Entropy Generation from Scale-Dependant Photon Absorption in an Isotropic Universe

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Abstract

We consider a box of gas filled with different groups of matter that absorb photons primarily in different and mostly distinct frequency ranges. As a group that absorbs primarily high frequency photons emits blackbody radiation, some photons with frequencies too low to be reabsorbed efficiently by that group are readily absorbed by a group that favours such low frequency photons. This causes the high frequency group to cool and the low frequency group to heat up and the high frequency group to cool down as energy is transferred between the groups. If an infinite ladder of such groups exists in an infinite box, then this process would cycle eternally with each group periodically cooling and reheating as a given group sheds low frequency photons to lower groups while cooling and higher energy groups dump photons into it, causing it to reheat. A cosmology for this scenario is then explored and compared quantitatively to measured data.

Heat Flow Between U-Groups

Consider a box filled with gas made up of two types of particles. The first type can only absorb a specific range of high frequency photons while the second type can only absorb a specific range of low frequency photons. The particles in each group are distributed evenly throughout the box. For simplicity, assume that the frequency ranges do not overlap, meaning that a given photon can only be absorbed by either type of particle, but not both. Therefore, the gasses are effectively transparent to each other. We will call these different groups of gas particles U-Groups.

Now suppose that the system is prepared such that the high frequency U-Group particles have a high average momentum while the low frequency group particles have a very low average momentum (i.e. the high frequency group starts with a high temperature and the low frequency group has a low temperature). If we now let the system evolve, the particles in both groups will emit photons with frequencies emitted according to the blackbody spectrum as they bounce around. Over time, the high energy group will randomly emit low frequency photons that the gas in the high frequency group cannot reabsorb, and those photons will instead be absorbed by the low frequency group. The low frequency group will also emit photons at energies too high to be absorbed by that group and will therefore be absorbed by the high energy group, but because of the skew of the blackbody spectrum, on average, more of the energy being emitted will be in the form of low frequency photons than high frequency. The net effect of this is that there will be an overall heat (and therefore momentum) transfer from the high frequency U-Group particles to the low frequency U-Group particles. Therefore, over time, the high frequency U-Group will cool while the low frequency U-Group will heat up. After sufficient time, the high frequency U-Group will become the low temperature group and the low frequency group will become the high temperature group.

Now suppose the box is infinitely large with an infinite ladder of U-Groups. Furthermore, assume that the initial temperatures of adjacent U-Groups alternate in temperature. So if we randomly choose some U-Group U_1 and impose that it begins with a low temperature, then the adjacent high energy group U_2 and Low energy U-Group $U_{1/2}$ will both start with high temperatures. Letting this system evolve, U_2 will cool as it sheds photons to U_1 while at the same time, $U_{1/2}$ will cool as it sheds photons to $U_{1/3}$. U_1 and $U_{1/3}$ will be reheated by U_2 and $U_{1/2}$ as a result of this. Because there is an infinite ladder of groups, this process will continue indefinitely. Each U-Group will heat up as it absorbs photons from the adjacent high frequency group that is cooling and then cool down as it sheds photons to the adjacent low frequency group, heating up that lower group.

In reality, however, these groups would not have precisely delineated lines between them. Instead, the absorption spectrums of each group, which would be related to the type of matter in that group, would be loosely analogous to the blackbody spectrum where each group has a peak range of frequencies it is most likely to absorb and the probability of absorption of higher and lower frequencies falls off exponentially. In our group, the majority of the matter available to absorb photons is in the form of molecular Hydrogen and Helium, so the absorption spectrum of our group would be primarily characterized by the spectrums of those molecules. Thus, all groups *can* absorb any frequency, but for a given group, relatively low frequency photons will be more likely to be absorbed by the lower energy groups while higher frequency photons will be more likely absorbed by the higher groups. The overall effect is a net heat flow from high energy groups to low energy groups as their temperatures oscillate and the total energy is conserved. And again, because the ladder is infinite, there will always be heat available to flow from a U_2 to a U_1 group. Globally, the net effect of this process is that the energy of a given amount of high frequency photons is broken up into an even greater number of low frequency photons. The energy is conserved, but the entropy of the entire Universe is increased due to the increased number of photons. What we get is a chain of heat pumps moving energy from large scales to smaller scales, with more total entropy being generated each cycle.

But what would be the differences in matter that would cause a difference in absorption spectrums in different groups? It is possible that there are different types of matter that have never been detected that might give rise to this effect. Alternatively, perhaps the fermions have ranges of properties within they're 'type' (the electrons in the box might have a range of masses for example). These ranges may be continuous or they may be discrete. Three such generations of matter are already known to exist, so perhaps there is an infinite number of such generations that we are simply unable to detect due to their energy scales. In this scenario, we can imagine that the bulk matter that gets produced (atoms, molecules, etc.) is made up of constituent particles with 'compatible' properties (atoms that we observe in nature are made of electrons and quarks with first generation properties). The high energy matter, corresponding to high frequency U-Groups, would be more spread out than the matter in our U-Group since the matter exchanges higher momentum photons on average, making the individual particles/molecules/bulk matter spread much farther apart. If each U-Group is less dense as one goes to higher and higher frequencies, then we would not necessarily detect their gravitational effects since the scale at which they distort spacetime would be so much larger than the scales we can currently measure. These high frequency groups may be the source of cosmic rays observed to come from random locations in space. Since the densities of the high frequency groups are so much lower than ours, we are not bathed in these rays, but there is still a non-zero chance that high energy photons produced regularly in higher U-Groups can randomly be absorbed by matter in our group, producing the cosmic rays. Photons from other U-Groups may also appear as variations in the Cosmic Microwave Background since, from our perspective, these photons would be emitted in random directions throughout space.

As for the lower energy groups, they would be perfect candidates for Cold Dark Matter. Since these groups absorb at such a low frequency range, they would effectively only be detectable gravitationally. Unlike the high energy groups, the low energy groups can be increasingly dense approaching infinite density in the limit of zero frequency. This does not however cause a gravitational problem (i.e. black holes) because the energy would be reduced as the density increased, so the curvature of spacetime would not become infinite.

We can make a useful analogy for this process by imagining a society with multiple economies. Each person in the society participates in only one of the economies. For one group of people, they can only buy products using paper bills, with the smallest bill being the one dollar bill. When they purchase products, they have to in a minimum of one dollar increments. But when the seller gives the buyer change, the seller will give that change as a mix of paper money but also with some of the money being returned as coins. But those coins cannot be spent in this economy because sellers here only accept bills. The coins become worthless to the buyers so they just toss out the coins. But there is a second economy that sells cheaper, lower quality versions of the products sold in the first economy. The people in this economy can purchase those products with coins. So as transactions occur in the first economy, some money will be transferred from the first economy into the second in the form of coins being discarded from the first economy. This can be expanded infinitely in both directions where there is a hierarchy of economies and the sellers in each economy only accept specific forms of money, but give change in many forms of money, some of which are not accepted in their economy, but are perfectly useful in one of the other economies. The total money is conserved over all the economies, but some of the wealth or buying power in each economy is shifted between economies by chance when purchases are made.

U-Group Cosmology

It is observed in our Universe that as the temperature drops, space expands. Based on that observation, we can conjecture that space at the scale of a particular U-Group will expand as the group sheds photons to lower groups and contracts when it is heated by a higher group. There does exists a cosmological model that seems particularly fit to describe this process: the internal Schwarzschild solution. So let us now examine the cosmology of a single U-Group.

Consider a collection of particles distributed as a 2D sphere in space such that any local interactions between the particles is negligible and it is surrounded externally and internally by vacuum. If this shell is made to uniformly expand or left to collapse, the particles will follow worldlines defined by the Schwarzschild metric. We shall examine a similar scenario where the spherically symmetric 'shell' is the entire Universe at a given time. Observation has shown that the Universe is:

- Spherically symmetric
- Homogenous in space
- Inhomogeneous across time.

We will also make one further assumption in this paper:

- The matter in a U-Group only ever occupies a single instant of Cosmic time and moves from one moment of cosmic time to the next where the time measured by observers between cosmic times depends on their respective motions. In other words, the 3D spatial distribution of energy in the Universe is physically moving through the time dimension from the past into the future, and matter only exists in the present. So if one were to view the Universe on a spacetime diagram, they would only see the Universe at one value of time with the rest of the diagram empty. A worldline in this scenario is like the dot of a laser pointer following a prescribed path as opposed to a drawn out line fixed in the spacetime.

This further assumption implies that the spherically symmetric U-Group is 'surrounded' by vacuum in the time dimension, analogously to how the aforementioned 2D shell was surrounded by a vacuum of space. Since the only spherically symmetric vacuum solution in General Relativity is the Schwarzschild metric, this assumption implies that the metric of the U-Group is the black hole metric, which will be referred to as the 'internal Schwarzschild metric' in this paper.

Next, consider the celestial spheres around an observer in the Universe. When we look out to distant events, we can use the redshift from these events to determine their distance from us. Events with the same distance from us can be thought of as residing on a celestial sphere, such that all these events are separated from us by the same magnitude of space and time. We can classify these spheres into three types:

1. Dynamic Spheres – These are the spheres that galaxies reside on. Objects on these spheres maintain a constant coordinate distance from us and move forward in time. We are able to move toward or away from objects on these spheres by moving through space. If we fix our sights on a particular galaxy, the light we see from that galaxy is being emitted later in time as we ourselves move through time.

2. Static Spheres – These are spheres fixed in time. The Cosmic Microwave Background is the most obvious example of these spheres. Light from the CMB sphere is always emitted from the same cosmological time, but as we ourselves move through time, we see light from that time emitted from farther and farther away from us in space, giving the impression that the CMB sphere is growing. We cannot move toward or away any objects on this sphere because it is frozen in time. Both metrics are able to capture this behaviour, but they do so in different ways.

3. The Dark Sphere – The Dark Sphere would be where current cosmological models would place the Big Bang. It is where the temperature of the U-Group becomes infinite. But no U-Group will actually reach this point as each U-Group would reach a maximum temperature before it starts cooling again.

These spheres are shown in terms of the internal Schwarzschild metric in Figure 1. Figure 1 shows the Schwarzschild coordinates of the internal metric plotted on the Kruskal-Szekeres coordinate plane. In these coordinates, space is the 't' coordinate emanating from the center of the diagram (Big Bang) and time is the 'r' coordinate depicted as hyperbolas (time is flowing forward as r goes toward zero). The upper right quadrant of this diagram represents a single fixed direction ($\theta = const, \phi = const$). So each bold line representing a sphere would be a point on each sphere over time. Note that light on this diagram travels on 45-degree lines.

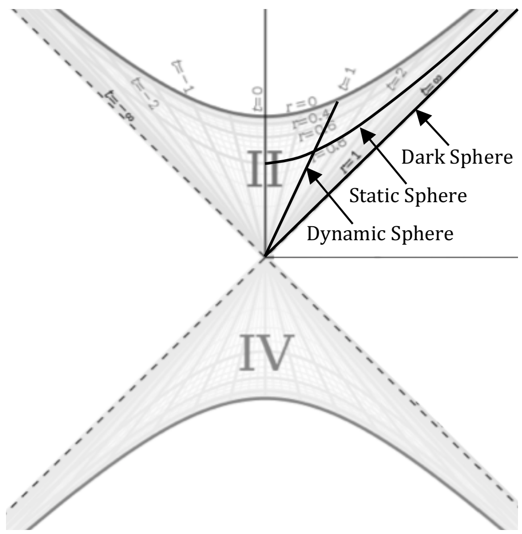

Figure 1 – Celestial Sphere Types on Kruskal-Szekeres Coordinate Chart[1]

Going forward, we will first examine the space time of the internal metric from the perspective of the inertial observer and compare results to experimental data. We will then examine the angular part of the metric more closely. Finally, we will tie the outgoing and incoming photons to and from outside U-Groups to the white and black hole singularities.

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[1] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

The Schwarzschild Metric

The Schwarzschild metric is the simplest solution to Einstein's field equations. It is a vacuum solution for the spacetime around a spherically-symmetric distribution of energy. The general form of the metric can be expressed as:

$$d\tau^2 = \frac{r}{u-r}dr^2 - \frac{u-r}{r}dt^2 - r^2 d\Omega^2 \tag{1}$$

Depending on the ratio $\frac{u}{r}$, we get three distinct descriptions of spacetime:

1. $u = 0$: This gives us the flat Minkowski metric of Special Relativity.
2. $\frac{u}{r} < 1$: This describes the metric for an *eternally* spherically-symmetric vacuum centered in space. This metric is also used to describe the vacuum outside a spherically symmetric object occupying a finite amount of space (like a star or planet).
3. $\frac{u}{r} \geq 1$: This describes the metric for a spherically symmetric vacuum centered on a point in *time*. Analogous to the second case, this metric should also describe a vacuum *of time* outside a spherically-symmetric object spanning *infinite space*. The center of the metric is *everywhere in space, but at a single point in time* (just like one could say that the vacuum described in the second case is centered at all times on a single point in space).

An important observation is that the internal metric describes a vacuum solution to the field equations. But the U-Group is clearly filled with energy, so how can this solution apply? In order to satisfy the requirements of the metric, the Universe must be "*a spherically-symmetric energy distribution occupying an infinite amount of space for a finite amount of time*". For this metric to be a cosmological description, it must be that Universe only truly exists in the present and in a very real sense moves into the future. The surrounding vacuum is the future and past.

Time being the radial dimension of the metric combined with the fact that the solution is a vacuum solution gives a mathematical justification for our intuitive notions of past, present, and future. The anisotropy along the radial direction gives us an arrow of time that distinguishes the 'past' and 'future' analogous to the way the external solution gives us an absolute distinction between 'up' and 'down'. And the vacuum as described above gives us a boundary between them, that boundary being the 'present' time.

Freefall Through Time

Let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates (θ, ϕ). There are infinitely many such lines, but since we have an isotropic, spherically symmetric

U-Group, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $\frac{u}{r} \geq 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 showed the $\frac{u}{r} \geq 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant *r* represent spacelike slices of constant cosmological time and the rays of *t* represent spatial distances. We will not be considering differences in angles until a later section in the paper, so we only need to consider the two halves of Figure 1. We will focus on the upper half where that half represents an observer pointed in a particular direction and the positive *t*'s represent the coordinate distance from the observer in that particular direction while the negative *t*'s represent coordinate distance in the opposite direction.

We must first determine the paths of inertial observers in the spacetime. For this we need the geodesic equations for the internal Schwarzschild metric [1] given in Equation 1. In these equations u represents a time constant that in the external metric would be the Schwarzschild radius (in Figure 1, the value of u is 1). The following equations are the geodesic equations for *t* and *r* $(r \leq u)$:

$$\frac{d^2t}{d\tau^2} = \frac{u}{r(u-r)}\frac{dr}{d\tau}\frac{dt}{d\tau} \tag{2}$$

$$\frac{d^2r}{d\tau^2} = \frac{u}{2r^2}\left[\frac{u-r}{r}\left(\frac{dt}{d\tau}\right)^2 - \frac{r}{u-r}\left(\frac{dr}{d\tau}\right)^2\right] - (u-r)\left(\frac{d\Omega}{d\tau}\right)^2 \tag{3}$$

In Equations 1, 2, and 3, we use units where $c = 1$ and equations 2 and 3 assume no angular motion. Looking at points $0 < r < u$, then by inspection of Equation 2 it is clear that an inertial observer at rest at *t* will remain at rest at *t* ($\frac{d^2t}{d\tau^2} = 0$ if $\frac{dt}{d\tau} = 0$). Also, we see that if an observer is moving inertially with some initial $\frac{dt}{d\tau}$, then if $\frac{dr}{d\tau} < 0$, the coordinate speed of the observer will be reduced over time (the coordinates are expanding beneath her) and if $\frac{dr}{d\tau} > 0$, the coordinate speed will be increased over time (the coordinates are collapsing beneath her).

Let us therefore examine Equation 3 for an observer with no angular motion. Combining Equations 1 and 3, equation 3 becomes:

$$\frac{d^2r}{d\tau^2} = -\frac{u}{2r^2}\left[1 + \left(\frac{d\Omega}{d\tau}\right)^2\right] - (u-r)\left(\frac{d\Omega}{d\tau}\right)^2 \tag{4}$$

For $\frac{d\Omega}{d\tau} = 0$, notice that the observer's acceleration through cosmological time is similar to the form of Newton's law of gravity, where *r* (a time coordinate) varies from *u* to 0 (If the Schwarzschild constant was 2*GM*, as it would be in the external solution, Equation 4 would be Newton's gravity). Also, anyone moving inertially starting with non-zero $\frac{dt}{d\tau}$ will experience the same acceleration through time as someone with zero $\frac{dt}{d\tau}$ since dt does not appear in Equation 4.

So we will first use Figure 1 to describe the freefall of the galaxies through the cosmological time dimension where galaxies (or galaxy clusters) follow lines of constant *t* (and any such observer can choose $t = 0$ as their coordinate). The U-Group cools and expands as it moves toward $r = 0$.

The Scale Factor

Expressions for the proper time interval along lines of constant *t* and Ω and the proper distance interval along hyperbolas of constant *r* and Ω from Equation 1 are:

$$\frac{dr}{d\tau} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{5}$$

$$\frac{ds}{dt} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{6}$$

Where *a* is the scale factor. First we should notice that neither Equation 5 nor 6 depend on the *t* coordinate. This is good because the *t* coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper velocity and proper distance only depends on the cosmological time *r*.

What is notable here is that in Schwarzschild coordinates, the scale factor is equal to the velocity through the time dimension for an observer at rest $\left(\frac{dt}{d\tau} = \frac{d\Omega}{d\tau} = 0\right)$. When $r = u$, Equations 5 and 6 are both 0. At this point, the proper velocity in time is zero. When crossing this point, expansion becomes collapse/reheat and vice versa. Crossing it has the same effect as reversing the direction of time.

At $r = 0$, both equations 5 and 6 are infinite. So when the worldlines enter or exit one of the $r = 0$ hyperbolas, they do so at infinite proper speed *through the time dimension*. If something is travelling through space at the speed of light, the proper distance between points in space is zero. In this case, since we have infinite proper velocity in the time dimension, the proper distance between points in space will be infinite, because you would traverse an infinite amount of time in order to move through an infinitesimal amount of space. What we see then is that at $r = 0$ space will be infinitely expanded and thus the scale factor is infinite and the temperature will be zero. But the U-Group matter will never reach this singularity because it will begin reheating (and therefore re-collapsing) before reaching this point. A plot of the scale factor vs. *r* (with $u = 1$) is given in Figure 2 below:

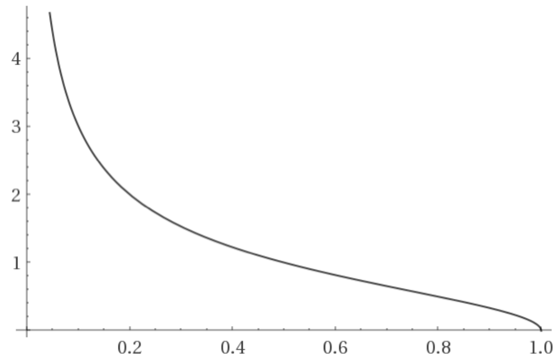

Figure 2 – Scale Factor vs. r for $u = 1$

Cosmological Parameters

In order to compare this model to cosmological data, we must solve for u and find our current position in time (r_0) in the model. Reference [3] gives us a 95% confidence interval for the measured transition redshift at $z_t = 0.426^{+0.27}_{-0.089}$. We can use the fact that $\sqrt{\frac{u-r}{r}}$ is the scale factor and get the expression for cosmological redshift caused by the expansion [1] (note that this Equation was derived from the FRW metric in the reference, but the internal metric, when setting $d\Omega = 0$, can be put in the same form as the FRW metric with a coordinate change, so the equation below is still valid for the internal metric):

$$z = \sqrt{\frac{r_{emit}}{(u-r_{emit})}}\sqrt{\frac{u-r}{r}} - 1 \tag{7}$$

We can see in Figure 2 that there is an inflection point that corresponds to the transition redshift in the model. To find this inflection point, we need to derive the Hubble parameter and deceleration parameter equations using the scale factor. The Hubble parameter is given by:

$$H = \frac{\dot{a}}{a} = \frac{d}{dr}\left(\sqrt{\frac{u-r}{r}}\right)\sqrt{\frac{r}{u-r}} = \frac{u}{2r(u-r)} \tag{8}$$

And the deceleration parameter is given by:

$$q = \frac{\ddot{a}a}{\dot{a}^2} = \frac{4r}{u} - 3 \tag{9}$$

The transition redshift occurs when $q = 0$, giving us $\left(\frac{r}{u}\right)_t = 0.75$. With this and Equation 7, we can find $\left(\frac{u}{r}\right)_0$:

$$z_t = 0.426^{+0.27}_{-0.089} = \sqrt{\frac{1}{\frac{1}{0.75}-1}}\sqrt{\left(\frac{u}{r}\right)_0 - 1} - 1 \rightarrow \left(\frac{u}{r}\right)_0 = \left(\frac{1.426^{+0.27}_{-0.089}}{\sqrt{3}}\right)^2 + 1 \tag{10}$$

Giving:

$$\left(\frac{u}{r}\right)_0 = 1.678^{+0.281}_{-0.082} \tag{11}$$

The current Hubble constant, as measured by the Planck mission was found to be $H_0 = 67.8 \pm 0.9\ (\mathrm{km/s})/\mathrm{Mpc}$ and from the Hubble Space telescope $H_0 = 73.48 \pm 1.66\ (\mathrm{km/s})/\mathrm{Mpc}$. With these and Equation 11, we can solve for limiting values of u and r_0 (after converting the units of H_0 so that u is measured in Gly):

$$H_0 = \left(\frac{u}{r}\right)_0 \left[\frac{1}{2u\left(1-\left(\frac{r}{u}\right)_0\right)}\right] \rightarrow u = \left(\frac{u}{r}\right)_0 \left[\frac{1}{2H_0\left(1-\left(\frac{r}{u}\right)_0\right)}\right] \tag{12}$$

Note that in Equation 12, H_0 is in units of $(Gy)^{-1}$. Before presenting the results, let us derive the expression for t vs. r along a null geodesic where the geodesic ends at the current time r_0. We can do this by setting $d\tau = rd\Omega = 0$ in Equation 1 and integrating:

$$t = \int_{r_0}^{r} \frac{r}{u-r} dr = u\ln\left(\frac{u-r_0}{u-r}\right) + (r_0 - r) \tag{13}$$

Table 1 below gives the values of u, r_0, a_0, q_0, r_t (coordinate time at transition redshift), H_t (Hubble constant at the transition redshift), and t_t (coordinate distance of transition redshift) given the measured bounds of z_t and H_0. All times are in Gy, distances are in Gly, and H are in $(\mathrm{km/s})/\mathrm{Mpc}$.

z_t	H_0	u	r_0	$u - r_0$	a_0	H_t	r_t	t_t	$r_t - r_0$	q_0
0.337	68.7	30.4	19.0	11.4	0.77	85.8	22.8	8.5	3.8	-0.5
0.337	66.9	31.2	19.5	11.7	0.77	83.6	23.4	8.8	3.9	-0.5
0.337	75.14	27.8	17.4	10.4	0.77	94.3	20.9	7.9	3.5	-0.5
0.337	71.82	29.1	18.2	10.9	0.77	89.5	21.8	8.1	3.6	-0.5
0.696	68.7	28.5	14.3	14.2	1.00	91.8	21.4	12.7	7.1	-1.0
0.696	66.9	29.3	14.7	14.6	1.00	89.3	22.0	13.0	7.3	-1.0
0.696	75.14	26.0	13.0	13.0	1.00	100.4	19.5	11.5	6.5	-1.0
0.696	71.82	27.3	13.7	13.6	1.00	95.8	20.5	12.1	6.8	-1.0

Table 1: Limiting Cosmological Parameter Values Based on z_t and H_0 Measurement

Note that these values cannot be calculated for the CMB because of lack of precision in z_t and H_0 measurements (The CMB is too close to $r = u$ to get meaningful values given the imprecise measurements). Table 2 has the proper times from $r = u$ to the transition redshift and current time for stationary, inertial observers ($dt = rd\Omega = 0$) by integrating Equation 1 (there is not enough precision in the measurements to calculate this for the CMB). The column τ_{tot} gives the time from $r = u$ to $r = 0$. The expression for τ_{tot} turns out to be quite simple[2]:

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[2] Thinking of τ_{tot} as a 'Universal Period' allows us to define a Universal constant $U = \frac{\pi}{2}u$ for time and space. Equation 14 is the maximum amount of time that can be measured between the Big Bang and $r = 0$. So if we set $U = \frac{\pi}{2}u = c = 1$ then we are working in units where space and time have the same units and all measurable times will be between 0 and 1. When working in these units, the constant in the interior Schwarzschild metric will be $u = \frac{2}{\pi}$.

$$\tau_{tot} = \frac{\pi}{2} u \tag{14}$$

The column τ_{remain} gives the time between $r = r_0$ and $r = 0$.

z_t	H_0	τ_0	τ_t	τ_{tot}	τ_{remain}
0.337	68.7	34.6	29.1	47.8	13.2
0.337	66.9	35.7	30.1	49.2	13.5
0.337	75.14	31.7	26.5	43.7	12.0
0.337	71.82	33.3	27.9	45.7	12.4
0.696	68.7	36.3	27.2	44.8	8.5
0.696	66.9	37.4	28.0	46.0	8.6
0.696	75.14	33.3	25.1	41.0	7.7
0.696	71.82	34.8	26.2	42.9	8.1

Table 2: Limiting Proper Times Based on z_t and H_0 Measurements (Time is in Gy)

Note that while the coordinate times for the current age of the U-Group ($u - r_0$) are close to current estimates (for high z_t), the proper time τ_0 is actually much larger. This is because at early times, observers are moving slower through the time dimension and therefore they accrue more proper time per unit coordinate time. But the speed through the time dimension increases over time such that even though we are presently only about halfway through the "coordinate life" of the U-Group (according to Table 1), the amount of proper time remaining is actually much less than the amount of proper time that has already passed (according to Table 2).

Next we would like to use the u and r_0 values found to create an envelope on a Hubble diagram to compare to measured supernova data. First we need to find r as a function of redshift. We can do this by solving for r_{emit} in Equation 7 where $a_0 \equiv \sqrt{\frac{u-r}{r}}$, the present value of the scale factor:

$$r = u \frac{z^2+2z+1}{a_0^2+z^2+2z+1} \tag{15}$$

Next we substitute Equation 15 into Equation 13 to get coordinate distance in terms of redshift:

$$t = u\left[\ln\left(\frac{r_0(a_0^2+z^2+2z+1)}{u}\right) - \frac{z^2+2z+1}{a_0^2+z^2+2z+1}\right] + r_0 \tag{16}$$

Finally, we convert Equation 16 to the distance modulus, μ, which is defined as:

$$\mu = 5 \log_{10}\left(\frac{t}{10}\right) \tag{17}$$

Where t in Equation 17 is in units of parsecs. A plot of distance modulus vs. redshift is shown in Figure 3 below plotted over data obtained from the Supernova Cosmology Project

[6]. Curves calculated from all combinations of u and r_0 in Table 1 are plotted, giving an envelope for the model's prediction of the true Hubble diagram.

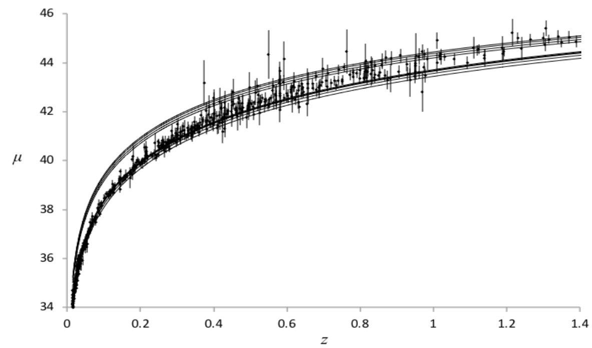

Figure 3 – Distance Modulus vs. Redshift Plotted with Supernova Measurements

Note that the lower curves correspond to the $z_t = 0.696$ data, suggesting that, if this model is correct, the true transition redshift is closer to 0.696 than 0.337.

In [7], the authors analyze a large sample of quasar data to obtain distance moduli at higher redshifts than is possible with supernova data. Although not definitive, the results of this analysis suggests that the "Dark Energy" density may be increasing with time, which does not fit with the ΛCDM model. However, the accelerated expansion predicted by the Schwarzschild solution *is* consistent with this type of expansion. Figure 4 shows the same predicted envelope from Figure 3 for the Hubble diagram plotted out to higher redshifts with the quasar data from [7] also shown with error bars. The black diamonds in the figure are the 18 high-luminosity XMM-Newton quasar points described in [7].

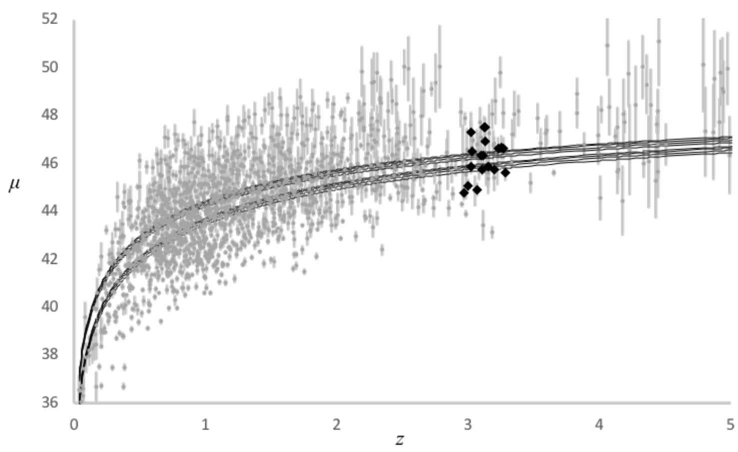

Figure 4 – Distance Modulus vs. Redshift Plotted with Quasar Measurements

The Angular Term of the Metric

Given that the radius of the internal Schwarzschild metric is the time dimension instead of the space dimension, we need a way to calculate the circumference of circles in this metric. Suppose an inertial observer at $t = 0$ and $r = R$ wants to measure a circle at coordinate distance $t = D$ away from them. Since they will see this circle in the past, they will be measuring the circle as it was at some time r_0 in the past found from Equation 13. To calculate the circumference of this circle using the metric, we first calculate the time r_1 at which an observer at $r = 0$ and $t = 0$ would see D using Equation 13. The circumference of this circle calculated from the metric will be $2\pi r_1$. We can then find the circumference for the observer at $r = R$ by multiplying the circumference measured by the $r = 0$ observer by the ratio of the scale factors at r_0 and r_1. The circumference measured by the observer at $r = R$ is therefore $C = \frac{a_0}{a_1} 2\pi r_1$.

The White Hole and Black Hole Singularities

The singularities at the top and bottom of Figure 1 provide a useful tool for depicting the photons that enter the U-Group during reheating as well as the photons that are removed from the U-Group during the cooling.

In that figure, let time move forward going from bottom to top. If we start near the bottom singularity, the U-Group will be at the beginning of the reheating phase, which happens in the bottom half of the diagram. During this phase, photons will appear randomly throughout space in all directions as the higher U-Group begins to cool, shedding photons into our group. Observationally, this will appear as though radiation is emerging from the vacuum, heating up the group. In Figure 1, this can be depicted as light coming out of the white hole singularity at the bottom of the diagram being absorbed by the U-Group as it moves up the diagram.

Once the group has reached maximum temperature, it will start cooling. In this phase, we are in the top half of the diagram with the matter continuing to move upwards. Unabsorbed low frequency light will be lost from the group in this phase, as if it disappeared into the vacuum. In Figure 1, this would be depicted as light entering the black hole singularity at the top of the figure.

References

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