THE REINTERPRETATION OF THE "MAXWELL EQUATIONS"

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ABSTRACT

This publication contains a mathematical approach for a reinterpretation of the “Maxwell equations” under the assumption of a magnetic field density. The basis for this is Faraday's unipolar induction, which has proven itself in practice, in combination with the calculation rules of vector analysis. The theoretical approach here is the assumption, according to Paul Dirac, that there is a magnetic field density. In this publication the “Maxwell equations” are recalculated in their entirety. It is shown that both the change in the magnetic field over time and the change in the electric field over time can be derived from a second level tensor (matrix), which can be interpreted as a spatial field distortion tensor. Likewise, both the magnetic field density and the electric field density are derived from the unipolar induction according to Faraday. The magnetic field density results from the fact that the \( \text{div} \vec{B} \) is equal to the \( (\text{Sp})\text{grad} \vec{B} \).

Another innovation are the two field gradients \( \text{grad} \vec{B} \), \( \text{grad} \vec{D} \) and the velocity gradient \( \text{grad} \vec{v} \), which can also be derived from Faraday's unipolar induction. These three gradients play an important role in the interpretation of spatially distorted fields.

1. INTRODUCTION

The “Maxwell equations” were defined in a simplified manner by Oliver Heaviside (1850-1925) in their current form. Since vector mathematics was still in its infancy at that time, the “Maxwell equations” were simplified by Oliver Heaviside using the methods of differential calculus and integral calculus at that time. He assumed that there was no magnetic field density. This was later questioned by Paul Dirac through a theoretical consideration. Therefore
this elaboration deals with the reinterpretation of the “Maxwell equations”, under the mathematical requirement of a magnetic field density and with the help of vector analysis. Faraday's unipolar induction serves as the basis.

2. IDEAS AND METHODS

2.1 IDEA FOR REINTERPRETATION OF THE “MAXWELL EQUATIONS”

The basic idea for the reinterpretation of the “Maxwell equations” is based on the discovery of magnetic “quasi-monopoles”, which cause a magnetic field density. These were demonstrated in the following experiments:


2. 2010, Paul-Scherrer-Institut, Formation of “quasi-monopoles” through synchronous radiation.


Based on Faraday's unipolar induction (equation 2.1.1) and the related analog equation (equation 2.1.2), the “Maxwell equations” can now be derived and reformulated, based on the mathematical requirement of a magnetic field density and with the aid of vector analysis will.

\[ \vec{E} = \text{electric field strength} \]
\[ \vec{v} = \text{velocity} \]
\[ \vec{B} = \text{magnetic flux density} \]
\[ \vec{H} = \text{magnetic field strength} \]
\[ \vec{D} = \text{electrical flux density} \]
\[ \times = \text{Cross product} \]
\[ \vec{s} = \text{distance} \]
\( t = \text{time} \)
\( \rho_e = \text{electrical space charge density} \)
\( \rho_m = \text{magnetic space charge density} \)
\( \delta = \text{Delta} \)
\( \text{rot} = \text{rotation} \)
\( \text{div} = \text{divergence} \)
\( \text{grad} = \text{gradient} \)

Faraday unipolar induction:
\[
\vec{E} = \vec{v} \times \vec{B} \quad (2.1.1)
\]

Unipolar induction for magnetic fields:
\[
\vec{H} = - (\vec{v} \times \vec{D}) \quad (2.1.2)
\]

### 2.2 Basics of Vector Calculation

In order to be able to derive the set of equations of the “Maxwell equations” from vector calculation, the basics of vector calculation used for this are described in this chapter.

First, three meta-vectors \( \vec{a} \), \( \vec{b} \) and \( \vec{c} \) are introduced at this point. The three meta-vectors will be used in the following basic mathematical description. In Equation 2.2.1, these three meta-vectors are used to map the cross product.

\[
\vec{c} = \vec{a} \times \vec{b} \quad (2.2.1)
\]

In equation 2.2.1, the rot-operator is now used on both sides of the equation. This results in equation 2.2.2.

\[
\text{rot} \vec{c} = \text{rot} (\vec{a} \times \vec{b}) \quad (2.2.2)
\]

Now the right side of equation 2.2.2 is rewritten according to the calculation rules of vector calculation. This results in equation 2.2.3.

\[
\text{rot} \vec{c} = \text{rot} (\vec{a} \times \vec{b}) = (\text{grad} \vec{a}) \cdot \vec{b} - (\text{grad} \vec{b}) \cdot \vec{a} + \vec{a} \cdot \text{div} \vec{b} - \vec{b} \cdot \text{div} \vec{a} \quad (2.2.3)
\]
On the right side, two vectorial gradients (grad) and two vectorial divergences (div) are created. If a minus sign is now used on all sides of equation 2.2.3, equation 2.2.3 changes to equation 2.2.4.

\[
\text{rot} (-\vec{a} \times \vec{b}) = - \text{rot} (\vec{a} \times \vec{b}) = - (\text{grad} \vec{a}) \cdot \vec{b} + (\text{grad} \vec{b}) \cdot \vec{a} - \vec{a} \cdot \text{div} \vec{b} + \vec{b} \cdot \text{div} \vec{a} \quad (2.2.4)
\]

### 2.3 UNIPOLAR INDUCTION FOR DESCRIBING ELECTRIC AND MAGNETIC FIELDS

The rot operator is calculated according to the calculation rules from Eq. 2.2.2, to Eq. 2.1.1 and Eq. 2.1.2 applied. Taking into account equation 2.2.4, the two expressions from equations 2.3.1 and 2.3.2 arise.

\[
\text{rot} \vec{E} = \text{rot} (\vec{v} \times \vec{B}) \quad (2.3.1)
\]

\[
\text{rot} \vec{H} = - \text{rot} (\vec{v} \times \vec{D}) \quad (2.3.2)
\]

In a next step, the right-hand side of equations 2.3.1 and 2.3.2 is rearranged according to the calculation rules from equations 2.2.3 and 2.2.4. This gives rise to the expressions from equations 2.3.3 and 2.3.4.

\[
\text{rot} \vec{E} = (\text{grad} \vec{v}) \cdot \vec{B} - (\text{grad} \vec{B}) \cdot \vec{v} + \vec{v} \cdot \text{div} \vec{B} - \vec{B} \cdot \text{div} \vec{v} \quad (2.3.3)
\]

\[
\text{rot} \vec{H} = -((\text{grad} \vec{v}) \cdot \vec{D} - (\text{grad} \vec{D}) \cdot \vec{v} + \vec{v} \cdot \text{div} \vec{D} - \vec{D} \cdot \text{div} \vec{v}) \quad (2.3.4)
\]

If equation 2.3.4 is simplified further, equation 2.3.5 arises.

\[
\text{rot} \vec{H} = -(\text{grad} \vec{v}) \cdot \vec{D} + (\text{grad} \vec{D}) \cdot \vec{v} - \vec{v} \cdot \text{div} \vec{D} + \vec{D} \cdot \text{div} \vec{v} \quad (2.3.5)
\]
2.4 DERIVATION OF THE “MAXWELL EQUATIONS”

2.4.1 “MAXWELL EQUATIONS”

First, the simplified forms of the “Maxwell equations” are listed by the equations 2.4.1, 2.4.2, 2.4.3 and 2.4.4, to which reference is made in this publication.

Gaussian law:
\[ \text{div} \; \vec{D} = \rho_{el} \] (2.4.1)

Gaussian law for magnetic fields:
\[ \text{div} \; \vec{B} = 0 \] (2.4.2)

Induction law:
\[ \text{rot} \; \vec{E} = -\frac{\delta \vec{B}}{\delta t} \] (2.4.3)

Flooding law:
\[ \text{rot} \; \vec{H} = \frac{\delta \vec{D}}{\delta t} + \vec{j} \] (2.4.4)

2.4.2 MATHEMATICAL DERIVATION OF THE “MAXWELL EQUATIONS”

In the following chapters, equations 2.4.2 and 2.4.3 are derived from equation 2.3.3. In addition, equations 2.4.1 and 2.4.4 are derived from equation 2.3.4. The derivation is based on the physical assumption that there is no magnetic field density. It is also assumed here that no distortions occur in the velocity vector field as well as in the magnetic field and in the electric field. As a result, the \( (\text{grad} \; \vec{v}) \) and the \( (\text{div} \; \vec{v}) \) have no influence on the overall result.

Furthermore, the two expressions \( \vec{v}(\text{grad} \; \vec{B}) \) and \( \vec{v}(\text{grad} \; \vec{D}) \) become \( \frac{\delta \vec{B}}{\delta t} \) and \( \frac{\delta \vec{D}}{\delta t} \).
2.4.3 DERIVATION OF GAUSSIAN LAW FOR MAGNETIC FIELDS AND THE LAW OF INDUCTION

\[ \text{rot} \vec{E} = (\text{grad} \vec{v}) \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div} \vec{B} - \vec{B} \text{ div} \vec{v} \quad (2.3.3) \]

First, the individual components from equation 2.3.3 are considered. Assuming a homogeneous velocity vector field, the \((\text{grad} \vec{v})\) and the \((\text{div} \vec{v})\) have no influence on the overall result and therefore assume the value 0. The \((\text{div} \vec{B})\) also assumes the value 0 according to the “Maxwell equations”. This results in equations 2.4.5, 2.4.6 and 2.4.2

\[
\begin{align*}
(\text{grad} \vec{v}) &= 0 \quad (2.4.5) \\
(\text{div} \vec{v}) &= 0 \quad (2.4.6) \\
(\text{div} \vec{B}) &= 0 \quad (2.4.2)
\end{align*}
\]

From the physical assumption that there is no magnetic field density, Gauss's law for magnetic fields follows directly from equation 2.4.2.

Under the conditions from equations 2.4.5, 2.4.6 and 2.4.2, Eq. 2.3.3 can be simplified to equation 2.4.7.

\[ \text{rot} \vec{E} = (\text{grad} \vec{v}) \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div} \vec{B} - \vec{B} \text{ div} \vec{v} \quad (2.3.3) \]

\[ \text{rot} \vec{E} = 0 \star \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \star 0 - \vec{B} \star 0 \quad (2.4.7) \]

If the terms that make no contribution to the overall result are eliminated in equation 2.4.7, the overall expression from equation 2.4.7 can be further simplified. This results in equation 2.4.8.

\[ \text{rot} \vec{E} = -(\text{grad} \vec{B}) \vec{v} \quad (2.4.8) \]

(\text{grad} \vec{B}) \vec{v} from equation 2.4.8 can be rewritten in the column notation. The changed notation is shown in equation 2.4.9.
\[-(\text{grad } \vec{B}) \cdot (\ddot{\vec{v}}) = \begin{bmatrix} \frac{\delta B_x}{\delta x} & \frac{\delta B_x}{\delta y} & \frac{\delta B_x}{\delta z} \\ \frac{\delta B_y}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_y}{\delta z} \\ \frac{\delta B_z}{\delta x} & \frac{\delta B_z}{\delta y} & \frac{\delta B_z}{\delta z} \end{bmatrix} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \] (2.4.9)

If now, in equation 2.4.9, the velocity vector \( \ddot{\vec{v}} \) is multiplied by \((\text{grad } \vec{B})\), equation 2.4.10 results.

\[-(\text{grad } (\vec{B})) \cdot \ddot{\vec{v}} = -\begin{bmatrix} \frac{\delta B_x}{\delta x} \cdot v_x + \frac{\delta B_x}{\delta y} \cdot v_y + \frac{\delta B_x}{\delta z} \cdot v_z \\ \frac{\delta B_y}{\delta x} \cdot v_x + \frac{\delta B_y}{\delta y} \cdot v_y + \frac{\delta B_y}{\delta z} \cdot v_z \\ \frac{\delta B_z}{\delta x} \cdot v_x + \frac{\delta B_z}{\delta y} \cdot v_y + \frac{\delta B_z}{\delta z} \cdot v_z \end{bmatrix} = \dddot{x}_{(\text{grad } \vec{B})\ddot{v}} \] (2.4.10)

The velocity vector \( \ddot{\vec{v}} \) can be rewritten in \( \frac{\delta \ddot{x}}{\delta t} \). Equation 2.4.11 shows this relationship.

\[\ddot{\vec{v}} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{\delta x}{\delta t} \\ \frac{\delta y}{\delta t} \\ \frac{\delta z}{\delta t} \end{bmatrix} = \begin{bmatrix} \frac{\delta B_x}{\delta x} \cdot \frac{\delta x}{\delta t} + \frac{\delta B_x}{\delta y} \cdot \frac{\delta y}{\delta t} + \frac{\delta B_x}{\delta z} \cdot \frac{\delta z}{\delta t} \\ \frac{\delta B_y}{\delta x} \cdot \frac{\delta x}{\delta t} + \frac{\delta B_y}{\delta y} \cdot \frac{\delta y}{\delta t} + \frac{\delta B_y}{\delta z} \cdot \frac{\delta z}{\delta t} \\ \frac{\delta B_z}{\delta x} \cdot \frac{\delta x}{\delta t} + \frac{\delta B_z}{\delta y} \cdot \frac{\delta y}{\delta t} + \frac{\delta B_z}{\delta z} \cdot \frac{\delta z}{\delta t} \end{bmatrix} \] (2.4.11)

If the modified expression from equation 2.4.11 is inserted into equation 2.4.10, equation 2.4.12 results.

\[-(\text{grad } (\vec{B})) \cdot \ddot{\vec{v}} = -\begin{bmatrix} \frac{\delta B_x}{\delta x} \cdot \frac{\delta x}{\delta t} + \frac{\delta B_x}{\delta y} \cdot \frac{\delta y}{\delta t} + \frac{\delta B_x}{\delta z} \cdot \frac{\delta z}{\delta t} \\ \frac{\delta B_y}{\delta x} \cdot \frac{\delta x}{\delta t} + \frac{\delta B_y}{\delta y} \cdot \frac{\delta y}{\delta t} + \frac{\delta B_y}{\delta z} \cdot \frac{\delta z}{\delta t} \\ \frac{\delta B_z}{\delta x} \cdot \frac{\delta x}{\delta t} + \frac{\delta B_z}{\delta y} \cdot \frac{\delta y}{\delta t} + \frac{\delta B_z}{\delta z} \cdot \frac{\delta z}{\delta t} \end{bmatrix} \] (2.4.12)
Assuming a distortion-free magnetic field, the magnetic flux density can only change in the respective effective direction. This simplifies the expression from equation 2.4.12 to equation 2.4.13.

\[-(\nabla (\vec{B})) \cdot \vec{v} = - \left( \begin{array}{c} \frac{\delta B_x}{\delta x} \cdot \frac{\delta x}{\delta t} + 0 + 0 \\ 0 + \frac{\delta B_y}{\delta y} \cdot \frac{\delta y}{\delta t} + 0 \\ 0 + 0 + \frac{\delta B_z}{\delta z} \cdot \frac{\delta z}{\delta t} \end{array} \right) \]  

(2.4.13)

Now \( \delta x \), \( \delta y \) und \( \delta z \) in equation 2.4.13 can be shortened and the total expression from equation 2.4.14 results.

\[-(\nabla (\vec{B})) \cdot \vec{v} = - \left( \begin{array}{c} \frac{\delta B_x}{\delta t} \\ \frac{\delta B_y}{\delta t} \\ \frac{\delta B_z}{\delta t} \end{array} \right) = \frac{\delta \vec{B}}{\delta t} \]  

(2.4.14)

Equation 2.4.14 depicts part of the law of induction. If equation 2.4.14 is now inserted into equation 2.4.8, equation 2.4.15 results.

\[\text{rot} \, \vec{E} = -(\nabla (\vec{B})) \cdot \vec{v} = -\frac{\delta \vec{B}}{\delta t} \]  

(2.4.15)

Equation 2.4.15 can now be simplified to equation 2.4.3, the result is the law of induction.

\[\text{rot} \, \vec{E} = -\frac{\delta \vec{B}}{\delta t} \]  

(2.4.3)

At this point, the note is inserted that the track of the magnetic flux density gradient, i.e. \( (\text{Sp})(\nabla \vec{B}) \), corresponds to the divergence of the magnetic flux density, i.e. \( \text{div} \, \vec{B} \). This mathematical requirement results in the fact that if the \( \text{div} \, \vec{B} \) is set equal to 0, the \( (\text{Sp})(\nabla \vec{B}) \) must also be set equal to 0. However, since the \( (\text{Sp})(\nabla \vec{B}) \) consists of
the individual components that ultimately become the expression $\frac{\delta \vec{B}}{\delta t}$ in equation 2.4.3,

the question arises which values the individual components of the expression $\frac{\delta \vec{B}}{\delta t}$ assume under these conditions and what results physically from this conclusion? These questions are dealt with from Chapter 2.5.

### 2.4.4 DERIVATION OF THE GAUSSIAN LAW AND THE FLOOD LAW

As in chapter 2.4.3, it is assumed in this chapter that neither the velocity vector field nor the vector field of the electric flux density experience any distortion. This means that the $(\text{grad} \vec{v})$ and the $(\text{div} \vec{v})$ have no influence on the overall result. In contrast to Chapter 2.4.3, however, the field divergence, i.e. $(\text{div} \vec{D})$, makes a contribution to the overall result. This means that there is an electric field density. These physical assumptions are shown in equations 2.4.5, 2.4.6 and 2.4.1.

$$(\text{grad} \vec{v}) = 0 \quad (2.4.5)$$

$$(\text{div} \vec{v}) = 0 \quad (2.4.6)$$

$${\text{div}} \vec{D} = \rho_{el} \quad (2.4.1)$$

From the assumption that there is an electric field density, Gauss' law follows directly from equation 2.4.1. Under the conditions of equation 2.4.5 and 2.4.6, equation 2.3.5 can now be simplified to equation 2.4.16.

$$\text{rot} \vec{H} = -(\text{grad} \vec{v}) \vec{D} + (\text{grad} \vec{D})\vec{v} - \vec{v} \cdot \text{div} \vec{D} + \vec{D} \cdot \text{div} \vec{v} \quad (2.3.5)$$

$$\text{rot} \vec{H} = -0 \cdot \vec{D} + (\text{grad} \vec{D})\vec{v} - \vec{v} \cdot \text{div} \vec{D} + \vec{D} \cdot 0 \quad (2.4.16)$$

If the terms that make no contribution to the overall result from equation 2.4.16 are eliminated, the overall expression from equation 2.4.16 can be further simplified. The result is equation 2.4.17.
rot \vec{H} = (\text{grad} \vec{D}) \vec{v} - \vec{v} \ast \text{div} \vec{D} \hspace{1cm} (2.4.17)

The term \((\text{grad} \vec{D}) \vec{v}\), from equation 2.4.17, can be rewritten in the form of equation 2.4.18.

\[
(\text{grad} \vec{D}) \cdot (\vec{v}) = \begin{vmatrix}
\frac{\delta D_x}{\delta x} & \frac{\delta D_x}{\delta y} & \frac{\delta D_x}{\delta z} \\
\frac{\delta D_y}{\delta x} & \frac{\delta D_y}{\delta y} & \frac{\delta D_y}{\delta z} \\
\frac{\delta D_z}{\delta x} & \frac{\delta D_z}{\delta y} & \frac{\delta D_z}{\delta z}
\end{vmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}
\] \hspace{1cm} (2.4.18)

If now, in equation 2.4.18, the velocity vector \(\vec{v}\) is multiplied by \((\text{grad} \vec{D})\), equation 2.4.19 results.

\[
(\text{grad}(\vec{D})) \cdot \vec{v} = \begin{vmatrix}
\frac{\delta D_x}{\delta x} \cdot v_x + \frac{\delta D_x}{\delta y} \cdot v_y + \frac{\delta D_x}{\delta z} \cdot v_z \\
\frac{\delta D_y}{\delta x} \cdot v_x + \frac{\delta D_y}{\delta y} \cdot v_y + \frac{\delta D_y}{\delta z} \cdot v_z \\
\frac{\delta D_z}{\delta x} \cdot v_x + \frac{\delta D_z}{\delta y} \cdot v_y + \frac{\delta D_z}{\delta z} \cdot v_z
\end{vmatrix} = \vec{x}_{(\text{grad} \vec{D}) \vec{v}}
\] \hspace{1cm} (2.4.19)

The velocity vector \(\vec{v}\) can, according to equation 2.4.11, be rewritten in \(\frac{\delta \vec{s}}{\delta t}\). This fact results in equation 2.4.20 from equation 2.4.19.

\[
\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} \frac{\delta x}{\delta t} \\ \frac{\delta y}{\delta t} \\ \frac{\delta z}{\delta t} \end{pmatrix}
\] \hspace{1cm} (2.4.11)
\[
(\nabla (\vec{D})) \cdot \vec{v} = \begin{pmatrix}
\frac{\delta D_x}{\delta t} \cdot \frac{\delta x}{\delta t} + \frac{\delta D_y}{\delta t} \cdot \frac{\delta y}{\delta t} + \frac{\delta D_z}{\delta t} \cdot \frac{\delta z}{\delta t} \\
\frac{\delta D_x}{\delta t} \cdot \frac{\delta x}{\delta t} + \frac{\delta D_y}{\delta t} \cdot \frac{\delta y}{\delta t} + \frac{\delta D_z}{\delta t} \cdot \frac{\delta z}{\delta t} \\
\frac{\delta D_x}{\delta t} \cdot \frac{\delta x}{\delta t} + \frac{\delta D_y}{\delta t} \cdot \frac{\delta y}{\delta t} + \frac{\delta D_z}{\delta t} \cdot \frac{\delta z}{\delta t}
\end{pmatrix}
\] (2.4.20)

Assuming that the electric field effect only changes in the respective effective direction, i.e. a distortion-free, electric flux density field is assumed, the expression from equation 2.4.20 changes to equation 2.4.21.

\[
(\nabla (\vec{D})) \cdot \vec{v} = \begin{pmatrix}
\frac{\delta D_x}{\delta t} \cdot \frac{\delta x}{\delta t} + 0 + 0 \\
0 + \frac{\delta D_y}{\delta t} \cdot \frac{\delta y}{\delta t} + 0 \\
0 + 0 + \frac{\delta D_z}{\delta t} \cdot \frac{\delta z}{\delta t}
\end{pmatrix}
\] (2.4.21)

The components \(\delta x\), \(\delta y\) and \(\delta z\) from equation 2.4.21 can now be reduced and equation 2.4.22 is formed.

\[
(\nabla (\vec{D})) \cdot \vec{v} = \begin{pmatrix}
\frac{\delta D_x}{\delta t} \\
\frac{\delta D_y}{\delta t} \\
\frac{\delta D_z}{\delta t}
\end{pmatrix} = \frac{\delta \vec{D}}{\delta t}
\] (2.4.22)

Equation 2.4.22 depicts part of the law of flow and can later be used in equation 2.4.4.

Flooding law:
\[
\text{rot} \vec{H} = \frac{\delta \vec{D}}{\delta t} + \vec{j}
\] (2.4.4)

If the relationships from equations 2.4.1 and 2.4.22 are now inserted into equation 2.4.17, equation 2.4.23 results.
\[ \text{div} \vec{D} = \rho_{el} \]  
\[ (\text{grad}(\vec{D})) \cdot \vec{v} = \frac{\delta D_x}{\delta t} = \frac{\delta \vec{D}}{\delta t} \]  
\[ \text{rot} \vec{H} = (\text{grad} \vec{D}) \vec{v} - \vec{v} \ast \text{div} \vec{D} \]  
\[ \text{rot} \vec{H} = (\text{grad} \vec{D}) \vec{v} - \vec{v} \ast \text{div} \vec{D} = \frac{\delta \vec{D}}{\delta t} - \vec{v} \ast \rho_{el} \]  
\[ \vec{j} = -\vec{v} \ast \rho_{el} \]  
\[ \text{rot} \vec{H} = \frac{\delta \vec{D}}{\delta t} + \vec{j} \]

2.5 THE REINTERPRETATION OF THE “MAXWELL EQUATIONS”

In order to be able to reinterpret the “Maxwell equations”, the framework conditions for them are first redefined. The first general condition is that it cannot be ruled out that both the vector field of the velocity and the two vector fields of the magnetic flux density and the electrical flux density can be subject to deformation. Accordingly, the velocity gradient \( \text{grad}(\vec{v}) \), cannot be equated with 0. In addition, the two field gradients \( \text{grad}(\vec{B}) \) and \( \text{grad}(\vec{D}) \) cannot be simplified, as in Chapters 2.4.3 and 2.4.4. All three the \( \text{div}(\vec{v}) \) and the \( \text{div}(\vec{B}) \) and the \( \text{div}(\vec{D}) \) are dependent on the trace (Sp) of the respective gradient.
From a mathematical point of view, these framework conditions result in equations 2.5.1, 2.5.2 and 2.5.3. Accordingly, the starting point for the reinterpretation of the “Maxwell equations” is equations 2.3.3 and 2.3.5.

\[ \text{rot} \vec{E} = (\text{grad} \vec{v}) \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div } \vec{B} - \vec{B} \text{ div } \vec{v} \] (2.3.3)

\[ \text{rot} \vec{H} = -(\text{grad} \vec{v}) \vec{D} + (\text{grad} \vec{D}) \vec{v} - \vec{v} \text{ div } \vec{D} + \vec{D} \text{ div } \vec{v} \] (2.3.5)

\[ (\text{Sp})(\text{grad} \vec{v}) = \text{div} (\vec{v}) \] (2.5.1)

\[ (\text{Sp})(\text{grad} \vec{B}) = \text{div} (\vec{B}) \] (2.5.2)

\[ (\text{Sp})(\text{grad} \vec{D}) = \text{div} (\vec{D}) \] (2.5.3)

When substances are deformed, the velocity gradient \( \text{grad} (\vec{v}) \) contributes to the overall result of equations 2.3.3 and 2.3.5 in the form shown in equation 2.5.4.

\[
(\text{grad} \vec{v}) = \begin{pmatrix}
\frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\
\frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\
\frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z}
\end{pmatrix}
\] (2.5.4)

Both in equation 2.3.3 and in equation 2.3.5, the velocity gradient is multiplied by the respective field size vector. For equation 2.3.3 this is \( \vec{B} \) and for equation 2.3.5 this is \( \vec{D} \). For the second term from equation 2.3.3, equation 2.5.5 can therefore be written. Similarly, for the second term from equation 2.3.5, equation 2.5.6 can be written.

\[ \text{rot} \vec{E} = (\text{grad} \vec{v}) \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div } \vec{B} - \vec{B} \text{ div } \vec{v} \] (2.3.3)
\[
\n(\text{grad} \vec{v}) \cdot (\vec{B}) = \begin{vmatrix}
\frac{\delta v_x}{\delta x} & \frac{\delta v_x}{\delta y} & \frac{\delta v_x}{\delta z} \\
\frac{\delta v_y}{\delta x} & \frac{\delta v_y}{\delta y} & \frac{\delta v_y}{\delta z} \\
\frac{\delta v_z}{\delta x} & \frac{\delta v_z}{\delta y} & \frac{\delta v_z}{\delta z}
\end{vmatrix} \cdot \begin{pmatrix}
B_x \\
B_y \\
B_z
\end{pmatrix}
\]

(2.5.5)

\[
\text{rot} \vec{H} = -(\text{grad} \vec{v}) \cdot \vec{D} + (\text{grad} \vec{D}) \cdot \vec{v} - \vec{v} \cdot \text{div} \vec{D} + \vec{D} \cdot \text{div} \vec{v}
\]

(2.3.5)

\[
(\text{grad} \vec{v}) \cdot (\vec{D}) = \begin{vmatrix}
\frac{\delta v_x}{\delta x} & \frac{\delta v_x}{\delta y} & \frac{\delta v_x}{\delta z} \\
\frac{\delta v_y}{\delta x} & \frac{\delta v_y}{\delta y} & \frac{\delta v_y}{\delta z} \\
\frac{\delta v_z}{\delta x} & \frac{\delta v_z}{\delta y} & \frac{\delta v_z}{\delta z}
\end{vmatrix} \cdot \begin{pmatrix}
D_x \\
D_y \\
D_z
\end{pmatrix}
\]

(2.5.6)

If the velocity gradient is now multiplied by the respective field vector, the expression from equation 2.5.7 results from equation 2.5.5 and equation 2.5.8 results for equation 2.5.6.

\[
(\text{grad} \vec{v}) \cdot (\vec{B}) = \begin{vmatrix}
\frac{\delta v_x}{\delta x} \cdot \vec{B}_x + \frac{\delta v_x}{\delta y} \cdot \vec{B}_y + \frac{\delta v_x}{\delta z} \cdot \vec{B}_z \\
\frac{\delta v_y}{\delta x} \cdot \vec{B}_x + \frac{\delta v_y}{\delta y} \cdot \vec{B}_y + \frac{\delta v_y}{\delta z} \cdot \vec{B}_z \\
\frac{\delta v_z}{\delta x} \cdot \vec{B}_x + \frac{\delta v_z}{\delta y} \cdot \vec{B}_y + \frac{\delta v_z}{\delta z} \cdot \vec{B}_z
\end{vmatrix} = \vec{x}_{(\text{grad} \vec{v})} \vec{B}
\]

(2.5.7)

\[
(\text{grad} \vec{v}) \cdot (\vec{D}) = \begin{vmatrix}
\frac{\delta v_x}{\delta x} \cdot \vec{D}_x + \frac{\delta v_x}{\delta y} \cdot \vec{D}_y + \frac{\delta v_x}{\delta z} \cdot \vec{D}_z \\
\frac{\delta v_y}{\delta x} \cdot \vec{D}_x + \frac{\delta v_y}{\delta y} \cdot \vec{D}_y + \frac{\delta v_y}{\delta z} \cdot \vec{D}_z \\
\frac{\delta v_z}{\delta x} \cdot \vec{D}_x + \frac{\delta v_z}{\delta y} \cdot \vec{D}_y + \frac{\delta v_z}{\delta z} \cdot \vec{D}_z
\end{vmatrix} = \vec{x}_{(\text{grad} \vec{v})} \vec{D}
\]

(2.5.8)

Under the assumption from equation 2.5.1, equation 2.5.8 yields a statement about the divergence of the velocity vector. This results in equation 2.5.9.

\[
(\text{Sp})(\text{grad} \vec{v}) = \text{div} \vec{v}
\]

(2.5.1)
If equation 2.5.9 is now multiplied by the respective field vector $\vec{B}$ or $\vec{D}$, equation 2.5.10 results for the fifth term from equation 2.3.3 and equation 2.5.11 results for the fifth term from equation 2.3.5.

$$\text{rot} \vec{E} = (\text{grad} \vec{v}) \cdot \vec{B} - (\text{grad} \vec{B}) \cdot \vec{v} + \vec{v} \cdot \text{div} \vec{B} - \vec{B} \cdot \text{div} \vec{v} \quad (2.3.3)$$

$$\vec{B} \cdot \text{div} (\vec{v}) = \begin{bmatrix} B_x(\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z}) \\ B_y(\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z}) \\ B_z(\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z}) \end{bmatrix} = \vec{x}_{B \cdot \text{div} \vec{v}} \quad (2.5.10)$$

$$\text{rot} \vec{H} = -(\text{grad} \vec{v}) \cdot \vec{D} + (\text{grad} \vec{D}) \cdot \vec{v} - \vec{v} \cdot \text{div} \vec{D} + \vec{D} \cdot \text{div} \vec{v} \quad (2.3.5)$$

$$\vec{D} \cdot \text{div} (\vec{v}) = \begin{bmatrix} D_x(\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z}) \\ D_y(\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z}) \\ D_z(\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z}) \end{bmatrix} = \vec{x}_{D \cdot \text{div} \vec{v}} \quad (2.5.11)$$

The electrical field density results from the mathematical prediction from equation 2.5.3. This relationship is shown in equation 2.5.12.

$$\text{Sp}(\text{grad} \vec{D}) = \text{div}(\vec{D}) \quad (2.5.3)$$

$$\text{Sp}(\text{grad} \vec{D}) = \text{div}(\vec{D}) = \frac{\delta D_x}{\delta x} + \frac{\delta D_y}{\delta y} + \frac{\delta D_z}{\delta z} \quad (2.5.12)$$

In order to get the fourth term from equation 2.3.5, the expression from equation 2.5.12 must now be multiplied by the velocity vector. The result is the electric current density $\vec{j}$. This fact is shown in equation 2.5.13.
\[ \text{rot} \vec{H} = - (\text{grad} \vec{\nu}) \hat{D} + (\text{grad} \hat{D}) \vec{\nu} - \vec{\nu} \text{ div} \hat{D} + \hat{D} \text{ div} \vec{\nu} \] (2.3.5)

\[
\vec{\nu} \text{ div}(\hat{D}) = \begin{bmatrix}
\frac{\delta D_x}{\delta x} + \frac{\delta D_z}{\delta y} & \frac{\delta D_x}{\delta y} + \frac{\delta D_z}{\delta z} & \frac{\delta D_x}{\delta z} + \frac{\delta D_z}{\delta x}
\end{bmatrix} = \vec{j}_{el} \quad (2.5.13)
\]

### 2.5.1 THE MAGNETIC FIELD DENSITY

From the mathematical requirement from equation 2.5.14 it follows that the divergence of the magnetic flux density, \( \text{div} \vec{B} \), which is directly related to the gradient of the magnetic flux density, \( \text{grad} \vec{B} \). The sum of the diagonals of the \( \text{grad} \vec{B} \), i.e. the trace (Sp) of the magnetic flux density gradient \( (\text{Sp})(\text{grad} \vec{B}) \), forms the \( \text{div} \vec{B} \). This applies to the matrix elements \( \frac{\delta B_x}{\delta x} \), \( \frac{\delta B_y}{\delta y} \) and \( \frac{\delta B_z}{\delta z} \). According to the “Maxwell equations”, the sum of these three elements must result in 0. However, since these three elements are an important part of equation 2.5.15, the following problem arises. Either \( \frac{\delta \vec{B}}{\delta t} \) or the sum of the individual elements from \( \frac{\delta \vec{B}}{\delta t} \) must be equated with 0. This is a contradiction to the law of induction.

\[
(\text{Sp})(\text{grad} \vec{B}) = \text{div}(\vec{B}) = \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} = 0 \quad (2.5.14)
\]

\[
\begin{bmatrix}
\frac{\delta B_x}{\delta x} & \frac{\delta B_x}{\delta t} \\
\frac{\delta B_y}{\delta y} & \frac{\delta B_y}{\delta t} \\
\frac{\delta B_z}{\delta z} & \frac{\delta B_z}{\delta t}
\end{bmatrix} = \frac{\delta \vec{B}}{\delta t} \quad (2.5.15)
\]

This results directly in one of the mathematical requirements from equations 2.5.16, 2.5.17, 2.5.18 or 2.5.19.
Either \( \frac{\delta \vec{B}}{\delta t} \) is equated with 0 or in the case of the theoretical movement of a point particle through a magnetic flux density, there is, in three-dimensional space, a dimensional direction of movement in which the flux density changes positively and two dimensional directions of movement, which add up to a negative one describe the change in the magnetic flux density. However, the condition for this is that the sum of all three magnetic flux density changes in the three possible dimensional directions of movement results in a 0. The resulting idea of the magnetic flux density and, ultimately, the idea of a magnetic field, does not coincide with the idea of the magnetic field in current physics.

The solution to this problem results from an approach by Paul Dirac that there is a magnetic field density. The calculation of this magnetic field density is shown in equation 2.5.20.

\[
\vec{v} \text{ div} (\vec{B}) = \begin{pmatrix}
\frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} \\
\frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} \\
\frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z}
\end{pmatrix} = \vec{j}_m
\]

**2.5.2 REFORMULATION OF THE “MAXWELL EQUATIONS”**

First, the equations 2.3.3 and 2.3.5 are written down again, since these two equations represent the fundamental statements for the reformulation of the “Maxwell equations”.

\[
\text{rot} \vec{E} = (\text{grad} \vec{v}) \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div} \vec{B} - \vec{B} \text{ div} \vec{v}
\]

(2.3.3)
The mathematical requirement from Chapter 2.5.1 that there is a magnetic field density.

Now the equations 2.4.10, 2.4.19, 2.5.7, 2.5.8, 2.5.10, 2.5.11, 2.5.13 and 2.5.20 are again written below one another for better clarity. The reason for this is that these equations are now used as individual components in equations 2.3.3 and 2.3.5. This set of equations has general validity, since it also offers an application possibility under the prerequisites that both the velocity vector field and the two vector fields of the magnetic flux density and the electric flux density can be subject to a deformation. In addition, equation 2.5.20 fulfills the mathematical requirement from Chapter 2.5.1 that there is a magnetic field density.

\[
\begin{align*}
\text{rot } \vec{H} &= - (\nabla \vec{v}) \cdot \vec{D} + (\nabla \vec{D}) \vec{v} - \vec{v} \cdot \nabla \vec{D} + \vec{D} \cdot \nabla \vec{v} & \quad (2.3.5) \\
\end{align*}
\]

Now the equations 2.4.10, 2.4.19, 2.5.7, 2.5.8, 2.5.10, 2.5.11, 2.5.13 and 2.5.20 are again written below one another for better clarity. The reason for this is that these equations are now used as individual components in equations 2.3.3 and 2.3.5. This set of equations has general validity, since it also offers an application possibility under the prerequisites that both the velocity vector field and the two vector fields of the magnetic flux density and the electric flux density can be subject to a deformation. In addition, equation 2.5.20 fulfills the mathematical requirement from Chapter 2.5.1 that there is a magnetic field density.
\[ \begin{align*}
\vec{B} \text{ div} (\vec{v}) &= B_x \frac{\delta v_x}{\delta x} + B_y \frac{\delta v_y}{\delta y} + B_z \frac{\delta v_z}{\delta z} \\
\vec{D} \text{ div} (\vec{v}) &= D_x \frac{\delta v_x}{\delta x} + D_y \frac{\delta v_y}{\delta y} + D_z \frac{\delta v_z}{\delta z}
\end{align*} \]  
(2.5.10)

\[ \begin{align*}
\vec{v} \text{ div} (\vec{D}) &= v_x \frac{\delta D_x}{\delta x} + v_y \frac{\delta D_y}{\delta y} + v_z \frac{\delta D_z}{\delta z} \\
\vec{v} \text{ div} (\vec{B}) &= v_x \frac{\delta B_x}{\delta x} + v_y \frac{\delta B_y}{\delta y} + v_z \frac{\delta B_z}{\delta z}
\end{align*} \]  
(2.5.11)

The equations 2.4.10, 2.4.19, 2.5.7, 2.5.8, 2.5.10, 2.5.11, 2.5.13 and 2.5.20 are now inserted into the equations 2.3.3 and 2.3.5. The result is equations 2.5.21 and 2.5.22. Another result is shown by equations 2.5.23 and 2.5.24.
The equations 2.5.21, 2.5.22, 2.5.23 and 2.5.24 therefore represent the simplified reformulation of the “Maxwell equations”. Equation 2.5.24 is the mathematical-physical expression, a magnetic field density.

3. DISCUSSION

1. It remains to be discussed whether the expression from equation 2.4.2, \( \text{div}(\vec{B}) = 0 \), is mathematically permissible, since the mathematical requirement from equation 2.5.2, \((\text{Sp})(\text{grad} \vec{B}) = \text{div}(\vec{B})\) consists. And if \( \text{div}(\vec{B}) = 0 \) is allowed, what does this mean for equation 2.5.14?

\[
\begin{align*}
(\text{Sp})(\text{grad} \vec{B}) &= \text{div}(\vec{B}) = \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} = 0 \\
(\text{Sp})(\text{grad} \vec{B}) &= \text{div}(\vec{B}) = \frac{\delta B_x}{\delta x} \cdot v_x + \frac{\delta B_y}{\delta y} \cdot v_y + \frac{\delta B_z}{\delta z} \cdot v_z
\end{align*}
\] (2.5.14)

2. What effects would a possible distortion of the velocity vector field \( \vec{v} \) have on the velocity gradient \( \text{grad} \vec{v} \)?

3. What effects would a possible distortion of the two flux density vector fields, the magnetic flux density and the electrical flux density, on whose two field gradients \( \text{grad} \vec{B} \) and \( \text{grad} \vec{D} \) have?

4. What effects do questions 1 to 3 have on equations 2.4.10, 2.4.19, 2.5.7 and 2.5.8?

\[
-(\text{grad}(\vec{B})) \cdot \vec{v} = - \left[ \frac{\delta B_x}{\delta x} \cdot v_x + \frac{\delta B_y}{\delta y} \cdot v_y + \frac{\delta B_z}{\delta z} \cdot v_z \right] = x_{(\text{grad} \vec{B})\vec{v}}
\] (2.4.10)
\[
(\nabla \vec{D}) \cdot \vec{v} = \begin{bmatrix}
\frac{\delta D_x}{\delta x} \cdot v_x + \frac{\delta D_y}{\delta y} \cdot v_y + \frac{\delta D_z}{\delta z} \cdot v_z \\
\frac{\delta D_x}{\delta x} \cdot v_x + \frac{\delta D_y}{\delta y} \cdot v_y + \frac{\delta D_z}{\delta z} \cdot v_z \\
\frac{\delta D_x}{\delta x} \cdot v_x + \frac{\delta D_y}{\delta y} \cdot v_y + \frac{\delta D_z}{\delta z} \cdot v_z
\end{bmatrix} = x_{(\nabla \vec{D})\vec{v}} \quad (2.4.19)
\]

\[
(\nabla \vec{v}) \cdot (\vec{B}) = \begin{bmatrix}
\frac{\delta v_x}{\delta x} \cdot \vec{B}_x + \frac{\delta v_y}{\delta y} \cdot \vec{B}_y + \frac{\delta v_z}{\delta z} \cdot \vec{B}_z \\
\frac{\delta v_x}{\delta x} \cdot \vec{B}_x + \frac{\delta v_y}{\delta y} \cdot \vec{B}_y + \frac{\delta v_z}{\delta z} \cdot \vec{B}_z \\
\frac{\delta v_x}{\delta x} \cdot \vec{B}_x + \frac{\delta v_y}{\delta y} \cdot \vec{B}_y + \frac{\delta v_z}{\delta z} \cdot \vec{B}_z
\end{bmatrix} = x_{(\nabla \vec{v})\vec{B}} \quad (2.5.7)
\]

\[
(\nabla \vec{v}) \cdot (\vec{D}) = \begin{bmatrix}
\frac{\delta v_x}{\delta x} \cdot \vec{D}_x + \frac{\delta v_y}{\delta y} \cdot \vec{D}_y + \frac{\delta v_z}{\delta z} \cdot \vec{D}_z \\
\frac{\delta v_x}{\delta x} \cdot \vec{D}_x + \frac{\delta v_y}{\delta y} \cdot \vec{D}_y + \frac{\delta v_z}{\delta z} \cdot \vec{D}_z \\
\frac{\delta v_x}{\delta x} \cdot \vec{D}_x + \frac{\delta v_y}{\delta y} \cdot \vec{D}_y + \frac{\delta v_z}{\delta z} \cdot \vec{D}_z
\end{bmatrix} = x_{(\nabla \vec{v})\vec{D}} \quad (2.5.8)
\]

5. What is the effect of equation 2.5.24 on the electromagnetic wave equation?

\[
\vec{v} \cdot \nabla (\vec{B}) = j_m \quad (2.5.24)
\]

6. Under what circumstances is the velocity vector field and the two vector fields, the magnetic flux density and the electrical flux density, deformed?

4. CONCLUSION

Under the mathematical requirement from equation 2.5.2, \( (\text{Sp}) (\nabla \vec{B}) = \nabla (\vec{B}) \), the physical requirement from equation 2.4.2, \( \nabla (\vec{B}) = 0 \), is only valid provided that \( (\text{Sp}) (\nabla \vec{B}) = 0 \). This means that either the physical conception of the magnetic field has to be reinterpreted or the assumption from equation 2.4.2 that \( \nabla (\vec{B}) = 0 \) is wrong.

By reinterpreting the “Maxwell equations” from equations 2.5.21, 2.5.22, 2.5.23 and 2.5.24, a mathematically and physically consistent approach for the calculation of electric and magnet-
ic fields was achieved. In addition, the distortions of the field quantities used in the equations were taken into account in these equations. A direct analogy between electric and magnetic fields was also derived mathematically. This analogy leads to the fact that the magnetic field density becomes a mathematical requirement when the \((\text{Sp})(\nabla \vec{B}) \neq 0\). It remains to be discussed under what circumstances this does not happen. It also remains to be discussed what influence the equations 2.5.21, 2.5.22, 2.5.23 and 2.5.24 have on other equations that are based on the “Maxwell equations” and which technical possibilities result from them.

5. CONFLICTS OF INTEREST

The author(s) declares that there is no conflict of interest relating to the publication of this article.

6. PROOF OF FINANCING

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