On the Inverse Fourier Transform of the Planck-Einstein-de Broglie law

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Abstract
Using results derived earlier by the author, in this letter after generalising $E = hf$, its inverse Fourier transform is calculated.

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In [1] I proposed the notion of angles-time space and the principle of maximum frequency based on the fundamental guiding principle that the so-called wave-particle duality is a purely classical result of spin motion of particles and argued that the Planck–Einstein relation $E = h\omega$ is nothing but $E = I\omega^2$ of classical mechanics with a change of dress. There I argued that the natural implementation of the principle of maximum frequency results in the following metric for angles-time space in absence of all interactions

$$d\Theta^2 = \omega_P^2 dt^2 - d\theta^2,$$

where $\omega_P$ is the Planck frequency. In this letter I prove that the principle of maximum frequency solves one of the foundational problems of quantum mechanics: Taking Planck-Einstein relation $E = h\omega$ as a fundamental law of nature and abiding by the principle that a fundamental law of nature must not depend on our choice of basis for the function space (Fourier basis), we expect the Inverse Fourier Transform of $E = h\omega$ to yield the energy of electromagnetic field as a function of time, viz.

$$E(t) = \frac{\hbar}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} \omega \, d\omega.$$
but this integral is wildly divergent, and I maintain that this is the root of the problem of renormalisation in QFT. I now prove that this problem is solved by implementing the principle of maximum frequency. As a result of the principle of maximum frequency the complete exact form of Planck–Einstein relation is given by[1]

\[
E(\omega) = \frac{2\hbar \omega^2}{\omega} \left( \frac{1}{\sqrt{1 - (\omega/\omega_P)^2}} - 1 \right)
\]

Note that \( E = \hbar \omega \) is an approximation to the above equation, by being the first term in the Taylor expansion of the angles-time space factor. Unlike \( E = \hbar \omega \) however, my proposed generalisation of Planck–Einstein relation is not plagued by infinities and it is perfectly possible to take its inverse Fourier transform. To this purpose, recall that

\[
\mathcal{F}\left[J_0(\omega_P t)\right] = 1 - \frac{2}{\sqrt{1 - (\omega/\omega_P)^2}},
\]

where \( J_0(t) \) is the zero-order Bessel function of the first kind. Note that according to the principle of maximum frequency

\[0 < \frac{\omega}{\omega_P} \leq 1,\]

therefore

\[
\mathcal{F}\left[J_0(\omega_P t)\right] = \frac{2}{\sqrt{1 - (\omega/\omega_P)^2}}.
\]

Now observe that for \( \omega \neq 0 \)

\[i\omega E(\omega) = 2i\hbar \omega^2 \left( \frac{1}{\sqrt{1 - (\omega/\omega_P)^2}} - 1 \right),\]

which is

\[
\mathcal{F}\left[\frac{d}{dt}E(t)\right] = i\hbar \omega^2 \mathcal{F}[J_0(\omega_P t) - 2\delta(t)],
\]

therefore

\[
E(t) = i\hbar \omega^2 \left( -2H(t) + \int_{-\infty}^{t} J_0(\omega_P \tau)d\tau \right)
\]

where \( H(t) \) is the Heaviside step function.

References