# The Planck Constant and its Relation to the Compton Frequency

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**Abstract**

The Planck constant is considered one of the most important universal constants of physics, but its physical nature still has not been fully understood. Further investigation and new perspectives on this quantity should therefore be of interest. We demonstrate that the Planck constant can be directly linked to the Compton frequency of one divided by the Compton frequency of one kg. This further implies that the Planck constant is related to the quantization of matter, not only energy. We will also show that the frequency of one, when expressed in relation to kg, depends on the observation time. This new interpretation of the Planck constant could be an important step towards more in-depth understanding its physical nature, and potentially explaining the origin of the mass-gap and the rest mass of a photon.

**Keywords**: Planck constant, Compton frequency, electron, proton count.

## 1 Background

The Planck constant is a cornerstone of modern physics, especially quantum mechanics where it plays an important role in the Heisenberg's uncertainty principle [1], the Schrödinger equation [2], the Klein–Gordon equation and the Dirac equation [3]. Max Planck introduced it to the scientific community in 1900 at the presentation of his derivation of the spectral distribution of black-body radiation, the so-called Planck's law [4]. The radiation, as a stream of photons of frequency $f$, comes in quanta of energy proportional to the Planck constant $h$, $E = hf$. Other ways of determining the Planck constant [14] include the Landauer quantization of conductance [5], photoemission spectroscopy [6] or a Kibble balance [7–9]. The last method links it to the kilogram, following the 2019 IS revision, which fixed the Planck constant's value at an exact number of $6.62607015 \cdot 10^{-34}$ J·Hz$^{-1}$ [12, 13].

We and several other authors note, however, that despite its omnipresence in the physical world, the Planck constant lacks in-depth understanding [10, 11]. In this paper we will propose a new, deeper perspective on this physical quantity, which provides its new definition as the frequency of one divided by the Compton frequency in an arbitrary amount of matter that we are used to call one kilogram, multiplied by $c^2$. The Planck constant is therefore linked to the frequency ratio, and most importantly to the Compton frequency of one. This approach will give us a new insight in energy and matter, and a new way to define and measure the Planck constant.

## 2 The Planck constant as the Compton frequency of one, divided by the Compton frequency of 1 kg multiplied by $c^2$

The reduced Compton frequency of a mass is simply the speed of light divided by the reduced Compton wavelength of this mass. For an electron we have

$$f_e = \frac{c}{\bar{\lambda}} \approx 7.76 \times 10^{20} \text{ per second} . \tag{1}$$

The hypothetical reduced Compton wavelength of one kilogram can be found by Compton's [18] wavelength formula

$$\bar{\lambda} = \frac{\hbar}{mc} = \frac{\hbar}{1kg \times c} = \frac{\hbar}{c} = 3.52 \times 10^{-43} \text{ m} . \tag{2}$$

As we will explain in section 4, composite masses can have a Compton wavelength shorter than the Planck length. This leads to a reduced Compton frequency of one kilogram

$$f_{1\text{kg}} = \frac{c}{\bar{\lambda}_{1\text{kg}}} \approx 8.52 \times 10^{50} \text{ per second} . \tag{3}$$

All masses expressed in kilograms can be described as reduced Compton frequencies divided by the reduced Compton frequency of one kilo, see Haug [15, 16]. This gives, for example, the kg mass of the electron equal to

$$m_e = \frac{f_e}{f_{1\text{kg}}} \approx \frac{7.76 \times 10^{20}}{8.52 \times 10^{50}} \approx 9.11 \times 10^{-31} \text{ kg} \tag{4}$$

A frequency divided by a frequency is a dimensionless number, so how did we arrive to a kg unit in the above calculations? First we need to ask what one kilogram is exactly. A kilogram of a mass is the amount of matter forming this mass relative to what has been decided upon as a kilogram of this matter. Our calculations correspond to the observed mass of the electron in terms of kg. This mass is in general observational time independent. Consequently, if we shrink the observational time window to half a second, both the reduced Compton frequency of the electron and the one kg will decrease by half, so their ratio will stay the same. However, if the observational window is close to or below the Compton time, then this mass becomes observational time dependent.

We will claim that the smallest observable mass inside a time interval is always linked to a reduced Compton frequency of 1. This is because observed frequencies must come in integer numbers, whereas fractions of frequencies do not make any sense from the observational point of view (non-integer frequencies can be linked to probabilities of observing events [15, 16], which are beyond the scope of this paper). If we are operating in an observational window of one second, the smallest observable reduced Compton frequency is one. To turn this into a kg of a mass, we need to divide one by the reduced Compton frequency of 1 kg, which gives

$$m_1 = \frac{f_1}{f_{1kg}} \approx \frac{1}{8.52 \times 10^{50}} \approx 1.17 \times 10^{-51} \text{ kg} . \tag{5}$$

We can calculate the energy equivalent of this mass using Einstein's energy–mass relation $E = mc^2$:

$$\frac{1}{f_{1kg}} c^2 = \frac{1}{8.52 \times 10^{50}} c^2 \approx 1.0545 \times 10^{-34} \text{ J} \tag{6}$$

The obtained value is equal to the reduced Planck constant, $\hbar$. In our view, this is not a coincidence. It shows that the Planck constant is linked not only to the quantization of energy, but also to the *quantization of matter*. We will further postulate that matter comes in discrete units linked to their reduced Compton frequency, and that the smallest frequency that can be observed is 1. At this point our considerations do not provide any new method for finding the Planck constant's value, as we already inserted it in our equations to find the reduced Compton wavelength of the one kilogram. However we will soon show how we can determine the Compton frequency of matter without any knowledge of the Planck constant.

As stated above, within a given time interval one can observe only an integer frequency, which means that we can observe only an integer number of occurrences of observed phenomena. The Planck constant's units are Joules per second, where Joule is kg·m$^2$·s$^{-2}$. Consequently, it is linked to kilograms, meters and seconds.

Let us now look more carefully at the reduced Compton frequency of one kilogram in relation to the exact 2019 NIST-defined value of the reduced Planck constant, $\hbar = h/(2\pi)$. We have

$$f_{1kg} = \frac{c}{\bar{\lambda}_{1kg}} = \frac{c}{\frac{\hbar}{1kg \times c}} = \frac{c^2}{\hbar} \left[ \frac{\text{kg} \cdot \text{m}^2/\text{s}^2}{\text{m}^2 \cdot \text{kg/s}} \right] = \frac{89,875,517,873,681,800}{1.054571817\ldots \times 10^{-34}} \text{ s}^{-1} \tag{7}$$

$$\approx 8522465366972893795814382170237724075267981488263 11.7503 \text{ s}^{-1} . \tag{8}$$

That is, the reduced Compton frequency of one kg in one second is given by only the speed of light and the reduced Planck constant. Since the speed of light and the Planck constant are both fixed at exact values, we obtain a non-integer reduced Compton frequency of one kg during an observational time window of one second. Fractional Compton frequencies cannot be observed, as explained above. The natural limitation on the observable frequency values, which is equal to one, invokes the problem of the mass gap [17]. Furthermore, the reduced Planck constant is equal to the frequency of 1 divided by the Compton frequency of a mass of 1 kg multiplied by $c^2$ (where we assumed that 1 kg physically represents a frequency ratio). A frequency of one divided by the reduced Compton frequency of one kilogram in the chosen observational window is then the kg definition of the mass gap. To turn it into the energy gap, we need to multiply it by $c^2$. An issue here is that the reduced Compton frequency that is linked to the Planck constant can't be observed in one second, because it is not an integer. An alternative and exact definition of the reduced Planck constant would therefore require us to decide on a new definition of a kilogram that is linked to an exact integer number in the reduced Compton frequency during a given time interval. For example we could round the current number up to exact

852246536697289379581438217023772407526798148826312

or down to

852246536697289379581438217023772407526798148826311

We could in fact decide on any integer number, if we did not mind obtaining a kilogram definition deviating significantly from the current value. However, it would make sense to round up the above number, even if it would enforce adjusting slightly the current definition of $c$ or $\hbar$ for the sake of remaining close to the current kilogram unit value. For instance, if we hold $c$ as it is and round up the reduced Compton frequency, the new value of the reduced Planck constant is

$$\begin{aligned} \hbar &= c^2 \frac{1}{852246536697289379581438217023772407526798148826312} \\ &\approx 1.054571816999999999999999999999999999999999999999999969\ldots \times 10^{-34} , \end{aligned} \tag{9}$$

and if we decide to round it up, it is

$$\begin{aligned} \hbar &= c^2 \frac{1}{852246536697289379581438217023772407526798148826311} \\ &\approx 1.054571817000000000000000000000000000000000000000000928\ldots \times 10^{-34} \end{aligned} \tag{10}$$

The later one is particularly convenient for defining a very accurate approximation of $\hbar \approx 1.054571817 \times 10^{-34}$, in cases where maximum precision is not needed.

Even if one ha chosen to operate with seconds in the SI unit system, this is an arbitrarily chosen time interval. If there is a fundamental time interval in nature it is likely the Planck time. The reduced Planck constant re-formulated in relation to Planck time would be

$$\begin{aligned} \hbar_p &= c^2 \frac{1}{852246536697289379581438217023772407526798148826312 \frac{l_p}{c}} \\ &= \frac{c}{852246536697289379581438217023772407526798148826312 l_p} \end{aligned} \tag{11}$$

or

$$\begin{aligned} \hbar_p &= c^2 \frac{1}{852246536697289379581438217023772407526798148826311 \frac{l_p}{c}} \\ &= \frac{c}{852246536697289379581438217023772407526798148826311 l_p} \approx 1954056587 = m_p c^2 \end{aligned} \tag{12}$$

The uncertainty of this definition would be high due to the uncertainty of the Planck length (which equals $1.616255(18) \cdot 10^{-35}$ m). Still, putting the CODATA value of the Planck length in the divisor, we obtain the Planck constant's value equal to the value of Planck mass energy. Again, however, the observable divisor value has to be integer. Should the current Planck constant value, as given by CODATA 2019 definition, turn out not to give an integer number at the Planck time for the reduced Compton frequency of one kg, the current definition is perhaps not fully consistent. The correction would be miniscule, anyway, as it would just require rounding e.g. 45994327.12 to 45994327. Still, the main problem here is that we do not know accurately the Planck length and, consequently, nor the Planck time. The Planck length is normally given by the formula first described by Max Planck as $l_p = \sqrt{\frac{G\hbar}{c^3}}$, which suggests that we need to know the Planck constant to find the Planck length [19, 20]. However, recent research has shown that it is possible to measure the Planck length without any knowledge of $G$ or $\hbar$, see [22, 23].

The quantisaztion of matter (and energy) through the Planck constant is given by one divided by the reduced Compton frequency in one kilogram. If measured over one second then this gives the standard value of the Planck contant, which correspond to a energy of $\hbar \times 1$, and a mass of $\frac{\hbar \times 1}{c^2}$. This is very close to the what has been suggested as the possible photon mass see [15, 21]. If we choose a time window of the Planck time, then a frequency of one divided by the Planck frequency per Planck time is the Planck mass. The smallest even hypothetical mass above zero is therefore observational time window dependent. , this ulike most masses that are observational time-independent. Well they are so as long as the observational time-window is above the Compton time. This is a new and somewhat controversial view, still we think it should be considered by the research community.

# 3 A new way to define and find the Planck constant

First we measure the Compton wavelength [18] of an electron by Compton scattering, which is given as

$$\lambda_e = \frac{\lambda_{\gamma,2} - \lambda_{\gamma,1}}{1 - \cos\theta} \tag{13}$$

We shoot a photon into an electron and measure the wavelengths of the photon when sent towards the electron and after the reflection, as well as the angle $\theta$ between the incident and reflected photon paths. We could also find the electron Compton wavelength from the kg mass of the electron by the well-known formula (also given by Compton in 1923),

$$\lambda_e = \frac{h}{mc} \ . \tag{14}$$

However, since we want to find the Compton wavelength in order to find the Planck constant, this formula naturally can't be used for our purpose. Once we find the Compton wavelength of the electron independent of the Planck constant, we can use the fact that the ratio of cyclotron frequencies a proton and an electron must be identical to the ratio of their Compton wavelengths. This cyclotron frequency is given by

$$f = \frac{qB}{2\pi m} \tag{15}$$

and, since the proton and electron charge is the same, we end up with

$$\frac{f_e}{f_P} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_P}} = \frac{m_P}{m_e} = \frac{\lambda_e}{\lambda_P} \approx 1836.15 \tag{16}$$

This is more than a theoretical value, as the cyclotron frequency has indeed been used as a method to find the proton–electron mass ratio, which is identical to their Compton length ratio, see for example [24, 25]. The CODATA 2019 value for the Compton wavelength of the electron and the proton is respectively $2.42631023867 \times 10^{-12}$ m and $1.32140985539 \times 10^{-15}$ m.

Now that we know the proton wavelength to be approximately equal

$$\lambda_P = \frac{\lambda_e}{\frac{\omega_e}{\omega_P}} \approx \frac{2.42 \times 10^{-12}}{1836.15} \approx 1.32 \times 10^{-15} \text{ m} \ , \tag{17}$$

we can decide how many protons we want in our kilogram definition. This approach is not new, as counting the number of atoms is one of important ways to define the Planck constant, see e.g. [26–28]. What is new here, however, is that at the deeper level we link it to the Compton frequency of matter. We can define it exactly as $6 \times 10^{26}$ protons plus $6 \times 10^{26}$ electrons. This means the Compton frequency per second for such a mass is

$$6 \times 10^{26} \times \left( \frac{c}{\lambda_P} + \frac{c}{\lambda_e} \right) \approx 1.36 \times 10^{50} \text{ times per second} \ . \tag{18}$$

If we take the frequency of one per second and divide by this value, we obtain

$$\frac{1}{1.36 \times 10^{50}} \approx 7.35 \times 10^{-51} \text{ kg} \ . \tag{19}$$

By assigning it the kilogram unit, we postulate that the kg mass is, at the deeper level, the frequency of the mass (energy) of interest divided by the Compton frequency in the arbitrary mass called kilogram. This view is discussed in more detail in [15], and it will also be discussed further in this paper. If we multiply the above frequency by $c^2$, we obtain energy,

$$hf = h \times 1 = \frac{1}{1.36 \times 10^{50}} c^2 \approx 6.6 \times 10^{-34} \text{ kg} \cdot \text{m}^2/\text{s}^2 \tag{20}$$

and we naturally have per definition $\hbar = \frac{h}{2\pi} \times 1 \approx 1.05 \times 10^{-34} kg \cdot m^2/s$. The value is very close to the Planck constant and the reduced Planck constant, and we could make it even closer by linking the Planck constant to the Compton frequency in today's kg definition. However, the main point of this derivation is not what the exact value of Planck constant should be, as we see that it is linked to an arbitrary quantity of matter, that we choose to call one kilogram. The Planck constant is always equal to one divided by the Compton frequency in this chosen quantity of matter, multiplied by $c^2$. The multiplication by $c^2$ is needed simply because the Planck constant is directly linked to the minimum physical amount (quantum) of energy and, therefore, indirectly to the minimum physical amount (quantum) of mass.

Let us now look at the quantization of energy in the context of the proposed Planck constant's definition. Energy comes in quanta, as expressed by the following formula

$$E = hf = h\frac{c}{\lambda} \tag{21}$$

The energy is thus $h$ multiplied by a frequency, where the frequency is measured per a time unit. On the other hand, the energy comes in Joules, namely kg·m$^2$·s$^{-2}$. Consequently, Joule is linked to time. In particular, if we look only at the frequency $f$, it is easy to see it is time dependent, as the Planck constant is just a constant, and the frequency is a frequency per second, $f = \frac{c}{\lambda}$.

## 4 Can composite masses have a Compton wavelength?

In section 3 of this paper we found the Compton wavelength of a proton, by first finding the Compton wavelength of an electron and then using the cyclotron frequency to estimate the Compton wavelength of a proton. One could argue that only elementary particles like the electron can have the Compton wavelength, whereas composite masses such as a proton cannot. We generally agree with such arguments. Yet, composite masses ultimately consist of aggregates of elementary particles. If each of these elementary particles has a real Compton wavelength, we should be able to somehow put them together to find the Compton wavelength of the composite particle. Naturally, the aggregated Compton wavelength of a composite mass must be consistent with standard mass aggregation, that is

$$m = m_1 + m_2 + m_3 + \cdots + m_n \tag{22}$$

Furthermore, since any kilogram mass can be represented mathematically as $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$, we can rewrite it as

$$\begin{aligned}
\frac{\hbar}{\bar{\lambda}}\frac{1}{c} &= \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_3}\frac{1}{c}\cdots + \frac{\hbar}{\bar{\lambda}_n}\frac{1}{c} \\
\frac{1}{\bar{\lambda}} &= \frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3}\cdots + \frac{1}{\bar{\lambda}_n} \\
\bar{\lambda} &= \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3}\cdots + \frac{1}{\bar{\lambda}_n}}
\end{aligned} \tag{23}$$

Consequently, any mass should have a Compton wavelength, even astronomical objects like planets, the Sun and other stars. Yet, any mass larger than the Planck mass has a Compton wavelength shorter than the Planck length, which suggests that it is not a physical Compton wavelength, since the Planck length is likely the shortest possible physical Compton wavelength. However, the aggregated Planck length could be shorter than the Planck length as it is not a physical length, but just an ancillary number that is very useful in calculations and in understanding the deeper constitution of reality. There is therefore nothing wrong in using the Compton wavelength of one kilogram as we have done above. Actually, recent research indicates that matter ticks at the Compton frequency, see for example [29, 30]. The interest for the Compton wavelength of the proton (a composite mass) goes back to at least to Levitt's paper [31] from 1958, and it has revived recently as Trinhammer and Bohr [32] have shown that the Compton wavelength of the proton could potentially be directly linked to the proton radius.

## 5 Conclusion

We have presented a new in-depth interpretation of the physical nature of the Planck constant as resulting from the quantum nature of mass and its Compton frequency. We derived its definition as equal to one divided by the Compton frequency of one kilogram, multiplied by $c^2$. By defining the kilogram as a given number of protons, we obtained the Compton frequency of one kilogram of matter as the sum of the Compton frequency of all the protons and electrons making up this matter. It is naturally important for our definition of the Planck constant that we can find the Compton frequency of a proton without knowing the Planck constant, and we demonstrated that it is fully possible. We believe that the presented physical picture gives a new insight in the nature of the Planck constant. Potentially it could help to explain the origin of the mass-gap as a consequence of the natural limit on the observable (integer by necessity) Compton frequency of one.

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