To Options

Equivalent ABC Conjecture Proved on Two Pages

A. A. Frempong

Abstract

By applying basic mathematical principles, the author proves an equivalent ABC conjecture, The equivalent ABC conjecture proved in this paper states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime of positive integers, with $A + B = C$, such that $C < K_{\varepsilon} rad(d)^{1+\varepsilon}$, where d is the product of distinct prime factors of A, B, and C, and K_{ε} is a constant. From the hypothesis, $A + B = C$, it was proved that $C < K_{\varepsilon} rad(d)^{1+\varepsilon}$.

 "The simplest solution is usually the best solution"---Albert Einstein

To First Page

Options

Back to Options

Option 1
Introduction

The equivalent conjectures states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime of positive integers, with $A + B = C$, such that $C < K_{\varepsilon} rad(d)^{1+\varepsilon}$ where d is the product of distinct prime factors of $A, B,$ and C, and K_{ε} is a constant.

If $A + B - C = 0$, $|A + B - C| = |0| = 0$. For a very small positive number, δ, $0 < \delta$, one can write $|A + B - C| < \delta$ From above, the hypothesis would be, $|A + B - C| < \delta$, and the conclusion would be $C < K_{\varepsilon} rad(d)^{1+\varepsilon}$.

Back to Options

Option 2

Equivalent ABC Conjecture Proved on Two Pages

The ABC equivalent conjecture, in this paper, states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime positive integers, with $A + B = C$, such that $C < K_\varepsilon rad(d)^{1+\varepsilon}$,where d is the product of distinct prime factors of A, B, and C, and K_ε is a constant.

Given: 1. $A + B = C$, where A, B and C are positive integers. with A, B and C being coprime.
2. d = product of the distinct prime factors of A, B and C.

Required: To prove that $C < K_\varepsilon rad(d)^{1+\varepsilon}$

Plan:

$$K_\varepsilon rad(d)^{1+\varepsilon} > C;$$

$$\log\{K_\varepsilon rad(d)^{1+\varepsilon}\} > \log C:$$

$$\log K_\varepsilon + \log\{rad(d)^{1+\varepsilon}\} > \log C:$$

$$\log K_\varepsilon + (1+\varepsilon)\log(rad(d)) > \log C:$$

$$\log K_\varepsilon + \log(rad(d)) + \varepsilon \log rad(d) > \log C:$$

$$\varepsilon \log rad(d) > \log C - \log K_\varepsilon - \log(rad(d))$$

$$\varepsilon > \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log rad(d)} \quad \text{or}$$

$$\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon \quad \text{(equivalent conclusion)}$$

Proof: One will apply the continued inequality method (condensed method) to handle the inequalities involved.

Step 1: $|A + B - C| < \delta \quad (\delta > 0)$ (hypothesis) (2)

One applies the absolute value symbol to the equivalent conclusion from above to obtain $\left|\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}\right| < \varepsilon$ (3).

(The above absolute value symbol will be removed in the last step)

The hypothesis $|A + B - C| < \delta$ is equivalent to

$$-\delta < A + B - C < \delta \quad \text{(hypothesis)} \quad (4)$$

The conclusion , $\left|\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))\}}\right| < \varepsilon$ is equivalent to

$$-\varepsilon < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon \quad \text{conclusion} \quad (5)$$

Step 2: Make the middle terms of (4) and (5) the same. Then (4) becomes.

$$\boxed{-\delta + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < A + B - C + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + \delta \text{ (hypoth)}} \quad (6)$$

and (5) becomes $\boxed{-\varepsilon + A + B - C < A + B - C + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon + A + B - C}$ (7)

Since (6) and (7) have the same middle terms, equate the left sides to each other and equate the right sides to each other. Then one obtains

$\boxed{-\delta + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} = -\varepsilon + A + B - C}$ and one solves for δ to obtain

$\boxed{\delta = \varepsilon + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} - A - B + C \text{ say } \delta_1}$ followed by solving

$\boxed{\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + \delta = \varepsilon + A + B - C}$ for δ to

obtain $\boxed{\delta = \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C \text{ say } \delta_2}$

$|A + B - C| < \delta$, implies that

$-\delta_1 \le -\delta < A + B - C < \delta \le \delta_2$ (hypothesis)

For $\varepsilon > 0$, choose $\delta = \min(\delta_1, \delta_2)$.

$-\delta < A + B - C < \delta$ (hypothesis) implies that

$-\delta_1 \le -\delta < A + B - C < \delta \le \delta_2$ (hypothesis) (8)

Step 3: Replace the left and right sides of (8) by

$\boxed{\delta = \varepsilon + \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} - A - B + C \text{ say } \delta_1}$ and

$\boxed{\delta = \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C \text{ say } \delta_2}$, from above, respectively to obtain

$$-\varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C < A + B - C < \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C \text{ (hyp) } \mathbf{(9)}$$

Break up inequality (**9**) into two simple inequalities and solve each one for $-\varepsilon$ and ε, respectively.

$-\varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C < A + B - C$ <------ Left part of the break-up

$\boxed{-\varepsilon < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}}$

Right part of the break-up --> $A + B - C < \varepsilon - \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} + A + B - C$

$\boxed{\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon}$

Back to Options

The combined solutions from the break-up of inequality (9) is

$-\varepsilon < \frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}$ **and** $\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}$; and this combination is equivalent to

$$\left|\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))}\right| < \varepsilon$$

Step 4: As was noted in Step 1, one will remove the absolute value symbol to obtain

$$\frac{\log C - \log K_\varepsilon - \log(rad(d))}{\log(rad(d))} < \varepsilon \quad \text{(equivalent conclusion)}$$

Therefore, if $|A + B - C| < \delta \ (\delta > 0)$ or $A + B = C$, $C < \{K_\varepsilon rad(d)\}^{(\varepsilon+1)}$, and the proof of the equivalent conjecture is complete.

Option 3

Discussion

In Step 1, (inequality (3)) the absolute value symbol was applied, and in Step 4, the symbol was removed. For analogy in elementary math, consider: **Factoring quadratic trinomials by the substitution method;**

.

Example: Factor $6x^2 + 11x - 10$.

In the first Step, **Multiply** the expression by the coefficient of the x^2-term.: $6(6x^2) + 6(11x) - 6(10)$; and in the last Step, **divide** by 6: $\frac{6(3x-2)(2x+5)}{6}$, and then the complete factorization of $6x^2 + 11x - 10$ is $(3x - 2)(2x + 5)$..

Conclusion

By applying basic mathematical principles, the author proved an equivalent ABC conjecture, The equivalent ABC conjecture proved states that for every positive real number ε, there exists only finitely many triples (A, B, C) of coprime of positive integers, with $A + B = C$, such that $C < K_\varepsilon rad(d)^{(1+\varepsilon)}$**,** where d is the product of distinct prime factors of A, B, and C, and K_ε is a constant. From the hypothesis, $A + B = C$, it was proved that $C < K_\varepsilon rad(d)^{(1+\varepsilon)}$, the conclusion. The continued inequality method (condensed method) was used in handling the inequalities involved in the proof.

PS: 1. A proof of the original ABC conjecture by the author is at viXra:2107.0094
2. For more on epsilon-delta proofs, see Lesson 5C, Calculus 1 & 2 by A. A. Frempong at Apple iBookstore.

Back to Options

Adonten