# The application of mean curvature flow into cosmology

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#### Abstract

In this paper, I used the application of something mathematically into physics. A mean curvature flow is a good candidate for the curvature of our universe; Therefore, I exerted Wilmore energy as dark energy and Hawking mass as dark matter. I also found out that there was space-time before the Big Bang. Moreover, I derived new Friedmann equations by adding Wilmore energy into the Einstein-Hilbert action to describe the evolution of our universe. Another discovery in this study was the unveiling of the fine-tuning problem.

#### 1 Introduction

It was once thought that time was an absolute quantity, and that gravity was only the force between two masses. However, this assumption was rejected by a genius named Albert Einstein. He realized that time and space were not independently absolute, and that gravitational force was the force associated with space-time. In other words, his field equation explained the relationship between the curvature of space-time and matter. This means that gravity refers to the curvature of space-time that causes the motion of planets, stars, and galaxies in our universe[12].

The exact solution of the Einstein field equation was proposed by Friedmann-Lemaitre-Robertson-Walker, called the FLRW model. This accelerated model and expansion of the universe generate a cosmological constant model. This model predicts the acceleration of our universe and is compatible with observations. However, there are some problems with this model; the most important problem is fine-tuning[1]. Fine-tuning is the large difference between the relativistic quantum mechanics and the cosmological constant model for calculating the vacuum density. In this article, I solved this problem by

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noting that dark energy has a geometric origin. This is not only a hypothesis, and I proved it in this article. To this end, I used the scalar curvature definition, the continuity equation, and some creativity in mathematical equations[2]. Then, I realized that the pressure to density ratio of the whole universe is equal to  $-\frac{2}{3}orw_{total}=-\frac{2}{3}=w_{\rm radiation}+w_{DE}+w_{DM}=\frac{1}{3}-1+0$  this data( $-\frac{2}{3}$ ) is compatible with observations in the cosmological constant model [2], and we can understand that dark energy has a geometrical origin. In my model, I used Wilmore energy as dark energy to solve the fine-tuning problem, and I added Wilmore energy instead of the cosmological constant into the Einstein-Hilbert action. The geometry of relativistic quantum mechanics is Minkowski's geometry [15]. In this geometry, there is no Wilmore energy because the space-time radius is constant (the space-time radius is the scale factor). My model can predict that before the Big Bang, the vacuum density was extremely high when the universe was in an infancy. Thus, we can conclude that relativistic quantum mechanics can only measure the vacuum density before the Big Bang. The reason for this result is that the radius in Minokowski's space-time is constant. My model can predict that the vacuum density is extremely low in the current time. In my model, I used the mean curvature flow to define the radius of the universe. In this model, the universe was initially shrinking. Before the Big Bang, the universe shrank to a point where the radius of the universe reached zero. Then, the Big Bang occurred and the universe was expanded. In my model, dark matter was made after the Big Bang, while ordinary matter was made before the Big Bang. In this model, dark matter is an imaginary number, while ordinary matter is a positive number. I used the Hawking mass formula to define dark matter and ordinary matter. Moreover, in my model, the dark matter mass and the radius of the universe after the Big Bang are imaginary quantities. My model can predict the age of the universe, which is slightly different from the cosmological constant theory. After applying Wilmore energy into the Einstein-Hilbert action [13], we can predict the density of our universe for each cosmic time. With this approach, quantum gravity is born. This model functions better than other models to expand our universe such as F(R) gravity and F(R,T) gravity [9],[10]. The reason is that this model has more universality and has simpler scalar curvature than all the modified theories. Definition of Wilmore energy in the Wilmore conjecture book: "Wilmore energy is a type of energy that is studied in differential geometry, physics, and mechanics. It was first introduced by Sim'eon-Denis Poisson and Marie-Sophie Germain, independently, at the beginning of the nineteenth century, but its complete formalism was due to Thomas Wilmore. In essence, this type of energy quantitatively measures the deviation of a surface from local sphericity. It is important to note that there is some ambiguity in the literature between the terms bending energy and Wilmore energy, and different sources provide different definitions." . [3]

## 2 Step 1: Finding pressure to the density ratio of the whole universe with a mathematical method

For this target, I use the scalar curvature definition and the continuity equation, and I assume that the pressure to density ratio of the whole universe is constant. Since this assumption is more successful than those proposed by the other models, the model is called the cosmological constant model . [4],[5]

$$w = \frac{P}{\rho},\tag{1}$$

 $P = P_{Dark matter} + P_{Dark energy} + P_{radiation}$ 

 $\rho = \rho_{Dark\text{matter}} + \rho_{Dark\text{energy}} + \rho_{\text{radiation}}$ 

 $w = w_{Dark\text{matter}} + w_{Dark\text{energy}} + w_{\text{radiation}}$ 

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{\alpha}}{\alpha}(w+1). \tag{2}$$

The stress-energy tensor in the FLRW model is defined by [5]:

$$\mathcal{T} = 3P - \rho = \rho(3w - 1) \to \rho = \frac{\mathcal{T}}{(3w - 1)}.$$
 (3)

density is defined by [4]:

$$\rho = \rho_0 \alpha^{-3(w+1)}.\tag{4}$$

$$\rho_{DM0}\alpha^{-3(w_{DM}+1)} + \rho_{DE0}\alpha^{-3(w_{DE}+1)} + \rho_{RA0}\alpha^{-3(w_{RA}+1)} = \rho_0\alpha^{-3(w+1)}$$

I apply (3) into (4), so scale factor is defined by: I apply (3) into (4), and thus, the scale factor is defined by:

$$\alpha = \left(\frac{\mathcal{T}}{\rho_0(3w-1)}\right)^{\frac{1}{-3(w+1)}}.$$
 (5)

Thus, the derivative of the scale factor is equal to:

$$\frac{d\alpha}{dt} = \dot{\alpha} = \frac{(3w-1)\dot{\rho}}{-3(w+1)} \left(\frac{\mathcal{T}}{\rho_0(3w-1)}\right)^{\frac{3w+4}{-3(w+1)}}.$$
 (6)

Then, I apply (6), (5) into (2).

$$\frac{1}{\rho} = (3w - 1)\left(\frac{\mathcal{T}}{\rho_0(3w - 1)}\right)^{\frac{-3(w+1)}{3(w+1)}},\tag{7}$$

$$\rho = \left(\frac{\rho(3w-1)}{\rho_0(3w-1)^2}\right) \to \rho_0 = \frac{1}{3w-1}.$$
 (8)

Thus, density and pressure as well as the scalar stress-energy tensor s of the whole universe can be obtained as follows:

$$\rho = \rho_0 \alpha^{-3(w+1)} = \frac{\alpha^{-3(w+1)}}{3w - 1},\tag{9}$$

$$P = w\rho_0 \alpha^{-3(w+1)} = \frac{w\alpha^{-3(w+1)}}{3w - 1},\tag{10}$$

$$\mathcal{T} = \frac{3w\alpha^{-3(w+1)}}{3w-1} - \frac{\alpha^{-3(w+1)}}{3w-1} = \alpha^{-3(w+1)},\tag{11}$$

$$(\rho(3w-1))^{\frac{1}{-3(w+1)}} = \alpha. \tag{12}$$

Now, we can obtain the pressure to density ratio of the whole universe using the scalar curvature definition in the Robertson-Walker metric [4] and the equation (12).

$$\mathcal{R} = -6\left(\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} + \frac{k}{\alpha^2}\right). \tag{13}$$

Thus, derivatives of the scale factor based on the equation (12) are equal to:

$$\dot{\alpha}^2 = \frac{(\rho(3w-1))^{\frac{(6w+8)}{-3(w+1)}}}{9(w+1)^2}\dot{\rho}^2,\tag{14}$$

$$\ddot{\alpha} = \frac{(3w+4)}{9(w+1)^2} (\rho(3w-1))^{\frac{6w+7}{-3(w+1)}} \dot{\rho}^2 + \frac{1}{-3(w+1)} (\rho(3w-1))^{\frac{(3w+4)}{-3(w+1)}} \ddot{\rho}.$$

Thus, the scalar curvature based on the equation (13) is equal to:

$$\mathcal{R} = -6\left(\frac{(3w+4)}{9(w+1)^2}(\rho(3w-1))^{-2}\dot{\rho}^2 + \frac{1}{-3(w+1)}(\rho(3w-1))^{-1}\ddot{\rho} + \frac{(\rho(3w-1))^{-2}\dot{\rho}^2}{9(w+1)^2} + k(\rho(3w-1))^{\frac{2}{(w+1)}}.$$
(16)

Now, we should replace  $\dot{\rho}$  and  $\ddot{\rho}$  in the equation (16).

$$\rho = \frac{\alpha^{-3(w+1)}}{3w - 1},\tag{17}$$

$$\dot{\rho} = \frac{-1}{3w - 1} 3(w + 1)\alpha^{-3(w+1)-1} \dot{\alpha} \to \dot{\rho}^2 = \frac{9(w + 1)^2 \alpha^{-6(w+1)-2} \dot{\alpha}^2}{(3w - 1)^2},$$

$$\ddot{\rho} = \frac{1}{3w-1} 3(w+1)((3w+4)\alpha^{-3(w+1)-2}\dot{\alpha}^2 - \frac{1}{3w-1} 3(w+1)\alpha^{-3(w+1)-1}\ddot{\alpha}.$$
(19)

Now, I apply (17), (18), and (19) into (16).

$$\mathcal{R} = -6\left(\frac{(3w+4)}{(3w-1)^2}\frac{\dot{\alpha}^2}{\alpha^2} - \frac{1}{3w-1}\left((3w+4)\frac{\dot{\alpha}^2}{\alpha^2} + \frac{1}{3w-1}\frac{\ddot{\alpha}}{\alpha} + \frac{1}{(3w-1)^2}\frac{\dot{\alpha}^2}{\alpha^2} + k\alpha^{-2}\right).$$
(20)

We can consider the equation (20) and the equation (13) as the same equations. Thus, we can make equality

$$-6\left(\left(\frac{3w+4}{(3w-1)^2} - \frac{3w+4}{3w-1} + \frac{1}{(3w-1)^2}\right)\frac{\dot{\alpha}^2}{\alpha^2} + \frac{1}{3w-1}\frac{\ddot{\alpha}}{\alpha} + k\alpha^{-2}\right) = -6\left(\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} + k\alpha^{-2}\right). \tag{21}$$

The coefficient of  $\frac{\dot{\alpha}^2}{\alpha^2}$  is equal to 1, thus:

$$\frac{(3w+4)}{(3w-1)^2} + \frac{1}{(3w-1)^2} - \frac{(3w+4)}{3w-1} = \frac{-9w^2 - 6w + 9}{(3w-1)^2} = 1, \quad (22)$$

$$-9w^2 - 6w + 9 = 9w^2 - 6w + 1, (23)$$

$$-18w^2 = -8, (24)$$

$$w^2 = \frac{8}{18} = \frac{4}{9} \to w = \pm \frac{2}{3}.$$
 (25)

Thus, the pressure to density ratio of the whole universe is obtained. However, we can accept  $w = -\frac{2}{3}$  because it is compatible with observations in the cosmological constant model [2]. I mean,

$$w_{total} = -\frac{2}{3} = w_{radiation} + w_{DE} + w_{DM} = \frac{1}{3} - 1 + 0.$$
 (26)

Thus, the existence of dark energy is not related to any physical interpretation, because I obtained the pressure to density ratio of the whole universe without matter and energy interpretation. Thus, all we have is math and geometry and, as a result, dark energy has a geometrical origin.

# 3 Step 2: Defining the radius of the universe as function of time using the mean curvature flow and finding the exact age of the universe with a mathematical method

What does curvature mean? A mean curvature flow is an example of a geometric flow of hypersurfaces in a Riemannian manifold. Spheres and cylinders are the easiest, and actually, some of the few nontrivial explicitly computable examples of mean curvature flows. Let me consider a sphere of radius  $\alpha(t)$ , which is a scale factor in the Robertson-Walker metric. In a mean curvature flow, the radius changes over time, similar to our universe that is expanding with the scale factor. I believe that our universe works like a mean curvature flow. Thus, I can conclude that our universe is a mean curvature flow. In addition to similarity, we can calculate the age of the universe, which is compatible with observations. The radius of the mean curvature flow is defined [6]:

$$\alpha(t) = \sqrt{\alpha_0^2 - 2nt},\tag{27}$$

where  $\alpha_0^2$  is the initial radius and can be calculated using the wavelength [1] and n is the dimension of surface  $(\sum)(n=2)$ ,

$$\lim_{t \to 0} \alpha(t) = \alpha_0 \tag{28}$$

Value of  $\alpha_0$  is equal to scale factor in the Minkowski metric

$$\alpha_0 = 1. \tag{29}$$

Hubble's law [1] can be written as follows:

$$H = \frac{\dot{\alpha}}{\alpha} = \frac{\frac{-n}{\sqrt{1 - 2nt}}}{\sqrt{1 - 2nt}} = \frac{-n}{1 - 2nt}.$$
 (30)

we should put Hubble constant into equation (30). that is based on observations .Then we can find age of universe . Hubble constant in the current time :  $H_0=1.18\times 10^{-61}$ .[11]

so age universe is equal to: We should put the Hubble constant into the equation (30), which is based on observations. Then, we can find the age of the universe. The Hubble constant in the current time:  $H_0 = 1.18 \times 10^{-61}$  [11]. Thus, the age of the universe is equal to:

$$t = \frac{2 + 1.81 \times 10^{-61}}{4 \times 1.81 \times 10^{-61}} \tag{31}$$

It is compatible with observations. There is a little difference between this model and the cosmological constant model [4]. This result is the prominent reason for using Wilmore energy and Hawking mass for describing dark energy and dark matter because A mean curvature flow is the milestone of this theory.

### 4 Step 3: Interpreting the scale factor or the radius of the universe

$$\alpha(t) = \sqrt{1 - 2nt}. (32)$$

First of all, it shows the existence of space-time before the Big Bang. The reason is that when the radius is equal to zero, time exists:

$$if: \alpha(t) = 0 \to t = \frac{1}{2n}. \tag{33}$$

It means that at the beginning of time until  $t = \frac{1}{2n}$ , our universe had shrunk and then the Big Bang occurred. After the Big Bang, the radius of the universe obtained an imaginary quantity because our universe had no center after the Big Bang. However, our universe had a center before the Big Bang, and the point is  $t = \frac{1}{4n}$ .

# 5 Step 4: Introducing a new candidate for dark matter using the Hawking mass and for dark energy using Wilmore energy

Willmore energy definition. [7]:

$$W(\Sigma) = \frac{1}{4} \int \mathcal{H}^2 d\Sigma. \tag{34}$$

where  $\mathcal{H}$  is the mean curvature. The scalar curvature in this model is equal to:

$$\mathcal{R} = -6\left(\frac{\ddot{\alpha}}{\alpha} + \frac{\dot{\alpha}^2}{\alpha^2} + \frac{k}{\alpha^2}\right) = -6\left(\frac{-n^2}{(1 - 2nt)^2} + \frac{n^2}{(1 - 2nt)^2} + \frac{k}{1 - 2nt}\right) = \frac{-6k}{1 - 2nt},$$
(35)

$$\mathcal{R} = \frac{-6k}{1 - 2nt} = \frac{-6k}{\alpha^2}.\tag{36}$$

As I previously stated, dark energy has a geometrical origin and I use Wilmore energy as dark energy. The reason is that Wilmore energy has an expanding nature [3]. The mean curvature is defined as follows [8]:

$$\mathcal{H} = \frac{n}{\alpha(t)}.\tag{37}$$

n is [8]:

$$n = -\dot{\alpha}(t)\alpha(t) = 2. \tag{38}$$

With applying the Equation (37), Wilmore energy is is obtained as follows:

$$\mathcal{W}(\Sigma) = \frac{1}{4} \int_{\Sigma} \left(\frac{n^2}{\alpha^2(t)}\right) d\Sigma = \frac{1}{4} \int_{\Sigma} \left(\frac{n^2}{\alpha^2(t)} 8\pi \alpha d\alpha,\right)$$
(39)

$$W(\Sigma) = \frac{1}{4} \int_{\Sigma} \frac{8\pi n^2}{\alpha} d\alpha = 2\pi n^2 \ln \alpha.$$
 (40)

The equation (40) is the dark energy equation. The constant of the integral in the equation (40) is zero because when the radius is equal to zero, dark energy is also equal to zero. Moreover, we should obtain the Hawking mass, which is the total mass in our universe defined by [7]:

$$m(\Sigma) = \frac{\Sigma^{\frac{1}{2}}}{(16\pi)^{\frac{3}{2}}} (16\pi - \int_{\Sigma} \mathcal{H}^2 d\Sigma = \frac{\Sigma^{\frac{1}{2}}}{(16\pi)^{\frac{3}{2}}} (16\pi - 4\mathcal{W}(\Sigma)).$$
 (41)

that  $(\Sigma = 4\pi\alpha^2)$  With applying the Equation (32) and (40), Hawking mass is obtained as follows:

$$m(\Sigma) = \frac{1}{2}\alpha - 4\alpha \ln \alpha = \frac{1}{2}\sqrt{(1-4t)} - \sqrt{(1-4t)} \ln \sqrt{(1-4t)}$$
 (42)

As can be observed, the Hawking mass is imaginary in the current time, meaning that the total mass created in the universe is imaginary in the current time. Furthermore, if I consider the early stages of the universe (before the Big Bang), I will obtain positive matter. It means that ordinary matter was created before the Big Bang whereas dark matter was created after the Big Bang.

### 6 Step 5: Unveiling the fine-tuning problem

In the fine-tuning problem, the vacuum density is the main issue. In this model, I should calculate the vacuum density as follows:

$$m_{vacuum}(\Sigma) = 0, (43)$$

$$\frac{\sum^{\frac{1}{2}}}{(16\pi)^{\frac{3}{2}}}(16\pi - 4\mathcal{W}_{vacuum}(\Sigma)) = 0, \tag{44}$$

$$W_{vacuum}(\Sigma) = 4\pi. \tag{45}$$

where V is the four-dimension volume of a ball with the radius  $\alpha$ 

$$\rho_{vacuum} = \frac{\mathcal{W}_{vacuum}(\Sigma)}{V} = \frac{4\pi}{\frac{\pi^2}{2}\alpha^4} = \frac{8}{\pi\alpha^4} = \frac{8}{\pi(1 - 2nt)^2} = \frac{8}{\pi(1 - 4t)^2}.$$
(46)

One will realize that the result of the equation (46) is compatible with observations if the current time is considered; however, if the time before the Big Bang is considered, relativistic quantum mechanics are predicted for the vacuum density. It means that the radius of the universe plays the main role, the geometry of the relativistic quantum mechanics is Minkowski space-time, and the radius is constant. Thus, relativistic quantum mechanics can only predict the vacuum density before the Big Bang.

# 7 Step 6: Adding Wilmore energy to the Einstein-Hilbert action to update the Friedmann equations to describe quantum gravity

$$S = \frac{1}{16\pi} \int \sqrt{-g} (\mathcal{R} + 2k^{-\frac{5}{2}} \mathcal{R}^{\frac{7}{2}} \tau^2 \mathcal{W}(\Sigma)) dx^4 + \int \sqrt{-g} \mathcal{L}_m dx^4.$$
 (47)

 $k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma)$  is dark energy term.

 $\tau$  is proper time in action above. Now I should take variation from action above but first of all I should know How can take variation from Wilmore energy: based on Eq (40) and Eq (36) we have where  $\tau$  is the proper time in the equation (47). Now I should take variation from the equation (47). However, first, I should know how to take variation from Wilmore energy. Based on the equations (40) and (36), we have

$$\mathcal{R} = \frac{-6k}{\alpha^2}, \mathcal{W}(\Sigma) = 2\pi n^2 \ln \alpha. \tag{48}$$

Since taking variation is partial, variation of Wilmore energy is as follows:

$$\delta \mathcal{W}(\Sigma) \approx 0. \tag{49}$$

The dimension of Wilmore energy can be explained from two aspects:

$$k^{-\frac{5}{2}} \mathcal{R}^{\frac{7}{2}} \tau^{2} \mathcal{W}(\Sigma) = \begin{bmatrix} L^{-7} \end{bmatrix} \begin{bmatrix} T^{2} \end{bmatrix} \begin{bmatrix} L^{5} \end{bmatrix} \begin{bmatrix} T^{-2} \end{bmatrix} = \begin{bmatrix} L^{-2} \end{bmatrix}$$
$$.k^{-\frac{5}{2}} \mathcal{R}^{\frac{7}{2}} \tau^{2} \mathcal{W}(\Sigma) = \begin{bmatrix} L^{5} \end{bmatrix} \begin{bmatrix} L^{-7} \end{bmatrix} \begin{bmatrix} T^{2} \end{bmatrix} \begin{bmatrix} T^{-2} \end{bmatrix} = \begin{bmatrix} L^{-2} \end{bmatrix}.$$
 (50)

where the first side k depends on the shape of the universe and the second side k is the spatial curvature.

As we know, the stress-energy tensor is defined by [16]:

$$\mathcal{T}_{\alpha\beta} = \frac{-2}{\sqrt{-g}} \frac{\partial (\sqrt{-g}\mathcal{L}_m)}{\partial g^{\alpha\beta}}.$$
 (51)

with the equation (47), we can take variation from the equation (47):

$$0 = \frac{1}{16\pi} \left( -\frac{1}{2} g_{\alpha\beta} \mathcal{R} + \mathcal{R}_{\alpha\beta} - g_{\alpha\beta} k^{-\frac{5}{2}} \mathcal{R}^{\frac{7}{2}} \tau^2 \mathcal{W}(\Sigma) + \frac{7}{2} \mathcal{R}^{\frac{5}{2}} \mathcal{R}_{\alpha\beta} 2k^{-\frac{5}{2}} \tau^2 \mathcal{W}(\Sigma) \right) - \frac{1}{2} \mathcal{T}_{\alpha\beta}.$$

$$(52)$$

Here, the equation changes to the new Einstein field equation:

$$(1 + 7\mathcal{R}^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^{2}\mathcal{W}(\Sigma))\mathcal{R}_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}\mathcal{R} - g_{\alpha\beta}k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^{2}\mathcal{W}(\Sigma) = 8\pi\mathcal{T}_{\alpha\beta}.$$
(53)

The interpretation of this equation is that gravity is a reaction to the expansion of the universe. Thus, Wilmore energy causes mass and energy bends space-time.

$$\gamma(\mathcal{R}) = 1 + 7\mathcal{R}^{\frac{5}{2}} k^{-\frac{5}{2}} \tau^{2} \mathcal{W}(\Sigma) 
\cdot \gamma(\mathcal{R}) \mathcal{R}_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} \mathcal{R} - g_{\alpha\beta} k^{-\frac{5}{2}} \mathcal{R}^{\frac{7}{2}} \tau^{2} \mathcal{W}(\Sigma) = 8\pi \mathcal{T}_{\alpha\beta}.$$
(54)

The new Einstein field equation has quantum gravity properties. One of the clues is the coefficient of  $\mathcal{R}_{\alpha\beta}$  and another is Wilmore energy. This equation is true, even when our universe was extremely small and present time becasue of background level.

Now, we can achieve new Friedmann equations:

$$(1 + 7\mathcal{R}^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^{2}\mathcal{W}(\Sigma))\mathcal{R}_{00} - \frac{1}{2}g_{00}\mathcal{R} - g_{00}k^{-\frac{5}{2}}\mathcal{R}^{\frac{7}{2}}\tau^{2}\mathcal{W}(\Sigma) = 8\pi\mathcal{T}_{00},$$
(55)

$$(1 + 7(\frac{-6k}{\alpha^2})^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma))\frac{\ddot{\alpha}}{\alpha} = -\frac{8\pi\rho}{3} + \frac{k}{\alpha^2} - \frac{1}{3}k^{-\frac{5}{2}}(\frac{-6k}{\alpha^2})^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma).$$
(56)

The radius property of the universe in the mean curvature is as follows:

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{\dot{\alpha}^2}{\alpha^2}.\tag{57}$$

Thus, the second Friedmann equation is as follows:

$$(1+7(\frac{-6k}{\alpha^2})^{\frac{5}{2}}k^{-\frac{5}{2}}\tau^2\mathcal{W}(\Sigma))\frac{\dot{\alpha}^2}{\alpha^2} = \frac{8\pi\rho}{3} - \frac{k}{\alpha^2} + \frac{1}{3}k^{-\frac{5}{2}}(\frac{-6k}{\alpha^2})^{\frac{7}{2}}\tau^2\mathcal{W}(\Sigma).$$
(58)

The equations (56) and (58) are new Friedmann equations.

#### 8 Conclusion

We can conclude that space-time existed before the Big Bang. In this period (before the Big Bang), the ordinary matter was created and space-time was shrinking. This means that dark energy was negative before the Big Bang, while after it, everything changed, positive dark energy appeared, and dark matter was imaginary. In this paper, I used the Hawking mass instead of dark matter and Wilmore energy instead of dark energy. Wilmore energy and the Hawking mass connected. Using the equation (41), meaning that dark energy and dark matter had the same origin. However, some questions remained unanswered in this article, including why dark matter and dark energy are connected, why space-time existed before the Big Bang. Achieving The age of the universe by this theory is the most prominent reason for using Wilmore energy and Hawking mass for describing dark energy and dark matter because A mean curvature flow is the milestone of this theory.

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