On R.T. Cahill's Re-analysis of the Michelson-Morley Experiment

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Abstract

R.T. Cahill has been advancing a new approach to the analysis of the Michelson-Morley experiment. He proposed that length contraction completely cancels the effect of absolute motion only in vacuum (n=1). He has proposed a re-analysis of the experiment by taking into account the refractive index of the medium (air), thereby explaining the origin of the small fringe shifts observed in the Michelson-Morley and the Miller experiments. In this paper we present two cases against Cahill's theory.

1. He may have overestimated the velocity predicted by his own theory by a factor of about 1.4.
2. His theory cannot explain the 'null' result of the Kennedy-Thorndike experiment, even in vacuum.

Introduction

R.T. Cahill has been advancing a new approach to the analysis of the Michelson-Morley (MM) experiment. He proposed that length contraction completely cancels the effect of absolute motion only in vacuum (n=1). He has proposed a re-analysis of the experiment by taking into account the refractive index of the medium (air), thereby explaining the origin of the small fringe shifts observed in the Michelson-Morley and the Miller experiments. Some researchers have been so intrigued by this idea that they have gone as far as testing it experimentally[1] and have written papers on it [3].

In this paper I present two cases against Cahill's theory. 1. It cannot explain the 'null' result of the Kennedy-Thorndike experiment, where the two arm lengths differ, even in vacuum. 2. He may have made a simple mistake while applying his own theory, predicting higher velocities by a factor of about 1.4. Moreover, an experiment done to test his theory did not confirm it[1].

Re-analysis of the Michelson-Morley experiment

I will start from Equation (10), page 5, in Cahill's paper [2]. I have used *u* instead of *v*.

$$\Delta t = \frac{2LV\sqrt{1-\frac{u^2}{c^2}}}{V^2 - u^2} - \frac{2L}{\sqrt{V^2 - u^2}}$$

where $V = c/n$ and u is the absolute velocity of the MM apparatus.

A more general form of the above formula will be:

$$\Delta t = \frac{2L_L V\sqrt{1-\frac{u^2}{c^2}}}{V^2 - u^2} - \frac{2L_T}{\sqrt{V^2-u^2}} \qquad . \;\; . \;\; . \;\; (1)$$

where L_L and L_T are the longitudinal and the transverse arm lengths, respectively.

This can be rewritten as:

$$\Delta t = \frac{2L_L}{V}\,\frac{\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{u^2}{V^2}} - \frac{2L_T}{V}\,\frac{1}{\sqrt{1-\frac{u^2}{V^2}}} \qquad . \;\; . \;\; . \;\; (2)$$

Since u << c , u << V (see Appendix)

$$\sqrt{1-\frac{u^2}{c^2}} \approx 1 - \frac{1}{2}\,\frac{u^2}{c^2}$$

$$\frac{1}{1-\frac{u^2}{V^2}} \approx 1 + \frac{u^2}{V^2}$$

$$\frac{1}{\sqrt{1-\frac{u^2}{V^2}}} \approx 1 + \frac{1}{2}\,\frac{u^2}{V^2}$$

Therefore,

$$\Delta t \approx \frac{2L_L}{V}\,(1 - \frac{1}{2}\frac{u^2}{c^2})\,(1 + \frac{u^2}{V^2}) - \frac{2L_T}{V}\,(1 + \frac{1}{2}\frac{u^2}{V^2})$$

$$\Delta t \approx \frac{2L_L}{V}\left(1 + \frac{u^2}{V^2} - \frac{1}{2}\frac{u^2}{c^2}\right) - \frac{2L_T}{V}\left(1 + \frac{1}{2}\frac{u^2}{V^2}\right) \text{, ignoring higher order terms}$$

Substituting $V = c/n$

$$\Delta t \approx \frac{2nL_L}{c}\left(1 + \frac{u^2n^2}{c^2} - \frac{1}{2}\frac{u^2}{c^2}\right) - \frac{2nL_T}{c}\left(1 + \frac{1}{2}\frac{u^2n^2}{c^2}\right)$$

$$\Delta t \approx \frac{2nL_L}{c}\left(1 + \frac{u^2}{c^2}\left(n^2 - \frac{1}{2}\right)\right) - \frac{2nL_T}{c}\left(1 + \frac{1}{2}\frac{u^2n^2}{c^2}\right) \quad . \; . \; . \quad (3)$$

First let us check this formula against Cahill's, for $L_L = L_T = L$.

$$\Delta t \approx \frac{2nL}{c}\left(1 + \frac{u^2}{c^2}\left(n^2 - \frac{1}{2}\right)\right) - \frac{2nL}{c}\left(1 + \frac{1}{2}\frac{u^2n^2}{c^2}\right)$$

$$\Delta t \approx \frac{2nL}{c}\left(\frac{u^2}{c^2}\left(n^2 - \frac{1}{2}\right) - \frac{1}{2}\frac{u^2n^2}{c^2}\right)$$

$$\Delta t \approx L\,n(n^2 - 1)\,\frac{u^2}{c^3} \quad . \; . \; . \quad (4)$$

which differs from Cahill's formula [2], Equation 11, Page 5:

$$\Delta t \approx -L\,(n - 1)\,\frac{u^2}{c^3} \quad . \; . \; . \quad (5)$$

Older ether theory predicts that, (with no length contraction), Equation 13, page 5 of Cahill's paper[2] (I obtained the same value except the minus sign):

$$\Delta t \approx -L \frac{u^2}{c^3} \quad . \quad . \quad . \quad (6)$$

Let us compare the velocity predicted by equation (4) with that predicted by equation (5).

Velocity predicted by equation (4):

Equating the expressions in equation (4) and equation (6):

$$L\, n(n^2-1)\frac{u^2}{c^3} = L \frac{{u_{MM}}^2}{c^3} \;\Rightarrow\; u = \frac{u_{MM}}{\sqrt{n(n^2-1)}} \quad . \; . \; . \quad (7)$$

Velocity predicted by equation (5):

Equating the expressions in equation (5) and equation (6):

$$-L\,(n-1)\frac{u^2}{c^3} = L \frac{{u_{MM}}^2}{c^3} \quad\Rightarrow\quad u = \frac{-\,u_{MM}}{\sqrt{n-1}} \quad . \; . \; . \quad (8)$$

Taking the ratio of equation (8) and equation (7), ignoring the negative sign:

$$\frac{\frac{u_{MM}}{\sqrt{n-1}}}{\frac{u_{MM}}{\sqrt{n(n^2-1)}}} = \sqrt{n(n+1)} \approx \sqrt{2}\ , for \quad n \approx 1$$

Therefore, Cahill may have overestimated the velocity predicted by his own theory by a factor of about 1.4. Jeremy Fiennes [3] has also obtained a different value of absolute velocity following Cahill's own approach.

The Kennedy-Thorndike experiment

As we have mentioned already, Cahill's theory cannot explain the 'null' result for the Kennedy-Thorndike experiment, in which the longitudinal and transverse arm lengths differ, even for vacuum (n=1).

From equation (3)

$$\Delta t \;\approx\; \frac{2nL_L}{c}\left(1+\;\frac{u^2}{c^2}\left(n^2-\frac{1}{2}\right)\right)-\;\frac{2nL_T}{c}\;\left(1+\frac{1}{2}\,\frac{u^2n^2}{c^2}\right)$$

If we assume n = 1

$$\Delta t \;\approx\; \frac{2L_L}{c}\left(1+\frac{1}{2}\,\frac{u^2}{c^2}\right)-\;\frac{2L_T}{c}\;\left(1+\frac{1}{2}\,\frac{u^2}{c^2}\right)$$

$$\Delta t \;\approx\; \frac{2}{c}\left(1+\frac{1}{2}\,\frac{u^2}{c^2}\right)(L_L-\;L_T\,)\quad .\;\;.\;\;.\quad (9)$$

For u = 0

$$\Delta t \;\approx\; \frac{2}{c}\left(1+\frac{1}{2}\,\frac{u^2}{c^2}\right)(L_L-\;L_T\;=\;\;\frac{2}{c}\,(L_L-\;L_T\,)$$

The maximum change in Δt is obtained by subtracting this value from the expression in Equation (9) :

$$\frac{2}{c}\left(1+\frac{1}{2}\,\frac{u^2}{c^2}\right)(L_L-\;L_T\,)-\;\frac{2}{c}\,(L_L-\;L_T\,)\;=\;\;\frac{2}{c}\,(L_L-\;L_T\,)\,(\,1+\frac{1}{2}\,\frac{u^2}{c^2}-1)$$

$$\text{Maximum } change \text{ in } \Delta t=\;\;\frac{2}{c}\,(L_L-\;L_T\,)\,\frac{1}{2}\,\frac{u^2}{c^2}\;=\;\;(L_L-\;L_T\,)\,\frac{u^2}{c^3}\quad .\;\;.\;\;.\qquad (10)$$

From equation (10) we can see that there would always be a change in Δt with change in u , hence a fringe shift, even in vacuum.

Therefore, Cahill's theory, which is a combination of length contraction and effect of refractive index of the medium(air), and which rejects time dilation, cannot explain the 'null' result of the Kennedy-Thorndike experiment.

References

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APPENDIX

For $-1 < x < 1$

$$\sqrt{1-x^2} = 1 - \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^6}{16} - \frac{5x^8}{128} - \quad . \ . \ .$$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + \ . \ . \ .$$

$$\frac{1}{\sqrt{1-x^2}} = 1 + \frac{x^2}{2} + \frac{3x^4}{8} + \frac{5x^6}{16} + \frac{35x^8}{128} + \quad . \ . \ .$$