Models that link and suggest data about elementary particles, dark matter, and the cosmos

Thomas J. Buckholtz

Ronin Institute for Independent Scholarship, Montclair, New Jersey 07043, USA

Abstract

We suggest progress regarding the following six physics opportunities. List all elementary particles. Describe dark matter. Explain ratios of dark matter to ordinary matter. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models. We use models based on Diophantine equations.

Keywords: Beyond the Standard Model, Dark matter, Galaxy formation, Neutrino masses, Evolution of the universe

Contents

Email address: Thomas.Buckholtz@RoninInstitute.org (Thomas J. Buckholtz)

1. Introduction

This unit previews physics results we propose, previews methods we use, and relates our work to other work in elementary particles, astrophysics, and cosmology.

1.1. Overview

This essay pursues the following two challenges. Describe new elementary particles and dark matter. Use descriptions of elementary particles and dark matter to explain astrophysics data and cosmology data.

Our explanations regarding large-scale data might help validate our descriptions of possible new elementary particles and our description of dark matter.

1.2. Research and results

Figure 1 diagrams flow - from bases through results - regarding our research. Our work has roots in the known elementary particles and in concordance cosmology. Our work features a hypothesis that nature includes six isomers of known elementary particles - of which only one isomer associates with ordinary matter and five isomers associate with most dark matter. Our work features new modeling based on Diophantine equations. We suggest new elementary particles and a specification for dark matter. We suggest insight regarding modeling gravity and regarding galaxy formation. We suggest explanations for known data for which - seemingly - other modeling does not offer explanations. We suggest data - about aspects of the universe - that people might be able to verify or refute.

Figure 2 alludes to all known elementary particles and to elementary particles that our work suggests. (Perhaps, preview table 10.) Discussion elsewhere in this essay indicates that - eventually - some successful modeling might not consider notions that our work associates with 3L and 4L to associate with elementary particles. (Perhaps, preview - for example - table 22 and the notion of QGD.)

Figure 3 shows quantitative ratios of dark matter effects to ordinary matter effects. (Perhaps, preview table 21.) Our work suggests quantitative explanations for the ratios. The explanations have bases in our specification for dark matter. The specification has roots in our hypothesis that associates with six isomers of known elementary particles. Each isomer of known elementary particles associates - approximately - with its own photon. Each isomer of known elementary particles associates with its own 0I, 0.5M, 0.5R, and 1J elementary particles.

Figure 4 suggests details about our explanation for the known ratio - five-plus to one - of dark matter density of the universe to ordinary matter density of the universe. (Perhaps, preview table 21.)

Figure 5 suggests eras in the evolution of the universe. (Perhaps, preview table 18.) As far as we know, direct observations and data associate only with the two multi-billion-years eras.

Figure 6 suggests eras in the evolution of - and various ratios of dark matter to ordinary matter for - galaxies. (Perhaps, preview table 20 and table 21.) People have observed galaxies that associate with each one of the suggested approximate ratios - one to zero-plus, five-plus to one, four to one, and zero-plus to one.

Figure 7 suggests possible relationships between physics properties. (Perhaps, preview table 11 and table 13.) This essay leaves open possible opportunities to use these possible relationships to envision possibly more - than this essay explores - fundamental aspects of nature.

Figure 8 suggests possible rest energies for all known elementary fermions (including neutrinos) and all suggested elementary fermions. (Perhaps, preview table 12, table 13, table 14, and table 15.) This essay associates each rest energy with direct use of or extrapolation from a few formulas. (Perhaps, preview table 13 and table 14.)

1.3. Methods

One goal of our modeling is to match and possibly extend a list of properties - of objects - that people infer or might infer based on observations based on so-called long-range interactions (or, so-called long-range forces). Long-range interactions include electromagnetism (which associates with notions of a spin-one boson - the photon), gravity (which associates with notions of a yet-to-be-found spin-two boson - the graviton), possibly interactions that would associate with a spin-three boson, and possibly interactions that would associate with a spin-four boson.

We find it convenient to divide elementary particles into three sets - carriers of long-range interactions, other elementary bosons, and elementary fermions. We associate, respectively with the three sets, the symbols LRI (as in long-range interaction or as in elementary boson that associates with a long-range interaction), SRI (as in short-range interaction or as in elementary boson that does not associate with a long-range interaction), and ELF (as in elementary fermion).

We associate the word simple - as in the phrase simple elementary particle - with the SRI elementary bosons and the ELF elementary fermions, but not with the LRI elementary bosons.

We develop mathematics modeling that outputs characteristics of long-range interactions and prop-

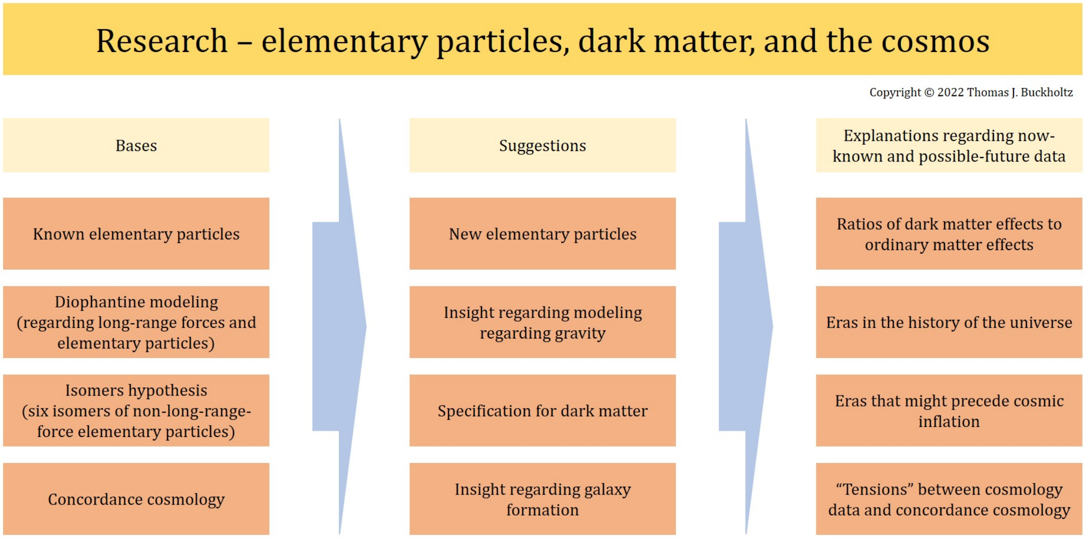

Figure 1: Research flow - from roots to results.

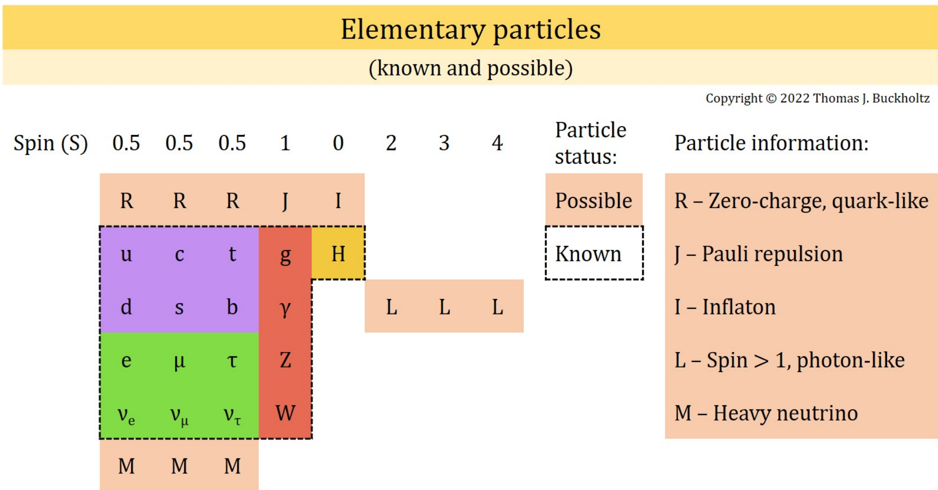

Figure 2: Known and suggested elementary particles. The table uses popular symbols for the known elementary particles. The following sentences introduce symbols for and notions about suggested new elementary particles. (The number S in a symbol $S\Phi$, associates with elementary-particle spin in units of $\hbar$.) 0.5R associates with three spin-one-half zero-charge analogs to quarks. 1J associates with a spin-one zero-charge boson that associates with Pauli repulsion. 0I associates with a spin-zero inflaton. 2L associates with a spin-two graviton. 3L associates with a spin-three relative of the photon and the graviton. 4L associates with a spin-four relative of the photon and the graviton. 0.5M associates with three spin-one-half heavy neutrinos.

Ratios of dark matter effects to ordinary matter effects

(Ratios that our modeling explains)

Aspect	DM:OM	Comment
Densities of the universe	5^+:1	Observed
Some galaxy clusters	5^+:1	Observed
Some absorption of CMB	1:1	Data observed; Association with DM proposed
Some early galaxies	0^+:1	Observed
Some later galaxies	0^+:1	Observed
Some early galaxies	1:0^+	Possibly too difficult to observe directly
Some later galaxies	1:0^+	Observed
Some later galaxies	~4:1	Observed
Many later galaxies	5^+:1	Observed

Legend:

DM – dark matter. OM – ordinary matter.

DM:OM – ratio of dark matter effects to ordinary matter effects.

Figure 3: Ratios of dark matter effects to ordinary matter effects. For each ratio except two of the ratios, the following three sentences pertain. People have observed the ratio. People attribute the so-called dark matter effects to dark matter. Our modeling explains the ratio. Regarding some specific depletion of cosmic microwave background radiation (or, CMB), people have observed the ratio, some people speculate that the effects that people might not attribute to ordinary matter are effects of dark matter, and our modeling suggests that non-ordinary-matter effects are effects of dark matter. We use the two-word term early galaxies to include galaxies observed at redshifts of at least (and possibly somewhat less than) seven. Most relevant data about later galaxies pertains to galaxies observed at redshifts considerably less than seven. The three-word term dark matter galaxy pertains to a galaxy for which the DM:OM ratio is one to zero-plus. Possibly, current techniques are not adequately sensitive to detect early dark matter galaxies.

Dark matter and ordinary matter

(Relative densities of the universe – dark matter : ordinary matter :: 5^+ : 1)

Isomer	DMAI	OMSE
0	0.06 DM (IGM is not EM self-interactive)	1.00 OM (IGM is EM self-interactive)
1	0.06 DM (IGM is not EM self-interactive)	1.00 DM (IGM is not EM self-interactive)
2	0.06 DM (IGM is not EM self-interactive)	1.00 DM (IGM is not EM self-interactive)
3	0.06 DM (IGM is not EM self-interactive)	1.00 DM (IGM might be EM self-interactive)
4	0.06 DM (IGM is not EM self-interactive)	1.00 DM (IGM is not EM self-interactive)
5	0.06 DM (IGM is not EM self-interactive)	1.00 DM (IGM is not EM self-interactive)

Legend:

Isomer – an isomer of all elementary particles except ΣL elementary particles (or, except [mainly] the photon and the graviton).

DM – dark matter. OM – ordinary matter.

DMAI – each isomer's 0.5R and 0.5M elementary particles are bases for stuff that measures as DM. {AI denotes "in all isomers."}

OMSE – denotes "ordinary-matter similar elementary particles" and includes all elementary particles except ΣL particles, 0.5R, and 0.5M.

Each of 0.06 and 1.00 is a relative density of the universe. The total relative densities are ~5.38 DM and 1.00 OM.

IGM – Intergalactic medium. EM – electromagnetically. "Might be" associates with – for example – details regarding collisions of galaxy clusters.

Figure 4: Dark matter density of the universe and ordinary matter density of the universe. The DM (or, dark matter) relative densities sum to approximately 5.38 times the OM (or, ordinary matter) relative density. Across isomers, the masses of similar elementary particles are identical. However, for charged leptons, associations between flavour and mass are not necessarily identical. Differences in associations between charged-lepton flavours and charged-lepton masses lead to differences in the evolution of stuff that associates with each isomer. The stuff that associates with at least four DM isomers of elementary particles evolves so that the associated IGM (or, intergalactic medium) does not interact electromagnetically much with itself, compared to the interactivity of OM IGM. The lack - across at least four DM isomers - of much IGM electromagnetic self-interaction might associate with observations regarding the Bullet Cluster collision of two galaxy clusters.

Eras regarding the rate of separating of large clumps

("rate of expansion of the universe")

Rate of separating	Duration	Name	←	Initiating force	RDF	Reach	PROP solutions
Is negative	?	NYN	↙	Attractive	r^{-6}	6	2g1`2`3`6`8x
Turns positive	?	NYN	↙	Repulsive	-	1	0g1`3`4`6
Increases rapidly	Fraction of a second	Inflation	↙	Repulsive	r^{-5}	1	2g1`2`3`4x
Decreases	Billions of years	NYN	↙	Attractive	r^{-4}	1	2g1`2`3
Increases	Billions of years	NYN	↙	Repulsive	r^{-3}	2	2g2`4

Legend:

The era that associates with a row in the table precedes eras that associate with subsequent rows in the table.

NYN – not yet named.

Initiating forces tend to gain prominence before - and dominate for at least early parts of - the respective eras.

RDF – radial dependence of force [r denotes the distance between two clumps.].

Reach (for one instance of the initiating force) – number of isomers. {The number of instances is six divided by the reach of one instance.}

PROP solution – 2g... denotes component(s) of gravity [x denotes more than one.]; 0g1`3`4`6 associates with Pauli repulsion (or, with the jay – or 1J – boson).

Figure 5: Suggested and known eras regarding the rate of expansion of the universe. The era that associates with a row in the table precedes eras that associate with subsequent rows in the table. For each row, the leftmost three columns describe aspects of the era. The rightmost four columns associate with a noteworthy cause for the era. Generally, the noteworthy cause gains prominence before the era starts. Our work proposes the first two eras to which the image alludes. Other work and our work suggest the era of inflation. Other work and our work model aspects of the two multi-billion-years eras. Our work might explain seeming difficulties that other work seems to exhibit regarding modeling aspects of the current multi-billion-years era of increasing rate of separation.

Eras regarding the formation of some galaxies

(explanations for DM:OM ratios of one to zero-plus, zero-plus to one, about-four to one, and five-plus to one)

Era	Phenomena	←	Initiating force	RDF	Reach	PROP solution
First	A one-isomer original clump (or, halo) forms	↙	Attractive	r^{-4}	1	2g1`2`3
First	The clump repels stuff associating with one other isomer	↙	Repulsive	r^{-3}	2	2g2`4
Second	The clump attracts stuff that associates with five isomers	↙	Attractive	r^{-2}	6	2g2
Third	Collisions result in a DM:OM ratio of 5^{+}:1	↙	Attractive	r^{-2}	6	2g2

Legend:

The stage that associates with a row in the table precedes stages that associate with subsequent rows in the table.

Initiating forces might tend to gain prominence before - and dominate for at least early parts of - the respective stages.

RDF – radial dependence of force [r denotes the distance between two significantly massive objects.].

Reach (for one instance of the initiating force) – number of isomers.

Some galaxies do not evolve beyond the first era. Some of these galaxies have essentially only ordinary matter. Some have essentially only dark matter.

During the second stage, the repulsive force component also repels some stuff that associates with the isomer that associates with the original clump.

Based on the repulsive force, some galaxies have nearby stuff that associates essentially with only five isomers.

Some galaxies do not evolve beyond the second era. A DM:OM ratio of ~4:1 can pertain. {DM – dark matter. OM – ordinary matter}

Collisions can merge stuff from galaxies for which the original-clump isomers differ.

Some galaxies might associate with more than one original clump and with more than one original-clump isomer.

Figure 6: Suggested eras and suggested DM:OM ratios for galaxies. The stage that associates with a row in the table precedes stages that associate with subsequent rows in the table. For each row, the leftmost two columns associate with aspects of the stage. The rightmost four columns associate with a noteworthy cause for the stage. The noteworthy cause might gain prominence before the stage starts. Some galaxies do not transit beyond some stages. Our work points to possible propensities for nature to form galaxies with DM:OM ratios of approximately one to zero-plus (that is, dark matter galaxies), five-plus to one, four to one, and zero-plus to one. Galaxies that both had more than one original clump and had three original-clump isomers might tend to cease star formation earlier than do some other galaxies.

Possible relationships between physics properties

(Based on properties of some elementary particles)

Possible relationship – for elementary bosons – between the properties of mass, spin, and charge

$$(\mathrm{Mass})^2 = ((\mathrm{Mass}_{\mathrm{Higgs}})^2/17) \times (j^2+l_{ms}-S^2-Q(Q+1))$$

Possible relationship between electromagnetism and gravity (interrelates one charge and two masses)

Define β: $(4/3) \times (\beta^2)^6 \equiv ((q_e)^2/(4\pi\varepsilon_0)) \,/\, (G_N(m_e)^2)$

Define β': $\beta' \equiv m_{tau}/m_e$

Posit: $\beta'=\beta$

Legend:

j = 4 for the Higgs boson (0H), 3 for the Z boson (1Z), 3 for the W boson ($1W_1$), and S^2 for all other elementary bosons.

S = spin (in units of $\hbar$).

l_{ms} = 1 for the Higgs boson (0H), 1 for the Z boson (1Z), 1 for the W boson ($1W_1$), and 0 for all other elementary bosons.

Q = |charge / (charge of the electron)|.

$((q_e)^2/(4\pi\varepsilon_0)) \,/\, (G_N(m_e)^2)$ is the ratio – for two electrons – of electrostatic repulsion to gravitational attraction.

$(\beta')^2$ is the ratio of the gravitational attraction between two tau to the gravitational attraction between two similarly (to the two taus) distanced electrons.

The exponent of 6 – in the definition for β – might associate (for some modeling) with the notion of 6 isomers of non-L-family elementary particles.

Figure 7: Possible relationships between physics properties. The boson-centric relationship might pertain for all known elementary bosons and for all new bosons that we suggest. The possible relationship between electromagnetism and gravity enables computing a tau mass that is compatible with experimental results.

Rest energies for elementary fermions

SΦ	Elementary particle	Approximate rest energy		Note
$0.5C_1$	Electron	0.5109989...	MeV	Exp
$0.5C_1$	Muon	105.658...	MeV	Exp
$0.5C_1$	Tau	1776.8400 ± 0.0115	MeV	Calc
$0.5Q_{2/3}$	Up (quark)	2.2	MeV	
$0.5Q_{1/3}$	Down (quark)	4.8	MeV	
$0.5Q_{1/3}$	Charm (quark)	1.27×10^3	MeV	
$0.5Q_{2/3}$	Strange (quark)	9.3×10^1	MeV	
$0.5Q_{2/3}$	Top (quark)	1.71×10^5	MeV	
$0.5Q_{1/3}$	Bottom (quark)	4.18×10^3	MeV	

SΦ	Elementary particle	Approximate rest energy		Note
0.5N	Neutrinos – two or three	3.4×10^{-2}	eV	
0.5N	Neutrinos – one or zero	$\leq 4.4 \times 10^{-4}$	eV	
0.5R	Arc	8.6 to 10.7	MeV	
0.5R	Arc	6.8 to 8.6	MeV	
0.5R	Arc	102 to 106	MeV	
0.5M	Heavy neutrino	$\geq 6 \times 10^3$	GeV	
0.5M	Heavy neutrino	$\geq 6 \times 10^3$	GeV	
0.5M	Heavy neutrino	$\geq 6 \times 10^3$	GeV	

Legend:

Known particle or value

Suggested particle or value

Exp – Result from experiments.

Calc – Result from a calculation: The standard deviation reflects the standard deviation of measurements of G_N.

Figure 8: Possible rest energies for elementary fermions. Eight standard deviations of the calculated tau rest energy fit within one standard deviation of the measured tau rest energy. The calculated quark rest energies comport with experimental results. Results regarding neutrinos comport with the notion of three (not heavy) neutrinos and with astrophysics data. Our work suggests possible ranges for the rest energies of each of the three possible arc elementary fermions. Our work suggests two possible lower limits for the rest energies of the three possible heavy neutrino elementary fermions. The figure shows one of those possible lower limits. The other possible lower limit is $\sim 2.5 \times 10^9$ GeV.

erties - of objects - that long-range interactions measure. The modeling features solving Diophantine equations.

LRI solutions generally come in pairs. For example, regarding electromagnetism, one so-called PROP solution associates with the property of charge. That PROP solution has a so-called CURR partner solution that associates with a current of charge. We think that each LRI pair of one PROP solution and a CURR partner can adequately associate with special relativity and can adequately associate with kinematics models that do not have bases in special relativity.

Our LRI modeling for electromagnetism has some similarities to modeling based on charge-and-current 4-vectors and has some parallels to modeling based on an electric field, a magnetic field, and Maxwell's equations. However, some differences pertain. For example, LRI modeling includes a PROP solution that associates with a component of magnetic field that associates with a notion of electromagnetic dipoles. That PROP solution has a CURR partner that associates with a current of electromagnetic dipoles.

Our LRI modeling for gravitation has similarities and differences with respect to gravitoelectromagnetism. (Regarding gravitoelectromagnetism, see references [1] and [2].)

SRI solutions and ELF solutions come in PROP and CURR pairs. Each known simple particle associates with a solution pair. Each not-yet-found simple particle that this essay suggests associates with a solution pair.

We think that modeling based on the LRI, SRI, and ELF aspects discussed above suffices to point to possibly relevant new physics. For example, we interpret our modeling as describing aspects of two possible eras - in the evolution of the universe - before the possible inflationary epoch. And, we interpret our modeling as suggesting a mechanism that leads to the recent multi-billion-years era increases in the rate of expansion of the universe.

However, modeling just based on LRI, SRI, and ELF aspects discussed above does not suffice to explain some data about ratios of dark matter to ordinary matter and does not suffice to explain the magnitude of the recent multi-billion-years era increases in the rate of expansion of the universe. (Regarding ratios of dark matter to ordinary matter, perhaps preview table 21. Regarding the magnitude of the recent multi-billion-years era increases in the rate of expansion of the universe, perhaps preview the notion of reach - or ρ_I - in table 18.)

To explain some data about ratios of dark matter to ordinary matter, we assume that nature includes six isomers of the set of simple (or, SRI and ELF) elementary particles. Ordinary matter associates with all of the simple particles in so-called ELPI0. The symbol ELPI denotes the three-word phrase elementary particle isomer. The integers l_I that associate with symbols of the form ELPIl_I range from zero to five. Dark matter associates with some yet-to-be-found ELF particles in EPLI0 and with all elementary particles in ELPI1 through ELPI5.

The symbol STUIl_I denotes stuff - such as hadron-like particles, atoms, and stars - made up of just (or essentially just) ELPIl_I elementary particles (plus LRI aspects that include electromagnetism and gravity).

We assume that, across isomers, the ELPI have similarities. We assume that the mass of each simple particle in any one isomer is the same as the mass of a counterpart simple elementary particle in each other isomer.

We assume that the six ELPI differ in at least one way. For each of isomer-zero and isomer-three, the flavour of the lowest-mass charged lepton equals the flavour for the two lowest-mass quarks. For each of the other four isomers, the flavour of the lowest-mass charged lepton does not equal the flavour of the two lowest-mass quarks. (Perhaps, preview table 17.) One possible other difference between isomers might be that ELPI0, ELPI2, and ELPI4 associate with left-handedness (for, at least, charged leptons) and ELPI1, ELPI3, and ELPI5 associate with right-handedness.

Regarding LRI, a so-called reach associates with each PROP-and-CURR pair. We use the symbol ρ_I to denote reach. Allowed values for ρ_I are one, two, and six. For relatively familiar physics - such as the physics of solar systems - the dominant gravitational PROP-and-CURR pair has a reach of six. The six STUI interact with each other via this component of gravity. Regarding electromagnetism, the reach that associates with the charge-and-charge-current PROP-and-CURR pair is one. The reach that associates with the electromagnetic-dipole-and-related-current PROP-and-CURR pair is one. Each one of the STUI has, in effect, its own instance of each of these two electromagnetic-centric PROP-and-CURR pairs. Each STUI does not interact with any other STUI via either of these two electromagnetic-centric PROP-and-CURR pairs.

The notion of six isomers and the notion of instances of LRI PROP-and-CURR pairs seem to suffice to explain ratios of dark matter to ordinary matter. (Perhaps, preview table 21.) The two notions might suffice to explain the size of the recent multi-billion-years era increases in the rate of expansion of the universe.

1.4. Our work and other work

We discuss relationships between our work and other work. Here, other work includes observational research and modeling-centric research.

1.4.1. We discuss relationships - between our work and other work - regarding elementary particles, physics constants, and physics properties.

We discuss other work that tries to suggest new elementary particles.

Reference [3] lists some types of modeling that people have considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Types of models associate with terms such as large extra dimensions, Kaluza-Klein (which associates with notions of gravity in more than four dimensions), grand unification, supersymmetry, and superstrings. Reference [4] provides information about some of these types of modeling. References [5], [6], and [7] provide some information about modeling and about experimental results. Reference [8] provides other information about modeling and about experimental results. (Perhaps, see reviews numbered 86, 87, 88, 89, 90, and 94.)

We discuss possible elementary particles that people have yet to find, that we suggest, and that other people might suggest.

Reference [9] suggests the notions of dark matter charges and dark matter photons. We suggest dark matter isomers of charged elementary particles and, in effect, dark matter components - such as components associating with electrostatics and magnetostatics - of electromagnetism.

Reference [10] suggests the notion of a so-called inflaton field. We suggest an inflaton elementary particle. (Perhaps, preview table 10 and note the 0I boson.)

People suggest the notion of a graviton. (See, for example, reference [11].) We suggest a graviton. (Perhaps, preview table 10.)

Reference [12] discusses notions of sterile neutrinos and heavy neutrinos. We suggest possible elementary particles that might associate with notions of heavy neutrinos. (Perhaps, preview table 10.)

We discuss possible elementary particles that people have yet to find, that we suggest, and for which other people might suggest that modeling rules out possible existence.

We suggest a spin-three analog to the photon and the could-be graviton. (Perhaps, preview table 6.) The spin-three analog might associate with nonzero anomalous magnetic moments for at least charged leptons. (Perhaps, preview table 4.) We suggest a spin-four analog to the photon and the could-be graviton. (Perhaps, preview table 6.) The spin-four analog might associate with notions - for at least neutrinos - of anomalous gravitational properties and mass mixing. (Perhaps, preview table 4 and table 14.) Some people suggest that modeling based on QFT (or, quantum field theory) implies that massless elementary particles cannot have spins that exceed two. Modeling based on QFT suggests - without assuming elementary particles with spins of more than one - values for some anomalous magnetic moments. We suggest the notion that a (possible but for now hypothetical) QFT that includes gravity might successfully estimate aspects that people associate with neutrino mass mixing. (Perhaps, preview table 22 and the notion of QGD.) Thus, our work might not be incompatible with notions that nature does not include zero-mass elementary bosons that have spins that exceed two. This essay de-emphasizes further discussing such a possible limit.

We discuss possible elementary particles that people have yet to find and our modeling seems not to suggest.

Reference [7] reviews modeling and experiments regarding so-called magnetic monopoles. Reference [7] notes that a symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. We suggest that nature might not include an interaction that would associate with magnetic monopoles. (Perhaps, preview table 3.) Reference [13] discusses a search - for magnetic monopoles - that did not detect magnetic monopoles.

Reference [5] reviews modeling and experiments regarding so-called axions. Reference [5] notes modeling that suggests that nature might include axions. We suggest that nature might not include axions. (Perhaps, preview table 7.) We suggest that phenomena that people might attribute to axions might not associate with axions. One such phenomenon could be electromagnetic interactions between ordinary matter and dark matter based on, for example, the so-called 1g1‘2‘4 component of electromagnetism. (Perhaps, preview table 3 and table 9.)

Reference [6] reviews modeling and experiments regarding so-called leptoquarks. We suggest that nature might not include leptoquarks. (Perhaps, preview table 7.)

We discuss prospectively some aspects, assuming that our work gains attention.

We discuss neutrino masses.

Reference [12] discusses modeling and data about neutrino masses and neutrino oscillations.

We suggest neutrino masses. (Perhaps, preview table 14.) As far as we know, our modeling is not incompatible with data that reference [12] discusses. Future experimentation might help validate or refute aspects of our work regarding neutrinos.

We discuss gravitation.

Reference [14] discusses experimental tests of theories of gravity.

We suggest effects - associating with isomers of elementary particles and with reaches of components of gravity - that suggest that other modeling regarding gravity would not be adequately accurate for some circumstances. This essay discusses some such circumstances. We are uncertain as to the extent to which aspects that reference [14], reference [15], or reference [16] discuss would tend to validate or refute aspects of our modeling that pertains to gravitation.

We use modeling - regarding gravity - that has some similarities to models that people associate with the term gravitoelectromagnetism. (References [1] and [2] discuss gravitoelectromagnetism.) Our modeling regarding gravity has some similarities to models that use classical physics perturbations regarding Newtonian gravity. (Reference [17] deploys modeling that associates with non-spherical distributions of mass.)

We discuss physics constants and properties.

Our work seems to interrelate some physics constants. (Perhaps, preview table 11 and table 13.) Our work seems to interrelate some properties, including via modeling that catalogs physics properties. (Perhaps, preview table 3 and table 9.)

We might offer new approaches to estimating some physics properties. This essay points to masses - that would comport with recent experimental results and that would have smaller standard deviations than standard deviations that associate with recent experiments - for each of the tau elementary fermion and the Higgs boson. (Perhaps, preview respectively table 13 and table 11.) This essay notes - regarding the anomalous magnetic dipole moment of the tau elementary fermion - a possible estimate that might approximate a Standard Model estimate. (Perhaps, preview discussion related to table 12 and table 13.) This essay notes - regarding the fraction of top quark decays that result in right-handed W bosons - a possible estimate that might approximate a Standard Model estimate.

1.4.2. We discuss relationships - between our work and other work - regarding cosmology.

We think that - with some exceptions - our work does not necessarily suggest significant changes - to concordance cosmology - regarding the large-scale evolution of the universe. (References [18], [19], and [20] review aspects of concordance cosmology.)

Each exception associates either with a possible aspect of nature for which people have no observations or with a known gap between observations and concordance cosmology.

One exception pertains regarding before inflation. One exception pertains regarding recent changes in the rate of expansion of the universe. In each case, we suggest noteworthy contributions by a gravitational force component for which each instance (of the component) has a reach that is greater than one isomer. (Perhaps, preview table 9.) For times associating with between the two cases, we suggest dominance by gravitational force components that have reaches of one isomer. For times associating with between the two cases, we do not propose significant incompatibilities between our work and large-scale concordance cosmology.

We discuss a possibility regarding times before inflation. (References [21] and [19] discuss inflation.)

We think that no direct observations pertain. We suggest two eras before inflation. (Perhaps, preview table 18.) The first of those two eras features aspects that the Standard Model and concordance cosmology do not include. One aspect is the so-called jay boson. (Perhaps, preview table 10 and table 18.) The other aspect is the set of so-called 2g1‘2‘3‘6‘8x components of gravity. (Perhaps, preview table 18.) An instance of each component has a reach of six isomers. For purposes of discussion, we assume that the universe transited those two eras. We assume that concordance cosmology can embrace the jay boson. For the first of those two eras, an extrapolation of concordance cosmology techniques might underestimate the strength of the key driver - the 2g1‘2‘3‘6‘8x components of gravity - by a factor of six.

We discuss phenomena during and after the lead-up to the current multi-billion-years era of increases in the rate of expansion of the universe.

People suggest that concordance cosmology underestimates increases in the rate of expansion. (References [20], [22], [23], [24], and [25] discuss relevant notions.)

We think that we point to a basis for the underestimates. Regarding times before that lead-up, we suggest dominance by an attractive quadrupole gravitational force component - 2g1‘2‘3. (Perhaps, preview table 18.) Each instance of that force component has a reach of one isomer. Before and during the recent multi-billion-years era, the 2g2‘4 gravitational force component gains prominence and then becomes dominant. Each instance of 2g2‘4 has a reach of two isomers. We suggest that concordance cosmology models that work well regarding times for which reach-one dominance pertains would not necessarily work well after those times. We suggest that extrapolating based on such concordance cosmology modeling would underestimate (conceptually by a factor of two) the strength of the driver for increases in the rate of expansion. We suggest that - to get good results via concordance cosmology modeling - people might adjust the equation of state. In general, for each relevant density, components of pressure that associate with repulsion need to increase.

Our suggested resolution regarding the underestimate seems to differ from possible resolutions based on concordance cosmology modeling. Our suggested resolution focuses on phenomena that would pertain at the times for which concordance cosmology modeling seems not to be adequate. Other possible resolutions might focus on phenomena early in the history of the universe. (See reference [20].)

1.4.3. We discuss relationships - between our work and other work - regarding astrophysics.

We think that our modeling is not necessarily incompatible with astrophysics data or with results based on concordance cosmology modeling. (Here, we assume that the two-word term concordance cosmology includes aspects that associate with dark matter, astrophysics, and effects of gravity on scales as small as one galaxy.)

We discuss properties of dark matter.

Reference [26] suggests the following notions. Most dark matter comports with notions of cold dark matter. Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. People suggest limits on the masses of basic dark matter objects. Observations suggest so-called small-scale challenges to the notion that all dark matter might be cold dark matter. People use laboratory techniques to try to detect dark matter. People use astrophysical techniques to try to infer properties of dark matter.

We think that our modeling regarding dark matter comports with such notions. For astrophysical phenomena (and not necessarily regarding the rate of expansion of the universe), components - that have reaches other than six - of gravity play roles locally; however, the impacts do not extend to cosmological scales. The dark matter isomer that might evolve similarly to ordinary matter might provide bases for resolving some of the so-called small-scale challenges.

We discuss observations and models regarding galaxy formation.

Reference [27] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Reference [27] discusses parameters by which people classify and describe galaxies.

We suggest that - regarding galaxies - observations of ratios of dark matter to ordinary matter might tend to cluster near some specific ratios. (Perhaps, preview table 21.) Our modeling seems to explain such ratios.

Our modeling suggests that ratios of dark matter to ordinary matter might reflect fundamental aspects - of nature - that concordance cosmology modeling does not include. Here, a key aspect is that of isomers. (Perhaps, preview table 21.)

Reference [27] seems not to preclude galaxies that have few ordinary matter stars. Reference [27] seems not to preclude galaxies that have little ordinary matter.

We think that dark matter to ordinary matter ratios that our modeling suggests are not necessarily incompatible with verified concordance cosmology modeling.

We discuss observations and models regarding interactions between galaxies.

Reference [28] suggests that concordance cosmology modeling might not adequately explain gravitational interactions between neighboring galaxies. We suggest that notions pertaining to reaches and isomers might help to bridge the gap between observations and concordance cosmology modeling.

We think that our work points to a possible opportunity to study harmony between results based on established kinematics models and results based on our notions of components of gravity.

2. Methods

This unit develops and deploys modeling that matches all known elementary particles; suggests new elementary particles; interrelates elementary particles, properties of individual objects, and properties of systems of objects; and provides specifications for dark matter.

The method that we develop here outputs solutions to equations that involve sums of integers.

Some solutions associate with modeling that has similarities to modeling based on Maxwell's equations, to modeling based on charge-and-current 4-vectors, or to modeling based on the notion of gravitoelectromagnetism. (References [1] and [2] discuss gravitoelectromagnetism.) Some solutions associate with electromagnetic fields - such as an electric field or a magnetic field - and with electromagnetic properties - such as charge and magnetic dipole moment - of systems. Some solutions associate with gravitational fields and with gravitational properties - such as mass.

Some solutions point to radial spatial dependences of potentials. (Regarding radial spatial dependences of potentials and forces, we use terminology that generally associates with Newtonian kinematics.) One such radial spatial dependence of potential is r^{-1} for a component of electromagnetism that associates with the charge of a system. Here, r denotes a distance away from the system that produces the component of electromagnetism. Another such radial spatial dependence of potential is r^{-2} for a component of electromagnetism that associates with the magnetic dipole moment of the system. (Generally, for radial spatial dependences other than r^{-1}, angular dependences also pertain.)

We associate the symbol 1g with solutions that associate with electromagnetism. Here, the one denotes the spin (in units of $\hbar$) of photons. We associate the symbol 2g with solutions that associate with gravitation. Here, the two denotes the spin (in units of $\hbar$) of (as yet not detected) gravitons. We associate the symbol 3g with solutions that might associate with a would-be spin-three elementary boson. We associate the symbol 4g with solutions that might associate with a would-be spin-four elementary boson. We associate the two-element term long-range force (or, the two-element term long-range interaction) with each one of 1g through 4g.

Other solutions associate mathematically with 0g. Some 0g solutions associate with elementary bosons, such as the Z boson and the W boson. Some 0g solutions associate with elementary fermions, such as quarks and charged leptons.

The method outputs solutions that seem to match all known elementary particles and that suggest new elementary particles.

We use the following method to catalog elementary particles. (Perhaps, preview table 10.) A symbol of the form $S\Phi$ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S associates with the expression $S(S+1)\hbar^2$ that associates with angular momentum. Values of S include 0, 0.5, 1, and 2 and might include 3 and 4. The symbol Φ associates with a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, this essay omits - from the symbols for families - the symbol Q. (Perhaps, preview table 10.)

2.1. Charge, mass, other properties, and elementary particles

This unit develops and deploys modeling that interrelates long-range forces (such as electromagnetism and gravity), properties of individual objects, properties of systems that include individual objects, and elementary particles.

2.1.1. We explore notions that point toward aspects of some modeling that our work uses.

We imagine two non-moving objects - object A and object B - that are located a distance r from each other. Each object has non-zero charge and non-zero mass. We consider the impacts of fields - such as electromagnetism or gravity - generated by object A on object B.

The electric potential that affects object B varies as r^{-1}. The gravitational potential that affects object B varies as r^{-1}.

We imagine hypothetical effects that associate with hypothetical interactions by object B with a hypothetical combination - produced by object A - of electric field and gravitational field. We imagine that the potential that associates with these interactions varies as r^{-1} times r^{-1}, which equals r^{-2}.

We discuss aspects of hypothetical particles that might intermediate interactions between object A and object B. We use the two-item term object C to denote such a hypothetical particle. We imagine that objects C traverse straight-line trajectories from object A to object B. We use the word axis to associate with the straight line.

We imagine objects C that have some similarities to and some differences from either an atom or a solar system. One or more components of an object C orbit a point that is central to object C. An object C exhibits orbitals. We imagine that, with respect to the axis that runs from object A to object B, each orbital associates with a unique magnitude $l_o\hbar$ of orbital angular momentum. Here, l_o is a positive integer. Up to one entity can associate with (or, occupy) an orbital. The integer l_{max} denotes the maximum value of l_0 that associates with an occupied orbital. Relative to the axis that runs from object A to object B, the angular momentum that associates with an occupied orbital is one of $-l_o\hbar$ and $+l_o\hbar$. (We exclude - for the occupied orbital that associates with l_o - values of l_i for which $-l_o < l_i < +l_o$.) The angular momentum that associates with an unoccupied orbital is $0\hbar$. (Regarding considering the object to be atom-like, the following notions pertain. The nucleus has zero spin. Entities that occupy orbitals have zero spin. Entities that occupy orbitals do not interact with each other.) Relative to the axis, the total angular momentum that associates with an object C is the sum - over the occupied orbitals - of the respective $\pm l_o\hbar$.

Regarding modeling that we discuss below, the following notions pertain.

- Individual objects C do not associate directly with elementary particles.
- Mathematical modeling regarding an object C associates with a so-called (mathematical) solution. Some sets of solutions associate with modeling for LRI (or, long-range interaction) elementary particles such as the photon. Some sets of solutions associate with modeling for SRI (or, short-range interaction) elementary bosons such as the W boson. Some sets of solutions associate with modeling for ELF (or, elementary fermion) elementary fermions such as the electron.
- Spatial dependences of potentials can depend on angular coordinates as well as on a radial coordinate such as r.

- We suggest that the modeling explains data (some of which other modeling seems not to explain), echoes useful modeling (that other people have developed), and predicts possibly reasonable data that might result from future observations and experiments. This essay does not necessarily point directly to a perhaps so-called fundamental basis for the modeling.

2.1.2. We develop a mathematical basis for modeling that this essay features.

We focus on mathematics that associates with the discussion about objects C and orbitals. We do not explore the notion of direct physics relevance for such objects. We do not explore the notion of such objects. For convenience, we continue to use the word orbital.

The discussion about hypothetical objects C suggests expressions of the form $\sum_{o \in O}(\pm l_o)$. Here, O associates with the set of occupied orbitals. The integer o denotes a member of O.

We use the symbol Γ to denote an ascending-order list of the relevant $o \in O$. Within a list Γ, we separate values of l_o by using the symbol ‘. The symbol l_{max} denotes the maximum value of l_o in Γ. For example, $\Gamma = 1‘3$ associates with $l_{max} = 3$ and with $1 \in O$, $2 \notin O$, and $3 \in O$.

We define l_Σ to be the sum of the various values of $\pm l_o$. The expression $l_\Sigma = \sum_{o \in O}(\pm l_o)$ pertains. We define Σ to be the absolute value of the sum of the various values of $\pm l_o$. The equation $\Sigma = |l_\Sigma| = |\sum_{o \in O}(\pm l_o)|$ pertains. We associate the word solution with the notion of $\Sigma = |l_\Sigma| = |\sum_{o \in O}(\pm l_o)|$. The term two-word Diophantine equations associates with the modeling that we pursue.

Table 1 alludes to all $l_\Sigma = \sum_{o \in O}(\pm l_o)$ expressions for which $1 \leq l_o \leq l_{max} \leq 4$ and no two values of l_o are the same. The rightmost five columns discuss solutions $\Sigma = |l_\Sigma| = |\sum_{o \in O}(\pm l_o)|$.

We use the symbol ΣgΓ to denote the combination of a list Γ and a relevant value of Σ. The letter g anticipates an association with electromagnetism and an association with gravity. (Perhaps, think of g as in gamma rays and g as in gravity. Perhaps, anticipate that 1gΓ associates with electromagnetism and that 2gΓ associates with gravity.)

We associate the symbol Σg with solutions of the form ΣgΓ. We associate the symbol Σg’ with Σg solutions for which $\Sigma \in \Gamma$. We associate the symbol Σg” with Σg solutions for which $\Sigma \notin \Gamma$.

2.1.3. We develop modeling that associates with intrinsic electromagnetic and gravitational properties of objects and with aspects of electromagnetic and gravitational fields.

We explore the notion that some solutions that table 1 lists associate with long-range interactions (or, LRI) and with properties - of physical objects such as planets or elementary particles - that people do infer or might infer via observations based on information carried by electromagnetic fields and gravitational fields.

Regarding observations - via electromagnetism - pertaining to an object with nonzero charge, people might infer both a size of a charge of the object and a velocity with which the object moves. (Other inferences, such as the magnetic moment of the object might also pertain.) We associate charge with a notion of intrinsic property and velocity with a notion of extrinsic property. For models based on special relativity, the notion of a charge-and-charge-current 4-vector pertains.

We explore the notion that some solutions that table 1 lists associate with intrinsic properties - such as charge - of objects. (Later, we explore extrinsic properties such as velocity.)

We assume the following associations. 1g associates with electromagnetism. 2g associates with gravitation. Each ΣgΓ solution (or, $\Sigma = |l_\Sigma| = |\sum_{o \in O}(\pm l_o)|$ solution) associates with two $l_\Sigma = \sum_{o \in O}(\pm l_o)$ expressions. We associate $l_\Sigma > 0$ with left-circular polarization. We associate $l_\Sigma < 0$ with right-circular polarization. (Elsewhere, for non-LRI elementary particles, we associate the notion of two expressions per solution with two handednesses. See discussion related to table 7. For LRI elementary particles, it is not necessarily inappropriate to associate left-circular polarization with left-handedness and right-circular polarization with right-handedness.)

Table 2 discusses interpretations - regarding properties of an object - regarding Σg’ solutions for which $1 \leq \Sigma \leq 2$ and $1 \leq l_{max} \leq 4$.

Table 2 suggests two uses for the words monopole, dipole, quadrupole, and octupole. One use associates with mathematics and with table 1. One use associates with physics and with the dependence of potentials that associate with the modeling of components of LRI-centric interactions (or, LRI forces).

We assume that a solution associates with a so-called RDP of the form $\Xi^{-n_{\Sigma g \Gamma}}$. RDP stands for radial dependence of potential. Here, we consider Newtonian modeling for potentials (as in potential energy) that associate with fields (such as the electromagnetic field and the gravitational field) that an object produces. For a solution other than a monopole solution, the potential can (and generally does) vary based on angular coordinates (as well as based on a radial coordinate). We assume that $\Xi^{-1} = r^{-1}$, in which r is the spatial distance from the object. (We provide a cautionary note regarding terminology. Per table 2, we associate the solution for which Σ is one and Γ is 1‘2‘4 with each one of the following: Ξ^{-3} and hence mathemati-

Table 1: $\Sigma = |l_\Sigma| = |\sum_{o\in O}(\pm l_o)|$ solutions, assuming that $1 \leq l_o \leq l_{max} \leq 4$ and that no two values of l_o are the same. The columns labeled l_1 through l_4 show contributions toward expressions $l_\Sigma = \sum_{o\in O}(\pm l_o)$. In those four columns, the symbol 0 is a placeholder for an unused pair, $-l_o$ and $+l_o$, of values. The symbol n_0 denotes the number of times the symbol 0 associates with an l_o for which $1 \leq l_o \leq l_{max} \leq 4$. The symbol n_Γ denotes the number of elements in the list Γ. For each row, there are 2^{n_Γ} possible ways to assign signs regarding the set of n_Γ terms. There are 2^{n_Γ} expressions of the form $l_\Sigma = \sum_{o\in O}(\pm l_o)$. Thus, there are $2^{n_\Gamma - 1}$ solutions $\Sigma = |l_\Sigma| = |\sum_{o\in O}(\pm l_o)|$. The Σ column shows values of Σ that associate with solutions. For example, for $l_{max} = 2$ and $\Gamma = 1$‘2, the two solutions feature, respectively, $\Sigma = 1$ (as in $1 = |-1+2|$) and $\Sigma = 3$ (as in $3 = |+1+2|$). (The two expressions that associate with $1 = |-1+2|$ are $-1+2$ and $+1-2$. The two expressions that associate with $3 = |+1+2|$ are $+1+2$ and $-1-2$.) The number $n_{\Sigma \text{g} \Gamma}$ equals $2^{n_\Gamma - 1}$ and states the number of solutions. The column for which the one-word label is notion refers to the number of solutions. For monopole, one solution pertains. For dipole, two solutions pertain. For quadrupole, four solutions pertain. For octupole, eight solutions pertain. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solutions. Regarding $\Sigma = 2$, $|-1+2-3+4| = 2 = |-1-2-3+4|$. Regarding $\Sigma = 4$, $|-1-2+3+4| = 4 = |+1+2-3+4|$.

l_{max}	Γ	l_1	l_2	l_3	l_4	Σ	n_0	n_Γ	$n_{\Sigma \text{g} \Gamma}$	Notion
1	1	±1	-	-	-	1	0	1	1	Monopole
2	2	0	±2	-	-	2	1	1	1	Monopole
2	1‘2	±1	±2	-	-	1,3	0	2	2	Dipole
3	3	0	0	±3	-	3	2	1	1	Monopole
3	1‘3	±1	0	±3	-	2,4	1	2	2	Dipole
3	2‘3	0	±2	±3	-	1,5	1	2	2	Dipole
3	1‘2‘3	±1	±2	±3	-	0,2,4,6	0	3	4	Quadrupole
4	4	0	0	0	±4	4	3	1	1	Monopole
4	1‘4	±1	0	0	±4	3,5	2	2	2	Dipole
4	2‘4	0	±2	0	±4	2,6	2	2	2	Dipole
4	3‘4	0	0	±3	±4	1,7	2	2	2	Dipole
4	1‘2‘4	±1	±2	0	±4	1,3,5,7	1	3	4	Quadrupole
4	1‘3‘4	±1	0	±3	±4	0,2,6,8	1	3	4	Quadrupole
4	2‘3‘4	0	±2	±3	±4	1,3,5,9	1	3	4	Quadrupole
4	1‘2‘3‘4	±1	±2	±3	±4	0,2,2,4,4,6,8,10	0	4	8	Octupole

Table 2: Interpretations - regarding properties of an object - regarding Σg’ solutions for which $1 \leq \Sigma \leq 2$ and $1 \leq l_{max} \leq 4$. We suggest the following notions. 1g1 associates with a component - of the electromagnetic field that the object produces - that associates with the object’s charge. The word scalar associates with this solution. 1g1‘2 associates with the object’s magnetic field. An axis associates with that field. The one-element term 3-vector associates with this solution. (For a bar magnet, the notions of charge and rotation do not necessarily pertain.) 1g1‘2‘4 associates with a combination of magnetic field and rotation (over time) of the axis of the magnetic field. (The Earth is an object for which the axis of rotation does not equal the axis of the magnetic field.) The one-element term 3-vector associates with that rotation. 2g2 associates with the object’s mass. The word scalar associates with this solution. 2g2‘4 associates with rotation of the object’s mass. An axis associates with that rotation. The one-element term 3-vector associates with this solution. (Regarding general relativity, this solution associates with aspects of rotational frame dragging.) 2g1‘2‘3 associates with a non-spherically symmetric distribution of mass. The one-element term 3-vector associates with each of a possible - axis and magnitude of a - minimal moment of inertia and a possible - axis and magnitude of a - maximal moment of inertia. 2g1‘2‘3‘4v associates with rotation (of a non-spherically symmetric distribution of mass) around a minor axis of moment of inertia. The one-element term 3-vector associates with that rotation. 2g1‘2‘3‘4w associates with rotation (of a non-spherically symmetric distribution of mass) around a major axis of moment of inertia. The one-element term 3-vector associates with that rotation. (Regarding general relativity, each of 2g1‘2‘3‘4v and 2g1‘2‘3‘4w might associate with aspects of rotational frame dragging.) For gravity produced by an object like the Sun, 2g’ solutions other than 2g2 associate with adjustments with respect to the gravity that associates with 2g2. Regarding large-scale gravitation, 2g’ solutions other than 2g2 can associate with gravitational effects that dominate gravitational effects that associate with 2g2.

Σ	Monopole	Dipole	Quadrupole	Octupole
1	1g1	1g1‘2	1g1‘2‘4	-
2	2g2	2g2‘4	2g1‘2‘3	2g1‘2‘3‘4v, 2g1‘2‘3‘4w

cal quadrupole, r^{-3} and hence a behavior of potential that associates with a notion of quadrupole, and a physics object that associates with a magnetic dipole that rotates around an axis that does not equal the axis that associates with the magnetic dipole. One way to think about the seeming tension between quadrupole and dipole is to associate the factor Ξ^{-1} that associates with $l_o = 4$ with $(ct)^{-1}$ instead of with r^{-1}. Here, c denotes the speed of light and t denotes the time that light takes to go from the magnetic-dipole object to the distance r from the object. This interpretation has consistency with the notion that the relevant quadrupole component of the electromagnetic field associates with an object that people might characterize as having the properties of a magnetic dipole.)

2.1.4. We extend our modeling to include extrinsic properties of objects.

We deploy the symbol PROP to associate with ΣgΓ solutions that we associate with intrinsic properties of objects. We deploy the symbol CURR to associate with ΣgΓ solutions that we associate with currents of properties.

We anticipate extending the notions of PROP and CURR to apply widely regarding modeling regarding LRI. We anticipate that, for each one of most LRI PROP solutions, there is an LRI CURR solution.

Notions of three degrees of freedom seem to pertain regarding solutions that table 2 shows.

The following examples - of three degrees of freedom - pertain regarding 1g' solutions. Regarding 1g1‘2, three degrees of freedom pertain. Two degrees of freedom associate with the orientation of the magnetic moment 3-vector. One degree of freedom associates with the magnitude of the magnetic moment 3-vector. Compared to 1g1‘2, 1g1‘2‘4 has three more degrees of freedom. Two degrees of freedom associate with the orientation of the angular velocity 3-vector. One degree of freedom associates with the magnitude of the angular velocity 3-vector.

Regarding each of the solutions that table 2 shows, $l_o = 4$ seems to associate - regarding rotation - with three degrees of freedom.

We suggest that - for some aspects of our modeling - three degrees of freedom, mathematics associating with two one-dimensional harmonic oscillators, and mathematics associating with the group $SU(2)$ associate with each other. (For integers l_i such that $l_i \geq 2$, reference [29] interrelates mathematics associating with l_i one-dimensional harmonic oscillators and mathematics associating with the group $SU(l_i)$.) Here, we consider that one oscillator might associate with boson-like excitations regarding a relevant aspect that associates with the relevant value of $-l_o$. The other oscillator might associate with boson-like excitations regarding a relevant aspect that associates with the relevant value of $+l_o$. The number of generators of the group $SU(2)$ is three.

To explore CURR solutions, we want to add three degrees of freedom that associate with the CURR aspects of PROP-and-CURR 4-somes. (We use the one-element term 4-some and not the one-element term 4-vector. For modeling centric to special relativity and scalar PROP aspects, the notion of 4-vector might be appropriate. However, the following notions pertain. This essay discusses PROP aspects that do not necessarily associate with the notion of scalar. This essay includes CURR aspects that - depending on choices regarding kinematics modeling - do not necessarily need to comport with special relativity.) We assume that modeling - regarding the three degrees of freedom - associates with one l_o.

The limit $l_{max} \leq 4$ does not allow for enough degrees of freedom. For example, regarding the PROP solutions 2g1‘2‘3‘4x (with x denoting either of v or w), one could not have CURR solutions.

We assume that $l_o = 6$ and $l_o = 8$ have relevance. For example, either one of 1g1‘2‘4‘6 and 1g1‘2‘4‘8 could be a CURR solution that associates with the PROP solution 1g1‘2‘4. We assume that - overall - $l_{max} \leq 8$ pertains and that each of $l_o = 5$ and $l_o = 7$ does not have relevance.

Each l_o in the series 1, 2, and 4 associates with the notion that $\log_2(l_o)$ is an integer. We explore notions regarding other values of l_o.

We discuss notions regarding $l_o = 3$. 1g1‘2 is a PROP solution that associates with intrinsic (nominal) magnetic moment. (See table 2.) Here, $\Sigma = 1$ and $\Gamma = 1‘2$ pertain. For $\Gamma = 1‘2$, $\Sigma = 3$ can pertain. We anticipate that a PROP 3g1‘2 solution associates with intrinsic anomalous magnetic moment. (Perhaps, preview table 4.) People measure anomalous magnetic moments for charged leptons (which are elementary fermions). For charged leptons, anomalous magnetic moments vary with fermion flavour. For some modeling, this essay associates - at least indirectly - $l_o = 3$ with (at least) the property of flavour for leptons. (The word leptons associates with some - but not all - elementary fermions. Perhaps, preview discussion related to table 22. This essay de-emphasizes, but does not rule out, notions that similar modeling associating with $l_o = 3$ might associate with aspects beyond lepton flavour.)

We discuss notions regarding $l_o = 6$ and $l_o = 8$. We assume that - for LRI PROP solutions - $l_o = 6$ can associate with interactions with elementary fermions and $l_o = 8$ can associate with interactions with elementary bosons. (Perhaps, preview table 7.)

2.1.5. We list and discuss Σg' solutions that associate with $l_{max} \leq 8$.

Table 3 extends table 2 and lists PROP and CURR Σg' solutions, for which $1 \leq \Sigma \leq 4$ and $l_{max} \leq 8$. Elsewhere, we discuss the extent to which 3gΓ solutions might associate with an elementary particle and the extent to which 4gΓ solutions might associate with an elementary particle. (Perhaps, preview table 9.)

2.1.6. We discuss a notion of cascades of solutions.

In table 3, for each CURR solution, the list Γ includes each of the elements from the list Γ for the associated PROP solution. Each CURR Γ includes exactly one element that does not occur in the Γ for the associated PROP solution. Such a CURR solution associates with the notion of associating - with the relevant PROP solution - a PROP-and-CURR 4-some. The three CURR components associate with notions of velocity.

We say that each such CURR solution cascades from the associated PROP solution.

For example, CURR 1g1'2 associates with linear movement of charge (which associates with 1g1) and (for some modeling) with a contribution to a magnetic field. Also, PROP solution 1g1'2 (occurs further down in the table and, for some modeling) associates with rotating charge (for which a notion of angular velocity can be appropriate) and (for some modeling) with a contribution to a magnetic field. PROP 1g1'2 also associates with effects of a bar magnet (in which a notion of angular velocity is not necessarily appropriate) and (for some modeling) with a contribution to a magnetic field.

We extend the notion of cascading to include the notion that a solution that appears as a CURR solution appears (usually further down) in table 3 as a PROP solution. The notion of velocity that pertains for the CURR use of the solution associates with a notion of angular velocity for the PROP use of the solution.

In table 3, the 1g' cascade starters are 1g1, 1g1'4'6, and 1g1'6'8. The 2g' cascade starters are 2g2 and 2g1'2'3. The 3g' cascade starters are 3g3 and 3g2'3'4. The 4g' cascade starters are 4g4 and 4g1'2'3'4x.

2.1.7. We discuss modeling that might associate with Σg' cascades.

For an elementary particle, the cascades that associate with 1g1, 2g2, 3g3, and 4g4 might be adequate to associate with relevant nominal intrinsic properties. These cascades associate with charge, nominal magnetic moment, mass, flavour (for elementary fermions), and spin. The cascades that associate with 1g1'4'6, 1g1'6'8, 2g1'2'3, 3g2'3'4, and 4g1'2'3'4x might have little relevance regarding modeling for intrinsic properties of elementary particles. Some cascades that associate with Σg" solutions might have relevance regarding modeling for anomalous intrinsic properties. (Regarding anomalous magnetic moments, see table 4.)

For an object (such as a proton or a galaxy) that includes more than one elementary particle, the cascades that associate with 1g1 and 2g2 can be relevant regarding modeling. The cascades that associate with 3g3 and 4g4 might not have (much or any) relevance. The cascades that associate with 1g1'4'6 and 1g1'6'8 might have relevance regarding modeling that associates with notions of orbital angular momentum and spin angular momentum. The cascade that associates with 2g1'2'3 might have relevance regarding modeling that associates with notions of stress-energy. This essay does not address the topic of the possible relevance of modeling that would associate with the cascades that associate with 3g2'3'4 or with 4g1'2'3'4x.

The end of each Σg' cascade associates with a row in table 3 for which no CURR solutions pertain. (The solutions 1g1'2'4'6'8x end the cascade that associates with 1g1, the cascade that associates with 1g1'4'6, and the cascade that associates with 1g1'6'8.) Regarding such a row, the PROP solution might associate with modeling regarding changes to properties that associate with the relevant cascade. For example, 2g1'2'3'4'6'8x might associate with notions that the energy or the stress-energy can change.

2.1.8. We list and discuss Σg" solutions that might associate with anomalous properties.

Table 4 lists some Σg" solutions. Table 4 previews the notion that 3g1'2 associates with modeling regarding anomalous magnetic moment. (Perhaps, preview table 9.) Table 4 previews the notion that 4g1'2'3 might associate with modeling regarding (anomalous) neutrino masses or regarding neutrino mass mixing. (Perhaps, preview table 9 and table 14.)

For each PROP Γ that table 4 shows, $1 \in \Gamma$ and $2 \in \Gamma$. A term such as the two-word term anomalous property might be more appropriate than a possible distinction between the three-word term anomalous magnetic moment and the three-word term anomalous gravitational property.

2.1.9. We provide perspective about modeling that associates with attractive and repulsive components of gravity.

We explore the extent to which each 2g' PROP solution associates with gravitational attraction and the extent to which each 2g' PROP solution associates with gravitational repulsion. We suggest results, based on generalizing from a series of cases.

We consider modeling regarding an object - object A - and the gravity that object A produces. We

Table 3: PROP and CURR Σg' solutions, for which $1 \leq \Sigma \leq 4$ and $l_{max} \leq 8$. The symbol $n_{\Gamma,\mathrm{PROP}}$ denotes the number of elements in the Γ that associates with PROP. Table 3 lists an RDF - or radial dependence of force - for each PROP solution. For a CURR ΣgΓ solution, the RDP (or, radial dependence of potential) equals Ξ^{-1} times the RDP for the associated PROP ΣgΓ solution. For each one of PROP and CURR, the RDF equals Ξ^{-1} times the RDP. For example, for each of 1g1 and 2g2, the RDF is Ξ^{-2}, which is r^{-2}. An x - as in ΣgΓx - denotes the notion that more than one solution pertains. Table 3 shows properties - of objects that produce ΣgΓ components of Σg - that associate with the PROP solution. The table attempts to use familiar symbols. The table associates the symbols with phrases. The symbol TBD denotes at least one of the three-word phrase to be determined and the notion that this essay discusses the aspect elsewhere.

Σ	ΣgΓ PROP	ΣgΓ CURR	$n_{\Gamma,\mathrm{PROP}}$	RDF PROP	PROP-associated properties
1	1g1	1g1‘2	1	Ξ^{-2}	q - Charge
1	1g1‘2	1g1‘2‘4	2	Ξ^{-3}	μ - Magnetic dipole moment (including from rotating charge)
1	1g1‘2‘4	1g1‘2‘4‘6, 1g1‘2‘4‘8	3	Ξ^{-4}	μ, ω - Magnetic dipole moment and internal angular velocity
1	1g1‘2‘4‘6x	1g1‘2‘4‘6‘8x	4	Ξ^{-5}	TBD
1	1g1‘2‘4‘8	1g1‘2‘4‘6‘8x	4	Ξ^{-5}	TBD
1	1g1‘2‘4‘6‘8x	None	5	Ξ^{-6}	TBD
1	1g1‘4‘6	1g1‘2‘4‘6x, 1g1‘4‘6‘8	3	Ξ^{-4}	TBD
1	1g1‘4‘6‘8	1g1‘2‘4‘6‘8x	4	Ξ^{-5}	TBD
1	1g1‘6‘8	1g1‘2‘6‘8x, 1g1‘4‘6‘8	3	Ξ^{-4}	TBD
1	1g1‘2‘6‘8x	1g1‘2‘4‘6‘8x	4	Ξ^{-5}	TBD
2	2g2	2g2‘4	1	Ξ^{-2}	m - Mass
2	2g2‘4	2g2‘4‘8	2	Ξ^{-3}	m, ω - Rotating (spherically symmetric aspects of) mass
2	2g2‘4‘8	None	3	Ξ^{-4}	TBD
2	2g1‘2‘3	2g1‘2‘3‘4v, 2g1‘2‘3‘4w, 2g1‘2‘3‘6, 2g1‘2‘3‘8	3	Ξ^{-4}	I_C - Moments of inertia
2	2g1‘2‘3‘4v	2g1‘2‘3‘4‘6x, 2g1‘2‘3‘4‘8x	4	Ξ^{-5}	I_C, ω - Rotating moments of inertia
2	2g1‘2‘3‘4w	2g1‘2‘3‘4‘6x, 2g1‘2‘3‘4‘8x	4	Ξ^{-5}	I_C, ω - Rotating moments of inertia
2	2g1‘2‘3‘6	2g1‘2‘3‘4‘6x, 2g1‘2‘3‘6‘8x	4	Ξ^{-5}	TBD
2	2g1‘2‘3‘8	2g1‘2‘3‘4‘8x, 2g1‘2‘3‘6‘8x	4	Ξ^{-5}	TBD
2	2g1‘2‘3‘4‘6x	2g1‘2‘3‘4‘6‘8x	5	Ξ^{-6}	TBD
2	2g1‘2‘3‘4‘8x	2g1‘2‘3‘4‘6‘8x	5	Ξ^{-6}	TBD
2	2g1‘2‘3‘6‘8x	2g1‘2‘3‘4‘6‘8x	5	Ξ^{-6}	TBD
2	2g1‘2‘3‘4‘6‘8x	None	6	Ξ^{-7}	TBD
3	3g3	3g3‘6	1	Ξ^{-2}	TBD (a function of elementary fermion flavour)
3	3g3‘6	None	2	Ξ^{-3}	TBD
3	3g2‘3‘4	3g2‘3‘4‘6x, 3g2‘3‘4‘8	3	Ξ^{-4}	TBD
3	3g2‘3‘4‘6x	3g2‘3‘4‘6‘8x	5	Ξ^{-5}	TBD
3	3g2‘3‘4‘8	3g2‘3‘4‘6‘8x	5	Ξ^{-5}	TBD
3	3g2‘3‘4‘6‘8x	None	6	Ξ^{-7}	TBD
4	4g4	4g4‘8	1	Ξ^{-2}	S - Angular momentum (scalar quantity)
4	4g4‘8	None	2	Ξ^{-3}	TBD
4	4g1‘2‘3‘4x	4g1‘2‘3‘4‘6x, 4g1‘2‘3‘4‘8x	4	Ξ^{-5}	TBD
4	4g1‘2‘3‘4‘6x	4g1‘2‘3‘4‘6‘8x	5	Ξ^{-6}	TBD
4	4g1‘2‘3‘4‘8x	4g1‘2‘3‘4‘6‘8x	5	Ξ^{-6}	TBD
4	4g1‘2‘3‘4‘6‘8x	None	6	Ξ^{-7}	TBD

Table 4: Some Σg" solutions. The table lists some Σg" PROP solutions ΣgΓ for which the following notions pertain. There is a positive Σ' - that is not equal to Σ - for which Σ'gΓ associates with a property. (That is, Σ'gΓ is a Σ'g' solution.) Σ is in the range $1 \leq \Sigma \leq 4$. For 3g1'2, two CURR solutions pertain.

Σ	PROP Γ	PROP $\Sigma = \ldots$	CURR $\Sigma = \ldots$	$n_{\Gamma,\mathrm{PROP}}$	Association
3	1'2	$\lvert +1+2 \rvert$	$\lvert +1-2+4 \rvert$, $\lvert -1-2+6 \rvert$	2	Anomalous magnetic moment
3	1'2'4	$\lvert +1-2+4 \rvert$	$\lvert +1-2-4+8 \rvert$	3	Anomalous magnetic moment
3	1'2'6	$\lvert -1-2+6 \rvert$	$\lvert -1+2-6+8 \rvert$	3	Anomalous magnetic moment
4	1'2'3	$\lvert -1+2+3 \rvert$	$\lvert -1-2+3+4 \rvert$	3	Anomalous gravitational property
4	1'2'3	$\lvert -1+2+3 \rvert$	$\lvert +1+2-3+4 \rvert$	3	Anomalous gravitational property
4	1'2'3	$\lvert -1+2+3 \rvert$	$\lvert -1+2-3+6 \rvert$	3	Anomalous gravitational property
4	1'2'3	$\lvert -1+2+3 \rvert$	$\lvert +1-2-3+8 \rvert$	3	Anomalous gravitational property

consider an object B that interacts with the gravity that object A produces. We assume that, relative to object B, object A does not move.

We include cases for which PROP solutions can pertain for components of object A or for the entire object.

One case associates with object A associating with the notion of a point mass. The PROP l_{max} is two. Object A has no internal components. The CURR solution 2g2'4 associates with the motion of object A and, per an assumption, is zero.

One case associates with the notion (regarding object A) of a non-rotating spherically symmetric distribution of mass. Object A has internal components. For each component, we assume that modeling based on a PROP l_{max} of two suffices. The 2g2 solution pertains. The CURR solution 2g2'4 associates with motion of the component. Across the components, the motions of components might help to overcome internal gravitational collapse. (For example, for some objects, one might consider that the motions associate with thermal energy.) Across the components, the contributions to 2g2'4 average to zero. The gravitational mass - which associates with 2g2 for the object - associates with the sum of the energies that associate with the PROP 2g2 solutions for the components. Object B senses no first-order effects that would associate with 2g2'4.

One case associates with the notion of a non-rotating non-spherically-symmetric distribution of mass. Here, we assume that three new (compared to the previous case) degrees of freedom associate with a magnitude and axis that associate with a maximal moment of inertia for object A and that three other new (compared to the previous case) degrees of freedom might associate with a magnitude and axis that associate with a minimal moment of inertia for object A. The PROP solutions 2g2 and 2g1'2'3 pertain regarding object A. Over time, object A might evolve to become more spherically symmetric. Energy that associates with having maintained at least one non-zero moment of inertia (which associates with 2g1'2'3) would drain from the 2g2 for object A. Independent of such possible evolution and compared to the previous case, this case illustrates the notion that the relevance of the 2g1'2'3 solution associates with increased (compared to the previous case) 2g2 and therefore with more (compared to the previous case) gravitational attraction (as experienced by object B).

The previous two cases illustrate the notion that for a 2g' PROP Γ solution for which $n_{\Gamma,\mathrm{PROP}} = 3$ associates with gravitational attraction.

One case associates with the notion of a uniformly rotating spherically symmetric distribution of mass. This (hypothetical) object A does not exhibit oblateness. For this case, one angular velocity ω pertains. The angular velocity is with respect to an axis that runs through the center of object A. The angular velocity pertains regarding each component of object A. The angular velocity associates with three degrees of freedom - two of which associate with the axis that associates with ω and one of which associates with the magnitude of ω. The PROP solutions 2g2 and 2g2'4 pertain. The CURR solutions 2g2'4 and 2g2'4'8 pertain. From the perspective of an object B that does not lie on an extension of the axis, the values of the 2g2'4'8 for the components do not necessarily sum to zero. The gravitational mass - which associates with 2g2 - for object A associates with the sum of the energies that associate with the PROP 2g2 solutions for the components.

If the hypothetical object A (from the previous case) existed in nature, object A would tend (over time) to become oblate. The transition process would release - from 2g2 - energy that, in effect, had maintained a lack of natural oblateness.

One case associates with the notion of the so-called natural oblateness that we just assumed.

The last two cases illustrate the notion that, for modeling based on 2g' PROP solutions, $n_{\Gamma,\mathrm{PROP}} = 2$ associates with reducing gravitational effects and thereby associates with gravitational repulsion.

Table 5 pertains regarding modeling that has bases in 2g' PROP solutions.

Table 5: 2g' PROP solutions and the extents to which gravitational attraction and gravitational repulsion pertain. An x - as in ΣgΓx - denotes the notion that more than one solution pertains. The symbol † associates with the notion that no CURR solutions pertain.

2g' $n_{\Gamma,\mathrm{PROP}}$	Gravitational ...	RDF PROP	N-pole	Examples of 2g' PROP solutions
1	Attraction	Ξ^{-2}	Monopole	2g
2	Repulsion	Ξ^{-3}	Dipole	2g2‘4
3	Attraction	Ξ^{-4}	Quadrupole	2g1‘2‘3
4	Repulsion	Ξ^{-5}	Octupole	2g1‘2‘3‘4x
5	Attraction	Ξ^{-6}	16-pole	2g1‘2‘3‘6‘8x
6	Repulsion †	Ξ^{-7}	32-pole	2g1‘2‘3‘4‘6‘8x

2.1.10. We discuss modeling that might associate with PROP solutions for which $8 \in \Gamma$.

We anticipate modeling - in which some $\Sigma = 0$ solutions associate with simple elementary particles - in which (for at least the known elementary particles) the following notions pertain regarding QFT (or, quantum field theories). (Perhaps, preview table 7 and table 8.) Simple particles for which PROP modeling associates with $8 \notin \Gamma$ can model (for some circumstances) as not entangled. Simple particles for which PROP modeling associates with $8 \in \Gamma$ seemingly do not model (for any circumstances) as not entangled.

For convenience, we associate the word free with aspects that associate with modeling that associates with PROP solutions for which $8 \notin \Gamma$. For convenience, we associate the word entwined with aspects that associate with modeling that associates with PROP solutions for which $8 \in \Gamma$. We chose the word entwined so as to avoid using the word entangled. (In some circumstances, free elementary particles model as entangled. An example features an electron that is part of an atom.)

2.1.11. We discuss possible limits on the applicability of Σg' solutions.

This essay suggests that - regarding Σg' solutions that might have relevance regarding modeling for LRI - that Σ does not exceed four. The suggestion associates with an extrapolation regarding data pertaining to the relative strengths of electromagnetism and gravity. (Perhaps, preview table 16.) That extrapolation suggests that a 5g5 solution would associate with a force strength of zero. We assume that, if 5g5 associates with zero force strength, each Σg' solution for which $\Sigma \geq 5$ is not relevant regarding LRI physics. The suggestion also associates with the notion that - for PROP solutions ΣgΓ for which $\Sigma \neq 0$ - $5 \notin \Gamma$. (See table 3.)

We discuss 3g' solutions and 4g' solutions.

Regarding 3g', table 3 suggests two cascades. One cascade starts with the 3g3 PROP solution. We suggest - regarding the 3g3‘6 PROP solution - that the lack of a partner CURR solution associates with a notion that non-zero effects that associate with 3g3 associate with a lack of structure that can exhibit nonzero angular velocity. We assume that 3g3 associates with interactions with (no more than) elementary particles. We assume that the 6 in the Γ for the CURR 3g3‘6 solution associates with three flavors and not necessarily with three degrees of freedom regarding motion. We assume that a 3g3 interaction would associate with the flavour of one elementary fermion. Based on such notions, we think that the cascade that starts with 3g2‘3‘4 does not have relevance for the physics modeling that this essay discusses.

Regarding 4g', table 3 suggests two cascades. Reasoning similar to reasoning that we just used regarding 3g' suggests the cascade that begins with the 4g1‘2‘3‘4x PROP solutions is not relevant regarding our physics modeling. Also, 4g4 might associate with interactions with single non-LRI elementary particles. Thus, a 4g4 interaction might associate with (no more than) the scalar S that associates with one elementary particle. Possibly, a notion that three values associates with the 8 in the Γ for the CURR solution 4g4‘8 associates with three allowed spins - (in units of $\hbar$) zero, one-half, and one - of simple elementary particles.

We discuss possible limits regarding 2g' solutions.

We consider two objects that are some distance apart from each other. We consider doubling linear dimensions - that is doubling the distance between the objects and doubling the diameters of the objects - while maintaining, for each object, a constant mass per unit volume. A PROP RDF Ξ^{-6} force after the doubling of linear dimensions equals the PROP RDF Ξ^{-6} force before the doubling of linear dimensions. Possibly, this invariance regarding scaling suggests reasons not to pursue - regarding interactions between pairs of large objects - modeling regarding PROP RDF Ξ^{-l_r} for which l_r exceeds six. Such a limit seems to be consistent with the notion that table 3 shows that no CURR solution partners with 2g1‘2‘3‘4‘6‘8x PROP solutions.

Regarding 2g', table 5 associates PROP RDF Ξ^{-6} solutions with gravitational attraction. PROP RDF Ξ^{-6} solutions might associate with a property that associates with energy. (Perhaps, preview table 9.) Possibly, the PROP RDF Ξ^{-7} solutions 2g1'2'3'4'6'8x associate with repulsion. (See table 5.) Possibly, the PROP RDF Ξ^{-7} solutions 2g1'2'3'4'6'8x associate with changes in the energies of objects.

This essay notes - but does not much discuss - the notion that the PROP solutions 2g1'2'3'4'6'8x might associate with spontaneous decay by elementary particles. (Perhaps, preview table 9. This essay notes - but does not much discuss - the notions that the PROP solutions 2g1'2'3'4'6'8x might associate with repulsion within a small object and might - along with solutions 1g1'2'4'6'8x - associate with aspects such as thermal radiation. Also, this essay does not further discuss possible associations of the notion of 2g1'2'3'4'6'8x repulsion with notions such as entropy and arrow of time.)

2.1.12. We list known and possible LRI elementary bosons.

Table 6 shows solutions that might associate with LRI (or, long-range interaction) elementary particles.

2.1.13. We develop modeling that matches and suggests all elementary particles (other than bosons that intermediate long-range forces) that this essay discusses.

We explore the notion that solutions for which $\Sigma = 0$ associate with known and possible non-ΣL elementary particles (or, non-LRI elementary particles). Based on arithmetic, for each $\Sigma = 0$ solution, n_Γ is at least three. We associate the symbol SRI (as in short-range interaction or as in elementary boson that does not associate with a long-range interaction) with non-ΣL elementary bosons. We associate the symbol ELF (as in elementary fermion) with fermion elementary particles.

For each $\Sigma = 0$ solution, there are two expressions of the form $0 = l_\Sigma = \sum_{o \in O}(\pm l_o)$. (See discussion related to table 1.) For each solution for which $\Sigma > 0$, there are two expressions. (See discussion related to table 2.) For each solution for which $\Sigma > 0$, we associate one expression with left-handedness and with left-circular polarization. For each solution for which $\Sigma > 0$, we associate one expression with right-handedness and right-circular polarization. For each $\Sigma = 0$ solution, we assume that one expression associates with the notion of left-handedness and the other expression associates with the notion of right-handedness.

Table 7 shows 0gΓ solutions that might associate with elementary bosons that are not ΣL bosons. (Reference [10] discusses the inflaton particle.)

Some Standard Model modeling suggests an association between a Higgs field and the notion that some elementary particles have nonzero masses. Elsewhere in this essay, table 11 interrelates the masses of elementary bosons. For elementary bosons, relationships feature squares of masses. Table 12 and table 13 interrelate the masses of some elementary fermions. For elementary fermions, relationships feature logarithms of masses. We note - but do not further discuss - the notion that - regarding the Higgs boson - differences between the CURR solution 0g1'2'3'4'8 and the CURR solution 0g1'2'3'4'6 might associate (from a perspective of modeling) with differences between the two types of relationships between elementary particle masses.

Table 8 shows 0gΓ solutions that might associate with elementary fermions.

Regarding table 8, we note - but do not discuss further - the notion that the number - one or two - of CURR solutions associates with the number - between particles and possible antiparticles - of handednesses. Similarly, the notion of Dirac fermions might associate with cases for which two CURR solutions pertain and the notion of Majorana fermions might associate with the case for which only one CURR solution pertains.

2.1.14. We preview aspects regarding the entwined simple elementary particles that we suggest and people have yet to find.

Each one of table 7 and table 8 shows simple particles that - within the context of our work - model as entwined.

After very early in the timeline that concordance cosmology features, 0.5R particles might exist only in hadron-like particles that include 1G (or, gluon) particles and that are somewhat analogs to known hadrons. In the very early universe, 0.5R and 1G particles might model as being components of so-called seas.

After very early in the timeline that concordance cosmology features, 0I particles might contribute effects that are negligible compared to effects that other bosons contribute. In the very early universe, 0I particles might fulfill the role that concordance cosmology posits for the so-called inflaton.

2.2. Isomers and dark matter

This unit suggests that most dark matter has bases in five isomers of the elementary particles that are not ΣL elementary particles and that ordinary matter has bases in one (other) isomer of most elementary particles that are not ΣL elementary particles.

2.2.1. We discuss the notion that, if nature includes only one isomer of each elementary particle, modeling might not suffice to explain known data about dark matter.

Discussion above points to two types of elementary particles that would measure as dark matter or that would provide bases for dark matter. 0.5M fermions associate with the notion of free and would

Table 6: Solutions that might associate with LRI (or, long-range interaction) elementary particles. The symbol n_{EP} denotes the number of elementary particles. Items that the table shows in parentheses might - depending on future data or on interpretations of vocabulary and modeling - associate with elementary particles. TBD denotes the three-word phrase to be determined.

ΣgΓ for PROP	ΣgΓ for CURR	Family	Boson	n_{EP}
1gΓ	1gΓ	1L	Photon	1
2gΓ	2gΓ	(2L)	(Graviton)	(1)
3gΓ	3gΓ	(3L)	(TBD)	(1)
4gΓ	4gΓ	(4L)	(TBD)	(1)

Table 7: Solutions that might associate with non-ΣL elementary (or, with SRI) bosons. Each column with label $0 = \ldots$ shows a calculation that produces the $\Sigma = 0$ that associates with a 0gΓ solution. Each integer that the calculation includes is a member of Γ. No other integer is a member of Γ. The symbol $n_{\Gamma,\text{PROP}}$ denotes the number of l_o that appear in the Γ for the PROP solution. The symbol n_{EP} denotes the number of elementary particles. For each of the Higgs boson, the Z boson, and the W boson, $(n_{\Gamma,\text{PROP}})^2$ associates with an aspect related to the mass of the boson. (Perhaps, preview the column - with the one-word label sum - in table 11.) The W boson is the only charged elementary boson. In table 7, only one PROP solution does not have - as members of Γ - all three of 1, 3, and 4. Based on the previous two sentences, we associate 0g1'2'3 with the W boson and we associate 0g1'3'4 with the Z boson. The symbol † associates with the possibility that - for each one of the Higgs boson and the Z boson - there might be differences between interactions with elementary bosons († b) and interactions with elementary fermions († f). For each of the aye boson, the jay boson, and the gluons, the parenthesized item in the leftmost column of table 7 suggests that the PROP solution cascades from a CURR solution in table 7. (Regarding the notion of cascading, see discussion regarding table 3.) We assume that the aye (or, 0I boson) associates with notions of an inflaton. Inflatons would be zero-mass zero-charge bosons that might have played key roles during a hypothesized inflationary epoch, early in the evolution of the universe. The table suggests that one CURR solution that associates with the Z boson equals the CURR solution that associates with the aye boson. We assume that the jay boson associates with notions of Pauli repulsion. Pauli repulsion associates with the notion that two fermions (whether elementary fermions or not elementary fermions) cannot occupy the same state. Pauli repulsion associates with repulsive aspects of the residual strong force. In the sense of discussion regarding table 3, the solutions that associate with the jay boson cascade to the solutions that associate with gluons. We suggest the possibility (but do not necessarily require) that some modeling might associate one of the gluon CURR solutions with aspects (of interactions) that erase - for example from a quark - one of three color charges and might associate the other of the gluon CURR solutions with aspects (of interactions) that paint - for example - onto a quark one of the three color charges.

$0 = \ldots$, re 0gΓ for PROP	$0 = \ldots$, re 0gΓ for CURR	$n_{\Gamma,\text{PROP}}$	Family	Bosons	n_{EP}
$\lvert +1-2-3+4 \rvert$	$\lvert +1-2-3-4+8 \rvert$ († b), $\lvert -1+2-3-4+6 \rvert$ († f)	4	0H	Higgs	1
$\lvert -1-3+4 \rvert$	$\lvert -1-3-4+8 \rvert$ († b) $\lvert +1-3-4+6 \rvert$ († f),	3	1Z	Z	1
$\lvert -1-2+3 \rvert$	$\lvert -1-2-3+6 \rvert$	3	1W$_1$	W	1
$\lvert -1-3-4+8 \rvert$ (1Z † b)	$\lvert +1-2-3-4+8 \rvert$	4	0I	Aye	1
$\lvert +1-3-4+6 \rvert$ (1Z † f)	$\lvert -1+3-4-6+8 \rvert$	4	1J	Jay	1
$\lvert -1+3-4-6+8 \rvert$ (1J)	$\lvert +1-2+3-4-6+8 \rvert$, $\lvert -1-2-3+4-6+8 \rvert$	5	1G	Gluons	8

Table 8: Solutions that might associate with elementary fermions (or, ELF elementary particles). We assume, regarding PROP 0g solutions, that $6 \in \Gamma$ associates with elementary fermions. (The might-be three degrees of freedom that might associate with l_6 might associate with three choices regarding flavours for elementary fermions.) Paralleling notions pertaining to non-ΣL elementary bosons, if, and only if, one of 1, 3, and 4 is not a member of a PROP Γ, an elementary particle that associates with table 8 has nonzero charge. The symbol n_{EP} denotes the number of elementary particles. The leftmost column in table 8 alludes to relevant PROP solutions and alludes to - in parentheses - a CURR solution from which a PROP solution cascades. (Regarding the notion of cascading, see discussion related to table 3.) We discuss solutions that associate with quarks. $0.5Q_{1/3}$ particles and $0.5Q_{2/3}$ particles are the only known particles for which $0 < Q < 1$. Here, Q denotes the magnitude of the charge, in units of $|q_e|$. q_e denotes the charge of the electron. The notion of PROP solution might associate with each of the following two solutions: $0 = |-1+2-3-6+8|$ and $0 = |-1+2-3-4+6|$. The notion of nonzero charge associates with the first of the two PROP solutions. The notion of $Q = 1$ associates with the first of the two PROP solutions. The notion of $Q = 0$ associates with the second of the two PROP solutions. The notions of $Q = 1/3$ and $Q = 2/3$ might associate with states that mix - and, regarding charge, lie between - $Q = 0$ and $Q = 1$. (Perhaps, preview aspects of table 12 and table 13.)

$0 = \ldots$, re 0gΓ for PROP	$0 = \ldots$, re 0gΓ for CURR	Families	Fermions	n_{EP}
$\|-1-2-3+6\|$ (1W)	$\|-1+2-3-4+6\|$	$0.5C_1$	Charged leptons	3
	$\|-1+2-3-6+8\|$			
$\|-1+2-3-4+6\|$ ($0.5C_1$)	$\|-1-2-3+4-6+8\|$	0.5M	Heavy neutrinos	3
	$\|+1-2+3-4-6+8\|$			
$\|+1-3-4+6\|$ (1Z)	$\|-1+3-4-6+8\|$	0.5N	Neutrinos	3
$\|-1+3-4-6+8\|$ (0.5N)	$\|-1-2-3+4-6+8\|$	0.5R	Arcs	3
	$\|+1-2+3-4-6+8\|$			
$\|-1+2-3-6+8\|$ ($0.5C_1$)	$\|-1-2-3+4-6+8\|$	$0.5Q_{y/3}$, with	Quarks	6
$\|-1+2-3-4+6\|$ ($0.5C_1$)	$\|+1-2+3-4-6+8\|$	$y = 1$ or 2		

measure as dark matter. 0.5R fermions associate with the notion of entwined. Hadron-like particles containing gluons and 0.5R fermions would contain no charged particles and would measure as dark matter.

We use the term DMAI to denote stuff that has bases in 0.5M elementary fermions or in 0.5R elementary fermions. DM abbreviates the two-word term dark matter. AI abbreviates the two-word term all isomers. (Here, we allude to a notion of multiple isomers of some elementary particles. For the moment we assume that nature includes just one isomer.)

We use notation of the form DM:OM to denote an inferred ratio of DM effects to OM effects. OM abbreviates the two-word term ordinary matter.

Measurements suggest seemingly significant DM:OM ratios. (For information about the ratios and for relevant references, perhaps preview table 21 and discussion related to table 21.) Ratios of approximately 5^+:1 pertain regarding densities of the universe, galaxy clusters, and many galaxies. Seemingly significant ratios of 1:0^+, 0^+:1, and ∼4:1 pertain regarding some galaxies. A ratio of 1:1 might pertain regarding some depletion of cosmic microwave background radiation (or, CMB). We know of no other such seemingly significant DM:OM ratios.

We suggest that, if DMAI is the only type of dark matter, DMAI might not suffice to explain various seemingly significant ratios of dark matter to ordinary matter. We suggest that the notion of DMAI might not suffice to explain dark matter.

2.2.2. We discuss the notion that nature includes six isomers of each elementary particle that does not intermediate a long-range force.

We suggest that nature includes six isomers of the SRI and ELF elementary particles - or, six isomers of the set of elementary particles that associates with all non-ΣL elementary bosons and all elementary fermions. (See table 7 and table 8.) One isomer associates with ordinary matter plus one isomer of DMAI. That one isomer of DMAI measures as dark matter. Each one of the other five isomers of the set of non-ΣL elementary particles measures as dark matter. Regarding densities of the universe, the five isomers of non-DMAI that measure as dark matter associate with the 5 in the DM:OM ratio of 5^+:1. The six isomers of DMAI associate with the $^+$ in the DM:OM ratio of 5^+:1.

We use a two-word phrase isomer *number* to denote one isomer. Here, *number* can be any one of zero, one, ..., and five. We associate the two-word term isomer zero with the isomer that includes ordinary matter. We use the two-word phrase alt isomer to denote any one of the five isomers that does not associate with ordinary matter.

2.2.3. We discuss modeling - regarding simple elementary particles - that might associate with the notion of six isomers of simple elementary particles.

Table 7 and table 8 point to all simple elementary particles that this essay features. For each PROP solution, each one of $1 \in \Gamma$ and $3 \in \Gamma$ pertains. We suggest that - relative to one of those two membership (in Γ) associations, the other membership (in Γ) association associates with three choices. (Here, the notion of three choices associates - for other circumstances - with notions that an l_o associates with three degrees of freedom. Perhaps, see discussion - regarding table 8 - regarding $6 \in \Gamma$ and three flavours.) Each PROP solution in table 7 and table 8 associates with two expressions. We assume that one expression associates with left-handedness and one expression associates with right-handedness. (Perhaps, preview discussion related to table 17.) We point to the possibility that the combination of three choices and two handednesses associates with six isomers. This essay uses this notion regarding the six isomers. (Perhaps, preview discussion related to table 17.)

2.2.4. We discuss modeling - for long-range interactions - that associates with the notion of six isomers of simple elementary particles.

All six isomers produce and interact with a common notion of gravity. We suggest that one instance of 2g2 mediates interactions between all six isomers. We say that one instance of 2g2 has a reach of six, as in six isomers. We suggest that each isomer associates with its own instance of 1g1 and its own instance of 1g1‘2. We say that each instance of 1g1 has a reach of one, as in one isomer. Each instance of 1g1‘2 has a reach of one. Each isomer - including the ordinary matter isomer - scarcely interacts with any other isomer via electromagnetism.

We address the topic of reach for each ΣgΓ to which table 1 alludes. Based on the reach of 1g1 and the reach of 1g1‘2, we suggest that $n_0 = 0$ associates with a reach of one. Based on the reach of 2g2, we suggest that $n_0 = 1$ associates with a reach of six. We assume that, for $n_0 \geq 1$, the reach (of one instance of a relevant PROP ΣgΓ) equals the number of generators of the group $SU(7)$ divided by the number of generators of the group $SU(2n_0+1)$. For an integer l_i that is at least two, the number of generators of the group $SU(l_i)$ is $(l_i)^2 - 1$. The reach that associates with $n_0 = 2$ is two. The reach that associates with $n_0 = 3$ is one. The number of instances of a PROP ΣgΓ component of a ΣL elementary particle is six divided by the reach that associates with the PROP ΣgΓ solution.

We assume that the reach of a CURR counterpart solution to a PROP ΣgΓ solution is the same as the reach of the PROP ΣgΓ solution.

We address the reach of the 2g1‘2‘3‘6‘8x PROP solutions. For 2g1‘2‘3‘6‘8, each of 1, 2, and 3 appears in Γ and 4 does not appear in Γ. We assume that $n_0 = 1$. The reach for each 2g1‘2‘3‘6‘8x is six. We assume that stress-energy is the relevant PROP-associated property. Stress-energy-and-velocity 4-somes can associate with stuff - such as a galaxy cluster - that associates with all of the six isomers. Stress-energy-and-velocity 4-somes can associate with stuff - such as a star - that associates with less than all of the six isomers.

We address the reach of the 2g1‘2‘3‘4‘6‘8x PROP solutions. For 2g1‘2‘3‘4‘6‘8, each of 1, 2, 3, and 4 appears in Γ. We assume that $n_0 = 0$. The reach for each 2g1‘2‘3‘4‘6‘8x is one. We might assume that abilities to change stress-energy associates with relevant PROP-associated properties. Stress-energy changes might - if modeling based on them has physics relevance - associate with single-isomer stuff - such as an atomic nucleus.

Table 9 shows the reach (ρ_I) for - and other information about - each one of some solutions that table 3 and table 4 list. Discussion that relates to table 3 and to table 6 suggests that some items that table 3 lists are not necessarily relevant to modeling that pertains to relevant physics. Table 9 does not include those not necessarily relevant items.

Regarding the notion of a reach, ρ_I, of two, there are three instances of the PROP solution. We number the isomers so that one instance of the 2g2‘4 solution intermediates interactions between isomer zero and isomer three. One instance of the 2g2‘4 solution intermediates interactions between isomer one and isomer four. One instance of the 2g2‘4 solution intermediates interactions between isomer two and isomer five.

We use notation of the form $\Sigma(\rho_I)$gΓ to denote a ΣgΓ solution and the reach ρ_I that associates with one modeling use that features an instance of the solution. For example, 2(2)g2‘4 pertains regarding 2g2‘4. We extend use of such notation to non-LRI elementary particles. For non-LRI elementary particles, the reach is one and notation of the form $S(1)\Phi$ pertains.

We assume that - for each Σ(2)gΓ solution - one instance of the solution intermediates interactions between isomer zero and isomer three. One instance of the solution intermediates interactions between isomer one and isomer four. One instance of the solution intermediates interactions between isomer two and isomer five.

2.2.5. We start to discuss the extent to which the properties of any one isomer's elementary particles differ from the properties of elementary particles that associate with other isomers.

If the stuff that associates with each of the five all-dark-matter isomers evolved similarly to ordinary matter, our suggestions regarding dark matter might not adequately comport with observations regarding the Bullet Cluster collision of two galaxy clusters. Elsewhere, we suggest that the isomers of ELF elementary particles differ sufficiently that our suggestions regarding dark matter do not necessarily disagree with observations pertaining to the Bullet Cluster. (Perhaps, preview discussion related to table 17.)

Table 9: Reaches and other information regarding some solutions that associate with electromagnetism, gravity, 3L, and 4L. ρ_I denotes reach. $\Sigma \in \Gamma$ associates with the symbol g'. $\Sigma \notin \Gamma$ associates with the symbol g". TBD denotes to be determined. NYN denotes not yet named. Discussion related to table 22 provides information regarding the notion of anomalous magnetic moment.

S	Σ	PROP solution	ρ_I	Solution type	PROP RDF	Properties or other associations
1	1	1g1	1	$\Sigma \in \Gamma$	Ξ^{-2}	Charge
1	1	1g1'2	1	$\Sigma \in \Gamma$	Ξ^{-3}	Magnetic moment
1	1	1g1'2'4	6	$\Sigma \in \Gamma$	Ξ^{-4}	Rotating (axis of) magnetic moment
1	1	1g1'4'6	2	$\Sigma \in \Gamma$	Ξ^{-4}	Charge, rotation, and velocity
1	1	1g1'4'6'8	2	$\Sigma \in \Gamma$	Ξ^{-5}	Charge, rotation, and angular velocity
1	1	1g1'2'4'6'8x	6	$\Sigma \in \Gamma$	Ξ^{-6}	Changes (re systems that include charged particles) re internal angular momenta
2	2	2g2	6	$\Sigma \in \Gamma$	Ξ^{-2}	Mass
2	2	2g2'4	2	$\Sigma \in \Gamma$	Ξ^{-3}	Rotating (spherically symmetric aspects of) mass
2	2	2g1'2'3	1	$\Sigma \in \Gamma$	Ξ^{-4}	Moments of inertia (stress-energy)
2	2	2g1'2'3'4v	1	$\Sigma \in \Gamma$	Ξ^{-5}	Rotation (associated with) moments of inertia
2	2	2g1'2'3'4w	1	$\Sigma \in \Gamma$	Ξ^{-5}	Rotation (associated with) moments of inertia
2	2	2g1'2'3'6'8x	6	$\Sigma \in \Gamma$	Ξ^{-6}	Stress-energy
2	2	2g1'2'3'4'6'8x	1	$\Sigma \in \Gamma$	Ξ^{-7}	Changes re stress-energy
3	3	3g3	2	$\Sigma \in \Gamma$	Ξ^{-2}	TBD (a function of elementary fermion flavour), NYN
3	3	3g1'2	1	$\Sigma \notin \Gamma$	Ξ^{-3}	Anomalous magnetic moment
3	3	3g1'2'4	6	$\Sigma \notin \Gamma$	Ξ^{-4}	Anomalous magnetic moment
3	3	3g1'2'6	2	$\Sigma \notin \Gamma$	Ξ^{-4}	Anomalous magnetic moment
4	4	4g4	1	$\Sigma \in \Gamma$	Ξ^{-2}	Elementary particle angular momentum (scalar quantity)
4	4	4g1'2'3	1	$\Sigma \notin \Gamma$	Ξ^{-4}	Anomalous gravitational property

2.2.6. We discuss notions regarding excitation and de-excitation of LRI fields (or, of ΣL elementary particles).

An excitation associates with a value of Σ and with a set of isomers. For example, consider an excitation that associates with active-gravitational properties of an ordinary matter star. The word active associates with the notion that the star generates gravity. The word gravitational associates with $\Sigma = 2$. The excitation might associate with the 2g2 solution, with the 2g2‘4 solution, or with another 2g solution. Because the star consists just of ordinary matter stuff, the set of isomers consists just of isomer zero.

A de-excitation associates with the notion of passive properties, with any same-Σ ΣL solution and with a set of isomers that associates with the original excitation. We continue the previous example. Regarding $\Sigma = 2$, the word passive associates with the notion that an object interacts with gravity that other objects actively produce. Because 2g2 has a reach of six, any object can de-excite, via 2g2, the excitation that the example features. Because 2g1‘2‘3 has a reach of one, only isomer zero stuff can de-excite, via 2g1‘2‘3, the excitation. Because 2g2‘4 has a reach of two, only isomer zero stuff or isomer three stuff can de-excite the excitation via 2g2‘4.

Generally, ten types of de-excitations exist. One type consists of de-excitations that associate with reach-six solutions. Three types consist of de-excitations that associate with reach-two solutions. One of the three types associates with isomer zero and isomer three. Another one of the three types associates with isomer one and isomer four. The other one of the three types associates with isomer two and isomer five. Six types consist of de-excitations that associate with reach-one solutions. Each one of the six types associates with exactly one isomer.

We discuss excitations and de-excitations that associate with long-range forces produced by a galaxy that consists of stuff that associates substantially with each of the six isomers. (Perhaps, preview table 21.) We discuss electromagnetism. A one-isomer distant observer would directly sense mostly aspects that associate with the distant observer's isomer. Here, most detection of photons would associate with the reach-one solutions 1g1 and 1g1‘2. (The observer might infer - from sensed data - aspects regarding the other five isomers and aspects regarding gravity.) Via aspects that associate with solutions such as 1g1‘2‘4 (for which the reach is six) and 1g1‘4‘6‘8 (for which the reach is two), the one-isomer observer might directly sense aspects that associate with isomers other than the observer's isomer. (For example, the observer might sense hyperfine transitions associating with one isomer other than the observer's isomer. Here, 1g(2)1‘4‘6‘8 might be relevant. The observer might identify hyperfine spectrum lines. The observer would not necessarily know directly the extent to which each of the two relevant isomers contributed to the data the observer collects.) We discuss gravitation. We assume that the observer has adequate means for directly detecting gravity that the galaxy produces. The one-isomer distant observer would sense all of the galaxy's stuff via 2g2 (for which the reach is six). But the sensing of subtleties (such as rotation of stuff or irregular distributions of stuff) might associate with solutions (such as 2g2‘4 and 2g1‘2‘3) for which the reaches are less than six. (Perhaps, see table 9.) The one-isomer observer would not necessarily sense directly via 2L all of the subtleties.

3. Results

This unit discusses explanations for known data and discusses suggestions regarding possible data that people have not yet measured. The discussion includes explanations and suggestions regarding elementary particles, dark matter, galaxies, and the cosmos.

3.1. Elementary particles

This unit lists elementary particles that associate with our modeling and discusses relationships between properties of elementary particles.

3.1.1. We list all elementary particles of which people know or that we suggest.

Table 10 consolidates and summarizes information about all elementary particles of which people know or that this essay suggests. (See table 3, table 7, and table 8.)

3.1.2. We explore relationships among properties of objects, elementary particles, and long-range interactions.

Table 11 discusses relationships between properties of elementary bosons. (Regarding the masses of the Higgs, Z, and W bosons, we used data that reference [8] provides.)

Table 11 points to possibly deeper (than people might otherwise suggest) relationships between the physics properties of spin, mass, and charge. (Also, regarding the non-zero-mass elementary bosons, a notion that non-zero spin might associate with - in effect - reduction in mass seems not to be incompatible with discussion related to table 5.)

We mention the following notions regarding anomalous properties. (See discussion related to table 4.) The notion - for the Higgs, Z, and W bosons - that l_{ms} is not zero might associate with a notion of anomalous property. To the extents that the proportionalities $(m_{\mathrm{Higgs}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{W}})^2$ do

Table 10: Elementary particles. The symbol Q associates with magnitude of charge. The columns labeled $Q > 0$ and $Q = 0$ have entries in the form of a name of one particle or a name of a set of more than one particle, followed (in parentheses) by a number of particles, followed by a symbol for the family of particles. NYN denotes not yet named. NYD denotes not yet detected. One might assert that people know of some NYD particles, at least indirectly. The word free associates with modeling that features PROP solutions for which $8 \notin \Gamma$. The word entwined associates with modeling that features PROP solutions for which $8 \in \Gamma$. For 1L, some modeling might associate with entwined. (See table 3 and discussion - regarding cascades - related to table 3.) For example, notions of entwined might pertain regarding electromagnetism within an atom or regarding light in a laser cavity. We associate the word mixed with ΣL for which some relevant components associate with PROP solutions for which $8 \notin \Gamma$ and some relevant components associate with PROP solutions for which $8 \in \Gamma$. For 2L, some modeling might associate with entwined. For example, notions of entwined might pertain regarding gravitation within a black hole. For each of 3L and 4L, notions of entwined might not pertain. (See discussion - regarding cascades - related to table 3.)

S	m	$Q > 0$	$Q = 0$	Status	Σ	Free / Entwined
0	>0	-	Higgs boson (1), 0H	Known	0	Free
1/2	>0	Charged leptons (3), $0.5\mathrm{C}_1$	Neutrinos (3), 0.5N	Known	0	Free
1/2	>0	-	Heavy neutrinos (3), 0.5M	NYD	0	Free
1	>0	W boson (1), $1\mathrm{W}_1$	Z boson (1), 1Z	Known	0	Free
1	=0	-	Jay boson (1), 1J	NYD	0	Free
1	=0	-	Photon (1), 1L	Known	1	Mixed
2	=0	-	Graviton (1), 2L	NYD	2	Mixed
3	=0	-	NYN (1), 3L	NYD	3	Free
4	=0	-	NYN (1), 4L	NYD	4	Free
0	=0	-	Aye boson (1), 0I	NYD	0	Entwined
1/2	>0	Quarks (3), $0.5\mathrm{Q}_{1/3}$	-	Known	0	Entwined
1/2	>0	Quarks (3), $0.5\mathrm{Q}_{2/3}$	-	Known	0	Entwined
1/2	>0	-	Arcs (3), 0.5R	NYD	0	Entwined
1	=0	-	Gluons (8), 1G	Known	0	Entwined

Table 11: Relationships between properties of elementary bosons. Q denotes the magnitude of charge, in units of $|q_e|$. m denotes mass, in units of $m_{\mathrm{Higgs}}/17^{1/2}$ or in units of $m_{\mathrm{Z}}/9^{1/2}$. S denotes spin, as in the expression $S(S+1)\hbar^2$. l_{ms} equals -1 for $m > 0$ and equals 0 for $m = 0$. The sum is the sum of the numbers in the preceding four columns. Each sum is the square of an integer. For each nonzero-mass particle, the integer equals $n_{\Gamma,\mathrm{PROP}}$. (See table 7.) There are no-nonzero mass elementary bosons for which the integer equals one or two. (For a Γ that includes just one value of l_o or that includes just two values of l_o, $\Sigma \neq 0$ pertains.) NYN denotes the three-word phrase not yet named. Of the non-zero masses to which table 11 alludes, the most accurately known mass is that of the Z boson. Using the mass of the Z boson and numbers in table 11, one can calculate a nominal mass for the Higgs boson and a nominal mass for the W boson. The calculated mass for the Higgs boson differs from the experimentally determined mass by less than two (experimental) standard deviations. The calculated mass for the W boson differs from the experimentally determined mass by less than four (experimental) standard deviations. To the extent that one uses the notion that ruling out an equality requires a difference of at least five standard deviations, experimental results do not seem to rule out relationships that table 11 states.

Bosons	Family	$Q(Q+1)$	m^2	S^2	l_{ms}	Sum
Higgs	0H	0	17	0	-1	16
Aye	0I	0	0	0	0	0
Z	1Z	0	9	1	-1	9
W	$1\mathrm{W}_1$	2	7	1	-1	9
Jay	1J	0	0	1	0	1
Gluons	1G	0	0	1	0	1
Photon	1L	0	0	1	0	1
Graviton	2L	0	0	4	0	4
NYN	3L	0	0	9	0	9
NYN	4L	0	0	16	0	16

not measure as exactly 17 : 9 : 7, anomalous properties might associate with variations from exactness. (Here, m denotes mass.)

We turn our attention to properties of elementary fermions.

We consider hypothetical elementary fermions for which $Q = 1$. For some value of mass, the gravitational attraction between two identical such hypothetical elementary fermions would equal the electrostatic repulsion between the two fermions. Our work shows that a mass - so-called $m(18, 3)$ - seems to have meaning beyond the notion that - for the mass $m(18, 3)$ - gravitational attraction between two $Q = 1$ identical elementary fermions would be three-quarters of the electrostatic repulsion between the two identical elementary fermions. (Perhaps, preview table 13 and table 16.)

Table 12 discusses relationships between properties of known charged elementary fermions. (Reference [8] provides the data that underlies table 12.)

Table 13 shows equations that underlie aspects of table 12. (Reference [8] provides the data that underlies table 13.)

3.1.3. We show modeling that might estimate the anomalous magnetic moment for the tau elementary particle.

We explore modeling regarding anomalous magnetic moments for $0.5\mathrm{C}_1$ elementary particles (or, charged leptons).

Table 4 associates two CURR solutions with the relevant (or, 3g1'2) PROP solution. The 3g1'2'6 CURR solution includes a 6 in Γ. We assume that the strength of 3g1'2'6 can vary based on elementary fermion flavour, but not based on charge. The 3g1'2'4 CURR solution does not include a 6 in Γ. We assume that the strength of 3g1'2'4 can vary based on charge, but not based on elementary fermion flavour.

We explore the notion that one can express a_{cl}, the anomalous magnetic moment for the cl charged lepton, via the expression $a_4 + a_6 t_{cl}$. Here, a_4 might vary only with charge and would be a constant with respect to a choice between $cl = e$ (for the electron), $cl = \mu$ (for the muon), and $cl = \tau$ (for the tau). Here, a_6 might vary only with fermion flavour. We assume that t_{cl} is $(\log(m_{\mathrm{cl}}/m_e))^2$. (Perhaps, compare with table 12 and with aspects - that comport with squares of properties - of table 13. The notion of squares of properties might associate with notions of self-interactions.) Based on data that reference [8] provides regarding the electron and the muon, we calculate a_4 and a_6. Then, we calculate a value, $a_{\tau,\mathrm{PM}}$, for a_τ. Here, PM denotes the two-word term proposed modeling. PM associates with our work. Reference [30] provides, based on Standard Model modeling techniques, a first-order result - which we call $a_{\tau,\mathrm{SM}}$ - for a_τ. Here, SM denotes the two-word term Standard Model. The value of $a_{\tau,\mathrm{PM}}$ results in a value of $(a_{\tau,\mathrm{PM}} - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ of approximately -0.00228. Each of $a_{\tau,\mathrm{PM}}$ and $a_{\tau,\mathrm{SM}}$ comports with experimental data that reference [8] provides.

Regarding anomalous magnetic moments, this essay does not explore quantifying aspects that associate with higher-order Standard Model terms or aspects that might associate with the PROP solutions 3g1'2'4 and 3g1'2'6. (Regarding the PROP solutions 3g1'2'4 and 3g1'2'6, see table 4.)

3.1.4. We discuss the masses of neutrinos.

Table 14 suggests rest energies that may pertain regarding the 0.5N neutrinos. This table extends aspects of table 12 and table 13. (Reference [8] provides data that underlies aspects of table 12, table 13, and table 14. Reference [12] discusses the notion of neutrino mass mixing.)

3.1.5. We discuss the masses of elementary fermions that we suggest.

Table 15 suggests rest energies that may pertain regarding the suggested 0.5R arcs. This table extends aspects of table 12 and table 13. (Reference [8] provides data that underlies aspects of table 12, table 13, and table 15.)

Table 12 and table 13 might point to possibly deeper (than people might otherwise suggest) relationships between the physics properties of mass and charge.

We explore two alternatives regarding values of $d'(0)$, $d'(1)$, and $d'(2)$. (See table 13.) Changing those numbers would impact the calculated masses for quarks and the calculated suggested masses for arcs. (Changing those numbers would not impact the calculated masses for charged leptons.) Regarding each of the two alternatives, if one excludes one of three methods for estimating the mass of the top quark, the calculated mass for each of the six quarks is within five standard deviations of the experimental mass. (Reference [8] discusses the three methods.) For the third method for estimating the mass of the top quark, the value that we calculate for the mass of the top quark would be less than eleven standard deviations below the mass people have calculated.

One alternative has bases in the notions of $d'(-1) = 0^2/2^2$, $d'(0) = 1^2/2^2$, $d'(1) = -2^2/2^2$, and $d'(2) = -(2 \times 3)/2^2$. For this alternative, the three arc rest energies would, respectively, be ≈ 8.14 MeV, $m(1, 3)c^2$, and $m(2, 3)c^2$.

The other alternative has bases in the notions of $d'(0) \approx 0.264825$, $d'(1) = -2^2/2^2$, and $d'(2) = -(2 \times 3)/2^2$. For this alternative, the three arc rest energies would, respectively, equal $m(1, 3)c^2$, $m(1, 3)c^2$, and $m(2, 3)c^2$. Across the three $0.5\mathrm{C}_1$ elementary fermions and the three 0.5R elementary fermions, $m(0, 3)c^2$ would pertain once, $m(1, 3)c^2$

Table 12: Values of $\log_{10}(m_{\text{particle}}/m_e)$ for known charged elementary fermions. Regarding "flavour," this table generalizes, based on terminology that associates with charged leptons and with neutrinos. For example, people use the term electron-neutrino. The symbol l_f numbers the three flavours. The "l_f (0.5C$_1$)" terms pertain for fermions in the 0.5C$_1$ family. The symbol 0.5Q$_{>0}$ denotes the pair 0.5Q$_{1/3}$ and 0.5Q$_{2/3}$. The "l_f (0.5Q$_{>0}$)" terms pertain for quarks (or, elementary particles in the two families 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$). l_m is an integer parameter. The domain $-6 \leq l_m \leq 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. The family 0.5C$_1$ associates with $Q = 1$. The family 0.5Q$_{2/3}$ associates with $Q = 2/3$. The family 0.5Q$_{1/3}$ associates with $Q = 1/3$. Regarding the rightmost four columns, items show $\log_{10}(m_{\text{particle}}/m_e)$ and - for particles that nature includes - the name of an elementary fermion. For each † case, no particle pertains. Each number in the column with label $Q = 1/2$ equals the average of the number in the $Q = 2/3$ column and the number in the $Q = 1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q = 2/3$ particle and the mass of the $Q = 1/3$ particle. Regarding each † case, a formula for $m(l_m, l_q)$ calculates this number. Regarding the formula, the domain $0 \leq l_q \leq 3$ pertains. Regarding table 12, $l_q = 3Q$ pertains. Table 13 shows the formula.

l_f (0.5C$_1$)	l_f (0.5Q$_{>0}$)	l_m	$Q = 1$	$Q = 2/3$	$Q = 1/2$	$Q = 1/3$
1 (Electron)	1 (Up, Down)	0	0.00 Electron	0.66 Up	0.80 †	0.94 Down
-	2 (Charm, Strange)	1	1.23 †	3.36 Charm	2.83 †	2.29 Strange
2 (Mu)	3 (Top, Bottom)	2	2.32 Muon	5.52 Top	4.72 †	3.92 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

Table 13: Equations that underlie aspects of table 12. This table shows equations that may pertain regarding all known charged elementary fermions, the known 0.5N neutrinos, and the suggested 0.5R arcs. (Regarding 0.5N neutrinos, perhaps preview table 14. Regarding 0.5R arcs, perhaps preview table 15.)

Topic	Note
Preliminary calculation	
	$\beta' = m_\tau/m_e$ - Defines β'. m_τ equals the mass of the tau particle (which is a charged lepton). m_e equals the mass of the electron.
	$(4/3) \times (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2)$ - Defines β. The right-hand side of the equation is the ratio of the electrostatic repulsion between two electrons to the gravitational attraction between the two electrons. The ratio does not depend on the distance between the two electrons.
	$\beta \approx 3477.1891 \pm 0.0226$ - This number results from data and the formula that defines β. The standard deviation reflects the standard deviation for G_N, the gravitational constant.
	$\beta' = \beta$ - We posit this equation.
	$m_{\tau,\text{ calculated}} \approx 1776.8400 \pm 0.0115$ MeV$/c^2$ - This number results from data and from $\beta' = \beta$.
Main calculation	
	These calculations produce numbers that table 12 shows.
	$l_q = 3Q$.
	$m(l_m, l_q) = m_e \times (\beta^{1/3})^{l_m + (j''_{l_m})d''} \times (\alpha^{-1/4})^{g(l_q)\cdot(1+l_m)+j'_{l_q}d'(l_m))}$.
	$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c)$ - Expression for α, the fine-structure constant.
	$j''_{l_m} = 0, +1, 0, -1$ for, respectively, $l_m \bmod 3 =$ $0, 1, 3/2, 2$; with $3/2 \bmod 3 \equiv 3/2$.
	$d'' = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840679 \times 10^{-2}$.
	$g(l_q) = 0, 3/2, 3/2, 3/2, 3/2$, for, respectively, $l_q = 3, 2, 3/2, 1, 0$.
	$j'_{l_q} = 0, -1, 0, +1, +3$ for, respectively, $l_q = 3, 2, 3/2, 1, 0$.
	$d'(0) \sim 0.324$, $d'(1) \sim -1.062$, $d'(2) \sim -1.509$ - Based on attempting to fit data.

Table 14: Rest energies that may pertain regarding the 0.5N neutrinos.

Topic	Note
$l_m = -1$	$m(-1,3) = m(-1,3/2)$ - Comports with the equation underlying the main calculation regarding the masses of charged elementary fermions.
Assumption	$m(l_m,3/2)$ pertains - regarding elementary fermions - for $l_m \leq -1$.
Neutrinos	We suggest masses for the three 0.5N neutrinos. People suggest - based on observations - that the sum of the three neutrino rest energies is at least approximately $0.06\mathrm{eV}/c^2$ and not more than approximately $0.12\mathrm{eV}/c^2$. We note two possibilities. • $mc^2 = m(-4,3/2)c^2 \approx 3.4 \times 10^{-2}$ eV pertains for each of the three neutrinos. • $mc^2 = m(-4,3/2)c^2 \approx 3.4 \times 10^{-2}$ eV pertains for each of two neutrinos. For one neutrino, one of $m(-6,3/2)c^2 \approx 4.2 \times 10^{-6}$ eV and $m(-5,3/2)c^2 \approx 4.4 \times 10^{-4}$ eV might pertain.
Neutrinos	We suggest aspects regarding possible differences between mass eigenstates and interaction eigenstates for the three 0.5N neutrinos. Regarding interactions between some ΣL and an elementary fermion, the following notions pertain. Interactions between 2L and an elementary fermion conserve the mass of the elementary fermion, but do not necessarily conserve the flavour of the elementary fermion. Interactions between 3L and an elementary fermion conserve the flavour of the elementary fermion, but do not necessarily conserve the mass of the elementary fermion. Interactions between 4L and an elementary fermion do not necessarily conserve the mass of the elementary fermion or the flavour of the elementary fermion. Standard Model notions of mass mixing might associate with effects that associate with the 4g" PROP solution 4g1'2'3 and some of the CURR solutions 4g1'2'3'4x, 4g1'2'3'6, and 4g1'2'3'8. (See table 4 and table 9.) A QFT (or, quantum field theory) that includes gravity might - within a limit (regarding ΣL) of $\Sigma \leq 2$ - associate mass mixing with a notion of anomalous gravitational properties and might successfully estimate aspects that people associate with mass mixing.

Table 15: Rest energies that may pertain regarding the suggested 0.5R arcs.

Topic	Note
Arcs	Our work suggests (but does not necessarily require) some specific masses for the three arc particles. $l_q = 0$ - This notion comports with the notion - for arcs - that $Q = 0$. $m(l_m,0) = m(l_m,1) \cdot (m(l_m,1)/m(l_m,2))$ - This essay assumes this equation. $m(0,0)c^2 \approx 10.7$ MeV, $m(1,0)c^2 \approx 6.8$ MeV, $m(2,0)c^2 \approx 102$ MeV.

would pertain twice, $m(2,3)c^2$ would pertain twice, and $m(3,3)c^2$ would pertain once.

We discuss possible masses for heavy neutrinos.

For purposes of estimating or calculating masses, neutrinos associate with a value of l_m for which $-6 \leq l_m \leq -3$. Charged leptons associate with $0 \leq l_m \leq 3$. If heavy neutrinos associate with $6 \leq l_m \leq 9$, a lower bound on rest energies for heavy neutrinos might be $m(6,3)c^2 \sim 6 \times 10^3$GeV, which might be large enough to comport with limits that associate with observations. (References [31] and [32] discuss limits that observations may set. People have not detected 0.5M particles.) To the extent the lower bound associates with $m(6,3/2)c^2$, the lower bound would be $\sim 2.5 \times 10^9$GeV.

3.1.6. We discuss a possible limit regarding the spins of elementary particles that intermediate long-range interactions.

Table 16 suggests the possibility that - for LRI elementary particles ΣL - Σ might be no greater than four.

A limit - for LRI elementary particles ΣL - of $\Sigma \leq 4$ seems to be consistent with other aspects of our modeling.

3.2. Dark matter

This unit suggests specifications for dark matter.

3.2.1. We discuss - for the six isomers - elementary-fermion masses, flavours, and handedness.

Regarding each l_I that is at least one, we assume that the elementary particles in isomer l_I match - with respect to mass - the elementary particles in isomer zero.

For $0 \leq l_I \leq 5$, we associate the quarks in isomer l_I with three values of l_m. (See table 12 and table 13.) The values are $3l_I+0$, $3l_I+1$, and $3l_I+2$. Across the six isomers, quarks associate with each value of l_m that is in the range $0 \leq l_m \leq 17$. Regarding quarks and flavours, we assume that - within isomer l_I - flavour 1 associates with $l_m = 3l_I$, flavour 2 associates with $l_m = 3l_I + 1$, and flavour 3 associates with $l_m = 3l_I + 2$.

Aspects of table 12 and table 13 point to the possibility that means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer zero does not have a charged lepton that associates with $l_m = 1$ and does have a charged lepton that associates with $l_m = 3$. We assume that - for each l_I - a charged lepton associates with each of $l_m = 3l_I + 0$, $l_m = 3l_I + 2$, and $l_m = 3l_I + 3$.

We assume that - for each isomer l_I such that $1 \leq l_I \leq 5$ - the charged-lepton flavour that associates with $l_m = 3(l_I) + 0$ equals the flavour that associates with the isomer $l_I - 1$ charged lepton that associates with the same value of l_m and - thus - with $l_m = 3(l_I - 1) + 3$. We assume that across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table 17 shows, for isomers of charged elementary fermions, matches between masses and flavours.

Beyond the topic of flavours, the topic of handedness exists. Ordinary matter associates with left-handedness. Our modeling suggests the possibility that isomers 0, 2, and 4 associate with left-handedness and that isomers 1, 3, and 5 associate with right-handedness.

Figure 9 interrelates elementary particles, isomers, ordinary matter, and dark matter.

3.2.2. We prepare to discuss the evolution of stuff that associates with each isomer.

We associate the symbol OMSE with all SRI elementary particles and all ELF elementary particles except 0.5M and 0.5R elementary particles. OMSE abbreviates the three-element phrase ordinary-matter-similar elementary particles. We associate the symbol DMAI with the 0.5M and 0.5R elementary particles. DMAI abbreviates the five-word phrase dark matter regarding all isomers. DMAI associates with the notion that - regarding isomer zero - these particles measure as being dark matter and do not measure as being ordinary matter.

We use the three-element term isomer *number* stuff to denote objects (including SRI elementary particles, ELF elementary particles, hadron-like particles, clumps of stuff, and stars) that associate with the isomer *number* set of simple elementary particles.

0.5R particles model as entwined. (See table 8.) We suggest that - at least after the inflationary epoch - 0.5R-based stuff consists of hadron-like particles. Each 0.5R-based-stuff hadron-like particle includes gluons and at least two arcs. (We de-emphasize discussing roles that jay bosons might play.) Our work does not suggest an extent to which 0.5R-based stuff might form primordial black holes. Our work does not necessarily suggest that a two-or-three-hadron hadron-like particle can include both at least one quark and at least one arc.

0.5M particles model as free. (See table 8.) Our work does not suggest an extent to which 0.5M-based stuff might form primordial black holes.

Regarding each one of the six isomers, we suggest that stuff made from DMAI behaves within bounds for dark matter that associate with concordance cosmology.

Table 16: The possibility that - for LRI elementary particles ΣL - Σ might be no greater than four.

Topic	Note
$l_m = 18$	$((q_e)^2/(4\pi\varepsilon_0))/(G_N(m(18,3))^2) = 4/3$.
Monopole properties	A force strength factor of 4 seems to associate with 1g1 and a force strength factor of 3 seems to associate with 2g2. (See, above, the equation $(4/3) \times (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2)$.) Possibly, other force strength factors would be 2 for 3g3, 1 for 4g4, and 0 (or, zero) for 5g5. Possibly, the notion of zero force strength regarding 5g5 associates with a lack of relevance for (and a lack of monopole properties that would associate with) solutions ΣgΣ for which $\Sigma \geq 5$ and with a lack of LRI elementary particles ΣL for which $\Sigma \geq 5$.

Table 17: Matches between masses and flavours, for isomers of charged elementary fermions. The symbol $0.5\mathrm{Q}_{>0}$ denotes the pair $0.5\mathrm{Q}_{1/3}$ and $0.5\mathrm{Q}_{2/3}$. The symbol l_f numbers the three flavours. (See table 12.)

Isomer	l_m ($0.5\mathrm{Q}_{>0}$)	Respective l_f ($0.5\mathrm{Q}_{>0}$)	l_m ($0.5\mathrm{C}_1$)	Respective l_f ($0.5\mathrm{C}_1$)
0	$0, 1, 2$	1,2,3	$0, 2, 3$	1,2,3
1	$3, 4, 5$	1,2,3	$3, 5, 6$	3,1,2
2	$6, 7, 8$	1,2,3	$6, 8, 9$	2,3,1
3	$9, 10, 11$	1,2,3	$9, 11, 12$	1,2,3
4	$12, 13, 14$	1,2,3	$12, 14, 15$	3,1,2
5	$15, 16, 17$	1,2,3	$15, 17, 18$	2,3,1

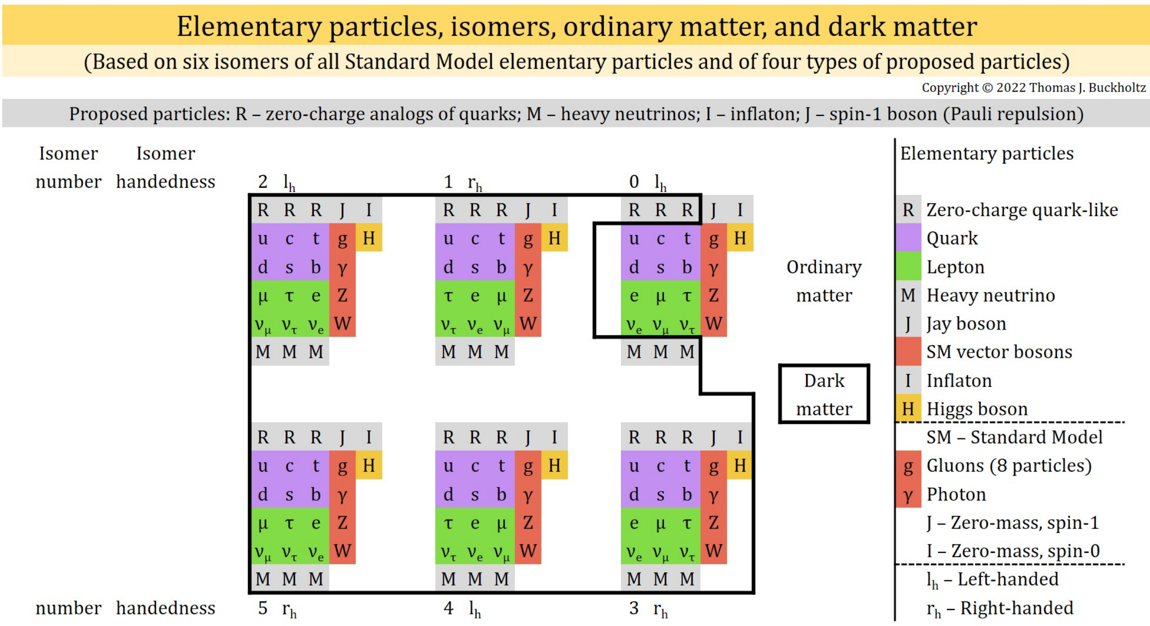

Figure 9: Elementary particles, isomers, ordinary matter, and dark matter. For counterpart elementary particles (across the six isomers), the masses are the same. Here, the word counterpart refers to positions in the six similar arrays of symbols for elementary particles. For counterpart elementary particles, the magnitudes of the charges are the same. For counterpart leptons, the flavours are not necessarily the same. For each known elementary particle, this figure uses popular symbols. Here, the lower-case letter g associates with gluons. Here, the symbol γ associates with the photon. The figure de-emphasizes the would-be graviton (or, 2L particle), the might-be 3L particle, and the might-be 4L particle. The figure de-emphasizes components - of the photon (or, 1L particle) for which the reaches exceed one.

3.2.3. *We discuss - for each dark matter isomer - the evolution of stuff that associates with that isomer.*

We discuss the evolution of isomer 1, 2, 4, and 5 OMSE stuff.

Here, we use the two-word term alt isomer to designate an isomer other than isomer zero and isomer three.

A charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, two tops and a bottom have a larger total mass than do one top and two bottoms.) Alt isomer flavour 3 charged leptons are less massive than isomer zero flavour 3 charged leptons. When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the alt isomer converts more charged baryons to zero-charge baryons than does isomer zero. Eventually, in the alt isomer, interactions that entangle multiple W bosons result in the alt isomer having more neutrons and fewer protons than does isomer zero. The sum of the mass of a proton and the mass of an alt isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer zero neutrons, alt isomer neutrons scarcely decay. The IGM (or, intergalactic medium) that associates with the alt isomer scarcely interacts with itself via electromagnetism.

We discuss the evolution of isomer three OMSE stuff.

The following two possibilities pertain. The evolution of isomer three OMSE stuff parallels the evolution of ordinary matter (or, isomer zero OMSE stuff). The evolution of isomer three OMSE stuff does not parallel the evolution of ordinary matter (or, isomer zero OMSE stuff). The second possibility might associate with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer three and isomer zero. (See discussion related to table 17.)

3.3. *Formation and evolution of the universe*

This unit suggests eras - two of which would precede cosmic inflation - in the rate of expansion of the universe and suggests mechanisms that associate with the eras.

3.3.1. *We discuss perspective regarding the rate of expansion of the universe.*

Concordance cosmology points to three eras in the so-called rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

This essay suggests using the notion of eras regarding the separating from each other of clumps - that, today, people would consider to be large - of stuff. Examples of such clumps might include galaxy clusters and possibly even larger clumps.

3.3.2. *We provide perspective regarding long-range interactions between objects.*

As two objects move away from each other, the relative effect of an RDF $\Xi^{-(k+1)}$ component decreases compared to the effect of an RDF Ξ^{-k} component. One might associate the two-word phrase time period with a time range in which an RDF Ξ^{-l_r} component provides dominant effects. Assuming that objects move away from each other and that one time period associates with $\Xi^{-(k+1)}$ and another time period associates with Ξ^{-k}, the time period that associates with $\Xi^{-(k+1)}$ comes before the time period that associates with Ξ^{-k}. Two smaller objects (such as galaxies) transit similar time periods more quickly than do two larger objects (such as galaxy clusters).

3.3.3. *We discuss known and suggested eras in the history of the universe.*

Table 18 discusses eras in the rate of separating of large clumps. (For discussion about the possible inflationary epoch, see references [33], [10], and [21]. For data and discussion about the two multi-billion-years eras, see references [34], [35], [36], and [37].)

Table 19 suggests details regarding eras to which table 18 alludes.

Figure 10 interrelates isomers of elementary particles, components of gravity, eras in the evolution of the universe, and eras in the evolution of galaxies. (Regarding galaxies, perhaps preview discussion related to table 20.)

3.4. *Formation and evolution of galaxies*

This unit suggests that our notions regarding long-range interactions and our specifications for dark matter combine to provide insight regarding galaxy formation and galaxy evolution.

3.4.1. *We suggest aspects regarding events leading to the formation of a galaxy.*

Reference [38] suggests that galaxies form around early clumps of stuff. The reference associates the word halo with such clumps.

Table 18 suggests that single-isomer stuff - such as stuff that features 0.5R particles - forms during an era in which PROP solutions 2g1‘2‘3‘6‘8x - which associate with attraction - dominate regarding prototype large clumps. Smaller-scale clumps might form before larger-scale clumps. Effects that associate with the PROP solution 2g1‘2‘3 - which is attractive might contribute to the formation of

Table 18: Eras regarding the rate of separating of large clumps. The rightmost two columns suggest eras. (Table 19 discusses aspects that associate with each of some eras.) In table 18, subsequent rows associate with later eras. The word inflation names the era that associates with the third row in the table. Regarding eras that would precede inflation, our modeling points to the possibility for the two eras that the table discusses. Concordance cosmology suggests inflation and the next two eras. Regarding inflation, people hypothesize this era. People suggest that the inflationary era started about 10^{-36} seconds after the Big Bang. People suggest that the inflationary era ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for this era. Observations support the notions of the two billions-of-years eras. TBD denotes to be determined. The symbol † denotes a possible association between the relevant era and the notion of a Big Bang. The leftmost four columns describe phenomena that our modeling suggests as noteworthy causes for the eras. (Regarding phenomena that associate with gravitation, table 18 echoes aspects - including aspects regarding attraction and repulsion - that table 5 and table 9 show.) An RDF associates with the PROP solutions. Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before an era starts.

Force	PROP solutions	RDF	ρ_I	$\rightarrow$	Rate of separating	Duration
Attractive	2g1‘2‘3‘6‘8x	Ξ^{-6}	6	$\rightarrow$	Is negative	TBD
Repulsive	0g1‘3‘4‘6	-	1	$\rightarrow$	Turns positive †	TBD
Repulsive	2g1‘2‘3‘4x	Ξ^{-5}	1	$\rightarrow$	Increases rapidly	Fraction of a second
Attractive	2g1‘2‘3	Ξ^{-4}	1	$\rightarrow$	Decreases	Billions of years
Repulsive	2g2‘4	Ξ^{-3}	2	$\rightarrow$	Increases	Billions of years
Attractive	2g2	Ξ^{-2}	6	$\rightarrow$	Would decrease	-

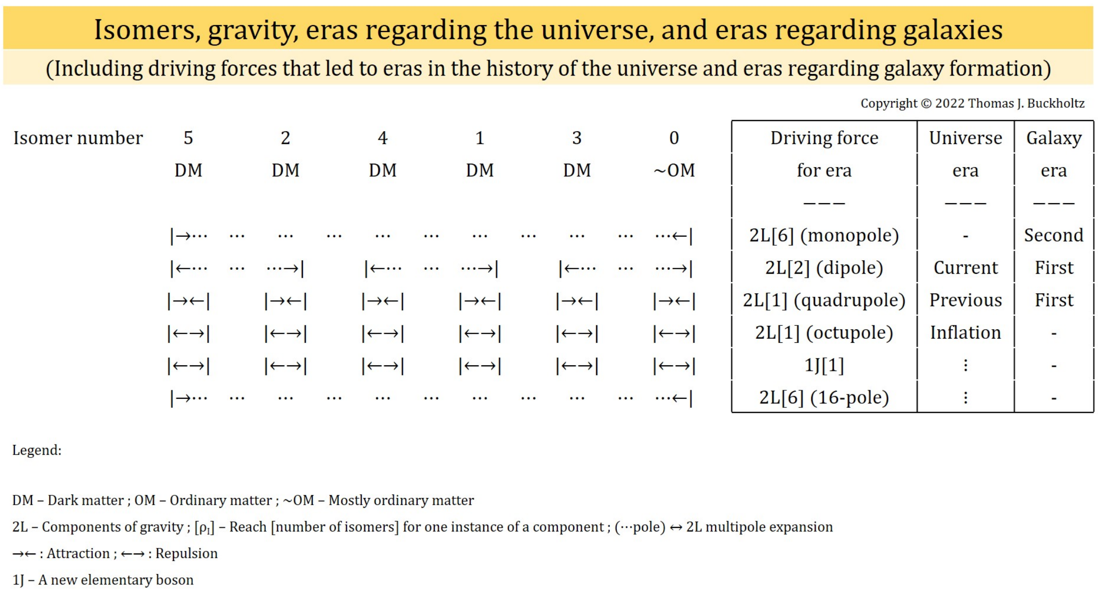

Figure 10: Isomers of elementary particles, components of gravity, eras in the evolution of the universe, and eras in the evolution of galaxies. Some current galaxies did not transit beyond the first era regarding the evolution of galaxies.

Table 19: Details regarding eras regarding the rate of separating of large clumps. Table 18 discusses the eras. Each of the symbols 2g1‘2‘3‘4x and 2g1‘2‘3‘4y denotes either or both of 2g1‘2‘3‘4v and 2g1‘2‘3‘4w. Our work does not necessarily specify the elementary fermions for which isomers form during the era that associates with the two-word phrase is negative. To the extent that the first significant appearance of most known elementary particles occurs during or just after the inflationary era, our work suggests that isomers of at least one of 0.5M and 0.5R form during the era that associates with the two-word phrase is negative. The symbol † associates with some aspects for which the involvement of 0.5M or 0.5R might pertain. We base 2L-aspects of table 19 on table 9. Other 2gΓ solutions might also pertain. (Perhaps, contrast table 3 with table 9.)

Rate of separating	Note
Is negative	Possibility: 2g1‘2‘3‘6‘8x and their compacting of “some form of energy” lead to conditions suitable for the universe to form and evolve. Possibility: The value of six for ρ_I (for 2g1‘2‘3‘6‘8x) associates with setting up a system for which roughly equal creation of isomers pertains. Possibility: Isomers of some elementary fermions and of 1J form. † Possibility: The following interactions might characterize this era. For each interaction, the net circular polarization for each of before and after the interaction might be zero. † Presumably, the formation of gluons (or, 1(1)G) could associate with the formation of arcs (or, 0.5(1)R)). • $2(6)\text{g}1‘2‘3‘6‘8\text{x} + 2(6)\text{g}1‘2‘3‘6‘8\text{x} \rightarrow 0.5(1)\text{M} + 0.5(1)\text{M}$. • $2(6)\text{g}1‘2‘3‘6‘8\text{x} + 2(6)\text{g}1‘2‘3‘6‘8\text{x} \rightarrow 0.5(1)\text{R} + 0.5(1)\text{R}$. • $2(6)\text{g}1‘2‘3‘6‘8\text{x} + 2(6)\text{g}1‘2‘3‘6‘8\text{x} \rightarrow 1(1)\text{J} + 1(1)\text{J}$. Possibility: The six isomers of the relevant elementary fermions populate approximately equally. † Possibility: Some clumps of relevant elementary fermion stuff serve - eventually - as seeds for galaxies. †
Turns positive	0g1‘3‘4‘6 associates with the 1J (or, jay) boson. The jay boson associates with the notion of Pauli repulsion. Possibility: 1J bosons stop the implosion of stuff that features relevant elementary fermion particles. † Possibility: Isomers of 0I form. The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction might be two. $1(1)\text{J} + 1(1)\text{J} \rightarrow 2(1)\text{g}1‘2‘3‘4\text{x} + 0(1)\text{I}$. Possibility: The six isomers of 0I populate approximately equally. Possibility: Aspects of this era associate with notions of a Big Bang.
Increases rapidly	Some concordance cosmology modeling suggests that inflatons provide a major component of stuff. Possibility: The following interaction might characterize this era. Here, the net circular polarization for each of before and after the interaction might be two. $0(1)\text{I} + 2(1)\text{g}1‘2‘3‘4\text{x} \rightarrow 0(1)\text{I} + 2(1)\text{g}1‘2‘3‘4\text{y}$.
Decreases	-
Increases	-
Would decrease	This essay does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repulsion based on 2g2‘4 to attraction based on 2g2 might occur.

smaller-scale clumps. The reach that associates with 2g1'2'3 is one.

We suggest that each one of many early halos associates with one isomer. We associate with such early halos the three-element term one-isomer original clump. We know of no reason why the six isomers would not form such clumps approximately equally. (Some concordance cosmology suggests that known elementary fermions form early in the era in which effects that associate with 2g1'2'3 dominate regarding large-scale phenomena. Per remarks above, we suggest that that era starts after the formation of halos. Also, we suggest that our scenario does not depend on whether or when 0.5M particles first form.)

Table 20 discusses suggestions regarding the formation and early evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Presumably, some galaxies form based on two or more clumps, for which all of the clumps associate with just one isomer. Presumably, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

3.4.2. We suggest aspects regarding the evolution of galaxies.

We suggest two eras regarding the evolution of galaxies. The first era associates with the first two rows in table 20. The second era associates with the 2g2 attractive force that associates with the third row in table 20.

Some galaxies do not exit the first era and do not significantly collide with other galaxies.

Many galaxies result from aspects associating with the 2g2 attractive force that associates with the third row in table 20. We discuss three cases. (Mixed cases and other cases might pertain.)

- Each of some era one galaxies does not collide with other galaxies. Such a galaxy accumulates (via 2g2 attraction) stuff associating with various isomers that have representation in nearby IGM (or, intergalactic medium). The galaxy becomes an era two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each of some era two galaxies merges (via 2g2 attraction) mainly just with galaxies that feature the same five isomers. The galaxy that merged, in effect, loses it status of being a galaxy. The resulting larger object is an era two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each of some era one or era two galaxies merges (via 2g2 attraction) with other galaxies. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era two galaxy. The galaxy might include stuff that significantly associates with as many as six isomers.

3.4.3. We suggest an explanation for the quenching of star formation within some galaxies and the stopping of the accrual of matter by some galaxies.

People report the notion that some galaxies seem to stop forming stars. (See reference [39] and reference [40].) Such so-called quenching might take place within three billion years after the Big Bang, might associate with a relative lack of hydrogen atoms, and might pertain to half of a certain type of galaxy. (See reference [40].) Reference [41] discusses a galaxy that seems to have stopped accruing both ordinary matter and dark matter about four billion years after the Big Bang.

We suggest that the quenching and the stopping of accruing nearby matter might associate with repulsion that associates with 2(2)g2'4. Quenching might associate with galaxies for which original clumps featured isomer zero stuff or isomer three stuff. The galaxy that reference [41] discusses might (or might not) associate with the notion of significant presence early on of one of isomers zero and three, one of isomers one and four, and one of isomers two and five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

3.4.4. We suggest an explanation for some data regarding stellar stream GD-1 in the Milky Way galaxy.

Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-ordinary-matter clump - in the Milky Way galaxy - with a mass of 10^6 to 10^8 solar masses. (For data and discussion regarding the undetected object, see references [42] and [43].) We suggest that the undetected object might be a clump of dark matter.

3.5. Ratios of dark matter effects to ordinary matter effects

This unit shows that our specification for dark matter seems to explain observed ratios of dark matter effects to ordinary matter effects.

Table 21 provides explanations for observed ratios of dark matter effects to ordinary matter effects. (For data and discussion regarding densities of the universe, see reference [8]. For data and discussion regarding galaxy clusters, see references [44], [45], [46], and [47]. For data and discussion regarding absorption of CMB, see references [48], [49], and [50]. For data and discussion regarding observed early galaxies, see references [51]

Table 20: Stages and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests stages, with subsequent rows associating with later stages. The rightmost one column describes aspects of the stage. The leftmost four columns in the table describe a component of 2L that is a noteworthy cause for the stage. (Regarding phenomena that associate with gravitation, table 20 echoes aspects - including aspects regarding attraction and repulsion - that table 5 and table 9 show.) The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before a stage starts. Table 20 associates with a scenario in which a galaxy forms based on one original clump and does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer.

Force	PROP solution	RDF	ρ_I	$\rightarrow$	Aspects of the stage
Attractive	2g1'2'3	Ξ^{-4}	1	$\rightarrow$	A one-isomer original clump forms.
Repulsive	2g2'4	Ξ^{-3}	2	$\rightarrow$	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.
Attractive	2g2	Ξ^{-2}	6	$\rightarrow$	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.

and [52]. Reference [51] influenced our choice of a time range to associate with the word early. For data and discussion regarding the combination of 0^+:1 and later, see references [53], [54], [55], [56], [57], and [58]. For data and discussion regarding observed dark matter galaxies, see references [38], [59], and [60]. Current techniques might not be capable of observing early dark matter galaxies. References [61] and [62] suggest, regarding galaxy clusters, the existence of clumps of dark matter that might be individual galaxies. Extrapolating from results that references [38] and [63] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM 1 : 0^+ later galaxies. For data and discussion regarding galaxies for which DM:OM ratios of $\sim$4:1 pertain, see references [64] and [65]. For data and discussion regarding later galaxies for which DM:OM ratios of 5^+:1 pertain, see reference [38]. References [66] and [67] provide data about collisions of galaxies.)

4. Discussion

This unit discusses some possibilities regarding specific possible elementary particles and regarding dark matter; discusses relationships between our work and other work; and discusses some aspects regarding so-called "tensions" regarding large-scale phenomena.

Here, other work includes observational research and modeling-centric research. We associate the two-word term extant modeling with modeling - including Standard Model modeling and concordance cosmology modeling - that other people developed.

4.1. Some hypothesized elementary particles

This unit discusses possibilities regarding the existence and properties of some hypothesized elementary particles.

4.1.1. We discuss the possible existence of axions.

This essay seemingly does not suggest an elementary boson that would associate with notions of an axion. People suggest that - under some circumstances - axions might convert into photons. We suggest that observations that people might associate with effects of axions might instead associate with the difference between our notion of 1(6)g1'2'4 and extant modeling notions that might associate with notions of 1(1)g1'2'4. Also, observations that people might associate with effects of axions might instead associate with interactions involving jay (or, 1J) bosons or aye (or, 1I) bosons. (See table 19.)

4.1.2. We discuss the possible existence of magnetic monopoles.

This essay does not suggest an elementary particle that would associate with notions of a magnetic monopole. Table 1 and table 3 seem not to suggest a 1L interaction with a monopole other than an electric monopole.

4.1.3. We develop modeling that might associate with the fraction of right-handed W bosons that one type of quark decay might produce.

Aspects related to table 13 and table 17 suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, our modeling does not suggest that $m(5,3)$ associates with the inertial mass of an isomer one charged lepton. However, perhaps such mass-like quantities associate with some measurable aspects of nature. For charged leptons and $0 \leq l_I \leq 4$ and $0 \leq l'_I \leq 2$, $m(3(l_I+1)+l'_I,3) = \beta m(3(l_I+0)+l'_I,3)$. One might conjecture that isomer zero observations of some aspects of isomer one phenomena associate with notions of non-inertial mass-like quantities that are β times the inertial masses for isomer zero elementary particles (and

Table 21: Explanations for observed ratios of dark matter effects to ordinary matter effects. DM denotes dark matter. OM denotes ordinary matter. DM:OM denotes a ratio of dark matter effects to ordinary matter effects. Inferences of DM:OM ratios come from interpreting data. Regarding densities of the universe, we assume that DMAI stuff associates with the plus in DM:OM $5^+ : 1$. Stuff - other than DMAI stuff - that associates with isomers one through five associates with the five in DM:OM $5^+ : 1$. Regarding some galaxy clusters, we assume that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe. The four-word phrase some absorption of CMB associates with the notion that people measured some specific depletion of CMB (or, cosmic microwave background radiation) and inferred twice as much depletion as people expected based solely on hyperfine interactions with hydrogen atoms. Possibly, half of the depletion associates with DM effects. We assume that isomer three hydrogen-like atoms account for the half of the absorption for which isomer zero (or, ordinary matter) hydrogen atoms do not account. The reach of an instance of 1g1'4'6'8 is two isomers. (See table 9.) The occurrence of $1 \in \Gamma$ associates with electromagnetism. The occurrence of $6 \in \Gamma$ associates with fermion aspects. The occurrences of $4 \in \Gamma$ and $8 \in \Gamma$ associate with spin and orbital angular momentum. (See table 3 and table 9.) We assume that 1g1'4'6'8 associates with hyperfine states. We assume that 1g1'2'4'6'8x associates with hyperfine transitions. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that people associate with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z. Here, z denotes redshift. The word later associates with the notion that observations pertain to objects later in the history of the universe. The three-word phrase dark matter galaxy denotes a galaxy that contains much less ordinary matter than dark matter. Possibly, people have yet to directly detect early dark matter galaxies.

Aspect	DM:OM	Comment
Densities of the universe	$5^+ : 1$	-
Some galaxy clusters	$5^+ : 1$	-
Some absorption of CMB	$1 : 1$	Half of the absorption might be via DM.
Some early galaxies	$0^+ : 1$	For each of some early galaxies, each original clump associates with isomer zero. Later, the galaxy might accumulate DM.
Some later galaxies	$0^+ : 1$	Some early DM:OM $0^+ : 1$ galaxies survive (without significant collisions with galaxies for which DM:OM is not $0^+ : 1$) until later times.
Some early galaxies	$1 : 0^+$	For each of some early galaxies, each original clump associates with an isomer other than isomer zero. Early on, the density of OM stars is small and people do not detect the galaxy. Later, the galaxy might accumulate enough OM to be visible. The term dark matter galaxy pertains.
Some later galaxies	$1 : 0^+$	Some early DM:OM $1 : 0^+$ galaxies survive (without significant collisions with galaxies for which the DM:OM is not $1 : 0^+$) until later times. The term dark matter galaxy pertains.
Some later galaxies	$\sim 4 : 1$	An original clump might associate with any isomer other than isomer three. (Isomer three repels OM stuff.) Eventually, the galaxy accumulates enough stuff (that does not associate with the isomer that associates with the original clump) to have a DM:OM ratio that is somewhat near $4 : 1$.
Many later galaxies	$5^+ : 1$	Over time, galaxies collide. Collisions tend to result in the formation of larger galaxies that include much stuff from smaller galaxies. A later galaxy that results from enough collisions is likely to associate with somewhat similar - across the six isomers - amounts of stuff from originally one- (or few-) isomer original clump galaxies.

that are β times inertial masses for the counterpart isomer one elementary particles).

Furthermore, isomer one might associate with right-handedness in a manner similar to the association of isomer zero with left-handedness. (See discussion related to table 17.)

Reference [68] discusses a fraction of decays - of ordinary matter top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+, is 3.6×10^{-4}. Reference [8] provides a confidence level of 90 percent that the rest energy of a would-be W_R (or, right-handed W boson) exceeds 715 GeV. (Perhaps, note also, reference [69].)

Based on notions of scaling that might calculate non-inertial mass-like quantities, one might conjecture that our modeling suggests that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that reference [68] discusses. A notion of $m_{\text{non-inertial},\text{W}_R\text{isomer one}}c^2 = \beta m_{\text{W}} c^2 \approx 2.8 \times 10^5$ GeV might pertain. Here, the notion of non-inertial mass-like quantity might associate with inferences that associate with 1L or 1W_1 and do not associate directly with 2L.

4.2. Interactions involving the jay boson

This unit discusses interactions that involve jay bosons.

4.2.1. We discuss interactions - that involve jay bosons - that might take place before or during inflation.

We consider interactions in which two jay bosons move in parallel, interact, and produce one aye boson plus something else. Here, we assume that conservation of angular momentum pertains and that one can de-emphasize angular momentum that is not intrinsic to the relevant elementary particles. We consider two cases. In the first case, the two jay bosons have the same (one of either right or left) circular polarization. Conservation of angular momentum allows an outgoing combination of one 2L particle and one 0I particle. Conservation of angular momentum might preclude producing one 1L particle and one 0I particle. In the second case, one jay boson has left circular polarization and the other jay boson has right circular polarization. Conservation of angular momentum allows the production of two 0I particles. Conservation of angular momentum might preclude producing one 1L particle and one 0I particle.

The two cases might comport with the notion that gravitation can be significant during inflation. The two cases might comport with the notion that jay bosons form before aye bosons form. (See table 19.)

The two cases might comport with a (possibly not relevant) notion that electromagnetism might become significant essentially only after inflation.

4.2.2. We discuss the notion of Pauli repulsion.

Extant modeling includes the notion that two identical fermions cannot occupy the same state. Regarding extant modeling, one notion is that repulsion between identical fermions associates with overlaps of wave functions. Another notion features wave functions that are anti-symmetric with respect to the exchange of two identical fermions.

Our modeling might be compatible with such aspects of extant modeling and, yet, not necessitate - for kinematics modeling - the use of wave functions. Modeling based on jay bosons might suffice.

Modeling based on jay bosons might suggest that prevention of two identical fermions from occupying the same state might associate with, in effect, trying to change aspects related to the fermions. Notions of changing a spin orientation or, for elementary fermions, changing a flavour might pertain.

4.2.3. We discuss so-called Pauli crystals.

Reference [70] and reference [71] report detection of Pauli crystals. We suggest that modeling based on the notion of jay bosons might help explain relevant phenomena.

4.2.4. We discuss a possible discrepancy - regarding energy levels in positronium - between extant modeling and observation.

Reference [72] and reference [73] discuss the transition - between two states of positronium - characterized by the expression $2^3S_1 \to 2^3P_0$. People discuss the energy that associates with the transition. Four standard deviations below the nominal observed value of energy approximately equals four standard deviations above the nominal value of energy that extant modeling suggests.

Perhaps, notions regarding jay bosons extend to explain the might-be discrepancy regarding positronium. Compared to extant modeling, a new notion of virtual charge exchange or a new notion of virtual flavour change might pertain.

To the extent that extant modeling does not suffice, modeling related to the jay boson might close the gap between observation and modeling.

4.3. Constraints regarding dark matter

This unit discusses the extent to which our notion of dark matter comports with constraints - about the nature of dark matter - that people associate with data about dark matter or with outputs from extant models that have bases in assumptions about dark matter.

4.3.1. *We discuss aspects related to cosmological models.*

Reference [38] summarizes some thinking about constraints on dark matter and about notions of dark matter. The article notes that so-called CDM (or, cold dark matter) might comport well with various models. Some models associate with the one-element term ΛCDM. The article notes that people have yet to determine directly whether nature includes CDM stuff. The article notes that people consider that notions of SIDM (or, self-interacting dark matter) might be appropriate regarding nature. People also use other terms, such as the three-word term warm dark matter, to note possible attributes of dark matter. Notions such as SIDM and WDM (or, warm dark matter) arose from modeling that differs from our modeling. We are reluctant to try to closely associate terms such as SIDM or WDM with our modeling. (We suggest that isomer zero 0.5R-based stuff, isomer zero 0.5M stuff, and all stuff associating with isomers one, two, four, and five might comport with some notions of CDM. We suggest that the remaining dark matter stuff - or, isomer three OMSE stuff - might associate with some notions of WDM and with some notions of SIDM.)

We suggest that our notion of dark matter is not necessarily incompatible with constraints - that have bases in cosmological models - on dark matter.

4.3.2. *We discuss aspects related to collisions of pairs of galaxy clusters.*

In particular we discuss the Bullet Cluster collision of two galaxy clusters. (Reference [74] discusses the Bullet Cluster.) Presumably, observations regarding other such collisions might pertain.

Observations suggest two general types of trajectories for stuff. Most dark matter - from either one of the clusters - exits the collision with trajectories consistent with having interacted just gravitationally with the other cluster. Also, ordinary matter stars - from either cluster - exit the collision with trajectories consistent with having interacted just gravitationally with the other cluster. However, ordinary matter IGM (or, intergalactic medium) - from either cluster - lags behind the cluster's ordinary matter stars and dark matter. That ordinary matter IGM interacted electromagnetically with the other cluster's ordinary matter IGM, as well as gravitationally with the other cluster.

Our work suggests that - regarding each cluster - essentially all dark matter - except isomer three IGM - passes through without interacting significantly electromagnetically with stuff from the other cluster. Our work suggests that isomer three IGM that associates with each cluster might interact significantly with isomer three IGM that associates with the other cluster. Isomer three IGM might follow trajectories similar to trajectories for isomer zero IGM.

We are uncertain as to the extent to which observational data might suggest that the amounts of dark matter that lags the bulk of dark matter are sufficiently small that our nominal notions regarding isomer three IGM do not comport with observations.

Should the actual fraction of lagging dark matter be too small, we might need to reconsider the extent to which isomer three differs from isomer one. We note some examples of possible reconsideration. For one example, possibly isomer three has right-handed elementary fermions but interactions involving such fermions model as retaining aspects of left-handed-centric interactions that associate with isomer zero. For another example, possibly isomer three does not evolve adequately similarly to isomer zero. To the extent that isomer three adequately differs from or does not evolve similarly to isomer zero, our explanation regarding CMB depletion via - in part - interactions with dark matter hydrogen-like atoms might be inaccurate (for example, based on an inaccurate estimate of the number of isomer three hydrogen-like atoms).

We suggest that our notion of dark matter is not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - on dark matter.

4.4. *"Tensions" regarding large-scale phenomena*

This unit suggests means to resolve so-called tensions - between data and extant modeling - regarding the rate of expansion of the universe and other large-scale phenomena.

4.4.1. *We explore the extent to which general relativity and the case of $n_I = 6$ are mutually compatible.*

Regarding general relativity and the case of $n_I = 6$, the notion of geodesic motion would not necessarily pertain.

For example, consider an isomer zero star and three possible planets. The planets are identical except that one planet associates with isomer zero, one planet associates with isomer one, and one planet associates with isomer three. The planets start out on identical orbits. We consider six orbit-centric cases. First, we assume that the star is spherically symmetric and does not rotate. Out of the 2g' components - only 2(6)g2 pertains. The three planets traverse identical orbits. Second, we assume that the star rotates. Here, 2(2)g2'4 associates with nonzero effects. The isomer one planet orbits as if 2g2'4 does not pertain. The isomer zero planet and the isomer three planet traverse a trajectory that differs from the trajectory that is common for the previous four cases.

Similarly, regarding general relativity and the notion that $n_I = 6$, the notion of a stress-energy tensor (and possibly non-zero cosmological constant) that does not consider isomers and that, in effect, governs all motion would not necessarily pertain. Lack of pertaining might echo the above discussion regarding a star and a planet.

4.4.2. We suggest an explanation for the notion that concordance cosmology underestimates recent increases in the rate of expansion of the universe.

Table 18 and table 19 discuss possible and known eras in the history of the universe.

People suggest that concordance cosmology modeling underestimates - for the second multi-billion-years era - increases in the rate of expansion of the universe. (See references [22], [23], [24], [25], [75], [76], and [77].)

We suggest the following explanation for such underestimates.

When using modeling based on general relativity, people might try to extend the use of an equation of state (or use of a cosmological constant) that works well regarding early in the first multi-billion-years era. Regarding that time, our modeling suggests dominance by attractive effects that associate with the 2g1‘2‘3 component of gravity. The notion of a reach of one pertains. The symbol 2(1)g1‘2‘3 pertains. Our modeling suggests that - later in the first multi-billion-years era - repulsive effects that associate with 2(2)g2‘4 become significant. Dominance by 2(2)g2‘4 pertains by the time the second multi-billion-years era starts. However, people's use of an equation of state that has roots in the time period in which 2(1)g1‘2‘3 dominates would - at best - extrapolate based on a notion of 2(1)g2‘4 (and not a notion of 2(2)g2‘4). That modeling would underestimate the strength of the key gravitational driver - of expansion - by a factor of two.

We point - conceptually - to the following possible remedy.

People might change (regarding the stress-energy tensor or the cosmological constant) the aspects that would associate with repulsion and the 2g2‘4 component of gravity. The contribution - to the pressure - that associates with 2g2‘4 needs to double (compared to the contribution that would associate with 2(1)g2‘4).

4.4.3. We suggest an explanation for the notion that concordance cosmology overestimates large-scale clumping of matter.

People suggest that concordance cosmology modeling overestimates large-scale clumping of matter - ordinary matter and dark matter. (For data and discussion, see references [78], [79], [80], and [25].)

We suggest that concordance cosmology modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. Our modeling suggests that 2(2)g2‘4 pertains. (That is, for each instance of 2g2‘4, a reach of two isomers pertains.) The additional (compared to concordance cosmology modeling) repulsion might explain the overestimating - of clumping, per concordance cosmology modeling - that people suggest.

4.4.4. We suggest an explanation for the notion that concordance cosmology might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies.

People suggest that concordance cosmology modeling might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (For data and discussion, see reference [28].)

We suggest that concordance cosmology modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. Our modeling suggests that 2(2)g2‘4 pertains. The additional (compared to concordance cosmology modeling) repulsion might explain at least some aspects of the observations that people report.

4.5. Our modeling and other modeling

This unit discusses some possible relationships between our modeling and types of extant models.

4.5.1. We suggest approximate relationships between some extant modeling and aspects of our modeling regarding properties.

Table 22 discusses approximate relationships between modeling that can deploy elementary-particle properties and aspects of our modeling.

4.5.2. We discuss aspects related to the value of two for reach (or, ρ_I).

This essay suggests that $\rho_I = 2$ pertains for some components of long-range interactions (or, LRI). This essay suggests that the notion of $\rho_I = 2$ might have importance regarding explaining data regarding the following - some depletion of CMB, large-scale clumping, the recent multi-billion-years era of increases regarding the rate of separation of large clumps, gravitational interactions between neighboring galaxies, and galaxy formation.

5. Conclusions

This unit summarizes aspects of our work and suggests perspective about our work.

Table 22: Approximate relationships between modeling that can deploy elementary-particle properties and aspects of our modeling. n_I denotes a number - one or six - of isomers. Extant modeling associates with $n_I = 1$. Each one of some of the items in the symbol column does not associate with an extant modeling symbol. CNC associates with charge-current 4-vectors and with Maxwell's equations. Compared to CNC, QED adds associations with magnetic fields created by other than charge currents and adds associations with anomalous magnetic moments. QCD associates with 1G, $0.5\mathrm{Q}_{1/3}$, and $0.5\mathrm{Q}_{2/3}$. We suggest the possibility that QCD might extend to associate with 0.5R. The symbol PEF associates with the three-word phrase Pauli exclusion force. We suggest that PEF associates with 1J, each 0.5Φ family, and fermions that are not elementary particles. WIP associates with $1\mathrm{W}_1$ and 1Z. The symbol † denotes a notion of a (currently) hypothetical analog to QED. Our modeling suggests that a modeling basis might need to encompass the notion of anomalous gravitational property and the notion of six isomers.

Modeling	Range of Σ	l_o PROP	n_I	Symbol
Newtonian gravity	2	2	1	NEW
Moments of inertia	2	1 - 3	1	MOI
Electrostatics	1	1	1	EST
Charge-and-current 4-vectors	1	1	1	CNC
Quantum electrodynamics	1, 3	1, 2, 4, 6, 8	1	QED
Quantum chromodynamics	0	1 - 4, 6, 8	1	QCD
Pauli exclusion force	0	1 - 4, 6, 8	1	PEF
Weak-interaction phenomena	0	1 - 4	1	WIP
Suggested by our modeling	0 - 4	1 - 4, 6, 8	6	PRM
Gravitational analog to QED †	2, 4	1 - 4, 6, 8	6	QGD

5.1. *Our modeling*

Our modeling features two bases.

One basis unifies and decomposes aspects of electromagnetism and gravity. For each of electromagnetism and gravity, the decomposition seems to associate well with properties - of objects - that people can measure and that extant modeling features. For electromagnetism, the properties include charge and magnetic moment. For gravity and kinematics related to mass, the properties include mass and moments of inertia.

One basis features isomers of elementary particles that do not intermediate long-range interactions and features instances of components of long-range interactions.

Our modeling extends from the two bases to do the following. Match all known elementary particles and suggest possible other elementary particles. Describe dark matter. Point to explanations for data that extant modeling seems not to explain. Suggest data that might associate with future observations.

We suggest the possibility that the notion that our work explains phenomena that extant modeling does not explain points to usefulness for our work. We explain quantitatively eight quantitative data points or approximate data ranges regarding observed ratios of dark matter effects to ordinary matter effects. (See table 21.) Some other explanations have quantitative bases but - to the extent that this essay uses the explanations - are qualitative. Presumably, people can use simulations to help verify or refute some of our qualitative explanations. Generally, we know of no cases in which our suggestions that address possible gaps between extant modeling and observations point - compared to extant modeling - in a wrong direction regarding closing gaps.

We suggest the possibility that the notion that our work suggests specifications and data that extant modeling does not suggest points to possible usefulness for our work. Our suggestions include a specification for dark matter, specifications for new elementary particles, and more (than current measurements provide) accurate masses for neutrinos and some other known elementary particles.

We suggest that the small set of bases for our modeling, the breadth of seemingly coherent scope of our modeling, the simplicity of relevant Diophantine equations, and the possible ease of integrating our modeling and extant modeling point to possible usefulness for our work.

5.2. *Our work*

Our work suggests augmentations - to physics modeling - that produce results that may provide progress regarding the following physics opportunities. Complete the list of elementary particles. Describe dark matter. Explain ratios of dark matter to ordinary matter. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models.

We use our modeling to match data that other modeling matches.

We use our modeling to suggest explanations for data that other modeling seems not to explain.

We use our modeling to suggest results regarding data that people have yet to gather.

The breadth and depth of the matched data might suffice to justify using our modeling.

The breadth and unity - within itself and with physics modeling that people use successfully - of our modeling might support the usefulness of our modeling.

Acknowledgments

The following people pointed, via personal contact, to topics or aspects that we considered for inclusion in the scope of our work: Andrea Albert, Raphael Bousso, Lance Dixon, Persis Drell, Immanuel Freedman, Ervin Goldfain, Vesselin Gueorguiev, Kamal Melek Hanna, Wick Haxton, Nick Hutzler, William Lama, Tom Lawrence, Surhud More, Holger Muller, J. Xavier Prochaska, Martin Rees, Harrison Rose, and Mak Tafazoli.

The following people provided comments regarding the effectiveness of drafts that led to parts of this essay: Immanuel Freedman, Ervin Goldfain, Vesselin Gueorguiev, William Lama, Tom Lawrence, and Mak Tafazoli.

The following people helped publish aspects leading to work that this essay describes: Charles K. Chui, Kamal Melek Hanna, Keith Jones, and Zeger Karssen. (Perhaps, note reference [81] and reference [82].) The following people provided or pointed to aspects regarding expressing or propagating the work: Elliott Bloom, Man Ho Chan, Maxwell Chertok, Charles K. Chui, Andrei Lucian Dragoi, Steven Frautschi, Carl Frederick, Ervin Goldfain, Vesselin Gueorguiev, Ioannis Haranas, Richard B. Holmes, Frank Hiroshi Ling, Davood Momeni, Michael Mulhearn, Richard A. Muller, Stephen Perrenod, Paul Preuss, Amal Pushp, Amir Sharif, and Wendy Shi. The following people suggested perspective, means, or suggestions regarding people with whom to try to have discussions: Vint Cerf, Yanbei Chen, James S. Clegg, Bill Daul, Jiggs Davis, George Djorgovski, Erica Ellingson, Mark Flegel, Ron Fredericks, Vesselin Gueorguiev, Tucker Hiatt, William Lama, Lianne La Reine, Robert Morgan, Doug Osheroff, Kennan Salinero, Jim Spohrer, Peter Walstrom, and Jon F. Wilkins.

References

[1] Jairzinho Ramos Medina. *Gravitoelectromagnetism (GEM): A Group Theoretical Approach*. PhD thesis, Drexel University, August 2006. Link: https://core.ac.uk/download/pdf/190333514.pdf. 1.3, 1.4.1, 2

[2] David Delphenich. Pre-Metric Electromagnetism as a Path to Unification. In *Unified Field Mechanics*. World Scientific, September 2015. Link: https://arxiv.org/ftp/arxiv/papers/1512/1512.05183.pdf. 1.3, 1.4.1, 2

[3] S. Gasiorowicz and P. Langacker. Elementary Particles in Physics. University of Pennsylvania. Link: https://www.physics.upenn.edu/ pgl/e27/E27.pdf. 1.4.1

[4] A. Hebecker and J. Hisano. 94: Grand Unified Theories. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/reviews/rpp2020-rev-guts.pdf. 1.4.1

[5] A. Ringwald, L. J. Rosenberg, and G. Rybka. 91: Axions and Other Similar Particles. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/web/viewer.html?file= 1.4.1

[6] S. Rolli and M. Tanabashi. 95: Leptoquarks. In P. A. Zyla and others (Particle data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/web/viewer.html?file= 1.4.1

[7] D. Milstead and E. J. Weinberg. 96: Magnetic Monopoles. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/web/viewer.html?file= 1.4.1

[8] P. A. Zyla et al. Review of Particle Physics. *PTEP*, 2020(8):083C01, 2020. Link: https://pdg.lbl.gov/2020/citation.html. 1.4.1, 3.1.2, 3.1.2, 3.1.2, 3.1.3, 3.1.4, 3.1.5, 3.1.5, 3.5, 4.1.3

[9] Lotty Ackerman, Matthew R. Buckley, Sean M. Carroll, and Marc Kamionkowski. Dark matter and dark radiation. *Physical Review D*, 79:023519, January 2009. Link: https://link.aps.org/doi/10.1103/PhysRevD.79.023519. 1.4.1

[10] Brian Green. *Until the End of Time: Mind, Matter, and Our Search for Meaning in an Evolving Universe*. Alfred A. Knopf, February 2020. Link: https://www.penguinrandomhouse.com/books/549600/until-the-end-of-time-by-brian-greene/. 1.4.1, 2.1.13, 3.3.3

[11] Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler. *Gravitation.* University of Princeton Press, October 2017. Link: https://press.princeton.edu/books/hardcover/9780691177793/gravitation. 1.4.1

[12] M. C. Gonzalez-Garcia and M. Yokoyama. 14: Neutrino Masses, Mixing, and Oscillations. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/reviews/rpp2020-rev-neutrino-mixing.pdf. 1.4.1, 3.1.4

[13] R. Abbasi, M. Ackermann, J. Adams, J. A. Aguilar, M. Ahlers, M. Ahrens, C. Alispach, A. A. Alves, N. M. Amin, R. An, et al. Search for Relativistic Magnetic Monopoles with Eight Years of IceCube Data. *Phys. Rev. Lett.*, 128:051101, February 2022. Link: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.128.051101. 1.4.1

[14] T. Damour. 21: Experimental Tests of Gravitational Theory. In P. A. Zla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/reviews/rpp2020-rev-gravity-tests.pdf. 1.4.1

[15] M. Kramer, I. H. Stairs, R. N. Manchester, N. Wex, A. T. Deller, W. A. Coles, M. Ali, M. Burgay, F. Camilo, I. Cognard, et al. Strong-Field Gravity Tests with the Double Pulsar. *Phys. Rev. X*, 11(4):041050, December 2021. Link: https://journals.aps.org/prx/abstract/10.1103/PhysRevX.11.041050. 1.4.1

[16] C. W. F. Everitt, D. B. DeBra, B. W. Parkinson, J. P. Turneaure, J. W. Conklin, M. I. Heifetz, G. M. Keiser, A. S. Silbergleit, T. Holmes, J. Kolodziejczak, et al. Gravity Probe B: Final Results of a Space Experiment to Test General Relativity. *Phys. Rev. Lett.*, 106:221101, May 2011. Link: https://link.aps.org/doi/10.1103/PhysRevLett.106.221101. 1.4.1

[17] Ioannis Haranas and Michael Harney. Detection of the Relativistic Corrections to the GravitationalPotential Using a Sagnac Interferometer. *Progress in Physics*, 3:3, July 2008. Link: http://www.ptep-online.com/complete/PiP-2008-03.pdf. 1.4.1

[18] K. A. Olive and J. A. Peacock. 22: Big-Bang Cosmology. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/web/viewer.html?file= 1.4.2

[19] J. Ellis and D. Wands. 23: Inflation. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/web/viewer.html?file= 1.4.2

[20] D. H. Weinberg and M. White. 28: Dark Energy. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/reviews/rpp2020-rev-dark-energy.pdf. 1.4.2

[21] Tao Zhu, Anzhong Wang, Gerald Cleaver, Klaus Kirsten, and Qin Sheng. Pre-inflationary universe in loop quantum cosmology. *Phys. Rev. D*, 96:083520, October 2017. Link: https://link.aps.org/doi/10.1103/PhysRevD.96.083520. 1.4.2, 3.3.3

[22] L. Verde, T. Treu, and A. G. Riess. Tensions between the early and late Universe. *Nature Astronomy*, 3(10):891–895, September 2019. Link: https://www.nature.com/articles/s41550-019-0902-0. 1.4.2, 4.4.2

[23] Johanna L. Miller. Gravitational-lensing measurements push Hubble-constant discrepancy past 5σ. *Physics Today*, 2020(1):0210a, February 2020. Link: https://physicstoday.scitation.org/do/10.1063/PT.6.1.20200210a/full/. 1.4.2, 4.4.2

[24] Thomas Lewton. What Might Be Speeding Up the Universe's Expansion? *Quanta Magazizne*, May 2020. Link: https://www.quantamagazine.org/why-is-the-universe-expanding-so-fast-20200427/. 1.4.2, 4.4.2

[25] Christopher Wanjek. Dark Matter Appears to be a Smooth Operator. *Mercury*, 49(3):10–11, October 2020. Link: https://astrosociety.org/news-publications/mercury-online/mercury-online.html/article/2020/12/10/dark-matter-appears-to-be-a-smooth-operator. 1.4.2, 4.4.2, 4.4.3

[26] L. Baudis and S. Profumo. 27: Dark Matter. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, 2019. Link: https://pdg.lbl.gov/2021/reviews/rpp2020-rev-dark-matter.pdf. 1.4.3

[27] Houjun Mo, Frank van den Bosch, and Simon White. *Galaxy Formation and Evolution*. Cambridge University Press, Cambridge, UK, 2010. Link: https://www.cambridge.org/us/academic/subjects/physics/astrophysics/galaxy-formation-and-evolution-1. 1.4.3

[28] Kyu-Hyun Chae, Federico Lelli, Harry Desmond, Stacy S. McGaugh, Pengfei Li, and James M. Schombert. Testing the Strong Equivalence Principle: Detection of the External Field Effect in Rotationally Supported Galaxies. *The Astrophysical Journal*, 904(1):51, November 2020. Link: https://iopscience.iop.org/article/10.3847/1538-4357/abbb96/meta. 1.4.3, 4.4.4

[29] Jean-Pierre Amiet and Stefan Weigert. Commensurate harmonic oscillators: Classical symmetries. *Journal of Mathematical Physics*, 43(8):4110–4126, August 2002. Link: https://sites.ifi.unicamp.br/aguiar/files/2014/10/P034ClassCommensurateOscillators2002.pdf. 2.1.4

[30] G. A. Gonzalez-Sprinberg and J. Vidal. Tau magnetic moment. *Proceedings of The International Conference On Nanoscience and Technology*, 912(1):012001, 2017. Link: http://stacks.iop.org/1742-6596/912/i=1/a=012001. 3.1.3

[31] P. Vogel and A. Piepke. Neutrino Properties. In P. A. Zyla and others (Particle Data Group), Prog. Theor. Exp. Phys, 083C01 (2020) and 2021 update, August 2019. Link: https://pdg.lbl.gov/2020/listings/rpp2020-list-neutrino-prop.pdf. 3.1.5

[32] E. Elfgren and S. Fredriksson. Mass limits for heavy neutrinos. *Astronomy and Astrophysics*, 479(2):347–353, December 2007. Link: https://www.aanda.org/articles/aa/pdf/2008/08/aa8898-07.pdf. 3.1.5

[33] Mark P. Hertzberg. Structure Formation in the Very Early Universe. *Physics Magazine*, 13(26), February 2020. Link: https://physics.aps.org/articles/v13/16. 3.3.3

[34] N. G. Busca, T. Delubac, J. Rich, S. Bailey, A. Font-Ribera, D. Kirkby, J.-M. Le Goff, M. M. Pieri, A. Slosar, E. Aubourg, et al. Baryon acoustic oscillations in the Lya forest of BOSS quasars. *Astronomy and Astrophysics*, 552(A96), April 2013. Links: https://www.aanda.org/2013-highlights/914-baryon-acoustic-oscillations-in-the-lyman-alpha-forest-of-boss-quasars-busca-et-al and https://arxiv.org/abs/1211.2616. 3.3.3

[35] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, Groom, et al. Measurements of Ω and Λ from 42 high-redshift supernovae Ω. *Astrophysical Journal*, 517(2):565–586, June 1999. Link: https://iopscience.iop.org/article/10.1086/307221/meta. 3.3.3

[36] Adam G. Riess, Alexei V. Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M. Garnavich, Ron L. Gilliland, Craig J. Hogan, Saurabh Jha, Robert P. Kirshner, et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astronomical Journal*, 116(3):1009–1038, September 1998. Link: https://iopscience.iop.org/article/10.1086/300499/meta. 3.3.3

[37] Adam G. Riess, Louis-Gregory Strolger, John Tonry, Stefano Casertano, Henry C. Ferguson, Bahram Mobasher, Peter Challis, Alexei V. Filippenko, Saurabh Jha, Weidong Li, et al. Type Ia Supernova Discoveries at z $>$ 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. *Astrophysical Journal*, 607(2):665–687, June 2004. Link: http://iopscience.iop.org/0004-637X/607/2/665. 3.3.3

[38] Joshua D. Simon and Marla Geha. Illuminating the darkest galaxies. *Physics Today*, 74(11):30–36, November 2021. Link: https://physicstoday.scitation.org/doi/10.1063/PT.3.4879. 3.4.1, 3.5, 4.3.1

[39] Ben Forrest, Marianna Annunziatella, Gillian Wilson, Danilo Marchesini, Adam Muzzin, M. C. Cooper, Z. Cemile Marsan, Ian McConachie, Jeffrey C. C. Chan, Percy Gomez, et al. An

Extremely Massive Quiescent Galaxy at z = 3.493: Evidence of Insufficiently Rapid Quenching Mechanisms in Theoretical Models. *Astrophysical Journal*, 890(1):L1, February 2020. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ab5b9f. 3.4.3

[40] Katherine E. Whitaker, Christina C. Williams, Lamiya Mowla, Justin S. Spilker, Sune Toft, Desika Narayanan, Alexandra Pope, Georgios E. Magdis, Pieter G. van Dokkum, Mohammad Akhshik, et al. Quenching of star formation from a lack of inflowing gas to galaxies. *Nature*, 597(7877):485–488, September 2021. Link: https://doi.org/10.1038/s41586-021-03806-7. 3.4.3

[41] David A. Buote and Aaron J. Barth. The Extremely High Dark Matter Halo Concentration of the Relic Compact Elliptical Galaxy Mrk 1216. *Astrophysical Journal*, 877(2):91, May 2019. Link: https://iopscience.iop.org/article/10.3847/1538-4357/ab1008. 3.4.3

[42] Ana Bonaca, David W. Hogg, Adrian M. Price-Whelan, and Charlie Conroy. The Spur and the Gap in GD-1: Dynamical Evidence for a Dark Substructure in the Milky Way Halo. *Astrophysical Journal*, 880(1):38, July 2019. Link: https://iopscience.iop.org/article/10.3847/1538-4357/ab2873. 3.4.4

[43] David Ehrenstein. Mapping Dark Matter in the Milky Way. *Physics Magazine*, 12(51), May 2019. Link: https://physics.aps.org/articles/v12/51. 3.4.4

[44] Ewa L. Lokas and Gary A. Mamon. Dark matter distribution in the Coma cluster from galaxy kinematics: breaking the mass-anisotropy degeneracy. *Monthly Notices of The Royal Astronomical Society*, 343(2):401–412, August 2003. Link: https://academic.oup.com/mnras/article/343/2/401/1038976. 3.5

[45] Elena Rasia, Giuseppe Tormen, and Lauro Moscardini. A dynamical model for the distribution of dark matter and gas in galaxy clusters. *Monthly Notices of The Royal Astronomical Society*, 351(1):237–252, June 2004. Link: https://academic.oup.com/mnras/article/351/1/237/1004623. 3.5

[46] Lawrence Rudnick. The Stormy Life of Galaxy Clusters: astro version. January 2019. Link: https://ned.ipac.caltech.edu/level5/March19/Rudnick/frames.html. 3.5

[47] Lawrence Rudnick. The stormy life of galaxy clusters. *Physics Today*, 72(1):46–52, January 2019. Link: https://physicstoday.scitation.org/doi/full/10.1063/PT.3.4112. 3.5

[48] Judd D. Bowman, Alan E. E. Rogers, Raul A. Monsalve, Thomas J. Mozdzen, and Nivedita Mahesh. An absorption profile centred at 78 megahertz in the sky-averaged spectrum. *Nature*, 555(7694):67–70, March 2018. Link: https://www.nature.com/articles/nature25792. 3.5

[49] Rennan Barkana. Possible interaction between baryons and dark-matter particles revealed by the first stars. *Nature*, 555(7694):71–74, March 2018. Link: https://www.nature.com/articles/nature25791. 3.5

[50] Paolo Panci. 21-cm line Anomaly: A brief Status. In *33rd Rencontres de Physique de La Vallee d'Aoste*, July 2019. Link: https://cds.cern.ch/record/2688533. 3.5

[51] Peter Behroozi, Risa Wechsler, Andrew Hearin, and Charlie Conroy. UniverseMachine: The correlation between galaxy growth and dark matter halo assembly from z = 0-10. *Monthly Notices of The Royal Astronomical Society*, 488(3):3143–3194, May 2019. Link: https://academic.oup.com/mnras/article/488/3/3143/5484868. 3.5

[52] R. Genzel, N. M. Forster Schreiber, H. Ubler, P. Lang, T. Naab, R. Bender, L. J. Tacconi, E. Wisnioski, S. Wuyts, T. Alexander, et al. Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago. *Nature*, 543(7645):397–401, March 2017. Link: https://www.nature.com/articles/nature21685. 3.5

[53] Pieter van Dokkum, Roberto Abraham, Jean Brodie, Charlie Conroy, Shany Danieli, Allison Merritt, Lamiya Mowla, Aaron Romanowsky, and Jielai Zhang. A High Stellar Velocity Dispersion and ˜100 Globular Clusters for the Ultra-diffuse Galaxy Dragonfly 44. *Astrophysical Journal*, 828(1):L6, August 2016. Link: http://iopscience.iop.org/article/10.3847/2041-8205/828/1/L6. 3.5

[54] Shannon Hall. Ghost galaxy is 99.99 per cent dark matter with almost no stars. *New Scientist*, August 2016. Link: https://www.newscientist.com/article/2102584-ghost-galaxy-is-99-99-per-cent-dark-matter-with-almost-no-stars/. 3.5

[55] Pavel E. Mancera Pina, Filippo Fraternali, Elizabeth A. K. Adams, Antonino Marasco, Tom Oosterloo, Kyle A. Oman, Lukas Leisman, Enrico M. di Teodoro, Lorenzo Posti, Michael Battipaglia, et al. Off the Baryonic Tully-Fisher Relation: A Population of Baryon-dominated Ultra-diffuse Galaxies. *Astrophysical Journal*, 883(2):L33, September 2019. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ab40c7/meta. 3.5

[56] Pavel E. Mancera Pina, Filippo Fraternali, Tom Oosterloo, Elizabeth A. K. Adams, Kyle A. Oman, and Lukas Leisman. No need for dark matter: resolved kinematics of the ultra-diffuse galaxy AGC 114905. *Mon. Not. R. Astron Soc.*, December 2021. Link: https://academic.oup.com/mnras/advance-article/doi/10.1093/mnras/stab3491/6461100. 3.5

[57] Qi Guo, Huijie Hu, Zheng Zheng, Shihong Liao, Wei Du, Shude Mao, Linhua Jiang, Jing Wang, Yingjie Peng, Liang Gao, et al. Further evidence for a population of dark-matter-deficient dwarf galaxies. *Nature Astronomy*, 4(3):246–251, November 2019. Link: https://www.nature.com/articles/s41550-019-0930-9. 3.5

[58] Pieter van Dokkum, Shany Danieli, Roberto Abraham, Charlie Conroy, and Aaron J. Romanowsky. A Second Galaxy Missing Dark Matter in the NGC 1052 Group. *Astrophysical Journal*, 874(1):L5, March 2019. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ab0d92. 3.5

[59] Charles Day. A primordial merger of galactic building blocks. *Physics Today*, 2021(1):0614a, June 2021. Link: https://physicstoday.scitation.org/do/10.1063/PT.6.1.20210614a/full/. 3.5

[60] Yuta Tarumi, Naoki Yoshida, and Anna Frebel. Formation of an Extended Stellar Halo around an Ultra-faint Dwarf Galaxy Following One of the Earliest Mergers from Galactic Building Blocks. *The Astrophysical Journal Letters*, 914(1):L10, June 2021. Link: https://iopscience.iop.org/article/10.3847/2041-8213/ac024e. 3.5

[61] Massimo Meneghetti, Guido Davoli, Pietro Bergamini, Piero Rosati, Priyamvada Natarajan, Carlo Giocoli, Gabriel B. Caminha, R. Benton Metcalf, Elena Rasia, Stefano Borgani, et al. An excess of small-scale gravitational lenses observed in galaxy clusters. *Science*, 369(6509):1347–1351, September 2020. Link: https://science.sciencemag.org/content/369/6509/1347. 3.5

[62] Maria Temming. Dark matter clumps in galaxy clusters bend light surprisingly well. *Science News*, September 2020. Link: https://www.sciencenews.org/article/dark-matter-clumps-galaxy-clusters-bend-light-surprisingly-well. 3.5

[63] Joshua D. Simon and Marla Geha. The Kinematics of the Ultra-faint Milky Way Satellites: Solving the Missing Satellite Problem. *Astrophys. J.*, 670(1):313–331, November 2007. Link: https://iopscience.iop.org/article/10.1086/521816. 3.5

[64] J. Jimenez-Vicente, E. Mediavilla, C. S. Kochanek, and J. A. Munoz. Dark Matter Mass Fraction in Lens Galaxies: New Estimates from Microlensing. *Astrophysical Journal*, 799(2):149, January 2015. Link: http://stacks.iop.org/0004-637X/799/i=2/a=149. 3.5

[65] J. Jimenez-Vicente, E. Mediavilla, J. A. Munoz, and C. S. Kochanek. A Robust Determination of the Size of Quasar Accretion Disks Using Gravitational Microlensing. *Astrophysical Journal*, 751(2):106, May 2012. Link: https://iopscience.iop.org/article/10.1088/0004-637X/751/2/106. 3.5

[66] Whitney Clavin. Rotating Galaxies Galore. April 2020. Link: https://www.caltech.edu/about/news/rotating-galaxies-galore. 3.5

[67] O. LeFevre, M. Bethermin, A. Faisst, P. Capak, P. Cassata, J. D. Silverman, D. Schaerer, and L. Yan. The ALPINE-ALMA [CII] survey: Survey strategy, observations and sample properties of 118 star-forming galaxies at 4<z<6. October 2019. Link: https://doi.org/10.1051/0004-6361/201936965. 3.5

[68] V. M. Abazov, B. Abbott, M. Abolins, B. S. Acharya, M. Adams, T. Adams, M. Agelou, J.-L. Agram, S. H. Ahn, M. Ahsan, et al. Search for right-handed W bosons in top quark decay. *Physical Review D*, 72:011104, July 2005. Link: https://link.aps.org/doi/10.1103/PhysRevD.72.011104. 4.1.3

[69] Paul Langacker and S. Uma Sankar. Bounds on the mass of W sub R and the W sub L - W sub R mixing angle. zeta. in general SU(2) sub L times SU(2) sub R times U(1) models. *Physical Review D*, 40(5):1569–1585, September 1989. Link: https://inspirehep.net/literature/277249. 4.1.3

[70] Marvin Holten, Luca Bayha, Keerthan Subramanian, Carl Heintze, Philipp M. Preiss, and Selim Jochim. Observation of Pauli Crystals. *Physical Review Letters*, 126:020401, January 2021. Link: https://link.aps.org/doi/10.1103/PhysRevLett.126.020401. 4.2.3

[71] Christie Chiu. Revealing a Pauli Crystal. *Physics*, 15(5), January 2021. Link: https://physics.aps.org/articles/v14/5. 4.2.3

[72] L. Gurung, T. J. Babij, S. D. Hogan, and D. B. Cassidy. Precision Microwave Spectroscopy of the Positronium $n = 2$ Fine Structure. *Physical Review Letters*, 125:073002, August 2020. Link: https://link.aps.org/doi/10.1103/PhysRevLett.125.073002. 4.2.4

[73] Matteo Rini. A Fine Positronium Puzzle. *Physics*, 13, August 2020. Link: https://physics.aps.org/articles/v13/s99. 4.2.4

[74] M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, W. Forman, C. Jones, S. Murray, and W. Tucker. Direct Constraints on the Dark Matter Self-Interaction Cross Section from the Merging Galaxy Cluster 1E 0657-56. *Astrophysical Journal*, 606(2):819–824, May 2004. Link: https://iopscience.iop.org/article/10.1086/383178. 4.3.2

[75] Natalie Wolchover. New Wrinkle Added to Cosmology's Hubble Crisis. *Quanta Magazine*, February 2020. Link: https://www.quantamagazine.org/new-wrinkle-added-to-cosmologys-hubble-crisis-20200226/. 4.4.2

[76] Wendy L. Freedman, Barry F. Madore, Taylor Hoyt, In Sung Jang, Rachael Beaton, Myung Gyoon Lee, Andrew Monson, Jill Neeley, and Jeffrey Rich. Calibration of the Tip of the Red Giant Branch (TRGB). *Astrophysical Journal*, 891(1):57, March 2020. Link: https://iopscience.iop.org/article/10.3847/1538-4357/ab7339. 4.4.2

[77] Vivian Poulin, Tristan L. Smith, Tanvi Karwal, and Marc Kamionkowski. Early Dark Energy can Resolve the Hubble Tension. *Physical Review Letters*, 122(22):221301, June 2019. Link: https://link.aps.org/doi/10.1103/PhysRevLett.122.221301. 4.4.2

[78] Charlie Wood. A New Cosmic Tension: The Universe Might Be Too Thin. *Quanta Magazine*, September 2020. Link: https://www.quantamagazine.org/a-new-cosmic-tension-the-universe-might-be-too-thin-20200908/. 4.4.3

[79] Khaled Said, Matthew Colless, Christina Magoulas, John R. Lucey, and Michael J. Hudson. Joint analysis of 6dFGS and SDSS peculiar velocities for the growth rate of cosmic structure and tests of gravity. *Monthly Notices of The Royal Astronomical Society*, 497(1):1275–1293, July 2020. Link: https://academic.oup.com/mnras/article-abstract/497/1/1275/5870121?redirectedFrom=fulltext. 4.4.3

[80] Supranta S. Boruah, Michael J. Hudson, and Guilhem Lavaux. Cosmic flows in the nearby Universe: new peculiar velocities from SNe and cosmological constraints. *Monthly Notices of The Royal Astronomical Society*, August 2020. Link: https://academic.oup.com/mnras/advance-article-abstract/doi/10.1093/mnras/staa2485/5894929?redirectedFrom=fulltext. 4.4.3

[81] Thomas J. Buckholtz. *Models for Physics of the Very Small and Very Large*, volume 14 of *Atlantis Studies in Mathematics for Engineering and Science*. Springer, 2016. Series editor: Charles K. Chui. Link: https://link.springer.com/book/10.2991/978-94-6239-166-6. 5.2

[82] Thomas J. Buckholtz. Predict particles beyond the standard model; then, narrow gaps between physics theory and data. In *Proceedings of the 9th Conference on Nuclear and Particle Physics (19-23 Oct. 2015 Luxor-Aswan, Egypt)*, May 2016. Link: http://www.afaqscientific.com/nuppac15/npc1509.pdf. 5.2

Manuscript date: February 12, 2022