Bell's theorem refuted: Einstein and locality prevail

Gordon Stewart Watson[1]

Abstract
In our terms, this is Bell's 1964 theorem, 'No local hidden-variable theory can reproduce exactly the quantum mechanical predictions.' Against this, and bound by what Bell takes to be Einstein's definition of locality, we refute Bell's theorem and reveal his error. We show that Einstein was right: the physical world is local; and we advance Einstein's quest to make quantum mechanics intelligible in a classical way. With respect to understanding, and taking mathematics to be the best logic, the author is as close as an email.

Keywords: Locality, completeness, free choice, causally independent, logically correlated, spooky action, Bell's error, Bell's inequality, Bell's theorem

1. Introduction

1.1. "... you make a very thorough analysis of EPR-Bell. As you still remain a 'realist' and refer to Bell's beables when you resolve Bell's dilemma, Bell might have liked your approach, who knows," Reinhold Bertlmann (2017), pers. comm. 26 June. Let's see.

1.2. We take this to be Bell's theorem, 'No local hidden-variable theory can reproduce exactly the predictions of quantum mechanics (QM),' after Bell (1964:195).

1.3. Bell's text, freely available (see §8.1), is taken as read. However, using P for probabilities, we replace Bell's expectation $P(\vec{a},\vec{b})$ by $E(AB)$ or $\langle AB\rangle$; etc. Then, beneath Bell 1964:(14), we identify the three unnumbered expressions as (14a)-(14c).

1.4. Our goal is a theory that matches the statistical predictions of QM, given two axioms. (i) *Locality*: 'the *real situation* of a physical system S_2 is independent of what is done with a physical system S_1 that is spatially separated from the former,' after Einstein; Bell (1964:200). (ii) *Completeness*: to analyze a physical system, we include every relevant element of physical reality; ie, every relevant Bell *beable*, as we understand him.

1.5. Then, again after Einstein, via Bell (1964:196), 'in a complete physical theory, the variables $[\lambda]$ have dynamical significance and related laws of motion.' We therefore allow the initial (pre-interaction) values of λ to be pairwise anti-correlated $[\lambda,-\lambda]$ via the pairwise conservation of total angular momentum.

1.6. Further. Under what we call *true realism*, we allow subsequent changes via local interactions. We therefore seek laws that associate the local transformation of one twin with that of the spatially-separated other. We also seek laws that relate polarizer inputs to outputs, commutation laws, etc. It follows that such searches take place in the context of

[1] eprb@me.com Correspondence welcomed. [Ex: 2020R.v0g, 20210911] Ref: 2020R.v1, 20210912

the Bohm-Aharonov experiment [β] that Bell (1964) studied. So, in our terms, here's β:

$$\pm 1 = A^{\pm} \Leftarrow \mathscr{A} \leftarrow {}_p(a^{\pm}) \leftarrow \Phi_a^{\pm} \leftarrow {}_p(\lambda) \leftarrow \bullet \rightarrow {}_p(-\lambda) \rightarrow \Phi_b^{\pm} \rightarrow {}_p(b^{\mp}) \rightarrow \mathscr{B} \Rightarrow B^{\mp} = \mp 1 \quad (1)$$

$$\ldots \quad \text{Alice's locale} \quad \rfloor \quad \lfloor \text{Source} \rfloor \quad \lfloor \quad \text{Bob's locale} \quad \ldots \qquad (2)$$

1.7. ${}_p(\lambda)$ and ${}_p(-\lambda)$ are paired [twinned] spin-half [spin $s = \frac{1}{2}$] particles with variables λ. Arrows [$\leftarrow, \rightarrow$] denote the flights of the particles from source $\bullet$ to interaction with their respective 2-channel linear-polarizers $[\Phi_a^{\pm}, \Phi_b^{\pm}]$, and beyond. Unit-vector a [b] denotes the principal-axis direction; freely and independently chosen by Alice [Bob].

1.8. Alice's [Bob's] polarizer transforms an input particle ${}_p(\lambda)$ [${}_p(-\lambda)$] to an output particle ${}_p(a^{\pm})$ [${}_p(b^{\mp})$], polarized either parallel or antiparallel to the related principal axis. Analyzer $\mathscr{A}$ [$\mathscr{B}$] faithfully identifies each polarized output. An arrow $\Leftarrow$ [$\Rightarrow$] shows that Alice's [Bob's] analyzer prints a single result from $A^{\pm} = \pm 1$ [$B^{\mp} = \mp 1$].

1.9. Since the particles are pairwise anti-correlated via λ [${}_p(\lambda), {}_p(-\lambda)$], it is a feature of β, (1), that if $a = b$, then $A^{\pm}B^{\mp} = -1$. That is, as the results are here anti–correlated, A^+B^+ and A^-B^- do not then occur. Hence the mnemonic sign convention [$\pm, \mp$] in (1).

2. Analysis

2.1. Introduced to Bell's theorem by Mermin (1988), we offer this heuristic in the context of (1)-(2): 'Without mystery, correlated tests on correlated things produce correlated results: so Bell's theorem is [most certainly] false,' author to David Mermin, 3 June 1989.

2.2. For, under locality, via (1)-(2) by observation, and taking mathematics to be the best logic: Alice's [Bob's] results $A^{\pm}$ [$B^{\mp}$] are functions of (a, λ) [$(b, -\lambda)$] alone. So now, *in the same instance* [see the line before Bell 1964:(1)], and after Bell 1964:(1) & (13), we can compare paired-results under a common polarizer-analyzer function A;

$$A^{\pm} = A(a, \lambda) = \pm 1, \; B^{\mp} = B(b, \lambda) = -A(b, \lambda) = A(b, -\lambda) = \mp 1. \qquad (3)$$

2.3. Now, from (1)-(3): $A^{\pm}$ [$B^{\mp}$] are directly related to polarizer outputs ${}_p(a^{\pm})$ [${}_p(b^{\mp})$]. So, given the heuristic in §2.1, we seek the transformation laws that match such correlations. To that end, via (1): on the elements ${}_p(\lambda)$ of $\Phi_a^{\pm}$'s domain [each paired with its twin ${}_p(-\lambda)$ in $\Phi_b^{\pm}$'s domain], let $\overset{a}{\sim}$ $[\overset{b}{\sim}]$ denote the equivalence relation *has the same output under* $\Phi_a^{\pm}$ $[\Phi_b^{\pm}]$. Then, *in the same instance*, by way of example:

$$\text{If } {}_p(\lambda) \rightarrow \Phi_a^{\pm} \rightarrow {}_p(a^+) \text{ then } {}_p(\lambda) \overset{a}{\sim} {}_p(a^+) \text{ for } {}_p(a^+) \rightarrow \Phi_a^{\pm} \rightarrow {}_p(a^+). \qquad (4)$$

$$\text{If } {}_p(-\lambda) \rightarrow \Phi_b^{\pm} \rightarrow {}_p(b^+) \text{ then } {}_p(-\lambda) \overset{b}{\sim} {}_p(b^+) \text{ for } {}_p(b^+) \rightarrow \Phi_b^{\pm} \rightarrow {}_p(b^+). \qquad (5)$$

2.4. So the laws we seek are similar to relations between polarized particles. Thus, for us, the heuristics now include: (i) The relation in §1.9. (ii) Malus' $\cos^2$ Law for polarized light $[s=1]$, adjusted for $s=\frac{1}{2}$. (iii) From (3), $A^{\pm}$ and $B^{\mp}$ are causally independent [the cause of one is not the cause of the other], and logically [thus law-like] anti-correlated via their λ-relations under the common function A. (iv) So the general product rule (for the probability of paired outcomes), applies: $P(XY)=P(X)P(Y\,|\,X)$.

2.5. Thus, for (4)-(5)'s particle-pair, via $P(Y\,|\,X)$ and the heuristics: here's a law that associates the local transformation of one twin with that of the spatially-separated other:

$$P\left[{}_p(\lambda)\overset{a}{\sim}{}_p(a^+)\,|\,{}_p(-\lambda)\overset{b}{\sim}{}_p(b^+)\right]=P(A^+\,|\,B^+)=\sin^2\tfrac{1}{2}(a,b);\text{ etc.}\tag{6}$$

$$\therefore P\left[{}_p(\lambda)\overset{a}{\sim}{}_p(a^-)\,|\,{}_p(-\lambda)\overset{b}{\sim}{}_p(b^+)\right]=P(A^-\,|\,B^+)=\cos^2\tfrac{1}{2}(a,b);\text{ etc.}\tag{7}$$

2.6. In passing, (3) is satisfied adequately (for now) by functions A, B:

$$A^{\pm}=A(a,\lambda)=a\cdot\left(\lambda\overset{a}{\sim}a^{\pm}\right)=\pm1,\,B^{\mp}=B(b,\lambda)=b\cdot\left((-\lambda)\overset{b}{\sim}b^{\mp}\right)=\mp1;\text{ with,}$$

$$\text{via Bell 1964:(3): }\langle a\cdot\sigma_1\,b\cdot\sigma_2\rangle=\left\langle a\cdot\left(\lambda\overset{a}{\sim}a^{\pm}\right)b\cdot\left((-\lambda)\overset{b}{\sim}b^{\mp}\right)\right\rangle=-a\cdot b;\text{ etc.}\tag{8}$$

2.7. Then, given the probabilistic relations, over *discrete* results, in (6)-(7): we now axiomatize the concept of expectation. Under β and after Whittle (1976), we use a probability-based definition of the expectation $E(AB)$. Thus, from (1):

$$\{A^+B^+,A^+B^-,A^-B^+,A^-B^-\}\text{ is the set of paired results.}\tag{9}$$

$$\therefore P(A^+B^+)+P(A^+B^-)+P(A^-B^+)+P(A^-B^-)=1.\text{ So, with the}\tag{10}$$

value $[\pm1]$ of each paired result weighted according to its probability:

$$E(AB)\equiv P(A^+B^+)-P(A^+B^-)-P(A^-B^+)+P(A^-B^-);\tag{11}$$

$$\text{with }P(A^+B^+)=P(A^+)P(B^+\,|\,A^+)\text{, etc, holding under §2.4(iv).}\tag{12}$$

Then, via LHS Bell 1964:(2) and RHS Bell 1964:(3), with

the "not possible" line thereunder, we have a Bell-certain inequality

$$\text{that is also Bell's theorem: }E(AB)\neq-a\cdot b,\text{ the QM expectation under }\beta.\tag{13}$$

3. Bell's theorem refuted

$$\begin{aligned}E(AB)&=P(A^+)P(B^+|A^+)-P(A^+)P(B^-|A^+)-P(A^-)P(B^+|A^-)+\\&\quad P(A^-)P(B^-|A^-)\text{, via (11)-(12).}&&(14)\\&=\tfrac{1}{2}\left[P(B^+|A^+)-P(B^-|A^+)-P(B^+|A^-)+P(B^-|A^-)\right]\text{, since}\\&\quad\lambda\text{ is a random variable, the marginals }P(A^{\pm})=\tfrac{1}{2}\text{ in (14).}&&(15)\\&=\tfrac{1}{2}\left[\sin^2\tfrac{1}{2}(a,b)-\cos^2\tfrac{1}{2}(a,b)-\cos^2\tfrac{1}{2}(a,b)+\sin^2\tfrac{1}{2}(a,b)\right]\text{ via (6)-(7).}&&(16)\\&=-\cos(a,b)=-a\cdot b;\text{ etc. So Bell's theorem, (13), is refuted. QED.}&&(17)\end{aligned}$$

3.1. Our *local* hidden-variable theory produces the quantum mechanical result exactly. Without mystery, correlated tests on correlated things do produce correlated results.

4. Bell's inequality refuted

4.1. Bell's proof of his theorem, (13), relies on his famous inequality, Bell 1964:(15): which we now reformat as **BI** for easy comparison with our similarly formatted (but irrefutable), **WI**. In this way, since **BI** and **WI** will have the same LHS, departure of RHS **BI** from RHS **WI** will signal a **BI** error. This, in turn will signal that Bell 1964:(15) is erroneous, and refuted. Thus, in preparation for comparison with **WI**:

$$\textbf{BI} \equiv |E(AB) - E(AC)| - 1 \leq E(BC) \text{ from Bell 1964:(15).} \tag{18}$$

4.2. Then, given the limits in (3), we see these expectations in (18):

$$-1 \leq E(AB) \leq 1,\ -1 \leq E(AC) \leq 1,\ -1 \leq E(BC) \leq 1. \tag{19}$$

$$\therefore E(AB[1+E(AC] \leq 1+E(AC);\ \text{for, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{20}$$

$$\therefore E(AB) - E(AC) - 1 \leq -E(AB)E(AC) \text{ from (20).} \tag{21}$$

$$\text{Similarly: } E(AC) - E(AB) - 1 \leq -E(AB)E(AC). \tag{22}$$

$$\text{So: irrefutably via (21)-(22), here's our never-false inequality } \textbf{WI}: \tag{23}$$

$$\textbf{WI} \equiv |E(AB) - E(AC)| - 1 \leq -E(AB)E(AC). \text{ So } \textbf{BI}\text{-(18) is refuted. QED.} \tag{24}$$

4.3. For **BI**-(18) and *irrefutable* **WI**-(24) have the same LHS. So with RHS **BI** differing from RHS **WI** almost everywhere: **BI**, and thus Bell 1964:(15), is false and refuted.

4.4. Then, to gauge the extent of Bell's error, here's a false (and low), Bell-bound. Let a,b,c be co-planar, not necessarily orthogonal to the particles' line of flight; with angles (a,b), (b,c), (a,c). Then, via (17) with $(a,c) = \frac{\pi}{2}, (a,b) = (b,c) = \frac{(a,c)}{2}$:

$$\text{RHS } \textbf{WI}\text{-(24)} = -E(AB)E(AC) = 0. \text{ RHS } \textbf{BI}\text{-(18)} = E(BC) = -\tfrac{\sqrt{2}}{2}. \text{ QED.} \tag{25}$$

5. Bell's error identified

5.1. False **BI** flows truly from Bell's (14b), so (14b) is false. So Bell's error is the false commutation from true (14a) to false (14b): using 1964:(1), as he says; which is our (3).

5.2. For, from either, *in the same instance*: pairwise correlated by the conservation of total angular momentum, (§1.5), paired results commute within instances; and not otherwise. (A commutation law, as foreshadowed in §1.6.) Thus, in our terms: $E(AB) = E(BA)$, and $E(AB)E(BA) \neq E(AA)E(BB) = 1$; using $E(AA) = E(BB) = -1$, from §1.9 or from Bell 1964:(8). So with '$\neq$' prevailing, we demonstrate Bell's commutation error:

$$\text{From false (14b): } E(AB)E(BC) \neq E(BB)E(AC) = -E(AC) \text{ in true (14a).} \tag{26}$$

$$\text{True RHS } \textbf{WI} = -E(AB)E(AC) \neq -E(AA)E(BC) = E(BC) = \text{false RHS } \textbf{BI}. \tag{27}$$

6. Conclusion

6.1. Bell's 1964 theorem and inequality are refuted; his error found and explained. Via independent locally-causal chains in Alice's and Bob's locales, see (1), our local hidden-variable Lorentz-invariant theory reproduces exactly the quantum mechanical predictions.

6.2. And it is unsurprising that experiments breach Bell inequalities that misstate irrefutably-valid bounds. Crucially, Bell's error has nothing to do with locality; a claim supported via similar analysis of other β-like experiments, including GHZ (1989).

6.3. Thus, via a notation consistent with Bell's, but avoiding error, we resolve Bell's (1990:84) locality dilemma, 'that in somehow distant things are connected, or at least not disconnected.' We show that Einstein was right: the physical world is local, with no spooky actions; we advance Einstein's argument for an intelligible quantum theory.

6.4. For further development under Einstein's classicality, via detour (8) and its direct links to QM, we have: (i) The second expectation therein is, via (9), $E(AB)$ as in (11). (ii) Interesting anti-parallels in Bob's locale: $\sigma_2 = -(\sigma_1), -\lambda = -(\lambda)$. (iii) Related laws.

6.5. Finally. Rejecting *naive* realism, we rely on true realism: some elements of physical reality change under local interactions; see (1) and §1.5. (For polarizers are not passive filters.) In this way, probability theory enters our analysis quite naturally. See Fröhner (1998) for its extension, and the consequent demystification of QM to 'quite some extent'.

7. Acknowledgment

7.1. It's a pleasure to acknowledge beneficial exchanges with Lazar Mayants (late of Amherst), Olivier Costa de Beauregard (late of Paris), Fritz Fröhner (late of Karlsruhe), Roger Mc Murtrie (Canberra), Gerardus 't Hooft (Utrecht), Reinhold Bertlmann (Vienna).

8. References

8.1. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200. http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf

8.2. Bell, J. S. (1990). "Indeterminism and Nonlocality [with Discussion]" in "Mathematical Undecidability, Quantum Nonlocality and the Question of the Existence of God." A. Driessen and A. Suarez (Eds.), Springer (Kluwer) 1997: 78-89.

8.3. Fröhner, F. H. (1998). "Missing link between probability theory and quantum mechanics: the Riesz-Fejér theorem." Z. Naturforsch. 53a, 637-654. http://zfn.mpdl.mpg.de/data/Reihe_A/53/ZNA-1998-53a-0637.pdf

8.4. GHZ (1989). "Going beyond Bell's theorem." in "Bell's Theorem, Quantum Theory and Conceptions of the Universe." M. Kafatos. Dordrecht, Kluwer Academic: 69-72. http://arxiv.org/pdf/0712.0921v1.pdf

8.5. Mermin, N. D. (1988). "Spooky actions at a distance: Mysteries of the quantum theory." The Great Ideas Today 1988. M. J. Adler. Chicago, Encyclopædia Britannica Inc: 2-53.

8.6. Whittle, P. (1976). Probability. London, John Wiley.