

# **Interaction of Complex Scalar Fields and Electromagnetic Fields in Klein-Gordon-Maxwell Theory in Cosmological Inertial Frame**

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## **ABSTRACT**

We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame.

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**Key words: Klein-Gordon-Maxwell Theory;**

**Cosmological Inertial Frame;**

**Complex Scalar fields;**

**Electromagnetic fields**

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## 1.Introduction

The Lagrangian  $L$  of complex scalar fields  $\phi, \phi^*$  and Electromagnetic fields  $F^{\mu\nu}, F_{\mu\nu}$  is Klein-Gordon-Maxwell theory in special relativity theory,

$$L = (\partial_\mu \phi + ieA_\mu \phi)(\partial^\mu \phi^* - ieA^\mu \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$\phi^*$  is  $\phi$ 's adjoint scalar,  $m$  is the mass of scalar fields  $\phi, \phi^*$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (1)$$

## 2. Equations of Interaction of Complex Scalar Fields and Electromagnetic Fields in Cosmological Inertial Frame

The Lagrangian  $L$  of interaction of complex scalar fields and Electromagnetic fields is Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$L = (\bar{\partial}_\mu \phi + ie\bar{A}_\mu \phi)(\bar{\partial}^\mu \phi^* - ie\bar{A}^\mu \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad (2-1)$$

We consider Type A of wave function and Type B of expanded distance,[1],[2],[3],[4]

$$\text{Type A of wave function: } r \rightarrow r\sqrt{\Omega(t_0)} \quad , \quad t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}}$$

Type B of expanded distance:  $r \rightarrow r\Omega(t_0), t \rightarrow t$

$$\bar{\partial}_\mu = (\sqrt{\Omega(t_0)} \frac{\partial}{c\partial t}, \frac{1}{\sqrt{\Omega(t_0)}} \vec{\nabla}), \bar{\partial}^\mu = (\sqrt{\Omega(t_0)} \frac{\partial}{c\partial t}, -\frac{1}{\sqrt{\Omega(t_0)}} \vec{\nabla})$$

$$\bar{A}_\mu = (\phi, \vec{A}\Omega(t_0)), \bar{A}^\mu = (\phi, -\vec{A}\Omega(t_0)), \bar{F}_{\mu\nu} = F_{\mu\nu}\Omega(t_0), \bar{F}^{\mu\nu} = F^{\mu\nu}\Omega(t_0)$$

$t_0$  is the cosmological time.  $\Omega(t_0)$  is the expanding ratio of universe in the cosmological time  $t_0$ .

$$(2-2)$$

Complex scalar field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_\mu \left( \frac{\partial L}{\partial(\bar{\partial}_\mu \phi)} \right) - \frac{\partial L}{\partial \phi} = (\bar{\partial}_\mu - ie\bar{A}_\mu)(\bar{\partial}^\mu \phi^* - ie\bar{A}^\mu \phi^*) + \frac{m^2 c^2}{\hbar^2} \phi^* = 0 \quad (3)$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_\mu \left( \frac{\partial L}{\partial(\bar{\partial}_\mu \phi^*)} \right) - \frac{\partial L}{\partial \phi^*} = (\bar{\partial}^\mu + ie\bar{A}^\mu)(\bar{\partial}_\mu \phi + ie\bar{A}_\mu \phi) + \frac{m^2 c^2}{\hbar^2} \phi = 0 \quad (4)$$

If operator  $\bar{\partial}_\mu, \bar{\partial}^\mu$  are in cosmological inertial frame,[1],[2],[3],[4]

$$\bar{\partial}_\mu' = \left( \frac{\partial}{c\partial t}, \frac{1}{\Omega(t_0)} \bar{\nabla} \right), \bar{\partial}^\mu' = \left( \frac{\partial}{c\partial t}, -\frac{1}{\Omega(t_0)} \bar{\nabla} \right)$$

$$\bar{F}^{\mu\nu}' = \bar{\partial}^\mu' \bar{A}^\nu' - \bar{\partial}^\nu' \bar{A}^\mu', \bar{F}_{\mu\nu}' = \bar{\partial}_\mu' \bar{A}_\nu' - \bar{\partial}_\nu' \bar{A}_\mu' \quad (5)$$

Electromagnetic field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_\nu' \left( \frac{\partial L}{\partial(\bar{\partial}_\nu' \bar{A}_\mu')} \right) - \frac{\partial L}{\partial \bar{A}_\mu'}$$

$$= \frac{1}{4} \bar{\partial}_\nu' (\bar{\partial}^\mu' \bar{A}^\nu' - \bar{\partial}^\nu' \bar{A}^\mu') - ie\phi(\bar{\partial}^\mu' \phi^* - ie\bar{A}^\mu') + ie\phi^*(\bar{\partial}^\mu' \phi + ie\bar{A}^\mu')$$

$$= \frac{1}{4} \bar{\partial}_\nu' \bar{F}^{\mu\nu}' - ie\phi(\bar{\partial}^\mu' \phi^* - ie\bar{A}^\mu') + ie\phi^*(\bar{\partial}^\mu' \phi + ie\bar{A}^\mu') = 0 \quad (6)$$

Hence,[5],

$$\bar{\partial}_\nu' \bar{F}^{\mu\nu}' = \frac{4\pi}{c} \bar{J}^{\mu'}$$

$$\bar{J}^{\mu'} = \frac{c}{\pi} ie[\phi(\bar{\partial}^\mu' \phi^* - ie\bar{A}^\mu') - \phi^*(\bar{\partial}^\mu' \phi + ie\bar{A}^\mu')] \quad (7)$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}^\nu' \left( \frac{\partial L}{\partial(\bar{\partial}^\nu' \bar{A}^\mu')} \right) - \frac{\partial L}{\partial \bar{A}^\mu'}$$

$$= \frac{1}{4} \bar{\partial}^\nu' (\bar{\partial}_\mu' \bar{A}_\nu' - \bar{\partial}_\nu' \bar{A}_\mu') + ie\phi^*(\bar{\partial}_\mu' \phi + ie\bar{A}_\mu') - ie\phi(\bar{\partial}_\mu' \phi^* - ie\bar{A}_\mu')$$

$$= \frac{1}{4} \bar{\partial}^\nu' \bar{F}_{\mu\nu}' + ie\phi^*(\bar{\partial}_\mu' \phi + ie\bar{A}_\mu') - ie\phi(\bar{\partial}_\mu' \phi^* - ie\bar{A}_\mu') = 0 \quad (8)$$

Hence,[5],

$$\bar{\partial}^\nu' \bar{F}_{\mu\nu}' = \frac{4\pi}{c} \bar{J}_\mu' = -4ie[\phi^*(\bar{\partial}_\mu' \phi + ie\bar{A}_\mu') - \phi(\bar{\partial}_\mu' \phi^* - ie\bar{A}_\mu')]$$

$$\bar{J}_\mu' = -\frac{c}{\pi} ie[\phi^*(\bar{\partial}_\mu' \phi + ie\bar{A}_\mu') - \phi(\bar{\partial}_\mu' \phi^* - ie\bar{A}_\mu')] \quad (9)$$

### 3. Conclusion

We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame.

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