Quantum Field Theory Models and the Generating Function Technique

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Abstract

Quantum Field Theory, or QFT, is a well-accepted set of theories used in particle physics that involves Lagrangian mechanics. An individual can generate a rich variety of Hamiltonian equation systems from the Lagrangian associated QFT to describe simultaneous or cofounding processes which occur in particle physics. Unfortunately, the equation systems associated with QFT are relatively hard to solve. This paper will show that the generating function technique (GFT) can be used to directly solve these equation systems while also producing renormalization results. The usage of the latter is necessary to display the consistency of the solutions and equation systems. Ultimately, an astute scientist in QFT can claim GFT is a valuable tool to be utilized in the field of particle physics.

1. *Introduction*

QFT is a combination of quantum mechanics, classical field theory, and special relativity [1]. It is commonly applied to particle physics, thus essential and in the formation of models within the realm of subatomic and condensed matter physics [1]. It heavily utilizes Lagrangian mechanics to display the interaction of particles, which are defined as quantum fields [2]. Since its advent in the 1920s and rebirth in the 1970s, QFT has had a prominent role in describing contemporary physics [3].

QFT was divided into at least three branches: quantum electrodynamics (QED), quantum flavordynamics (QFD), and quantum chromodynamics (QCD). QED was primarily developed by Dirac in 1927 which built upon the concept of canonical quantization [4]. It dealt with the interaction of fermionic and electromagnetic fields [5]. QFD was the study of electroweak nuclear force, such as bosons Z^0 and $W^{\pm}$ activities, while QCD involved strong nuclear interactions, generally mediated gluon fields [6,7]. It is not uncommon to find situations where certain branches, like QED and QCD, crossed over or encroached on each other.

The generating function technique (GFT) is a novel method for solving [nonlinear] PDEs [8]. It assumes there is a general solution to the PDE of interest already exists; thus, solving the PDE requires one to determine the appropriate degree[s] of the solution, then (s)he computes the necessary constants to obtain the solution. Even though the processes of GFT are simple to comprehend, it requires a computer to carry out the steps.

This paper utilizes GFT in the derivation of sets of exact solutions to a set of new QFT models. The study is reduced into three more sections. Section two deals with the ascertainment of critical Hamiltonian equations from the Lagrangian of more extensive QFT models. Section two also provides a brief description of GFT that is used to derive sets of exact solutions for the Hamiltonian equations and an easier way to generate renormalization results using the solutions derived from GFT. Section three describes several QFT scenarios in which GFT and the new renormalization method are implemented to produce solutions and mass-energies for particle fields. Finally, section five gives a synopsis of the QFT models and the implications of the efficiency of GFT to generate solutions and renormalization results.

2. Models and Methodology

2.1. A variation of the Yukawa interaction

A Yukawa interaction is a type of QCD model which involves the relationship between a gauge boson and fermion fields [9]. The former field can be [partially] self-interacting; in other words, the constant λ in the equation is not null. The principle of least action for a gauge boson ϕ_i which either decays into or generates from a fermion ψ_j and its antiparticle $\psi_j^\dagger$ is expressed as follows:

$$\mathcal{S}[\dot{\phi} i, \psi j] = \int dx^4 \sum_j \left(-\phi_i \delta_j \psi_j \psi_j^\dagger + \frac{\lambda \phi_i^3}{3} + \frac{1}{2}\phi_i^2 \left(m_{\phi_i}^2 + \partial_\mu \partial^\alpha\right) + \delta_j \psi_j \psi_j^\dagger \left(m_\varphi^2 + \partial_\mu \partial^\mu\right) \right),$$

where $m_{\phi i}$ and $m_{\psi j}$ are the invariant masses of the gauge boson ϕ_i and fermion ψ_j, respectively, δ_j is not a Kronecker product and equals ± 1 depending upon whether the field occurs before or after the gauge boson ϕ_i, and λ is a coupling constant. The above equation can be converted to a Hamiltonian:

$$\mathcal{H}[\phi i, \psi j] = \int dx^4 \sum_j \left(\dot{\phi}_i^2 + i \delta_j \psi_j \dot{\psi}_j - \frac{1}{3}\lambda \phi_i^3 - \frac{1}{2}\phi_i^2 \left(m_{\phi_i}^2 + \partial_\mu \partial^\mu\right) - \delta_j \psi_j \psi_j^\dagger \left(m_\psi^2 + \partial_\mu \partial^\mu\right) + \phi \delta_j \psi_j \psi_j^\dagger \right).$$

With Poisson brackets [10], an individual can derive time evolution equations for the gauge boson ϕ_i:

$$\ddot{\phi}_i = \left\{ \dot{\phi}_i, \int d\mathrm{x}^4 \sum_j \left(\dot{\phi}_i^2 + i \dot{\psi}_j \psi_j^\dagger + \phi_i \delta_j \psi_j \psi_j^\dagger - \frac{1}{3}\lambda \phi_i^3 - \frac{1}{2}\phi_i^2 \left(m_{\phi_i}^2 + \partial_\mu \partial^\mu\right) - \psi \delta_j \psi_j^\dagger \left(m_{\psi_j}^2 + \partial_\mu \partial^\mu\right) \right) \right\},$$

or

$$\ddot{\phi}_i = -\lambda\phi_i^2 - \phi_i(m_\phi^2 - \Delta) + \sum\nolimits_j \delta_j\psi_j\psi_j^\dagger.$$

By placing all terms on the left side of the equation, the individual yields:

$$\ddot{\phi}_i - \Delta\phi_i + \lambda\phi_i^2 + \phi_i m_{\phi_i}^2 - \sum_j \delta_j\psi_j\psi_j^\dagger = 0.$$

To obtain a comparable equation for fermion field ψ_j, the same individual again must use Poisson brackets:

$$\delta_l\ddot{\psi}_l = \left\{\delta_l\dot{\psi}_l^\dagger, \int dx^4 \left(\dot{\phi}_i^2 + i\delta_j\dot{\psi}_j\psi_j^\dagger - \phi_i\delta_j\psi_j\psi_j^\dagger - \frac{1}{3}\lambda\phi_i^3 - \frac{1}{2}\phi_i^2\left(m_{\phi_i}^2 + \partial_\mu\,\partial^\mu\right) - \delta_j\psi_j\psi_j^\dagger\left(m_{\psi_j}^2 + \partial_\mu\,\partial^\mu\right)\right)\right\},$$

or

$$\ddot{\psi}_l = i\dot{\psi}_l - \phi_i\psi_l - \psi_l\left(m_{\psi_l}^2 - \Delta\right),$$

where *l* is an element of the *j*-th fermion field pair. Note: fermion fields in a *j*-th pair do not have to be the same entity. By placing all terms on the right side of the equation, the individual obtains:

$$i\dot{\psi}_l - \ddot{\psi}_l - \phi_i\psi_l + \Delta\psi_l - \psi_l m_{\psi_l}^2 = 0.$$

The above equation is a variation of the Schrodinger equation.

2.2. A more extensive QCD model

The principle of least action for a gauge boson ϕ_i which decays into or produces from another gauge boson ϕ_j and its antiparticle $\overline{\phi}_j$ is expressed as follows:

$$\mathcal{S}[\phi i, \phi j] = \int dx^4 \sum_j \left(-\phi_i\delta_j\phi_j\overline{\phi_j} + \delta_j\phi_j\overline{\phi_j}\left(m_{\phi_j}^2 + \partial_\mu\,\partial^\mu\right) + \frac{\lambda\phi_i^3}{3} + \frac{1}{2}\phi_i^2\left(m_{\phi_i}^2 + \partial_\mu\,\partial^\mu\right)\right),$$

where $m_{\phi i}$ and $m_{\phi j}$ are the invariant masses of the gauge boson ϕ_i and ϕ_j, respectively, δ_j again is not a Kronecker product and is equal to ± 1 depending upon whether the field occurs before or after the gauge boson ϕ_i, and λ is a coupling constant. The above equation can be converted to a Hamiltonian:

$$\mathcal{H}[\phi\mathrm{i},\phi j]=\int d\mathrm{x}^4\sum_j\left(\delta_i\overline{\phi}_j\dot{\phi}_j+\phi_i\delta_j\phi_j\overline{\phi_j}-\delta_j\phi_j\overline{\phi_j}\left(m_{\phi_j}^2+\partial_\mu\,\partial^\mu\right)-\frac{1}{3}\lambda\phi_i^3\right.$$
$$\left.-\frac{1}{2}\phi_i^2\left(m_{\phi_i}^2+\partial_\mu\,\partial^\mu\right)\right).$$

With Poisson brackets, an individual can derive time evolution equations for the gauge bosons ϕ_i and ϕ_j:

$$\ddot{\phi}_i=\left\{\phi_i,\int d\mathrm{x}^4\sum_j\left(\phi_i\delta_j\phi_j\overline{\phi_j}-\delta_j\phi_j\overline{\phi_j}\left(m_{\phi_j}^2+\partial_\mu\,\partial^\mu\right)+i\dot{\phi}_i^2+\phi_j\dot{\phi}_j-\frac{1}{3}\lambda\phi_i^3\right.\right.$$
$$\left.\left.-\frac{1}{2}\phi_i^2\left(m_{\phi_i}^2+\partial_\mu\,\partial^\mu\right)\right)\right\},$$

or

$$\ddot{\phi}_i=-\sum_j\left(-\delta_j\phi_j\overline{\phi_j}+\lambda\phi_i^2+\phi_i\left(m_{\phi_i}^2-\Delta\right)\right).$$

By placing all terms on the left side of the equation, the individual generates:

$$-\sum_j\delta_j\phi_j\overline{\phi_j}+\ddot{\phi}_i-\Delta\phi_i+\lambda\phi_i^2+\phi_i m_{\phi_i}^2=0.$$

To obtain an equation for gauge boson ϕ_j, the same individual again must use Poisson brackets:

$$\delta_l\overline{\phi}_l=\left\{\dot{\phi}_j,\int d\mathrm{x}^4\sum_j\left(\phi_i\delta_j\phi_j\overline{\phi_j}-\delta_j\phi_j\overline{\phi_j}\left(m_{\phi_j}^2+\partial_\mu\,\partial^\mu\right)+i\dot{\phi}_i^2+\phi_j\dot{\phi}_j-\frac{1}{3}\lambda\phi_i^3\right.\right.$$
$$\left.\left.-\frac{1}{2}\phi_i^2\left(m_{\phi_i}^2+\partial_\mu\,\partial^\mu\right)\right)\right\},$$

or

$$-\dot{\phi}_l+\ddot{\phi}_l-\phi_i\phi_l+\phi_l(m_{\phi_l}^2-\Delta)=0,$$

where l is an element of the j-th gauge boson field pair. Note: gauge boson fields in a j-th pair do not have to be the same entity. By placing all terms on the right side of the equation, the individual derives:

$$-\dot{\phi}_l+\ddot{\phi}_l+\phi_i\phi_l-\Delta\phi_l+\phi_l m_{\phi_l}^2=0.$$

The above equation is a telegraph equation.

2.3. GFT

GFT is a method for solving [non]linear PDEs via the utilization of a general solution u_g that comprises Laurent series sets of combinatorial number or trigonometric-based generating functions [16]. An individual determines the maximal and minimal power degree, or p_s, through which the Laurent series is eventually truncated. Then, one solves a linear auxiliary/characteristic ordinary differential equation to yield a function f is plugged into the transformed general solution U_g, or:

$$U(\xi) = \sum_{i=1}^{2}\sum_{j=-p_s}^{p_s}(a_{ij}(\sum_{k=0}^{\infty} 2f(\xi)^k S_k(0)^i)^j + b_{ij}(\sum_{k=0}^{\infty} 2C_k(0)^i f(\xi)^k)^j),$$

or

$$U(\xi) = \sum_{i=1}^{2}\sum_{j=-p_s}^{p_s}(\mathrm{dl_{ij}}(\sum_{k=0}^{\infty} 2\mathrm{C}_k(0)^i f(\xi)^k)^j + \mathrm{cl_{ij}}(\sum_{k=0}^{\infty} 2f(\xi)^k S_k(0)^i)^j),$$

where the expression $S_k(0)$ is the square root of the k-th Fibonacci number at/about zero, or

$$S_k(0) = \sin\left(\frac{\pi k}{2}\right),$$

the expression $C_k(0)$ is the k-th Chebyshev U number at/about zero, or

$$C_k(0) = \cos\left(\frac{\pi k}{2}\right),$$

and the transformed variable ξ for a (3+1) system is defined as:

$$\xi = \alpha t + \beta_l x + \beta_2 y + \beta_3 z.$$

For this article, the arbitrary constants a_{ij} and b_{ij} are used for the primary gauge boson field while the arbitrary constants cl_{ij} and dl_{ij} are used for secondary gauge boson and or fermion fields where $l = 1, 2, \dots, n$ and n is the total number of secondary items.

2.4. The generation of renormalization results

The basic formula for renormalization is defined as [17]:

$$m_p = \frac{1}{2}\int d\mathrm{V}p(x)p^*(x),$$

where m_p is the mass-energy of particle field p, p^* is the conjugate of particle field p, and V is the volume that contains the particle field p. If the volume for particle field f is equivalent to the following expression [18]:

$$V = \frac{\pi x^3}{6},$$

then the formula for renormalization becomes:

$$m_p = \int_0^{\infty} \frac{1}{4} \pi x^2 p(x) p^*(x) dx.$$

The above expression can be simply redefined as:

$$m_p = \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{4} \pi x^2 p(x) p^*(x) dx .$$

In terms of cross distance [19], the mass-energy of particle field p can also be expressed as:

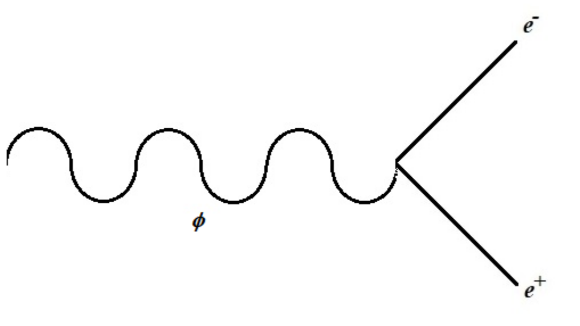
$$m_p = \frac{1}{8} \pi \langle x p(x), x p(x) \rangle.$$

3. *Examples*

The supplementary materials contain Mathematica (R) spreadsheets that pertain to the QFT models described in this paper.

3.1. *Mesonic decay and photoproduction*

Assume one is dealing with a simple Feynman diagram where a meson decays and gives rise to an electron and position pair:



The principle of least action for this system is given by the following equation:

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$$\mathcal{S}[\phi,\psi] = \int dx^4 \left(\frac{\lambda\phi^3}{3} + \psi\psi^\dagger\left(m_\psi^2 + \partial_\mu\,\partial^\mu\right) + \frac{1}{2}\phi^2\left(m_\phi^2 + \partial_\mu\,\partial^\mu\right) + \psi\phi\psi^\dagger\right).$$

The above expression can be used to derive the Hamiltonians for all particles of interest:

$$\ddot{\phi} - \Delta\phi + \lambda\phi^2 + \phi m_\phi^2 + \psi\psi^\dagger = 0,$$

and

$$i\dot{\psi} - \ddot{\psi} + \Delta\psi - \psi m_\psi^2 - \psi\phi = 0.$$

Next, GFT and renormalization are to generate solutions and mass-energies of particles. GFT is used to generate the solutions to the Hamiltonians:

$$\phi(t,x,y,z) = -\frac{12m^2\exp\left(\frac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta1^2+\beta2^2)(3c\lambda+c)^2-4m^4+2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta1x + 2\beta2y\right)}{(3\lambda+1)\left(1+\exp\left(\frac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta1^2+\beta2^2)(3c\lambda+c)^2-4nt^4+2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta1x + 2\beta2y\right)\right)^2},$$

and

$$\psi_e(t,x,y,z) =$$
$$-((6\sqrt{(\lambda+1)m^4}\exp\left(\frac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta1^2+\beta2^2)(3c\lambda+c)^2 - 4m^4 + 2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta1x + 2\beta2y\right)$$
$$(-1+\exp\left(\frac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta1^2+\beta2^2)(3c\lambda+c)^2 - 4m^4 + 2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta1x + 2\beta2y\right)))/$$
$$(\sqrt{(3\lambda+1)^2}$$
$$(1+\exp\left(\frac{2im^2t}{3\lambda+1} + \frac{z\sqrt{-4(\beta1^2+\beta2^2)(3c\lambda+c)^2 - 4m^4 + 2c^2(3\lambda+1)m^2}}{\sqrt{(3c\lambda+c)^2}} + 2\beta1x + 2\beta2y\right))^2)),$$

while the results of renormalization of the same particles can be expressed as the following if one sets the constant λ to null and the speed of light c to unity:

$$m_\phi = 0.0949744|m|^4,$$

and

$$m_{\psi_e} = 0.536761|m|^4.$$

Using the above results, one can calculate the needed center-of-mass, or$\sqrt{s}$, for electron-positron collision to produce a particular meson. For instance, (s)he first must set m_ϕ to well-known mass-energy and solve for m, then (s)he can calculate the $\sqrt{s}$ for the particle by plugging m into $m_{\psi e}$. The following table shows the predicted $\sqrt{s}$ for various mesons:

meson	mass-energy (eV)	center-of-mass (eV)
neutral pion	*1.34*10^8*	*7.57*10^8*
neutral kaon	*4.98*10^8*	*2.81*10^9*
neutral D meson	*1.86*10^9*	*1.05*10^10*
neutral B meson	*5.28*10^9*	*2.98*10^10*

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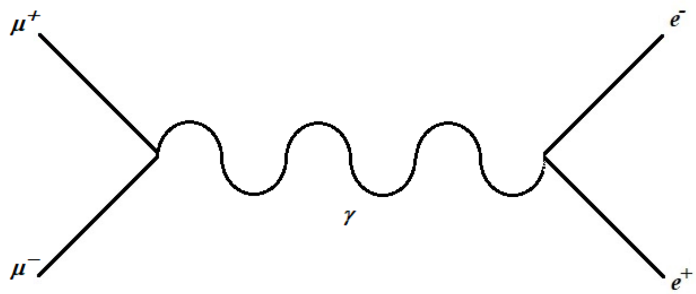
3.2. Lepton pair decay and production

Assume one is dealing with a simple Feynman diagram where [anti]muon pair decay into a photon and the photon gives rise to an electron and position pair:



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The principle of least action for this system is given by the following equation:

$$\mathcal{S}[\phi,\psi] = \int dx^4 \left(\frac{\lambda\phi^3}{3} - \psi_1\psi_1^\dagger\left(m_{\psi_1}^2 + \partial_\mu\,\partial^\mu\right) + \psi_2\psi_2^\dagger\left(m_{\psi_2}^2 + \partial_\mu\,\partial^\alpha\right) + \frac{1}{2}\phi^2\left(m_\phi^2 + \partial_\mu\,\partial^\mu\right) - \psi_1\phi\psi_1^\dagger + \psi_2\phi\psi_2^\dagger\right).$$

The above expression can be used to derive the Hamiltonians for all particles of interest:

$$\ddot{\phi} - \Delta\phi + \lambda\phi^2 + \phi m_\phi^2 - \psi_1\psi_1^\dagger + \psi_2\psi_2^\dagger = 0,$$

$$i\dot{\psi}_1 - \ddot{\psi}_1 + \Delta\psi_1 - \psi_1 m_{\psi_1}^2 - \psi_1\phi = 0,$$

and

$$i\dot{\psi}_2 - \ddot{\psi}_2 + \Delta\psi_2 - \psi_2 m_{\psi_2}^2 - \psi_2\phi = 0,$$

where the gauge boson ϕ is equal to photon γ.

Next, GFT and renormalization are to generate solutions and mass-energies of particles. GFT is used to generate the solutions to the Hamiltonians:

$$\gamma(t,x,y,z) = -\frac{3}{2}c^2m^2\sec^2\left(c^2m^2t + \frac{1}{2}i\left(z\sqrt{-4(\beta 1^2+\beta 2^2) - 4c^2m^4 - c^2m^2} + 2\beta 1x + 2\beta 2y\right)\right),$$

$$\psi_1(t,x,y,z) = -\mathrm{d}1(1{,}4)\left(\tan\left(c^2m^2t + \frac{1}{2}iz\sqrt{c^2(-m^2)(4m^2+1) - 4(\beta 1^2+\beta 2^2)} + i\beta 1x + i\beta 2y\right) + i\right)^2,$$

and

$$\psi_2(t,x,y,z) = \frac{1}{2}\sqrt{4\mathrm{d}1(1{,}4)^2 - 9c^4(\lambda-1)m^4}(\tan\,(c^2m^2t + \frac{1}{2}iz\sqrt{c^2(-m^2)(4m^2+1) - 4(\beta 1^2+\beta 2^2)} + i\beta 1x + i\beta 2y) + i)^2,$$

while the results of renormalization of the same particles can be defined as the following if one sets the constant λ to null and the speed of light c to unity:

$$m_\gamma = 0.379898|m|^4,$$

$$m_e = 0.168843|\mathrm{d}1(1{,}4)|^2,$$

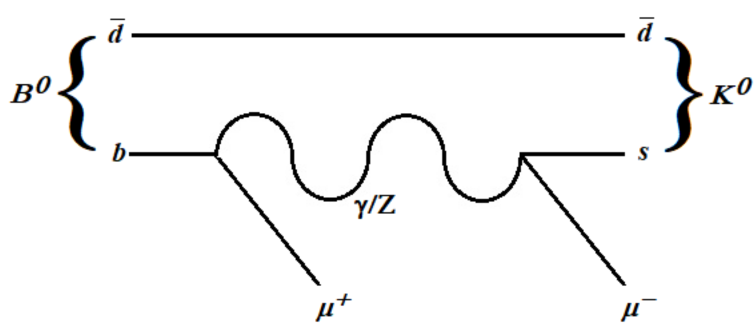
and

$$m_\mu = 0.0422108|9m^4 - 4.\,\mathrm{d}1(1{,}4)^2|.$$

Finally, an individual would use the results from renormalization to obtain the value of constants m and $d1_{14}$ and prove the consistency of the results. By setting m_μ and m_e to $1.05*10^8$ and $5.11*10^5$ eV, respectively, (s)he produces an m and $d1_{14}$ equal to *–128.781* and *24937.5*, also respectively. The mass-energy m_γ equals *104.489* MeV, which is consistent with the difference between m_μ and m_e.

3.3. Mesonic decay into other mesons

Assume one is dealing with a simple Feynman diagram where a B meson decays into a [anti]muon pair and kaon:



The principle of least action for this system is given by the following equation:

$$\mathcal{S}[\phi,\psi] = \int dx^4 (\frac{\lambda\phi^3}{3} + \psi_1\psi_3^\dagger(m_\psi^2 + \partial_\mu\,\partial^\mu) - \psi_2\psi_3^\dagger(m_\varphi^2 + \partial_\mu\,\partial^\mu) -$$
$$\psi_4\psi_4^\dagger(m_\psi^2 + \partial_\mu\,\partial^\mu) + \frac{1}{2}\phi^2(m_\phi^2 + \partial_\mu\,\partial^\mu) - \psi_1\phi\psi_3^\dagger + \psi_2\phi\psi_3^\dagger + \psi_4\phi\psi_4^\dagger).$$

The above expression can be used to derive the Hamiltonians for all particles of interest:

$$\ddot{\phi} - \Delta\phi + \lambda\phi^2 + \phi m_\phi^2 - \psi_1\psi_3^\dagger + \psi_2\psi_3^\dagger + \psi_4\psi_4^\dagger = 0,$$

$$i\dot{\psi}_1 - \ddot{\psi}_1 + \Delta\psi_1 - \psi_1 m_{\psi_1}^2 - \psi_1\phi = 0,$$

$$i\dot{\psi}_2 - \ddot{\psi}_2 + \Delta\psi_2 - \psi_2 m_{\psi_2}^2 - \psi_2\phi = 0,$$

$$i\dot{\psi}_3 - \ddot{\psi}_3 + \Delta\psi_3 - \psi_3 m_{\psi_3}^2 - \psi_3\phi = 0,$$

and

$$i\dot{\psi}_4 - \ddot{\psi}_4 + \Delta\psi_4 - \psi_4 m_{\psi_4}^2 - \psi_4\phi = 0.$$

where gauge boson ϕ is γ/Z, fermion ψ_1 is a beauty quark, fermion ψ_2 is a strange quark, fermion ψ_3 is an antidown quark, and fermion ψ_4 is the [anti]muon particle.

Next, GFT and renormalization are to generate solutions and mass-energies of particles. GFT is used to generate the solutions to the Hamiltonians:

$$\gamma/Z(t,x,y,z) = \frac{3}{2}m^2\sec^2\left(m^2 t + \frac{i\left(z\sqrt{-4c^2(\beta1^2+\beta2^2) - 4m^4 - c^2m^2} + 2\beta1cx + 2\beta2cy\right)}{2c}\right),$$

$$\psi_1(t,x,y,z) = -\mathrm{d1}(1,4)\left(\tan\left(m^2 t + \frac{iz\sqrt{-4c^2(\beta1^2+\beta2^2) - 4m^4 - c^2m^2}}{2c} + i\beta1x + i\beta2y\right) + i\right)^2,$$

$$\psi_2(t,x,y,z) =$$
$$-((\mathrm{d1}(1,4) - \frac{4\mathrm{d4}(1,4)^2 + 9(\lambda+1)m^4}{4\mathrm{d3}(1,4)})(\tan\,(m^2 t + \frac{iz\sqrt{-4c^2(\beta1^2+\beta2^2) - 4m^4 - c^2m^2}}{2c} + i\beta1x + i\beta2y) + i)^2)’$$

$$\psi_3(t,x,y,z) = -\mathrm{d3}(1,4)\left(\tan\left(m^2 t + \frac{iz\sqrt{-4c^2(\beta1^2+\beta2^2) - 4m^4 - c^2m^2}}{2c} + i\beta1x + i\beta2y\right) + i\right)^2,$$

and

$$\psi_4(t,x,y,z) = -\mathrm{d4}(1,4)\left(\tan\left(m^2 t + \frac{iz\sqrt{-4c^2(\beta1^2+\beta2^2) - 4m^4 - c^2m^2}}{2c} + i\beta1x + i\beta2y\right) + i\right)^2,$$

while the results of renormalization of the same particles can be expressed as the following if one sets the constant λ to null and the speed of light c to unity:

$$m_{\gamma/Z} = 0.379898|m|^4,$$

$$m_b = 0.168843|\mathrm{d1}(1,4)|^2,$$

$$m_s = 0.0105527\left|\frac{(9m^4 + 4\mathrm{d4}(1,4)^2 - 4\mathrm{d1}(1,4)\mathrm{d3}(1,4))^2}{\mathrm{d3}(1,4)^2}\right|,$$

$$m_{\overline{d}} = 0.168843|\mathrm{d3}(1,4)|^2,$$

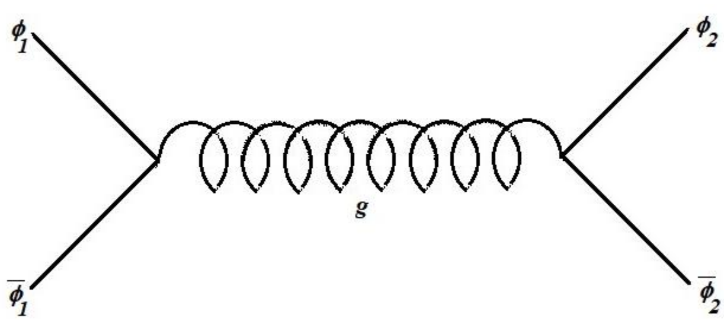
and

$$m_\mu = 0.168843|\mathrm{d4}(1,4)|^2.$$

Finally, an individual would use the results from renormalization to obtain the value of constants m, $d1_{14}$, $d3_{14}$, and $d4_{14}$ and prove the consistency of the results. By setting m_b, m_s, m_d, and m_μ to $4.18*10^9$, $9.60*10^7$, $4.70*10^6$, and $1.05*10^8$ eV, respectively, (s)he obtains the value of constants m, $d1_{14}$, $d3_{14}$, and $d4_{14}$ equal to *76.9*, $1.57*10^5$, $5.25*10^3$, and $2.49*10^5$, also respectively. The mass-energy $m_{\gamma/Z}$ equals *13.289* MeV for the system to be consistent.

3.4.) Glueball prediction

Assume one is dealing with the following Feynman diagram:



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The principle of least action for this system is given by the following equation:

$$S[\mathrm{g}, \phi 1, \phi 2] = \int d\mathrm{x}^4 \left(2\mathrm{g}\phi_1\overline{\phi_1} - 2g\phi_2\overline{\phi_2} + \phi_1\overline{\phi_1}(m_{\phi_1}^2 + \partial_\mu\,\partial^\mu) - \phi_2\overline{\phi_2}(m_{\phi_2}^2 + \partial_\mu\,\partial^\mu) + \frac{g^3\lambda}{3} + \frac{1}{2}g^2(m_g^2 + \partial_\mu\,\partial^\mu)\right).$$

The above expression can be used to derive the Hamiltonians for all particles of interest:

$$-2\phi_1\overline{\phi_1} + 2\phi_2\overline{\phi_2} + \ddot{g} + g^2\lambda - \Delta g + gm_g^2 = 0,$$

$$-\dot{\phi}_1 + \ddot{\phi}_1 - \Delta\phi_1 - g\phi_1 + \phi_1 m_{\phi_1}^2 = 0,$$

and

$$-\dot{\phi}_2 + \ddot{\phi}_2 - \Delta\phi_2 - g\phi_2 + \phi_2 m_{\phi_2}^2 = 0.$$

where gauge boson g is glueball based upon gluons, gauge boson ϕ_1 is a vector kaon, and gauge boson ϕ_2 is a π meson.

Next, GFT and renormalization are to generate solutions and mass-energies of particles. GFT is used to generate the solutions to the Hamiltonians:

$$g(t,x,y,z) = -\frac{3}{2}m^2 \operatorname{sech}^2\left(m^2 t + \frac{iz\sqrt{4c^2(\beta1^2+\beta2^2) - 4m^4 + c^2m^2}}{2c} - \beta1x - \beta2y\right),$$

$$\phi_1(t,x,y,z) = \mathrm{d1}(1,4)\left(-1 + \tanh\left(m^2 t + \frac{iz\sqrt{4c^2(\beta1^2+\beta2^2) - 4m^4 + c^2m^2}}{2c} - \beta1x - \beta2y\right)\right)^2,$$

and

$$\phi_2(t,x,y,z) = \frac{1}{2}\sqrt{4\mathrm{d1}(1,4)^2 - \frac{9}{2}(\lambda-1)m^4}\left(-1 + \tanh\left(m^2 t + \frac{iz\sqrt{4c^2(\beta1^2+\beta2^2) - 4m^4 + c^2m^2}}{2c} - \beta1x - \beta2y\right)\right)^2.$$

while the results of renormalization of the same particles can be expressed as the following if one sets the constant λ to null and the speed of light c to unity:

$$m_g = 0.379898|m|^4,$$

$$m_{\phi_1} = 0.168843|\mathrm{d1}(1,4)|^2,$$

and

$$m_{\phi_2} = 0.0211054|9m^4 + 8.\mathrm{d1}(1,4)^2|.$$

Finally, an individual can use the results from renormalization to obtain the value of constants m and $d1_{14}$ and prove the consistency of the results. By setting $m_{\phi 1}$ and $m_{\phi 2}$ to $8.91*10^8$ and $1.40*10^8$ eV, respectively, (s)he produces a m and $d1_{14}$ equal to *-177.375+177.375**i*** and 7.27*10^4, also respectively. The mass-energy m_g equals *1.50* GeV, which shows mass-energy conservation for the particles inside the system. If $m_{\phi 1}$ is a ρ or a ϕ meson, then m_g equals *1.27* or *1.76* GeV, respectively.

4. *Conclusion*

4.1. QFT can be used to generate a large variety of equation systems describing particle physics.

Section three provides several examples which implemented some variation of the QFT models provided by section two. The equation systems produced by the Lagrangian for the examples are novel, wide-ranging, and highly descriptive. In other words, there is practically no limit to the Lagrangian or equation systems one can contemplate in QFT.

4.2. GFT can easily derive solutions and renormalization results to many equation systems in QFT.

Section three and supplementary material also showed relative ease of solving particle fields and generating renormalization results from the solution via GFT. In other words, only a few steps are needed to produce solutions to both fermion and gauge boson fields involved in each QFT model. Ultimately, GFT is an ideal tool for solving QFT models.

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