A note on the muon's anomalous magnetic dipole moment

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Abstract

We consider the computation of the muon's anomalous magnetic moment within the theoretical framework proposed in [1], in which field theory is only an approximation of a more fundamental description of the physical world. We discuss how the hadron contribution to the electromagnetic coupling strength is larger than in the Standard Model, while the other contributions remain unchanged. The extra amount precisely fills the gap between theoretical estimate and experimental value.

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1 Introduction

The computation of the muon's anomalous magnetic moment is an important test for the theory of elementary particles. The Standard Model accounts only in part for its value. The deviation from the experimental value is small, but, relative to the highly accurate field theoretical calculations, significant. The anomalous magnetic dipole moment is usually quoted in the form:

$$\frac{g-2}{2} = \frac{\alpha}{2\pi}\,, \tag{1.1}$$

where g is the gyromagnetic ratio, and α the electromagnetic coupling. In the case of the muon, the quantity of interest is therefore α_μ. The most recent experimentally measured value, as reported by [2] (derived from [3]–[22]), is given by:

$$\alpha_\mu^{\rm exp} = 116592061(41) \times 10^{-11}\,. \tag{1.2}$$

Since errors and uncertainties are large, and subject to continuous slight modification, we report here also the previous value as reported in [23]:

$$\alpha_\mu^{\rm exp} = 116592089(63) \times 10^{-11}\,, \tag{1.3}$$

whereas the computation within the Standard Model of elementary particles gives:

$$\alpha_\mu^{\rm SM} = 116591810(43) \times 10^{-11}\,. \tag{1.4}$$

This value is smaller than the experimental coupling, the difference being larger than the intrinsic uncertainty of the computation. This fact is usually interpreted as a signal of "new physics" at play: there must exist new degrees of freedom whose contribution to the correction of the coupling fills the gap. From this assumption one can derive constraints on models of elementary particle physics "beyond the Standard Model", either in new models (see for instance [24]), or in (supersymmetric) extensions of the Standard Model (see for instance [25]–[30]). There are however other possibilities: it has also been proposed that this result could be explained by a better understanding of low-energy hadronic physics [31]. In this note, instead of looking for further elementary states within a traditional field-theoretical approach, I consider the computation of α_μ in the theoretical framework introduced and discussed in [1], [32]–[34], a scenario proposed to be behind, underlying and unifying, general relativity and quantum mechanics. Thanks to the high predictive power of this theory, the elementary particle masses and several cosmological parameters can be correctly computed out of the only free parameter of the theory, the age of the universe. It is therefore worth challenging this scenario also with a refined computation such as the anomalous magnetic moment of the muon.

2 Feynman diagrams for the effective theory

The scenario we are going to consider is a non-field theoretical framework, in which the space of the expanding universe is of finite extension, and there is a minimal distance, the Planck length. The appendix at page 7 is a brief introduction to the setup, in particular to concepts we will refer to, such as "entropy of geometries" and "entropy of string constructions" (for a more detailed description we refer the reader to Ref. [34]). Although the framework is non-field theoretical, the dynamics of local experiments, such as those relevant for this discussion, can nevertheless be approximately described by a superstring-derived field theoretical effective action, in which masses and couplings are treated as external inputs, and are not introduced by a Higgs mechanism [1]. They depend on the age of the universe. In the limit in which the space becomes infinitely extended, the spectrum becomes massless. Therefore, although the introduction of masses as external parameters without a Higgs mechanism explicitly breaks gauge symmetry, it makes sense to approximate the dynamics by an effective gauge field theory with massless fields, because the approximation gets better and better as the universe evolves. At any finite time, we need not care about cancellation of infinities because we know that the theory is finite, being just an approximate description of a theory defined on a space of finite extension and with a minimal distance. It can therefore be treated as if it were a gauge theory, with the following important modification: the results of Feynman diagram computations performed on gauge field theory with the same massless spectrum can be used, except for the dynamics of the Higgs field, which in our context does not exist. In practical computations, it is like working with a very heavy Higgs field. The latter can be considered decoupled to all effects, and the only term that matters is its coupling to the matter states. This just results in mass terms, because all what pertains the dynamics of the Higgs field is suppressed: the Higgs propagation is neglected here.

A key point of this scenario is that quantization is introduced as an implementation of the fact that the world at any instant is a collective effect in which any observable receives contribution from all possible geometries of space-time. As a consequence, differently from the field theory approach, the expansion in Feynman diagrams is here a representation of paths that contribute to the statistics, and therefore to the entropy of the physical process, with vertices weighted by coupling amplitudes, which in our framework measure the relative weights of interactions. The usual field theory computations through Feynman diagrams are valid for the determination of the amplitude also in our framework only as long as one just considers a purely perturbative phase of the theory, in which all the couplings are small. However, since in the underlying scenario the S-duality is only "softly" broken, i.e., there is a contribution from the S-dual phases that is not always statistically negligible, the total contribution to the scattering amplitudes and, in general, to the physical quantities is obtained by also taking into account the additional contributions originating from the S-dual phases of the couplings. This is a rather non-trivial aspect, which proves to be the key difference enabling this scenario to allow a correct evaluation of quantities such as the α_μ which is the object of this note. A discussion of couplings and a presence of S-dual phases is therefore necessary in this context.

3 The couplings

As discussed in Ref. [33], in the most entropic string geometries, those that shape the physical world, S-duality is broken at the Planck energy scale. Below this scale, the spectrum admits an approximate description in terms of fields and couplings of an effective quantum field theory with gauge interactions. The symmetry group of the matter states splits into two factors: one weakly coupled, giving rise to the $U1) \times SU(2)$ electro-weak interaction, and one which, below the Planck scale, *is always at the strong coupling*. Since the effective theory is characterized by the highest amount of symmetry breaking, the states of this sector are charged under the minimal symmetry group. Once interpreted as gauge symmetry, the latter has a negative beta function. From the analysis of the content (i.e. by counting the number of degrees of freedom and symmetry of the charged states) we recognize this as an $SU(3)$ symmetry, that we identify with the color symmetry that glues quarks together. However, differently from the usual field theoretic approach to chromodynamics, the color sector is strongly coupled (i.e. coupling larger than one) at any energy scale. As we will discuss, a color coupling smaller than one exists only as a quantum fluctuation in a transitory state. In this approach, Feynman diagrams are useful tools for estimating the statistics of the channels that contribute to the statistical weight of the process. Our formulation corresponds to the physics "on-shell". The amplitudes computed in our framework (corresponding always to physical amplitudes) can be compared

[1]The scattering resonances usually referred to as evidence of the production of a Higgs boson receive in this scenario a different interpretation (for a detailed discussion, see Ref. [34], chapter 4, section 5.2).

with the results of the usual computations through Feynman diagrams only when the latter contribute to a physical, observable quantity. For instance, this is the case of the electro-magnetic coupling, which is directly related to the amplitude of the tree $\psi\psi A$ vertex (ψ can be an electron or any other charged particle). Similarly, we have a relation between the weak coupling and the amplitude for the $n \to p+e+\nu$ decay (beta-decay). Nothing alike can be said of the color coupling α_s, which is only indirectly related to asymptotic states [2]. The coupling α_s that we obtain in this framework has no direct relation to the strong coupling as it is reported and used in usual field theoretical computations. In our case, the coupling is larger than one, whereas in QCD it is smaller than one but runs toward increasing values as the energy scale decreases. This difference is fundamental for the calculation of the anomalous magnetic moment of the muon: the evaluation of statistical weights reveals that there are extra contributions compared to the ordinary QCD evaluation. These are due to an extra multiplicity introduced by the rigid symmetry which arises from the "freezing" of a gauge symmetry at strong coupling.

S-dual phases

In our framework, couplings are ratios of occupation volumes in the phase space and are therefore directly related to the statistical weight of an interaction. They admit two interpretations, related by S-duality: 1) as gauge couplings, with value lower than one; 2) as larger-than-one values. In this case they correspond to mass/energy ratios of states related by a rigid symmetry. Both these S-dual interpretations of the coupling strength exist because of the relation (A.1), which implies that S-duality is only "entropically", i.e. statistically broken.

The electroweak and color couplings arise from S-dual sectors of the string-theory. The dominant situation is the one in which the electroweak coupling is "weak" and the color coupling is strong. In this case, the color symmetry is no more a continuous gauge symmetry mediated by propagating boson fields: everything is rigidly, tightly bound. However, in this theoretical framework, the Heisenberg uncertainty encodes the contribution of all possible geometries (see the appendix on page 7 and Ref. [34] for a more detailed explanation). Therefore, the entire physical content of a quantum experiment is not exhausted by the canonical quantization of the fields of just one single effective representation. Mean values of observables may therefore receive contribution from all the dual phases of the theory, i.e. from both the strong and weak coupling phase of any interaction. In particular, the shorter is the time of the experiment, the higher is the uncertainty and correspondingly the contribution of otherwise suppressed phases. This implies that, during the short time of an experiment, such as for instance a high-energy collision, also S-dual phases may show up. In Refs. [33, 34] we analysed the mass hierarchy, obtaining a chain of relations that reproduces in first approximation the mass ratios of the spectrum as functions of the $SU(2)$ coupling strength. We also saw how, under circumstances, the S-dual of the electromagnetic coupling can bind temporary excitations. Due to the increased statistical weight, a strong-coupling phase of the electromagnetic interaction contributes to the enhancement of cross sections, *resulting in the resonances usually interpreted as a Higgs field signal* (see footnote 1). For the color coupling α_s, which in our scenario is always strong (i.e. > 1) at sub-Planckian energies, an S-dual phase occurs during the short time of a scattering, when barions can be viewed as consisting of partons, which also emit soft gluons. Of course, no free quark or free propagating gluon is detected as final state, because such situations are only transitory and, after a short time, everything recombines into strongly coupled color singlets. Nevertheless, owing to this phenomenon, it is legitimate to introduce an effective "weak" color coupling, which is dual to α_s. This is what in our scenario substitutes the usual color coupling of QCD [3].

[2] The emission of soft gluons in high energy collisions does not constitute a counter-example, because the final and initial state do not involve free quarks, i.e. free states charged under the color symmetry, but only color singlets.

[3] In principle, something similar should hold for α_s as has been proven for $SU(2)$ and the electromagnetic $U(1)$ couplings. Namely, also the $\alpha > 1$ value should be at the basis of a mass hierarchy. Indeed, if its value is around ~ 10 at the up-down quark mass scale, we should expect a pion mass of order $\sim 10 \times (m_u + m_d)$ and, further α^{-1} factor above, the mass of the barions made of three such quarks, such as neutron and proton. This is what seems roughly to occur. However, computations of this kind are expected to be deeply affected by corrections due to the higher statistical weight of the neutron and proton mass scale, and should be taken only as rather approximate indications.

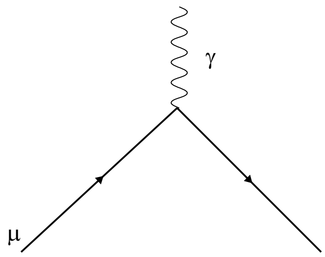

Figure 1: The tree-level contribution to α_μ.

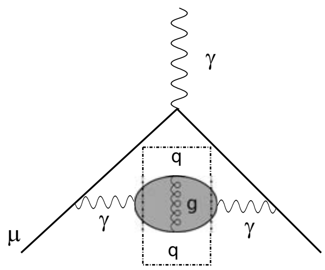

Figure 2: The HVP correction to α_μ.

4 QCD corrections to the muon's anomalous magnetic dipole moment revised

For what we said in the previous section, the entire perturbative part of the corrections to α_μ goes through practically unchanged as compared to the field theory approach. Strong electromagnetic phases are not expected to be of relevance (they would be detected as "Higgs-like" resonances of the scattering amplitude). For this part of the corrections we can therefore use the results obtained in field theory. On the other hand, care must be taken for the color coupling. The profound difference that distinguishes this scenario from the field theory is evident in the contributions that are more sensitive to the strong coupling. Let us have a closer look at the terms involving color-interacting quarks.

The coupling of the muon to the photon from which the value of α_μ is derived is illustrated in figure 1. Typical QCD corrections, based on gluon exchange between quark pairs, can be classified into two classes of contributions: HVP (hadron vacuum polarisation) and Hlbl (hadron light-by-light), illustrated in figures 2 and 3 respectively. Since the degrees of freedom of the effective theory are the same also in our scenario, this pattern is valid also in our case. The usual QCD computations, based on values of the running parameters tuned on the meson physics (QCD singlets build out of two coupled quarks), correctly account for the contribution due to the strong coupling of the two quarks of a vacuum polarisation diagram, such as the one appearing in figure 2. We should therefore expect that the traditional QCD approaches give an accurate estimate of the HVP contributions. However, when more quarks are involved in the process, like in the Hlbl contributions, the contribution based on the combinatorial of two-by-two gluon exchange does not account for the whole amount. In this case, our scenario statistically admits also an $\alpha_s > 1$ quark binding allowing the temporary formation of singlets out of three quarks, thereby making them basically three-by-three interchangeable. The presence of this contribution is not expected in ordinary QCD, therefore let us have a closer look at what is happening. The ordinary QCD approach to this kind of corrections to the scattering amplitude assumes that, for the short time of the interaction, before the scattering products set down once again to color singlets (as also the incoming states were), energy is high and the color coupling can be considered to be small, making QCD corrections perturbative. This approach does not admit the possibility that, while some quarks are indeed of high energy, other ones in the loop have a small energy/momentum, so that they interact strongly by forming virtual singlets which have, instead, a high energy. The running coupling $\alpha_s(Q)$ is considered fixed to a single scale Q, the typical scale of the process, for all the channels and sub-channels of the perturbative expansion. However, we know that the rest energy of a color singlet made out of three quarks, such as a neutron or a proton, is in general much higher than the one of the constituent quarks. High energy in one of the virtual propagator lines of this scattering can therefore be the result of strongly interacting low-energy quarks, such as $(q_b q_c q_d)$ of figure 5. Although excluded in ordinary field-theoretical QCD, this situation is perfectly legitimate in our framework, in which Feynman diagrams basically serve only as a way of evaluating the statistics of the contributions to the amplitude of a process. So the situation we are talking about is similar to the "electromagnetic strong coupling phase", that occurs as soon as the CM energy allows the creation of a strongly coupled singlet. The difference is that here we are talking of an energy/momentum that runs in a virtual loop, and that the minimal energy for which such a quark condensate is possible is much lower than the one at which an electromagnetic condensate may appear. In the case of a lepton's electromagnetic interaction, this occurs at an energy of order $\alpha_\gamma^{-1} \times m_\ell \sim \mathcal{O}(10^2\,\mathrm{GeV})$). In this case, we expect something of order $\alpha_s^2 \times m_q \sim \mathcal{O}(\mathrm{GeV})$, where q is a quark of the first family. Heavier quarks can be neglected, because their contribution to the loop is anyway suppressed by powers of $(m_{u/d}/m_q)$ from the propagators. From a practical point of view the conditions allowing this situation can therefore be satisfied along almost the entire range of momenta integrated in the loop.

For $\alpha_s \geq 1$ the "gauge group" as such no more exists (not even in the approximation of our scenario, which, we recall, is not a field theory scenario). There are no more propagating intermediate fields carrying the interaction. A coupling larger than one means that the elements of the orbit of the group are effectively identified. Because of this, we do not need to worry about an (impossible) calculation of the virtual amplitude via non-perturbative resummation of all the gluon-driven interactions among the quarks b, c, d of figure 5: for a coupling that we can consider larger than one, the color symmetry is no more a gauge symmetry, but a rigid combinatorial symmetry. The quark-colors b, c, d must in fact be considered as "identified" and interchangeable [4]. In the case of more than two quarks, strong coupling means therefore a combinatorial factor in front of amplitudes, that accounts for the replication of the scenario by exchange of the strongly coupled elements. This case is illustrated in figure 3. Although the factor for the strong color coupling of two-by-two quarks (illustrated in figure 4) is accounted for in usual computations, the usual QCD calculations of the Hlbl contribution lack the extra combinatorial factor due to the strong coupling singlets built from the three quarks, namely:

$$\binom{4}{3} \;=\; \frac{4!}{3!} \;=\; 4\,. \tag{4.1}$$

For each of these possibilities we have the usual QCD correction accounting for the color coupling of the quark triplet to the remaining quark. As a result, the Hlbl contribution should be a factor 4 larger than currently estimated. The situation is illustrated in figure 5. Alternatively, one can think that for each of the four diagrams of figure 4 there is a further contribution of the type of figure 5, which is a three-times replica of 4. The factor three is the volume of the discrete symmetry group exchanging three of the four quarks, arising in the $\alpha > 1$ phase of the color interaction. The contribution of these diagrams can then be evaluated from the already available computation of the Hlbl contributions. We use here the average of the values reported in Ref. [23]:

$$(\mathrm{Hlbl})_{\mathrm{QCD}} \;\approx\; 91(17) \times 10^{-11}\,. \tag{4.2}$$

According to our discussion, this leads to:

$$\mathrm{Hlbl} \;=\; 4 \times (\mathrm{Hlbl})_{\mathrm{QCD}} \;\approx\; 364(68) \times 10^{-11}\,. \tag{4.3}$$

This means, an amount

$$\Delta \;\approx\; 273(51) \times 10^{-11} \tag{4.4}$$

higher than what usually estimated, to be compared with the difference $\Delta\alpha_\mu \equiv \alpha_\mu^{\mathrm{exp}} - \alpha_\mu^{\mathrm{SM}}$, that we derive from 1.3 and 1.4:

$$\Delta\alpha_\mu \;\approx\; 279(53) \times 10^{-11}\,, \tag{4.5}$$

or from 1.2 and 1.4:

$$\Delta\alpha_\mu \;\approx\; 251(52) \times 10^{-11}\,. \tag{4.6}$$

[4] Notice that this virtual color singlet has fractional charge that, being the sum of three terms in which $Q(b) = -Q(a) = -Q(c)$, is opposite to the charge of the fourth quark running in the loop.

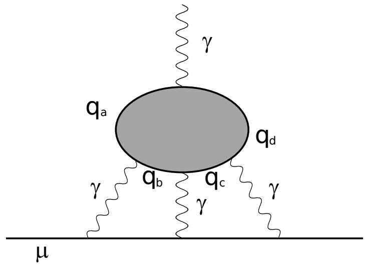

Figure 3: The hadron-light-by-light (Hlbl) diagram of the correction to α_μ.

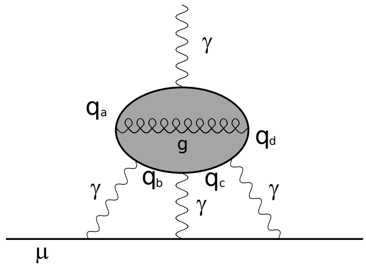

Figure 4: The typical gluon exchange of the interaction among quarks in the Hlbl loop.

Therefore, the amount Δ we computed fills the gap between the experimentally measured α_μ and its theoretic estimate.

Conclusion

We have considered the computation of α_μ within the theoretical framewok described in Ref. [34], obtaining an agreement between theoretical computation and experimental results. The agreement is not achieved by introducing extra particles and/or fields in order to provide further interaction channels and fill thereby the gap between theoretical computation of the correction to α_μ and the experimental value: our evaluation is made within a scenario that does not introduce extra degrees of freedom in the world of elementary particles, besides the already known particles and fields of the Standard Model. The missing amount in the value of α_μ comes from a different way of evaluating the parts involving strongly interacting hadrons. Besides the usual QED and QCD terms, in our theoretical framework there is also a contribution from terms in which a part of the quark loop is at the pure strong coupling, a scenario not allowed in field theory, even if ”string-derived”. Here it is possible because the theoretical framework goes beyond field theory, being based on a different approach to the foundations of the physics of elementary particles, in which field theory, and even string theory, show up only as approximate descriptions. In our case, the experimental value of the muon’s anomalous magnetic moment does not indicate the presence of so-called “new physics”, but provides instead

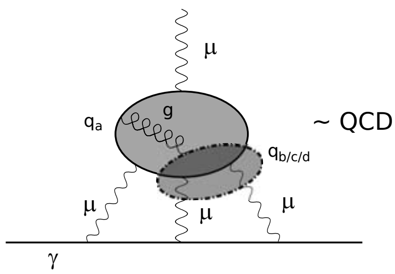

Figure 5: The “anomalous” interaction of a quark with a virtual quark condensate in the Hlbl loop.

a further strong support to the proposed theoretical scenario.

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Appendix: the universe of codes in summary

Let us briefly recapitulate the key aspects of the theoretical framework used in this paper. As discussed in Ref. [1], the basic formulation is given in terms of mappings from the set of natural numbers to a product of discrete spaces. The mapping from a discrete set to a product of discrete spaces can be interpreted as an assignment of units of what we may call "energy" to a set of "positions". We build up in this way a discrete geometry: in the limit to the continuum one can introduce an energy density, and a curvature related to the energy density according to Einstein's equation. The total number of energy units which are going to be distributed by such a mapping can be interpreted as the total energy of this geometry. Let us call E this total amount. It is clear that, if we consider the set of all the possible mappings distributing E energy units, this contains as subgeometries the set of all the possible mappings distributing $E' < E$ energy units. Therefore, in the space of all such mappings for any value of E we can introduce an order by grouping the geometries according to their total energy content. As surprising as it may seem, the ordering of these sets by increasing E has all the characteristics of a temporal order [34]. Any set $\{\Psi\}(E)$ containing all the geometries $\Psi(E)$ corresponding to a distribution of E energy units is a "universe" at total energy E, and the E-ordered sequence of sets $\{\Psi\}(E)$ a history of the universe. We are used to view the physical universe as corresponding, at any time in its history, to just *one* geometry; how can a collection of geometries correspond then to the universe? In Ref. [34] we introduced the concept of superposition of geometries. There is no room here for going into details, but let us at least take a glimpse of this point, which is a fundamental concept of the entire construction. According to this approach, what an observer sees is not one geometry, but an "ensemble of geometries leading to weighted averages of observables". For instance, the amount of energy measured around a certain point in the space is the result of observing a set of geometries, each one with its own energy distribution. The average observed effect is a weighted sum, in which each geometry contributes to the detected energy according to its statistical weight in the space of all the geometries: geometries that occur more often contribute more. What determines the weight of a geometry is the number of equivalent times it can be realized, i.e. the number of times in which distributing a certain amount of energy units produces the same geometry. This is the order of the symmetry group of the geometry. The basic expression in this scenario is the sum over all the possible energy distributions:

$$\mathcal{Z}(E) = \sum_{\Psi(E)} \mathrm{e}^{S(\Psi(E))} \, . \tag{A.1}$$

where $\Psi(E)$ is a geometry that corresponds to the distribution along the space of a total amount of energy E. $S(\Psi(E))$ is the entropy of the geometry $\Psi(E)$ in the phase space $\{\Psi\}(E)$ of all the geometries of energy E. It is related to the volume of occupation in the phase space, $W(\Psi)$, in the usual way: $S = \log W$ [5]. Contact with the real world is established by identifying the size of the unit cell in the discrete space with the Planck length, the energy unit with the Planck mass times c^2, and taking the limit to the continuum. Indeed, the sum A.1 is not restricted to three-dimensional spaces. Nevertheless, three dimensional space arises as the most entropic, i.e. the most frequently realized, dimension within the class of the most symmetric geometries. Owing to the interpretation of the path through sets of geometries ordered according to their energy content E as a history of the universe, with an appropriate conversion of units [6], up to a multiplicative constant we can also identify the total energy E with a time coordinate $\mathcal{T}$, that we call "age of the universe". As astonishing as

[5] This volume is related to the volume of the symmetry group of the energy distribution. Since the sum A.1 is always regularized to discrete spaces of finite volume, the volume of the symmetry group turns out to correspond to the order of a finite group.

[6] Based as usual on the constants c, $\hbar$, and the Planck length.

it may be, this leads, in the continuum limit, to a universe with the physical and geometrical properties of the universe we live in, with a three-dimensional space governed by relativistic quantum mechanics. Indeed, this setup leads in a natural way to the concept of mean value of an observable, and to an uncertainty principle: if we consider as "classical space" the most entropic geometry at any age $\mathcal{T}$, we have that, as seen from the classical space, the contribution to the mean energy around a certain point is not just the value assigned by the most entropic geometry, the "classical" value, but is corrected by an amount Δ, that collects the contribution of all the other geometries. Since any physical observation, any experiment, can only occur during a certain non vanishing interval of time, we cannot observe an "instant value" of energy. It only makes sense to speak of a variation of energy during a certain interval of time. It can be shown that ΔE and $\Delta \mathcal{T}$ are inversely proportional, leading to a relation that from a formal point of view coincides with the Heisenberg's uncertainty relation. Indeed, as discussed in Ref. [34], this gives us an idea about why the physical world is necessarily quantum mechanical, even before introducing the notion of light and measurements made through wavelike probes: in this scenario quantum mechanics is an expression of the fact that we can only observe mean values, because the universe does not consist of a single geometry. Once the notion of classical geometry is appropriately introduced, also the concept of propagation of information, and the fact that the speed of light is a universal constant can be shown to result as a consequence of the fundamental setup.

Till now we have spoken only of energy. Indeed, since what we have called energy is what shapes the geometry, and *anything* in the universe is in the end a deformation of space, we are not separating here a background space in which other degrees of freedom (particles, fields, whatever else) live in. The set of geometries are not the space where other elements live in: they *are* the universe. The universe *consists* of geometries. The sum A.1 can therefore be considered the "partition function", or the functional generating all the observables of the theory. The dynamics is intrinsic in A.1: the time evolution is uniquely given by the entropy-weighted sum as a function of $E \propto \mathcal{T}$. Any type of "force" or interaction is therefore entropic by definition. In this scenario, there is neither reversibility nor conservation of energy. Although not surprising for what we just said, the statement about energy conservation may seem to contradict one of the most established principles of physics. However, here we are talking about the non-conservation of the total energy of the universe, a phenomenon that occurs at the cosmological scale. Indeed, as discussed in [34], this seems to agree with several cosmological observations. The physics at the scale of a typical local experiment is on the other hand approximately described by an effective theory obtained by neglecting the cosmological evolution. In this case the usual energy conservation principle is recovered. Indeed, the sum of A.1 over all energies turns out to be a generalization of the Feynman's path integral, in which the paths are the geometries. Although the measure of the sum is not formulated in terms of exponential of an action but exponential of entropy, it can be shown that the sum $\sum_E \mathcal{Z}(E)$ implies the Langragian formulation in the continuum limit, a non-trivial result for which we refer the reader to Ref. [34] (chapter 3, section 5) [7].

The effective theory

At any age of the universe the most entropic geometry turns out to be a three-sphere with radius proportional (after appropriate conversion of units) to the age of the universe itself. Therefore, what we introduced as classical space, namely the space corresponding to the most entropic geometry, is always of finite extension. From A.1 one derives that the history of the universe progresses toward increasing size of the space, which can be interpreted as a classical space expanding proportionally to the age, to serve as base for a quantum world provided with a minimal distance. On the continuum, this scenario can be approximated by string theory. The physical content of the world implied by A.1 can be investigated by considering string constructions corresponding to a compactification to four dimensions, in which however the space-time coordinates are considered of infinite extension only for convenience of the analysis. Indeed, from a physical point of view, they too are compactified. However, differently from the other string coordinates,

[7]The "weird" formulation of the measure reflects the profound difference between this approach and other based on a discretization of fields and/or the space they live in (such as for instance the spin foam models (see [35], and [36] for a review). In its basic formulation our approach does not rely on any kind of Langrangian/Hamiltonian formulation, in A.1 there is neither time evolution nor quantization. It is pure statistics, and as such it deeply differentiates also from approaches with which it shares somehow some concepts, such as Wheeler's geometrodynamics [37].

they are not frozen at the Planck scale, but extended as much as the classical space of the universe. These coordinates expand therefore as a function of the age of the universe. As it was for the geometries, also in the case of string compactifications one must proceed in analogy to (A.1), and investigate string constructions by introducing an entropy in the space of all string compactifications, allowing to weight the compactifications, and identify those of highest entropy. This can be done, and an entropy can be introduced and put in relation to the symmetry of the string construction [8].

There is no single string compactification that, alone, exactly accounts for the physical content, and allows to investigate the spectrum of elementary particles and interactions. The latter can be investigated by looking at the class of string compactifications whose geometries corresponds to the highest amount of symmetry reduction, because they are those that contribute the most. Indeed, as we said, what are usually called compactifications to four dimensions, namely constructions in which four coordinates are infinitely extended, do not appropriately correspond to the real physical situation, in which also space-time is compact. This point is not a negligible aspect: precisely the compactness of space gives rise to non-vanishing sub-Planckian masses as lowest momenta [9]. It should by now be clear that, like field theory, in this context also string theory is only a useful tool for investigating certain properties, but not the whole story. Anyway, although complicated, the analysis of the physical content can nevertheless be done. What comes out is that supersymmetry is broken at the Planck scale [10], and the spectrum of the elementary particles and their interactions is the already known one of the Standard Model, except for the Higgs field, absent here. Not only masses, but also couplings, and as a consequence all physical quantities, turn out to depend on the extension, and therefore on the age, of the universe, which is the only independed parameter of the theory. At any chosen age of the universe, the entire physical content, the masses and the interaction couplings, are uniquely determined. This theoretical framework is therefore extremely predictive. Its predictive power is a "double-edged sword": on one side any mismatch with the experimental results can potentially rule out the entire scenario; on the other side, any agreement is a strong support of the entire construction.

In this scenario, reversibility only exists as an approximation, in an effective action derived for any point of the evolution of the universe, in which one deals with masses and couplings as they were constant. At a fundamenzal level, everything is ruled by entropy: the concept of causality itself is here substituted by an evolution toward more entropic states. Once the cosmological evolution is neglected it is possible to approximate the dynamics with a field theoretical action built out of the degrees of freedom of the spectrum. In principle, any instant of the age of the universe should corresponds to a different effective action. However, since the cosmological evolution is encoded in the dependence of masses and couplings on the age of the universe, while the fundamental degrees of freedom remain the same, it is possible to consider an effective action in which masses and couplings are inserted as external parameters, without caring about their origin, and neglecting their dependence on the age of the universe. Since at present time the dependence of these quantities on the age of the universe is very mild, as long as one is not interested in phenomena occurring at the cosmological scale, the approximation obtained by freezing them to a fixed, present age, makes sense. The dynamics implied in the effective action will approximately describe the time evolution in a neighbor of the present age, with a time parameter that does not encode the whole time evolution. As a consequence, such an effective theory does not need to be consistent as gauge theory in the massive sector (no Higgs mechanism). It does not need to provide a fundamental description of time-reversal and therefore also of CP violation either: in this theoretical framework, these phenomena must be investigated outside of field theory. The effective theory only works as a truncated theory, in which the computation of amplitudes through Feynman diagrams makes only sense once endowed with a different interpretation, that implements the statistical approach derived from the sum (A.1).

[8]It is not necessary to introduce it in an absolute way: what we are interested in is just the comparison between different constructions, therefore their *relative* degree of symmetry. In the case of orbifold constructions this is simple to find out, because they are derived by modding out flat space by different symmetry groups. Luckly, orbifolds turn also out to be enough for the purpose of investigating the spectrum.

[9]Compactness of space does not necessarily imply non-vanishing ground modes of the momenta. However, here the space is a bit more complicated: in the string orbifold language, it is "shifted".

[10]Differently from what one would naively expect, despite of being supersymmetry broken at the Planck scale, the vacuum energy is not of order of the Planck mass, but has a mild dependence on the inverse of the age of the universe, leading to the correct estimation of the cosmological constant (see Ref. [34]).

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