scales. Galaxies can exist in clusters and superclusters all with probably very different cutoffs, but we can get a very crude average of what it might be. Current thinking is that there are $\approx 10^{12}$ galaxies, and for simplicity let us assume they are all of the same average mass. This is far from true, but it will help us build an average picture to start with. We will also assume a homogenous and isotropic distribution of them, which is also not very accurate. We can enclose them in spherical bubbles that touch each other, all of equal radius related to the scale factor. (We are assuming no gravitational binding between
these galaxy bubbles, no clusters or superclusters etc. so it can only be approximate.) To keep things simple assume a horizon radius of $\approx 4 \times 10^{10} \text{ly}$ so that $10^{12}$ equal bubbles will have a radius of $\approx 4 \times 10^6 \text{ly} \approx 2.23 \times 10^7 \text{lp}$. At this point we could, as an example, use the Milky Way which has a baryonic mass of $m \approx 10^{10} m_p$. If we calculate the bubble size (as in Figure 10.1.2) of background cosmic density that contains the Milky Way mass, it is very close to the above average sphere we calculated at approximately four million light years. So our cutoff radius must be something less than this, and we will try a simple exponential cutoff in a possible covariant metric. Because the effect of an $\alpha^2 / r^2$ term (as in section 8) is completely irrelevant at galactic scales, we do not need to add unnecessary complexity by including it in this proposed galactic metric.

A possible covariant metric is

$$g_{rr}^{-1} = g_{00} = 1 - \frac{2m}{r} + 2\sqrt{ma_0} \log \left[1 - \exp \left(\frac{-r}{r'}\right)\right]$$

(11.1.5)

The cutoff radius is $r'$ and we will use the same procedures as in sections 5.2.1 & 5.2.2, plus the general reasoning behind the derivation of Eq. (5.2.8). We will also use $\beta$ as the radial inward velocity of a small mass, with no angular momentum in the same rest frame as the galaxy centre, and with zero initial velocity at a point where $g_{00} = 1$. It does not relate with rotational velocities as in Keplarian or Newtonian dynamics. Doing it this way relates directly with the Lorentz invariance of the combination of $\gamma$ & $\beta$ as follows:

$$1 + \gamma^2 \beta^2 = \gamma^2 = \frac{1}{g_{00}} = \frac{1}{1 - \frac{2m}{r} + 2\sqrt{ma_0} \log \left[1 - \exp \left(\frac{-r}{r'}\right)\right]}$$

$$\gamma^2 \beta^2 = \gamma^2 - 1 = \frac{1}{g_{00}} - 1 = \frac{1 - g_{00}}{g_{00}}$$

$$\beta^2 = \frac{\gamma^2 - 1}{\gamma^2} = 1 - g_{00} - 1 - \left[1 - \frac{2m}{r} + 2\sqrt{ma_0} \log \left[1 - \exp \left(\frac{-r}{r'}\right)\right]\right] = -2 \times \text{Potential Energy.}$$

$$\beta^2 = \frac{2m}{r} \left(1 - \frac{a_0}{m} \log \left[1 - \exp \left(\frac{-r}{r'}\right)\right]\right)$$

and using Milgrom’s critical radius $r_c = \sqrt{\frac{m}{a_0}}$

$$\beta^2 = \frac{2m}{r} \left(1 - \frac{r}{r_c} \log \left[1 - \exp \left(\frac{-r}{r'}\right)\right]\right) = -2\Phi$$

(11.1.6)

In Eq.(5.2.8) we showed $\frac{\Delta \rho_{G\phi_{\text{min}}}}{\rho_{G\phi_{\text{min}}}} \approx 2 \frac{m}{r}$ and later in Equ’s. (5.2.11) & (5.2.12) we showed $\beta^2 = \frac{2m}{r}$ when the graviton coupling constant is $\alpha_G$ as in non MOND regions. Also using Eq.(5.2.13) (and the reasoning immediately prior to it) in the above, we can show it is covariant. Thus Eq. (11.1.6) implies an increased $\alpha'_G$ where:

$$\frac{\alpha'_G}{\alpha_G} = 1 - \frac{r}{r_c} \log \left[1 - \exp \left(\frac{-r}{r'}\right)\right]$$

(11.1.7)
11.1.2 Different galaxy masses in relation to MOND

Using the same arguments as those leading up to Figure 10.1.2 for galaxy masses different to the Milky Way, we can say that, averaged over many galaxies, the bubble radius \( r_b \propto m^{1/3} \)
where \( m \) is the galaxy mass; but the critical radius \( r_c \propto m^{1/2} \) and thus:

\[
\frac{\text{Bubble radius}}{\text{Critical radius}} = \frac{r_b}{r_c} = \frac{m^{1/3}}{m^{1/2}} = \frac{1}{m^{1/6}} \tag{11.1.8}
\]

This ratio, which is proportional to the MOND region, gets larger as the 1/6 power of the galaxy mass. Lower mass galaxies should be more MOND-like than massive galaxies.

\[
\Phi = \frac{g_\infty - 1}{2} = -\frac{m}{r} + \sqrt{ma_0} \log \left[ 1 - \text{Exp} \left( \frac{-r}{r'} \right) \right] \quad \text{and} \quad \nabla \Phi = \frac{m}{r^2} + \frac{\sqrt{ma_0}}{r} \left( \frac{r'}{r'} \times \text{Exp} \left( \frac{-r}{r'} \right) \right)
\]

\[
\frac{\text{Predicted behaviour}}{\text{MOND behaviour}} = \frac{\left( \frac{r'}{r'} \times \text{Exp} \left( \frac{-r}{r'} \right) \right)}{\left( 1 - \text{Exp} \left( \frac{-r}{r'} \right) \right)} \tag{11.1.9}
\]

We will try \( r' = 0.2r_{\text{bubble}} \) as a cutoff radius.

\[
\nabla \Phi \approx \frac{m}{r^2} + \frac{\sqrt{ma_0}}{r} \approx \frac{m}{r^2} (1 + \sqrt{\frac{a_0}{m}}) \approx \frac{m}{r^2} (1 + \frac{a_0}{r_c}) \approx \frac{m}{r^2} (1 + \frac{\nabla \Phi_0}{\nabla \Phi})
\]

Where \( \frac{\nabla \Phi_0}{\nabla \Phi} \approx \frac{a_0}{a} \approx \frac{\text{Milgroms critical acceleration}}{\text{Local acceleration}} \) in MOND region only.

We have used \( r' = 0.2r_{\text{bubble}} \) in Figure 11.1.1 below, but it needs to be calibrated over a range of galaxies. These plots for a large range of masses only represent predicted average behaviours. They are all based on a central mass for simplicity and would differ for disc shaped mass.

**Figure 11.1.1** Plots Eq. (11.1.9) for a large range of galaxy masses illustrating lower mass galaxies have more MOND-like behaviour (on average). Because our units are \( r/r_{\text{Critical}} \), massive galaxies with larger critical radii counter-intuitively appear to have smaller bubble radii. The Milky Way cutoff radius \( r' = 0.2r_{\text{bubble}} \approx 8 \times 10^3 \text{ly} \) is close to estimates of its halo radius.
11.2 Accelerating Cosmic Expansion

We can rewrite Eq. 11.1.1 as follows:

\[
\frac{\Delta \alpha_g}{\alpha_g} = \frac{\Delta \alpha_g}{\alpha_g} = -\frac{r}{r_c} \log \left[ 1 - \exp \left( \frac{-r}{r'} \right) \right]
\]

(11.1.10)

Where \( \Delta \alpha_g \) is the local increase in the graviton coupling constant. It is also proportional to the extra local expansion in the metric, and also the extra \( k_{\text{min}} \) gravitons it causes. All of this behaves as if there has been an extra mass \( \Delta m \propto \Delta \alpha_g \) that is added at that point, and extra mass (or more \( k_{\text{min}} \) gravitons) will cause the horizon area to expand. When scale factors are calculated, it is assumed that the mass inside the expanding scale factor box remains constant while the galaxy spacing expands. So our extra \( \Delta m \propto \Delta \alpha_g \) forces an extra expansion on the scale factor. We have for simplicity assumed spherical symmetry with a central mass and even though most galaxies are disc shaped we can get an indication of what might happen. We can imagine spherical shells with:

\[
A \text{ phantom extra mass } dm \propto \frac{-r}{r_c} \log \left[ 1 - \exp \left( \frac{-r}{r'} \right) \right] r^2 dr
\]

(11.1.11)

**Figure 11.1.2** plots Eq. (11.1.11) for a typical galaxy with \( r' = 20r_c \) or 0.2 bubble radius.

If we numerically integrate Eq. (11.1.11) we can show that the total extra mass is

\[
\text{Total phantom mass } m' \propto \frac{(r')^4}{r_c} \propto \left( \text{Scale factor} \right)^4
\]

(11.1.12)

Critical radius \( r_c = \sqrt{\frac{m}{a_0}} \) and always \( \propto \sqrt{R_{\text{OH}}} \) for a constant galaxy mass if Eq. (11.1.4) is true.

\[
\text{Total phantom mass } m' = k \times R_{\text{OH}}^{1/2} \times \left( \text{Scale factor} \right)^4 \text{ where } k \text{ is some constant.}
\]

(11.1.13)

The mass of normal matter inside a horizon scale box of side length \( a(t) \) and volume \( a^3(t) \) is constant, so any effective increase (or phantom/virtual) mass inside this box must increase the expansion of the scale factor as follows: Using Eq. (5.2.9) the universe density is

\[
\rho_u \approx 0.8823 k_{\text{min}}^2 \propto \frac{1}{R_{\text{OH}}} \quad \text{but } \rho_u \text{ is also } \propto \frac{1 + k \times R_{\text{OH}}^{1/2} \times a(t)^4}{a^3(t)} \propto k_{\text{min}}^2
\]

\[
a^3(t) \propto \frac{1 + k \times R_{\text{OH}}^{1/2} \times a(t)^4}{k_{\text{min}}^2} \propto k_{\text{min}}^2 \text{ and using Eq. (5.2.9) again and putting } R = R_{\text{OH}}
\]
\[ a^3[t] \propto R^2 (1 + k \times R^{-1/2} \times a(t)^4) \]

Scale factor \( a[t] \propto R^{2/3} (1 + k \times R^{-1/2} \times a(t)^4)^{1/3} \) \hspace{1cm} (11.1.14)

The Hubble parameter \( H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{d}{dt} \left( \frac{R^{2/3} (1 + k \times R^{-1/2} \times a(t)^4)^{1/3}}{R^{2/3} (1 + k \times R^{-1/2} \times a(t)^4)^{1/3}} \right) \)

but using Eq.(5.3.12) in flat space \( H(t) = \frac{\text{Horizon Hubble flow velocity}}{\text{Horizon radius}} = \frac{V-1}{R} = \frac{dR}{dt} \frac{-1}{R} \) and we can equate these to get:

\[ H(t) = \frac{d}{dt} \left( \frac{R^{2/3} (1 + k \times R^{-1/2} \times a(t)^4)^{1/3}}{R^{2/3} (1 + k \times R^{-1/2} \times a(t)^4)^{1/3}} \right) \]

But can also say that

\[ H(t) = \frac{d}{dt} \left( \frac{R^{2/3} (1 + k \times R^{-1/2} \times a(t)^4)^{1/3}}{R^{2/3} (1 + k \times R^{-1/2} \times a(t)^4)^{1/3}} \right) \]

We have two simultaneous differential equations where the flat space horizon velocity \( dR / dt = V = 3 \) in the matter dominated era when \( t \to 0 \). Table 11.2.1 lists possible solutions at three parameter \( k \) values assuming \( H_o \approx 72.5(km/s)/Mpc \). These possible values of \( k \) are proportional to the average increase in the graviton coupling constant \( \alpha_g \) over the cosmos (i.e. all MOND regions). The late stage cosmic acceleration is much more aggressive than in the \( \Lambda CDM \) with more negative deceleration parameters, and this reduces the horizon radius compared to the \( \Lambda CDM \) value of 46.5 billion light years. Possible cosmic ages are less than the \( \Lambda CDM \) value of \( \approx 13.8 \) billion years. If the Hubble parameter is closer to \( H_o \approx 75(km/s)/Mpc \) as found in a recent survey by Caramera and Mara \[20] this further reduces the horizon radii in Table 11.2.1 by about 3%. This survey also found a value for the local deceleration of \( q_0 \approx -1.08 \pm 0.29 \), about twice as negative as the \( \Lambda CDM \) model, but in the central region of our possible solutions. Figure 11.1.3 plots the velocity, acceleration and radius of the horizon as a function of cosmic time for \( k = 0.14 \). Figure 11.1.4 and Figure 11.1.5 plot the deceleration as a function of cosmic time and as a function of redshift both for \( k = 0.14 \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( k = 0.13 )</th>
<th>( k = 0.14 )</th>
<th>( k = 0.15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hubble horizon velocity</td>
<td>( V_H \approx 2.85 )</td>
<td>( V_H \approx 2.97 )</td>
<td>( V_H \approx 3.1 )</td>
</tr>
<tr>
<td>Horizon radius</td>
<td>( R_{OH} \approx 38 \times 10^7 \text{ly} )</td>
<td>( R_{OH} \approx 40 \times 10^7 \text{ly} )</td>
<td>( R_{OH} \approx 42 \times 10^7 \text{ly} )</td>
</tr>
<tr>
<td>Cosmic age</td>
<td>( \approx 12 \times 10^9 \text{ years} )</td>
<td>( \approx 12.4 \times 10^9 \text{ years} )</td>
<td>( \approx 12.8 \times 10^9 \text{ years} )</td>
</tr>
<tr>
<td>Deceleration parameter</td>
<td>( q_0 \approx -0.95 )</td>
<td>( q_0 \approx -1.165 )</td>
<td>( q_0 \approx -1.38 )</td>
</tr>
<tr>
<td>Transion redshift</td>
<td>( z_s \approx 0.32 )</td>
<td>( z_s \approx 0.37 )</td>
<td>( z_s \approx 0.42 )</td>
</tr>
</tbody>
</table>

Table 11.2.1 Possible solutions to Equ’s. (11.1.15) & (11.1.16) assuming a Hubble parameter of \( H_o \approx 72.5(km/s)/Mpc \).
For simplicity, the above analysis assumes that the mass in galaxies is reasonably constant, and this could be so in the last few billion years which is possibly the most important period when accelerations become relevant. We have almost certainly oversimplified all this to facilitate easier computation, and the actual formation of ICM’s, galaxies, clusters and superclusters is bound to be far more complex. We have not attempted to address the effect of all the above in the radiation era where MOND makes a correct prediction for the second peak of the CMB power spectrum but dark matter had to be adjusted to fit. However, MOND does not get all peaks correct. The effects of an infinitesimally modified GR as in Eq. (10.1.1) and the bubbles of Figure 10.1.2 that we have used in the matter dominated era (as in the above modified MOND analysis) may give a better CMB outcomes.
11.2.1 What is the cosmic density for this solution?

Our solution puts $R_{OH} \approx 40 \times 10^9 $ly $\approx 2.34 \times 10^{61}$ Planck lengths if $k = 0.14$. Using Eq. (5.3.20)

$$\rho_U \approx \frac{1.114 \alpha_G}{R_{OH}^2} \approx \frac{1.114 \alpha_G}{(2.34 \times 10^{61})^2} \approx 2.03 \times 10^{-123} \alpha_G$$

in Planck units. (11.1.17)

This is equivalent to $\rho_U \approx 10.4 \times 10^{-27} \alpha_G \text{kg/m}^3$.

As the proton mass is $\approx 1.67 \times 10^{-27} \text{kg}$, the density is equivalent to $\approx 6 \alpha_G \text{protons/m}^3$.

Current estimates put normal matter as 0.2 to 0.25 atoms per cubic metre which suggests that $\alpha_G^{-1} \approx 24$ if we ignore the potential extra density of about one billion neutrinos per proton. This is unlikely to be very significant as neutrino mass is thought to be in the very low electron volt range.

Inserting $\alpha_G^{-1} \approx 24$ into Eq. (5.3.18)

$$\gamma^2 = k_{min}^2 R_{OH}^2 \approx 1.26 \alpha_G \approx 0.052$$

$$\gamma = k_{min} R_{OH} \approx 0.223$$

$$\gamma_{Max} = k_{min}^{-1} \approx 4.48 R_{OH} \approx 180 \times 10^9 \text{ly.}$$

11.3 Further Issues not Already Covered.

11.3.1 Preferred frames

It might seem that we have been arguing in earlier sections for a preferred frame. But there is really no difference in what we are proposing compared to current physics. In comoving frames the cosmic microwave background is isotropic. At peculiar velocity $\beta_p$, it is no longer isotropic, and the average background temperature increases by $\gamma_p^2$, exactly the same increase as $k_{min}$ to $k'_{min} = \gamma_p k_{min}$; that is if we could measure it, which is most unlikely. We have frequently talked in this paper about local observers measuring $k_{min}$, but only as a thought experiment, and the average (over all directions) background temperature can be used to measure either $\gamma_{peculiar}$ or $\gamma_{metric}$ at any particular cosmic time, provided we already know its value in flat comoving coordinates which is from Eq. (5.3.19) $k_{min} \approx 0.223 R_{OH}^{-1}$. There are no other changes in physics in this comoving frame; it is exactly as Einstein originally postulated, an important experimentally verified feature of GR. However, it does make everything we have done here much simpler if we work in comoving coordinates. All the mass moving at peculiar velocities in random directions has no effect on the average universe density of either $k_{min}$ gravitons or the $k_{min}$ action density that they require. We calculated the average density of $k_{min}$ action from the horizon in these comoving coordinates. But if we think in terms of four volume, or 4D $k_{min}$ action density invariance, then whether we are in a non-comoving frame, or in a non-flat metric, it makes no difference; and is why we can use 4 vector notation for the extra $k_{min}$ gravitons around mass concentrations.
11.3.2 Action principles and the Einstein field equations

The field equations of GR can be derived from an invariant action principle $\delta I = 0$ where

$$ I = \frac{1}{16\pi G} \int R \sqrt{-g} \cdot d^4x + I_m(\psi_m, g_{\mu\nu}) $$

and $R$ is the Ricci scalar, with $I_m$ the matter action which depends on matter fields $\psi_m$ coupled to the metric $g$. Varying the action with respect to $g_{\mu\nu}$, we obtain Einstein’s field equations $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$. This paper suggests, however, that a more general cosmic action invariance (or conservation) applies to the volume inside the horizon limiting the domain in which the above field equations of GR are accurate.

11.3.3 Gravitational waves and 4 volume invariance

We showed in section 5.4.1 that the 4 volume $k_{\text{min}}$ graviton density at any cosmic time $\rho_{4D_{\text{grav}}}$ is invariant in all coordinates and in any metric. But the metric can oscillate and not change this invariance, with such disturbances travelling at the speed of light. We can imagine extra gravitons around a mass concentration and the background gravitons (if there is accelerating mass as in binary pairs) generating real transversely polarized gravitons. This has some parallels to what we found in the Kerr metric, but now with real gravitons. But the intensity, or probability density, of these real gravitons will drop as the inverse radius squared, at least when far away. We can also show from Eqs.(5.1.9) that most of these gravitons are close to the locally measured value of the $k_{\text{min}}$ wavenumber, about 96% are between $k_{\text{min}}$ & $5k_{\text{min}}$. Thus, most of this radiated energy is near $k_{\text{min}}$. The frequency of the radiated wave is twice the orbital frequency of the binary pair source, typically hundreds of orbits per second. Typical wavelengths are in the thousands of kilometres or roughly $10^{41}$ Planck lengths. As most of the energy in the wave is in quanta near $k_{\text{min}}$ there is no connection with the frequency of the radiated wave as in spin 1 photons and electromagnetism. The wavelength of $k_{\text{min}}$ gravitons is $1/k_{\text{min}} \approx 4.5R_{\text{pl}} \approx 10^{62}$ Planck lengths, with the ratio between these two wavelengths of the order of $\approx 10^{20}$. This ratio is inverse to the binary pair orbital frequency. It could only approach one if the binary orbital period is approximately twice the age of the universe.

11.3.4 Black holes, the firewall paradox and possible spacetime boundaries

Several recent papers [42-46] have discussed the black hole firewall paradox. In section 5.2.2 we use the fact that outside observers see infalling mass remaining on the horizon. In fact, if we look carefully at the analyses in sections 5.2.1 & 5.2.2 we see they may suggest that there is a cutoff for GR at the black hole horizon; one of the possible firewall paradox implications. The equations we derived do not appear to work inside the horizon. Our argument that a constant graviton scalar $K_{\text{grav}}$ is consistent with GR is questionable inside the horizon. ($k_{\text{min}}$ quanta that go in to build superpositions would not return in time $\Delta T \approx k_{\text{min}}^{-1} \approx T$). Is it possible that the horizon of a black hole could be some sort of spacetime boundary?
11.3.5 **Higgs boson**

It is not clear whether the Higgs boson is a spin zero superposition so it is not in Table 2.2.1. However, if it is, it would be some superposition of infinite superpositions with a total angular momentum vector summing to zero, just as two spin $\frac{1}{2}$ fermion superpositions can for example.

11.3.6 **Constancy of fundamental charge**

It has always been fundamental that the electromagnetic charge of protons and electrons is precisely equal and opposite to get a neutral universe. In section 4.2 we showed that the probability of superpositions was $sN \cdot dk(1+\varepsilon)/k$ where the infinitesimal $\varepsilon$ is proportional to rest mass squared and thus different for various particles. We used this probability to determine interaction coupling strengths in section 3.3. This suggests that the probability of virtual photon emission is also proportional to the probability $sN \cdot dk(1+\varepsilon)/k$ of each superposition, and would not be precisely equal for electrons and protons due to small variations in $\varepsilon$ of the order of $\approx 10^{-45}$ between electrons and quarks. If, however, we look closely at Eq.(4.2.3) and the following equations, by adding the amplitude for gravity at right angles we effectively added the probabilities of spin 2 gravity generated superpositions to those of spin 1 colour and electromagnetic superpositions. If, somehow, only those superpositions generated by spin 1 electromagnetic and colour interact with spin 1 photons, this would cancel any minute difference in charge. If this is not so, then there are infinitesimal differences in charge of the order of $\approx 10^{-45}$ which would surely have shown up in some form by now, unless there are minute differences in the total number of electrons and protons.

11.3.7 **Superpositions, Feynman’s strings and possible resonances**

As we have already mentioned in section 5.4, over a century ago there were models of the electron with the Abraham-Lorenz probably the best known [28], [29] but all of them suffered from electromagnetic mass in the field being 4/3 times the relativistic mass. Poincare showed that bursting forces due to charge balanced by stresses (or forces) in the same rest frame as the particle could cancel the extra 1/3 figure, and restore covariance [30]. In chapter 29 Volume II of his famous lectures on physics, Feynman, probably jokingly, suggested that if the electron is held together by strings, their resonances could explain the muon mass [47]. He just may have been right. The dominant $n = 6$ mode of the electron family of superpositions is held in orbit by a squared vector potential $Q^2A^2 = 16\hbar^2k^4r^2$. The bursting force is a scalar potential of order $\alpha$, a small perturbation in comparison. Perhaps we could imagine some sort of cubic equation with three solutions for rest mass, and something similar for quarks, but with larger perturbing forces.
Discussion

This paper began by forming the fundamental particles from infinite superpositions. Apart from some infinitesimal differences, the proposed model agrees with the most basic version of the SM. These infinitesimal differences however, have some profound consequences at cosmic scale. Like Einstein, our proposals look at gravity not as a force, but purely as a consequence of warped spacetime, or invariant four volume (4D) $k_{\text{min}}$ graviton densities with particles following geodesics. Perhaps the most significant consequence is that there is a maximum wavelength for what we have called $k_{\text{min}}$ gravitons. The action quanta they require come from the distant horizon, implying the density of matter has to be proportional to the inverse horizon area ($\rho_0 \propto R_{\text{OH}}^2$) at all times. When mass is distributed evenly as a dust these gravitons have a uniform three volume (spatial) density throughout a sphere of radius $R_{\text{OH}}$ and space is flat everywhere. If any of this mass is moved to a central location it increases the three volume density of $k_{\text{min}}$ gravitons around this mass and space has to expand locally in agreement with Einstein’s equations, apart from an infinitesimal difference only effective at cosmic radii. There is also a quantum correction near the Schwarzschild radius of order $m^2/r^2$ with parallels to QED corrections close to point charges. Except when close to black holes these disagreements are irrelevant near mass concentrations up to solar system scale. These QED type black hole corrections may, however, make current mass estimates in binary mergers appear slightly larger; depending on their angular momentum. Of course, QM can only control the density and expansion rate of the cosmos in the manner described with some changes to the Friedmann equations. We have argued however, that our infinitesimally modified Einstein equations, over very large homogenous and isotropic regions of space, make the 16 components of these modified tensors average zero, thus allowing QM to control the expansion regardless of $\Omega$.

The constant rotation velocities in galaxies is thought by most physicists to be due to dark matter, but despite huge effort no clear experimental evidence for it has been found. Mordehai Milgrom proposed MOND as an alternative, and forty years of surveys confirm many of his predictions. As David Merritt [50] summarizes in his book “MOND predicts, and dark matter accommodates”. MOND becomes effective below a critical acceleration, and expressed as an inverse length it is half the maximum wavelength of our $k_{\text{min}}$ gravitons. The SM coupling constants $\alpha_1, \alpha_2, \alpha_3$ change their values at the energy levels of massive bosons, but this paper proposes an increasing graviton coupling constant when the inverse length of the potential energy gradient is greater than half the wavelength of $k_{\text{min}}$ gravitons, and all happening at the opposite end of the energy spectrum. This paper proposes a covariant metric agreeing with MOND and cutting off exponentially in a way that agrees with our infinitesimally modified GR equations. Any increase in the graviton coupling constant has the same effect as adding mass to galaxies, which in turn requires more action quanta from the horizon, increasing its radius accordingly. As the scale factor increases so do the MOND regions, and in this respect it all behaves similarly to what has been labelled dark energy. Very simplified mathematical models of this predict a deceleration of $q_0 \approx -1.16$ which is
close to some recent surveys (Caramera and Mara [20]) and a smaller radius than the $ΛCDM$ of about 40 billion light years assuming a Hubble parameter of $H_0 \approx 72.5 (km/s/Mpc$.

As noted in our introduction, Merritt points out that dark energy responsible for accelerating expansion in the $ΛCDM$ is an auxiliary hypothesis, added to the model around 1998. However, since that time astronomers have adjusted its properties, as needed, to match whatever new observational data relating to the universe’s large scale structure becomes available, under the assumptions that Einstein’s theory of gravity is correct. (D. Merritt, personal communication with the author’s brother, October 20, 2018). And in this regard, the Reiss team claim their latest figures for the Hubble parameter provide “stronger evidence for physics beyond the $ΛCDM$ ” (Reiss et al., 2019 p.1)[41]

On a final note, the primary interactions involved in forming the fundamental particles are simple, allowing simple mathematics. Indeed, we have purposely avoided exotic mathematics throughout this paper in the belief that, while powerful and elegant, it can hide the wood for the trees so to speak. The theoretical physicist Hossenfelder expresses a similar sentiment in the title of her recent book [3] Lost in Math: How Beauty Leads Physics Astray. If “Mother Nature” can be described simply, why not do so?

The ideas presented, although radical, at a fundamental level are also very simple. Superpositions are a signature component of quantum mechanics and we have merely extended them to building the fundamental particles. The proposals form a consistent package conforming with both quantum mechanics and SR. They also suggest new physics that may facilitate progress in a field considered by many to be in difficulty, or even in crisis [1-5].

13 Conclusions

If the fundamental particles are built from infinite superpositions as we propose, they must have at least an infinitesimal mass that is always inversely proportional to the horizon radius. And this may have some very significant consequences:

- Because cosmic wavelength gravitons vastly outnumber all other particles, the invariance of the action quanta they borrow from the horizon directly relates with infinitesimally modified Einstein field equations.

- This infinitesimal modification limits the range of GR to much smaller than horizon scale and also makes the Einstein equations average zero over isotropic and homogeneous regions.

- The Freidman equation components also average zero, and space is flat. QM controls the expansion of space regardless of $Ω$, with or without inflation.

- Just as QED modifies fields near electric charges, QM also introduces a similar small correction very near black holes.
• Milgrom’s critical acceleration has an inverse length of \( \approx \frac{\lambda_{\text{max}}}{4\pi} \) of \( k_{\text{min}} \) gravitons.

• Galaxies behave as if the graviton coupling constant \( \alpha_g \) increases inversely to \( \nabla \Phi \) in regions where \( \nabla \Phi \ll \frac{4\pi}{\lambda_{\text{max}}} \), with an exponential cutoff radius \( \propto (\text{Galaxy mass})^{1/3} \).

• On average, lower mass galaxies are more MOND-like than massive ones.

• This increase in \( \alpha_g \) behaves as if mass is added to galaxies.

• Even though galaxies are not gravitationaly bound, this phantom added mass changes the metric in vast bubbles surrounding them that expands with the scale factor.

• The effect on cosmic expansion is very similar to what dark energy is purported to do.

• If all the above is true there is a chameleon like nature to this phantom added mass. It can behave as if it is dark matter in galaxies. It can also behave as if it is dark energy and accelerate cosmic expansion.

Acknowledgements

I would like to thank my brother Maynard for his valuable assistance in the preparation of this paper, particularly the cosmology section. He convinced me of the importance of David Merritt’s paper discussing Karl Popper’s rules for avoiding “conventionalist” methods in the practice of science, and the implications of Popper’s arguments for the current \( \Lambda CDM \) cosmological model. I had always been somewhat skeptical about the importance of philosophy in science, while both Maynard and my other brother David felt differently on such matters. His way with words is also much better than mine.

14 References


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