A Common Myth about Mechanical Resonance

Roman Vinokur

Abstract

The coincidence of the excitation and resonant frequencies is sufficient to create resonances only for a 1-DOF (one degree of freedom) mechanical system. For multi-DOF mechanical systems, this condition is necessary but may not be sufficient. The spatial distribution and orientation of the oscillating forces is important as well. This is important for many practical applications and therefore should be taken into account by design engineers to avoid potential failures with new projects.

Generally speaking, myths are more legend than fact, but if not pushed too far, they approximately describe the world behavior in practical terms. Ancient Mayans believed the skies were populated with cosmic serpents and dragons serving as vehicles for deities. Based on this hypothesis, Mayan priests were actually able to predict solar and lunar eclipses. Certainly, such primitive science could not help people navigate to the Moon nor invent a thermonuclear reaction by analyzing the burning of hydrogen in the Sun's core.

When Apollo 8 mission astronaut Bill Anders said "I think Isaac Newton is doing most of the driving now," he meant that spaceship movement was governed by Isaac Newton's laws of mechanics. However, Newton was more vigilant in describing his own achievements: "To myself I am only a child playing on the beach, while vast oceans of truth lie undiscovered before me."

The best way to engineering success is to continue studying new things and examine common knowledge that may not fit present-day requirements. In this context, analysis of common "myths" can be very effective. Famous composer Igor Stravinsky said "I have learned . . . chiefly through my mistakes and pursuits of false assumptions, and not by my exposure to founts of wisdom and knowledge."

Dual Condition for Resonance. In many handbooks, mechanical resonance is generally defined as: *a large vibration caused by an oscillating force whose frequency coincides with one of the natural frequencies of the resonating body*. Strictly speaking, resonance occurs at a so-called resonant frequency that may differ from the natural frequency, but in most practical cases the difference is minor.

However, the coincidence of the excitation and resonant frequencies is not even mentioned in the existing standard ANSI S2.1-2000/ISO 2041:1990: *Vibration and Shock – Vocabulary*. It states: *Resonance of a system in forced oscillation exists when any change, however small, in the frequency of excitation causes a decrease in a response of the system*. Why? Because the coincidence of the excitation and resonant frequencies is sufficient to create resonances only for a 1-DOF (one degree of freedom) mechanical system. For multi-DOF mechanical systems, this condition is necessary but may not be sufficient. The spatial distribution and orientation of the oscillating forces is important as well.

For example, consider the 2-DOF mechanical system of two similar lumped masses interconnected with a massless spring, shown in Figures 1a and 1b. (Note that a damper is not shown for simplicity.) The black arrows indicate displacement vectors, while the white arrows indicate force vectors. Such a system has two natural modes: "opposite-phase vibration" mode (1a; the masses oscillate in opposite directions around the spring center, which remains stationary) and "rigid-body" mode (1b; the two masses move in phase with no spring deformation). The natural frequency of the first mode is calculated as:

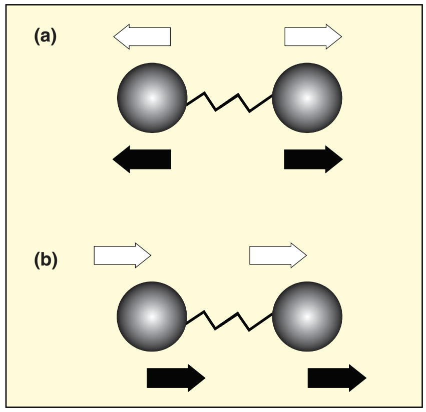

Figure 1. Vibration modes of a two DOF mechanical system with force vectors (white) and displacement vectors (black).

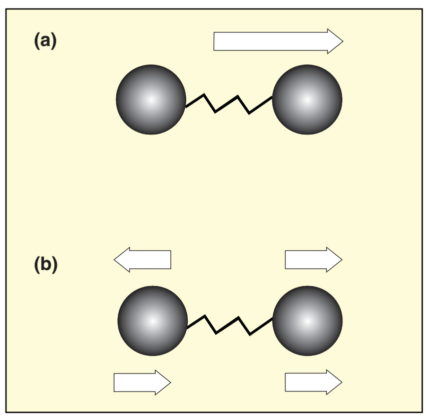

Figure 2. Decomposing an arbitrary force into modal components.

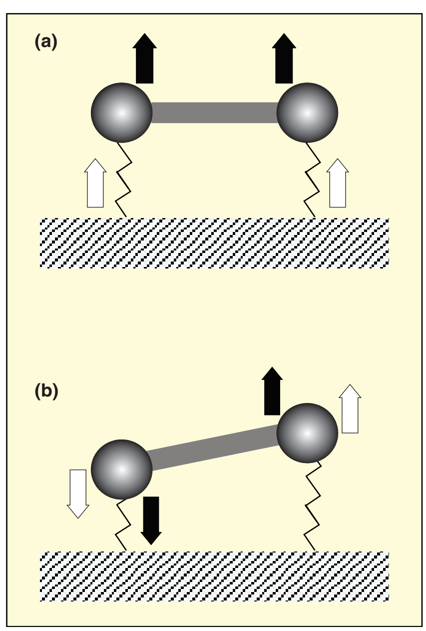

Figure 3. Vibration modes of a two DOF mechanical system with differing force and displacement vectors.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

where k and m are the spring stiffness and mass, respectively. As noted previously, the resonant frequency is close to the natural frequency in most practical cases. On the other hand, the resonance cannot be excited by two similar "in-phase" oscillating forces (Figure 1b). Such forces will only vibrate the whole system back and forth as a rigid body at any frequency. Therefore, to excite the resonance in such a mechanical system, two coincidental conditions should take place simultaneously:

- The oscillating frequency must be equal to the resonant frequency of the "opposite-phase vibration" mode.
- The oscillating forces must include a pair of "opposite-phase vibration" components.

Indeed, the second condition can be fulfilled in a multitude of practical situations. In particular, if the oscillating force F is applied to the second mass and no force acts upon the first mass (Figure 2a), such a spatial arrangement is equivalent to a combination of two pairs of oscillating forces with an amplitude of 0.5 F (Figure 2b); the upper pair excites opposite-phase vibration, and the lower pair shows rigid-body motion.

Spatial Force Distribution. The 2-DOF mechanical system shown in Figures 3a and 3b consists of two identical masses m supported by identical vertical springs with stiffness k and firmly attached to the ends of a perfectly rigid and massless rod. The rod length is $2L$. Such a system has two natural modes:

- "Piston" mode – in Figure 3a, both masses move "in phase" up and down with no rotation about the rod center, which is the system center of mass.
- "Rocking" mode – in Figure 3b, the masses rotate around the center of mass, which remains in equilibrium.

The natural frequency of the piston mode calculates:

$$f_{pist} = \frac{1}{2\pi}\sqrt{\frac{2k}{2m}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The natural frequency of the rocking mode calculates:

$$f_{rock} = \frac{1}{2\pi}\sqrt{\frac{2kL^2}{J}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where $J = 2mL^2$ is the moment of inertia of the system. The natural frequencies for both modes coincide. However, the system vibrates in the piston mode if the two oscillating forces acting upon the masses are identical (Figure 3a) and in the rocking mode if the oscillating forces have the same magnitude but opposite directions (Figure 3b).

Which Coincidence Condition? Resonance of an ideal 1-DOF mechanical system is excited due to the coincidence of the oscillating forcing frequency and resonance frequency. In this case, the force is always applied along the system axis, and just one natural vibration mode exists. In an idealized thin plate of infinite span, where bending waves are excited by incident sound waves in air, a powerful resonance occurs if the wavelength of the "along-plate" component of an incident sound wave coincides with that of a free bending (flexural) wave propagating in the plate (Figure 4). Here the frequencies of the incident and bending waves should coincide automatically as a consequence of the energy conservation law. Since in this case, the incident wave plays the role of the exciting force, the resonance is driven by a "force distribution coincidence."

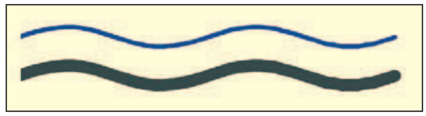

Figure 4. coincidence frequency of an acoustically-excited structure.

In real multi-DOF mechanical systems, both frequency and force distribution coincidence effects are of practical importance. The frequency coincidence may become more important because of the input vibration energy redistribution between the system degrees of freedom, in particular at high frequencies. But at relatively low frequencies, such a randomization is less significant.

Importance of Understanding Practical Mechanical Resonances. According to Albert Einstein, "intellectuals solve problems, geniuses predict them." Many industrial products, particularly in automotive and aerospace industries may exhibit significant structural reliability even under vibration fatigue and shock loading, so NVH engineers must be geniuses predicting and reducing the risks of structural failure. Even with extensive computer modeling, it is difficult to segregate all the discrepancies. Sometimes computer simulations do not predict actual performance characteristics. Many contemporary FEA specialists are extremely good at analytical computations but lack "hands on" experience with the machinery they model. This can severely handicap their ability to interpret the simulation results obtained.

Design engineers can never be 100% certain of the structural reliability of their creations until a prototype or pre-production sample is built to "speak for itself" during vibration fatigue and shock testing. Simulation test conditions must be close to real operation conditions to avoid wrong predictions. Occasionally, a new high-speed vehicle successfully passes all the simulation and laboratory testing and promptly fails in real-world operation.

Since most structural failures occur at mechanical resonances, it is important to test the prototypes under real-life resonant conditions. Measured acceleration of a system tested on a shaker may be high, and the resonance frequency may be carefully explored and correlated to the design FE model. But the environment encountered in real life can present different loading patterns that excite untested vibration modes. A clear understanding of resonances in multi-DO F mechanical systems is an effective tool to predict and fix structural problems even before a vehicle is tested under real road or flight conditions. In combination with true experimental data, this should help create reliable products.

The author can be reached at: rvinokr@aol.com.