Quantum Field Theory Models and the Generating Function Technique

Robert L. Jackson, M.D., Stilwell, Kansas

Abstract

Quantum Field Theory, or QFT, is a well-accepted set of theories used in particle physics that involves Lagrangian mechanics. An individual can generate a rich variety of Hamiltonian equation systems from the Lagrangian associated QFT to describe simultaneous or cofounding processes which occur in particle physics. Unfortunately, the equation systems associated with QFT are relatively hard to solve. This paper will show that the generating function technique (GFT) can be used to directly solve these equation systems while also producing renormalization results. The usage of the latter is necessary to display the consistency of the solutions and equation systems. Ultimately, an astute scientist in QFT can claim GFT is a valuable tool to be utilized in the field of particle physics.

1. *Introduction*

QFT is a combination of quantum mechanics, classical field theory, and special relativity [1]. It is commonly applied to particle physics, thus essential and in the formation of models within the realm of subatomic and condensed matter physics [1]. It heavily utilizes Lagrangian mechanics to display the interaction of particles, which are defined as quantum fields [2]. Since its advent in the 1920s and rebirth in the 1970s, QFT has had a prominent role in describing contemporary physics [3].

QFT was divided into at least three branches: quantum electrodynamics (QED), quantum flavordynamics (QFD), and quantum chromodynamics (QCD). QED was primarily developed by Dirac in 1927 which built upon the concept of canonical quantization [4]. It dealt with the interaction of fermionic and electromagnetic fields [5]. QFD was the study of electroweak nuclear force, such as bosons Z^0 and W^{+}_{-} activities, while QCD involved strong nuclear interactions, generally mediated gluon fields [6,7]. It is not uncommon to find situations where certain branches, like QED and QCD, crossed over or encroached on each other.

The generating function technique (GFT) is a novel method for solving [nonlinear] PDEs [8]. It assumes there is a general solution to the PDE of interest already exists; thus, solving the PDE requires one to determine the appropriate degree[s] of the solution, then (s)he computes the necessary constants to obtain the solution. Even though the processes of GFT are simple to comprehend, it requires a computer to carry out the steps.

This paper utilizes GFT in the derivation of sets of exact solutions to a set of new QFT models. The study is reduced into three more sections. Section two deals with the ascertainment of critical Hamiltonian equations from the Lagrangian of more extensive QFT models. Section two also provides a brief description of GFT that is used to derive sets of exact solutions for the Hamiltonian equations and an easier way to generate renormalization results using the solutions derived from GFT. Section three describes several QFT scenarios in which GFT and the new renormalization method are implemented to produce solutions and mass-energies for particle fields. Finally, section five gives a synopsis of the QFT models and the implications of the efficiency of GFT to generate solutions and renormalization results.

2. *Models and Methodology*

2.1. *A variation of the Yukawa interaction*

A Yukawa interaction is a type of QCD model which involves the relationship between a gauge boson and fermion fields [9]. The former field can be [partially] self-interacting; in other words, the constant λ in the equation is not null. The principle of least action for a gauge boson ϕ_i which either decays into or generates from a fermion ψ_j and its antiparticle $\psi_j$is expressed as follows:

$$\mathcal{S}[\phi_i, \psi_j] = \int dx^4 \sum_j \left(-\phi_i \delta_j \psi_j \psi_j^* + \frac{\lambda \phi_i^3}{3} + \frac{1}{2}\phi_i^2 \left(m_{\phi_i}^2 + \partial_\mu \partial^\alpha \right) + \delta_j \psi_j \psi_j^* \left(m_\psi^2 + \partial_\mu \partial^\mu \right) \right),$$

where m_{fi} and m_{yj} are the invariant masses of the gauge boson f_i and fermion y_j, respectively, d_j is not a Kronecker product and equals ± 1 depending upon whether the field occurs before or after the gauge boson f_i, and l is a coupling constant. The above equation can be converted to a Hamiltonian:

$$\mathcal{H}\left[\phi_i, \psi_j\right] = \int dx^4 \sum_j \left(\frac{\partial \phi_i}{\partial t}^2 + i\delta_j \psi_j^* \frac{\partial \psi_j}{\partial t} + i\delta_j \frac{\partial \psi_j^*}{\partial t} \psi_j - \frac{1}{3}\lambda \phi_i^3 - \frac{1}{2}\phi_i^2 \left(m_{\phi_i}^2 + \partial_\mu \partial^\mu \right) - \delta_j \psi_j^* \psi_j \left(m_\psi^2 + \partial_\mu \partial^\mu \right) + \phi \delta_j \psi_j \psi_j \right).$$

With Poisson brackets [10], an individual can derive time evolution equations for the gauge boson :

$$\frac{\partial^2 \phi_i}{\partial t^2} = \left\{ \frac{\partial \phi_i}{\partial t}, \int dx^4 \sum_j \left(\frac{\partial \phi_i}{\partial t}^2 + i\delta_j \psi_j^* \frac{\partial \psi_j}{\partial t} + i\delta_j \frac{\partial \psi_j^*}{\partial t} \psi_j - \frac{1}{3}\lambda \phi_i^3 - \frac{1}{2}\phi_i^2 \left(m_{\phi_i}^2 + \partial_\mu \partial^\mu \right) - \delta_j \psi_j^* \psi_j \left(m_{\psi_j}^2 + \partial_\mu \partial^\mu \right) \right) \right\},$$

or

$$\tfrac{\partial^2 \phi_i}{\partial t^2} = -\lambda \phi_i^2 - \phi_i (m_\phi^2 - \Delta) + \textstyle\sum_j \delta_j \psi_j \psi_j^*.$$

By placing all terms on the left side of the equation, the individual yields:

$$\frac{\partial^2 \phi_i}{\partial t^2} - \Delta \phi_i + \lambda \phi_i^2 + \phi_i m_{\phi_i}^2 - \sum_j \delta_j \psi_j \psi_j^* = 0.$$

To obtain a comparable equation for fermion field ψ_j, the same individual again must use Poisson brackets:

$$\delta_l \frac{\partial^2 \psi_l}{\partial t^2} = \left\{ \delta_l \frac{\partial \psi_j^*}{\partial t}, \int dx^4 \sum_j \left(\frac{\partial \phi_i}{\partial t}^2 + i\delta_j \psi_j^* \frac{\partial \psi_j}{\partial t} + i\delta_j \frac{\partial \psi_j^*}{\partial t} \psi_j - \frac{1}{3}\lambda \phi_i^3 - \frac{1}{2}\phi_i^2 \left(m_{\phi_i}^2 + \partial_\mu \partial^\mu \right) - \delta_j \psi_j^* \psi_j \left(m_{\psi_j}^2 + \partial_\mu \partial^\mu \right) \right) \right\},$$

or

$$\frac{\partial^2 \psi_l}{\partial t^2} = i \frac{\partial \psi_l}{\partial t} - \phi_i \psi_l - \psi_l \left(m_{\psi_l}^2 - \Delta \right),$$

where l is an element of the j-th fermion field. Note: fermion fields in a j-th pair do not have to be the same entity. By placing all terms on the right side of the equation, the individual obtains:

$$i \frac{\partial \psi_l}{\partial t} - \frac{\partial^2 \psi_l}{\partial t^2} - \phi_i \psi_l + \Delta \psi_l - \psi_l m_{\psi_l}^2 = 0.$$

The above equation is a variation of the Schrodinger equation. It is highly similar to the equation derived by Arbab and Yassein (et al., 2011). In future analysis, m_{ϕ_i} is set to m while m_{ψ_j} is set to null or *0.0*.

2.2. GFT

GFT is a method for solving [non]linear PDEs via the utilization of a general solution u_g that comprises Laurent series sets of combinatorial number or trigonometric-based generating functions [8]. An individual determines the maximal and minimal power degree, or p_s, through which the Laurent series is eventually truncated. Then, one solves a linear auxiliary/characteristic ordinary differential equation to yield a function f is plugged into the transformed general solution U_g, or:

$$U(\xi) = \sum_{i=1}^{2} \sum_{j=-p_s}^{p_s} (a_{ij}(\sum_{k=0}^{\infty} 2f(\xi)^k S_k(0)^i)^j + b_{ij}(\sum_{k=0}^{\infty} 2C_k(0)^i f(\xi)^k)^j),$$

or

$$U(\xi) = \sum_{i=1}^{2} \sum_{j=-p_s}^{p_s} (dl_{ij}(\sum_{k=0}^{\infty} 2C_k(0)^i f(\xi)^k)^j + cl_{ij}(\sum_{k=0}^{\infty} 2f(\xi)^k S_k(0)^i)^j),$$

where the expression $S_k(0)$ is the square root of the k-th Fibonacci number at/about zero, or

$$S_k(0) = \sin\left(\frac{\pi k}{2}\right),$$

the expression $C_k(0)$ is the k-th Chebyshev U number at/about zero, or

$$C_k(0) = \cos\left(\frac{\pi k}{2}\right),$$

and the transformed variable x for a (3+1) system is defined as:

$$\xi = \alpha t + \beta_1 x + \beta_2 y + \beta_3 z.$$

For this article, the arbitrary constants a_{ij} and b_{ij} are used for the primary gauge boson field while the arbitrary constants cl_{ij} and dl_{ij} are used for secondary gauge boson and or fermion fields where $l = 1, 2, \ldots, n$ and n is the total number of secondary items.

2.3. The generation of renormalization results

The basic formula for “self-interacting" renormalization is defined as:

$$m_p = \frac{1}{2} \int dV p(x) p^*(x),$$

where m_p is the mass-energy of particle field p, p^* is the conjugate of particle field p, and V is the volume that contains the particle field p. If the spherical volume for particle field p is equivalent to the following expression, assuming one is working with Manhattan/taxicab-like distance ξ [11]:

$$V = \frac{\pi \xi^3}{6},$$

then the formula for renormalization becomes:

$$m_p = \int_0^{\infty} \frac{1}{4}\pi\xi^2 p(\xi)\, p^*(\xi)\, d\xi.$$

The above expression can be simply redefined as:

$$m_p = \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{4}\pi\xi^2 p(\xi)\, p^*(\xi)\, d\xi.$$

In terms of the inner product, the mass-energy of particle field p can also be expressed as:

$$m_p = \frac{1}{8}\pi \langle \xi p(\xi), \xi p(\xi) \rangle .$$

For renormalization that is based on mixing interactions, consider the following expression:

$$m_{p_{1-2}} = \int_0^{\infty} \frac{1}{4}\pi\xi^2 p_1(\xi)\, p_2^*(\xi)\, d\xi = \frac{1}{8}\pi \langle \xi p_1(\xi), \xi p_2(\xi) \rangle ,$$

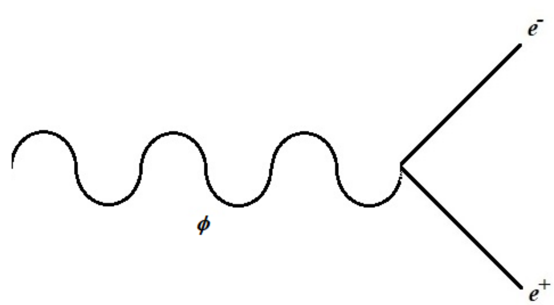
where particles p_1 and p_2 are different quantum fields.

3. *Examples*

The supplementary materials contain Mathematica (R) spreadsheets that pertain to the QFT models described in this paper.

3.1. Mesonic decay and photoproduction

Assume one is dealing with a simple Feynman diagram where a meson decays and gives rise to an electron and position pair:



The principle of least action for this system is given by the following equation:

$$\mathcal{S}[\phi, \psi] = \int dx^4 \left(\frac{\lambda\phi^3}{3} + \psi\psi^* \left(m_\psi^2 + \partial_\mu\partial^\mu \right) + \frac{1}{2}\phi^2 \left(m_\phi^2 + \partial_\mu\partial^\mu \right) + \psi\phi\psi^* \right).$$

The above expression can be used to derive the Hamiltonians for all particles of interest:

$$\frac{\partial^2\phi}{\partial t^2} - \Delta\phi + \lambda\phi^2 + \phi m_\phi^2 + \psi\psi^* = 0,$$

and

$$i\frac{\partial\psi}{\partial t} - \frac{\partial^2\psi}{\partial t^2} + \Delta\psi - \psi m_\psi^2 - \psi\phi = 0.$$

Next, GFT and renormalization are to generate solutions and mass-energies of particles. GFT is used to generate the solutions to the Hamiltonians:

$$\phi(t,x,y,z) = -\frac{12m^2\exp(\frac{2im^2t}{3\lambda+1}+\frac{z\sqrt{-4(\beta_1^2+\beta_2^2)(3c\lambda+c)^2-4m^4+2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}}+2\beta_1x+2\beta_2y)}{(3\lambda+1)(1+\exp(\frac{2im^2t}{3\lambda+1}+\frac{z\sqrt{-4(\beta_1^2+\beta_2^2)(3c\lambda+c)^2-4nt^4+2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}}+2\beta_1x+2\beta_2y))^2},$$

and

$$\begin{gathered}
\psi_e(t,x,y,z) = \\
-((6\sqrt{(\lambda+1)m^4}\exp(\tfrac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta1^2+\beta2^2)(3c\lambda+c)^2-4m^4+2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta_1x + 2\beta_2y) \\
(-1+\exp(\tfrac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta_1^2+\beta_2^2)(3c\lambda+c)^2-4m^4+2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta1x + 2\beta2y)))/ \\
(\sqrt{(3\lambda+1)^2} \\
(1+\exp(\tfrac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta_1^2+\beta_2^2)(3c\lambda+c)^2-4m^4+2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta_1x + 2\beta_2y))^2)),
\end{gathered}$$

while the results of renormalization of the same particles can be expressed as the following if one sets the constant λ to null and the speed of light c to unity:

$$m_\phi = 0.0949744|m|^4,$$

and

$$m_{\psi_e} = 0.536761|m|^4.$$

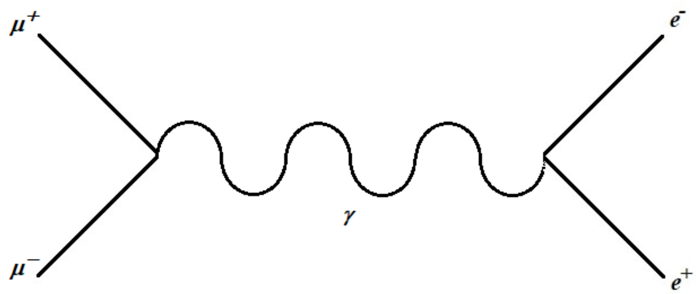
Using the above results, one can calculate the needed center-of-mass, or $\sqrt{s}$, for electron-positron collision to produce a particular meson. For instance, (s)he first must set m_ϕ to well-known mass-energy and solve for m, then (s)he can calculate the $\sqrt{s}$ for the particle by plugging m into m_{ψ_e}. The following table shows the predicted $\sqrt{s}$ for various mesons:

meson	***mass-energy (eV)***	***center-of-mass (eV)***
neutral pion	*1.34∗10ˆ8*	*7.57∗10ˆ8*
neutral kaon	*4.98∗10ˆ8*	*2.81∗10ˆ9*
neutral D meson	*1.86∗10ˆ9*	*1.05∗10ˆ10*
neutral B meson	*5.28∗10ˆ9*	*2.98∗10ˆ10*

.

3.2. *Lepton pair decay and production*

Assume one is dealing with a simple Feynman diagram where [anti]muon pair decay into a photon and the photon gives rise to an electron and position pair:



The principle of least action for this system is given by the following equation:

$$\mathcal{S}\left[\phi,\psi_1,\psi_2\right] = \int dx^4 \left(\frac{\lambda\phi^3}{3} - \psi_1\psi_1^*\left(m_{\psi_1}^2 + \partial_\mu\partial^\mu\right) + \psi_2\psi_2^*\left(m_{\psi_2}^2 + \partial_\mu\partial^\alpha\right) + \frac{1}{2}\phi^2\left(m_\phi^2 + \partial_\mu\partial^\mu\right) - \psi_1\phi\psi_1^* + \psi_2\phi\psi_2^*\right).$$

The above expression can be used to derive the Hamiltonians for all particles of interest:

$$\frac{\partial^2\phi}{\partial t^2} - \Delta\phi + \lambda\phi^2 + \phi m_\phi^2 - \psi_1\psi_1^* + \psi_2\psi_2^* = 0,$$

$$i\frac{\partial\psi_1}{\partial t} - \frac{\partial^2\psi_1}{\partial t^2} + \Delta\psi_1 - \psi_1 m_{\psi_1}^2 - \psi_1\phi = 0,$$

and

$$i\frac{\partial\psi_2}{\partial t} - \frac{\partial^2\psi_2}{\partial t^2} + \Delta\psi_2 - \psi_2 m_{\psi_2}^2 - \psi_2\phi = 0,$$

where the gauge boson ϕ is equal to photon γ.

Next, GFT and renormalization are to generate solutions and mass-energies of particles. GFT is used to generate the solutions to the Hamiltonians after setting the masses of the spin-½ fermions to null:

$$\gamma\left(t,x,y,z\right) = -\frac{3}{2}c^2m^2\sec^2\left(c^2m^2t + \frac{1}{2}i\left(z\sqrt{-4\left(\beta_1^2+\beta_2^2\right) - 4c^2m^4 - c^2m^2} + 2\beta_1x + 2\beta_2y\right)\right),$$

$$\psi_1\left(t,x,y,z\right) = -d1\left(1,4\right)\left(\tan\left(c^2m^2t + \frac{1}{2}iz\sqrt{c^2\left(-m^2\right)\left(4m^2+1\right) - 4\left(\beta_1^2+\beta_2^2\right)} + i\beta_1x + i\beta_2y\right) + i\right)^2,$$

and

$$\psi_2(t,x,y,z) =$$
$$\tfrac{1}{2}\sqrt{4d1(1,4)^2 - 9c^4(\lambda-1)m^4}(\tan(c^2m^2t + \tfrac{1}{2}iz\sqrt{c^2(-m^2)(4m^2+1) - 4(\beta 1^2 + \beta 2^2)} + i\beta_1x + i\beta_2y) + i)^2,$$

while the results of renormalization of the same particles can be defined as the following if one sets the constant *l* to *2* and the speed of light *c* to unity:

$$m_\gamma = 0.379898|m|^4,$$

$$m_{\psi_e} = 0.168843|d1\,(1,4)\,|^2,$$

and

$$m_{\psi_\mu} = 0.0422108|9m^4 - 4.d1(1,4)^2|.$$

Finally, an individual would use the results from renormalization to obtain the value of constants m and $d1_{14}$ and prove the consistency of the results. By setting $m_{\psi_\mu}$and m_{ψ_e} to *1.05∗10ˆ8* and *5.11∗10ˆ5* eV, respectively, (s)he produces an m and $d1_{14}$ equal to *–128.781* and *24937.5,* also respectively. The mass-energy m_γ equals *104.489* MeV, which is consistent with the difference between $m_{\psi_\mu}$and m_{ψ_e}.

3.3. Glueball prediction via possible spin-½ Majorana fermions model

Assume the following principle of least action is true:

$$\mathcal{S}[\phi_i, \psi_{j,\{1,2\}}, \chi_j] = \int dx^4 \sum_j \Big(\phi_i \delta_j \psi_{j,1} \chi_j^H + \delta_j \psi_{j,1} \chi_j^H \left(M^2_{\psi^2_{j,1}-\chi_j} + \partial_\mu \partial^\mu\right) + \phi_i \delta_j \chi_j \psi^*_{j,2} + \frac{\lambda \phi_i^3}{3} + \frac{1}{2}\phi_i^2 \left(m^2_{\phi_i} + \partial_\mu \partial^\mu\right) + \delta_j \chi_j \psi^*_{j,2} \left(M^2_{\psi_{j,2}-\chi_j} + \partial_\mu \partial^\mu\right)\Big),$$

where χ is Majorana fermion and M is some association of the fermion ψ and χ invariant masses. Like typical Dirac fermions, some Majorana particles are spin-½ particles. The Hamiltonian equations for the above equation are defined as:

$$\frac{\partial^2 \phi_i}{\partial t^2} + \delta_j \psi_{j,1} \chi_j^H - \Delta \phi_i + \lambda \phi_i^2 + \phi_i m^2_{\phi_i} + \delta_j \chi_j \psi_{j,2} = 0,$$

$$-\frac{\partial \psi_{j,\{1,2\}}}{\partial t} + \frac{\partial^2 \psi_{j,\{1,2\}}}{\partial t^2} - \phi_i \psi_{j,\{1,2\}} + \psi_{j,\{1,2\}} M^2_{\psi_{j,\{1,2\}}-\chi_j} - \Delta \psi_{j,\{1,2\}} = 0,$$

and

$$-\frac{\partial \chi_j}{\partial t} + \frac{\partial^2 \chi_j}{\partial t^2} - \phi_i \chi_j - \Delta \chi_j + \chi_j M^2_{\psi_j,\{1,2\}-\chi_j} = 0,$$

where $\psi_{j,\{1,2\}}$is either the Dirac fermion particle $\psi_{j,1}$ or $\psi_{j,2}$ while χ is a Majorana fermion, which is equal to its Hermitian χ^H. Setting λ and the association M to null, the solutions for the particles via GFT are the following:

$$\phi(t,x,y,z) = -\frac{3}{2}c^2 m^2 \sec^2\left(c^2 m^2 t + \frac{1}{2} i \left(z\sqrt{-4\left(\beta_1^2 + \beta_2^2\right) - 4c^2 m^4 - c^2 m^2} + 2\beta_1 x + 2\beta_2 y\right)\right),$$

$$\psi_1(t,x,y,z) = -d1(1,4)\left(tan\left(c^2 m^2 t + \frac{1}{2} i z\sqrt{c^2\left(-m^2\right)\left(4m^2+1\right) - 4\left(\beta_1^2 + \beta_2^2\right)} + i\beta_1 x + i\beta_2 y\right) + i\right)^2,$$

$$\psi_2(t,x,y,z) = -\left(\left(\frac{9c^4(\lambda-1)m^4}{4\,d3(1,4)} - d1(1,4)\right)\left(tan\left(c^2 m^2 t + \frac{1}{2} i z\sqrt{c^2\left(-m^2\right)\left(4m^2+1\right) - 4\left(\beta_1^2 + \beta_2^2\right)} + i\beta_1 x + i\beta_2 y\right) + i\right)^2\right),$$

and

$$\chi(t,x,y,z) = -d3(1,4)\left(tan\left(c^2 m^2 t + \frac{1}{2} i z\sqrt{c^2\left(-m^2\right)\left(4m^2+1\right) - 4\left(\beta_1^2 + \beta_2^2\right)} + i\beta_1 x + i\beta_2 y\right) + i\right)^2.$$

Using the mass-energy equation, one can derive the following solutions:

$$m_\phi = 1.5|m|^4,$$

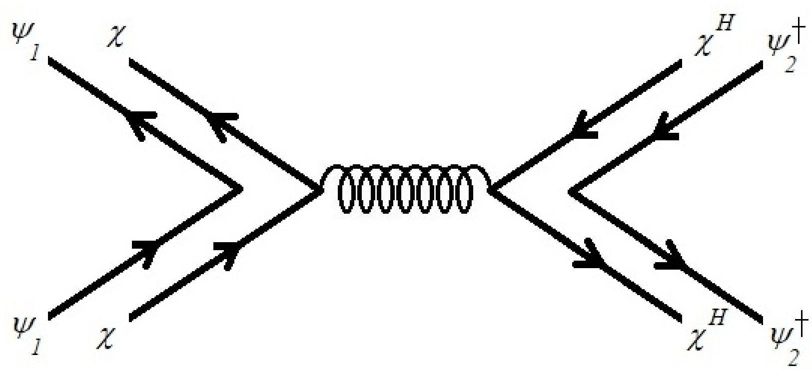
$$m_{\psi_1} = 0.667|\, d1(1.,4.)|^2,$$

$$m_{\psi_2} = 0.0417\left|\frac{\left(9.m^4+4.d1(1.,4.)d3(1.,4.)\right)^2}{d3(1.,4.)^2}\right|,$$

and

$$m_\chi = 0.667|\, d3(1.,4.)|^2.$$

One may also assume that the interaction between two Majorana fermions could form a glueball. Some scientists claim the interaction of spin-½ Majorana fermions can generate spin-1 liquid, thus the following diagram may be correct:



.

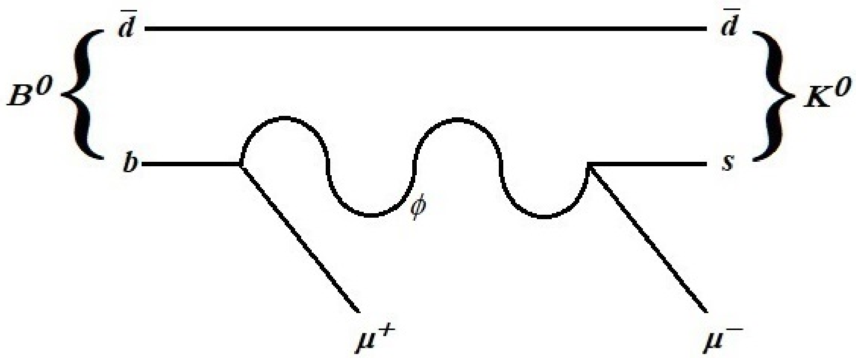
The Feynman aspect of the picture is the gluon interaction formed between the purposed Majorana fermions. In other words, the mass of a glueball would constitute the mass-energy of Majorana spin-½ fermion. A table of predicted glueball masses derived from various meson decays is featured below:

meson	**m_ϕ *(eV)***	**m_{ψ_1} *(eV)***	**m_{ψ_2} *(eV)***	**m_χ*/glueball(eV)***
charged pion	*1.40∗10ˆ8*	*2.20∗10ˆ6*	*4.70∗10ˆ6*	*1.47∗10ˆ9*
neutral kaon	*4.98∗10ˆ8*	*4.70∗10ˆ6*	*9.60∗10ˆ7*	*1.73∗10ˆ9*
charmed eta	*2.98∗10ˆ9*	*1.28∗10ˆ9*	*1.28∗10ˆ9*	*1.73∗10ˆ9*
bottom eta	*9.30∗10ˆ9*	*4.18∗10ˆ9*	*4.18∗10ˆ9*	*5.17∗10ˆ9*
J/psi	*3.10∗10ˆ9*	*1.28∗10ˆ9*	*1.28∗10ˆ9*	*1.88∗10ˆ9 [15]*

.

3.4 New Physics analysis

Assume one is dealing with the following Feynman diagram:



,

The principle of least action for this system is given by the following equation:

$$\mathcal{S}[\phi_i, \psi_{j,\{1,2\}}, \chi_j] =$$

$$\int dx^4 \sum_j \left(\phi_i \delta_1 \psi_{1,1} \chi_1^H + \phi_i \delta_2 \psi_{2,1} \chi_2^H + \delta_1 \psi_{1,1} \chi_1^H \left(M^2_{\psi_{1,1}-\chi_1} + \partial_\mu \partial^\mu\right) + \right.$$

$$\delta_2 \psi_{2,1} \chi_2^H \left(M^2_{\psi_{2,1}-\chi_2} + \partial_\mu \partial^\mu\right) + \phi_i \delta_1 \chi_1 \psi^*_{1,2} + \phi_i \delta_2 \chi_2 \psi^*_{2,2} + \frac{\lambda \phi_i^3}{3} + \frac{1}{2} \phi_i^2 \left(m^2_{\phi_i} + \partial_\mu \partial^\mu\right)$$

$$\left. + \delta_1 \chi_1 \psi^*_{1,2} \left(M^2_{\psi_{1,2}-\chi_1} + \partial_\mu \partial^\mu\right) + \delta_2 \chi_2 \psi^*_{2,2} \left(M^2_{\psi_{2,2}-\chi_2} + \partial_\mu \partial^\mu\right) + \phi_i \delta_3 \psi_3^* \psi_3 + \delta_3 \psi_3^* \psi_3 \left(m^2_{\psi_3} + \partial_\mu \partial^\mu\right)\right).$$

The above expression can be used to derive the Hamiltonians for all particles of interest:

$$\frac{\partial^2 \phi_i}{\partial t^2} + \delta_1 \psi_{1,1} \chi_1^H + \delta_2 \psi_{2,1} \chi_2^H - \Delta \phi_i + \lambda \phi_i^2 + \phi_i m^2_{\phi_i} + \delta_1 \chi_1 \psi_{1,2} + \delta_2 \chi_2 \psi_{2,2} = 0,$$

$$-\frac{\partial \psi_{j,\{1,2\}}}{\partial t} + \frac{\partial^2 \psi_{j,\{1,2\}}}{\partial t^2} - \phi_i \psi_{j,\{1,2\}} + \psi_{j,\{1,2\}} M^2_{\psi_{j,\{1,2\}}-\chi_j} - \Delta \psi_{j,\{1,2\}} = 0,$$

and

$$-\frac{\partial \chi_j}{\partial t} + \frac{\partial^2 \chi_j}{\partial t^2} - \phi_i \chi_j - \Delta \chi_j + \chi_j M^2_{\psi_{j},\{1,2\}-\chi_j} = 0,$$

Next, GFT and renormalization are to generate solutions and mass-energies of particles. GFT is used to generate the solutions to the Hamiltonians after setting the association M to null and λ to *-1.0*:

$$\phi(t,x,y,z) = -\frac{3}{2} m^2 \sec^2\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right),$$

$$\psi_{1,1}(t,x,y,z) = -d1(1,4)\left(tan\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right) + i\right)^2,$$

$$\psi_{1,2}(t,x,y,z) = -d2(1,4)\left(tan\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right) + i\right)^2,$$

$$\psi_{2,1}(t,x,y,z) = -d4(1,4)\left(tan\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right) + i\right)^2,$$

$$\psi_{2,2}(t,x,y,z) = -\frac{1}{4\,d6(1,4)}\left(4\,d3(1,4)(d1(1,4)+d2(1,4)) - 4\left(d4(1,4)d6(1,4)+d7(1,4)^2\right) - 9(\lambda-1)m^4\right)$$

$$tan\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right) + i,$$

$$\psi_3(t,x,y,z) = -d7(1,4)\left(tan\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right) + i\right)^2,$$

$$\chi_1(t,x,y,z) = -d3(1,4)\left(tan\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right)^2 + i\right),$$

and

$$\chi_2(t,x,y,z) = -d6(1,4)\left(tan\left(m^2 t - \frac{iz\sqrt{-4c^2\left(\beta_1^2+\beta_2^2\right)-4m^4-c^2m^2}}{2c} + i\beta_1 x + i\beta_2 y\right) + i\right)^2.$$

while the results of renormalization of the same particles can be expressed as the following if one sets the constant λ to unity and the speed of light c to unity:

$$m_\phi = 0.38|m|^4,$$

$$m_{\psi_{1,1}} = 0.169|\,d1(1.,4.)|^2,$$

$$m_{\psi_{1,2}} = 0.169|\,d2(1.,4.)|^2,$$

$$m_{\psi_{2,1}} = 0.169|\,d4(1.,4.)|^2,$$

$$m_{\psi_{2,2}} = 0.0422\left|\frac{\left(9.m^4-2.d7(1.,4.)^2+2.d1(1.,4.)d3(1.,4.)+2.d2(1.,4.)d3(1.,4.)-2.d4(1.,4.)d6(1.,4.)\right)^2}{d6(1.,4.)^2}\right|,$$

$$m_{\psi_3} = 0.169|\,d7(1.,4.)|^2,$$

$$m_{\chi_1} = 0.169|\,d3(1.,4.)|^2,$$

and

$$m_{\chi_2} = 0.169|\,d6(1.,4.)|^2.$$

If $m_{\psi_{1,1}}$, $m_{\psi_{1,2}}$, and the sum of the mixing interaction mass-energies $m_{\psi_{1,1}}$-m_{χ_1} and $m_{\psi_{1,2}}$-m_{χ_1} are equal to *4.18∗10ˆ9*, *4.70∗10ˆ6*, and *5.28∗10ˆ9 eV*, respectively, then the mass-energy of Majorana particle c_1 is *6.24∗10ˆ9 eV*. On the other hand, if $m_{\psi_{2,1}}$, $m_{\psi_{2,2}}$, and the sum of the mixing interaction mass-energies $m_{\psi_{2,1}}$-m_{χ_2} and $m_{\psi_{2,2}}$-m_{χ_2} are equal to *9.60∗10ˆ7*, *4.70∗10ˆ6*, and *4.98∗10ˆ8 eV*, respectively, then the mass-energy of Majorana particle c_2 is *1.73∗10ˆ9 eV*. Next, one would find the mass-energy of particle f is *2.34∗10ˆ9 eV*. Note: this is the value produced if an individual takes the geometric mean of the mass-energies of the prospective (Nambu partner to the) Higgs boson (*3.25∗10ˆ11 eV*) and particle *X17* (*1.68∗10ˆ7 eV*) [12,13].

4. *Conclusion*

4.1. QFT can be used to generate a large variety of equation systems describing particle physics.

Section three provides several examples which implemented some variation of the QFT models provided by section two. The equation systems produced by the Lagrangian for the examples are novel, wide-ranging, and highly descriptive. In other words, there is practically no limit to the Lagrangian or equation systems one can contemplate in QFT.

4.2. GFT can easily derive solutions and renormalization results to many equation systems in QFT.

Section three and supplementary material also showed relative ease of solving particle fields and generating renormalization results from the solution via GFT. In other words, only a few steps are needed to produce solutions to both fermion and gauge boson fields involved in each QFT model. Ultimately, GFT is an ideal tool for solving QFT models.

References

[1] Peskin, M.; Schroeder, D. (1995). An Introduction to Quantum Field Theory. Westview Press.
[2] Sutherland, R.I. (2006). Lagrangian Description for Particle Interpretations of Quantum Mechanics – Entangled Many-Particle Case. arXiv-1509.02442
[3] Shifman, M. (2012). Advanced Topics in Quantum Field Theory. Cambridge University Press.
[4] Dirac, P. A. M. (1 February 1927). "The Quantum Theory of the Emission and Absorption of Radiation". Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. The Royal Society. 114 (767): 243–265.
[5] Feynman, R. (1985). QED: The Strange Theory of Light and Matter. Princeton University Press.
[6] Griffiths, D. (2009). Introduction to Elementary Particles. pp. 59–60.
[7] Lincoln, D. (2016). QCD: Quantum Chromodynamics. United States: N. p., 2016. Web.
[8] Jackson, R.L. (2019) "A possible theory of partial differential equations" vixra.org/1910.0064.
[9] Yukawa, H. (1935). "On the interaction of elementary particles". Proc. Phys. Math. Soc. Jpn. 17: 48.
[10] Arnold, V. I. (1989). Mathematical Methods of Classical Mechanics (2nd ed.). New York: Springer. ISBN 978-0-387-96890-2.
[11] Colakoglu, H.B. (2009). Volume of a Tetrahedron in the Taxicab Space. Missouri J. Math. Sci. 21(1): 21-27.
[12] Osipov, A.A. (2021). Top-Bottom Condensation Model: Symmetries and Spectrum of the Induced 2HDM. Symmetry, 13(7); 10.3390/sym13071130.
[13] Krasznahorkay, A.J. (2016). Observation of Anomalous Internal Pair Creation in ^{8}Be: A Possible Indication of a Light, Neutral Boson. *Physical Review Letters*. 116 (42501): 042501.
[14] Sarantsev, A.V. (2021). Scalar isoscalar mesons and the scalar glueball from radiative J/ψ decays. arXiv:2103.09680v1.