In Situ Experiment on Fractal Corresponds with Cosmological Observations and Conjectures

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Blair D. Macdonald © 2021,09,04

fractalnomics@gmail.com

Abstract

Fractal geometry is an accepted mathematical description of nature. One of the great questions in cosmology—along with what is the 'dark energy' and the other cosmic anomalies—is whether the universe is also fractal? The 2012 WiggleZ Dark Energy Survey found, in agreement with fractal-cosmology proponents, the small-scale observable universe is fractal, the large-scale is not fractal. Fractals have not been modelled from the perspective of being within a growing one. Can a (different) fractal model explain all cosmological observations and conjectures, and if so, are we are modelling the fractal universe incorrectly? An experiment was conducted on a 'simple' (Koch snowflake) fractal, testing the perspective of an in-situ observer within a growing/emergent fractal — 'looking back' in iteration-time to its origin. New triangle sizes were held constant allowing earlier triangles in the set to expand as the set iterated. Classical kinematic equations of velocities and accelerations were calculated for the total area total and the distance between points. Hubble-Lemaitre's Law and other cosmological observations and conjectures were tested for. Results showed area(s) expanded exponentially from an arbitrary starting position; and as a consequence, the distances between points — from any location within the set — receded away from the 'observer' at increasing velocities and accelerations. It was concluded, at the expense of the cosmological principle, that the fractal is a geometrical match to the cosmological problems, including the inflation epoch, Hubble-Lemaitre and accelerated expansion; inhomogeneous (fractal) galaxy distribution on the small and homogenous on large scales; and other problems — including the cosmological catastrophe. The fractal may offer a direct mechanism to the cosmological problem and can further explain the quantum problem — unifying the two realities as being two aspects of the same geometry.

Keywords Fractal-cosmology, Dark Energy, Inflation, Hubble- Lemaitre's Law, Quantum Mechanics

1 INTRODUCTION

At the time of writing this paper the standard model of cosmological (ΛCDM), with its accelerated 'dark energy' expansion[2],[3] from a 'big bang'/ 'inflationary'[4] beginning and underlying general relativity, by all accounts remains in a self-titled 'state of crisis'. Nobody—it is claimed—has any idea how to 'make sense of it', let alone be able to marry it in any 'simple' way with its equal enigma, our quantum reality. One geometric candidate put forward in the 1980's—soon after its conception—was fractal geometry, and a new field of cosmology, fractal-cosmology[5] was formed. Fractals are well accepted as a mathematical description of our reality, explaining classically the likes of clouds, trees, and market prices, and more[6]; but, cosmology experts are not entirely convinced. As small-scale astronomical observations improved revealing clustering and super-clustering [7],[8], [9],[10], [11],[12] of galaxies, fractal-cosmologists claimed the universe may be fractal, and a debate surrounding this claim immediately ensured[13]. Proponents argued even larger galactic structures would be discovered out beyond the current observed. The debate came to a head with the findings of the 2012 WiggleZ Dark Energy Survey[14] and others[15] where it was concluded—but granted—that the universe does indeed show direct evidence of small-scale fractal galaxy distribution for distances less than 70 to 100 Mega parsecs away (3 billion light-years); however, the universe is assumed overall 'smooth', homogenous and isotropic, beyond this on large-scales (Figure 1 A and B), and that the cosmological principle and thus general relativity and the standard (ΛCDM) model holds. Today, for cosmology, 'fractals are out'[16].

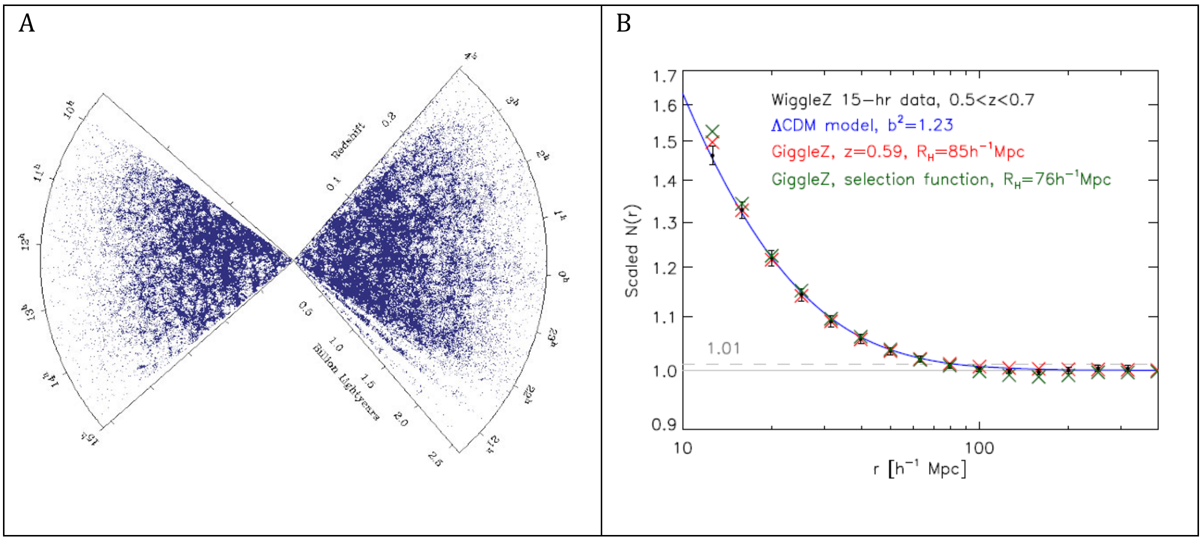

Figure 1. 2dF Galaxy Redshift Surveys and Evolution. A, 2003 2df Redshift Survey map showing small scale 'fractal' clustering[17]; and B, the WiggleZ Dark Energy Survey figure 13, page 16, corresponding to 'A' and revealing changing galaxy distributions from small-scale to large-scale [14].

Notwithstanding the discovery by improved large-scale surveys after the WiggleZ paper was released—contradicting all of the above fractal rebuttals—of the 'very large', 'thin', and old beyond the small-scale granted fractal in the assumed 'smooth universe' (the 4 billion light-years sized Huge 'Large Quasar Group'[18] and the 10 billion light-years sized Hercules Corona Borealis Great Wall [19]): is it that we have the fractal model wrong and rough to smooth and expanding from a point beginning is what we would expect to see if observing within a fractal? Can a different perspective of the fractal help explain all of the cosmic observations and conjectures?

Current fractal cosmology studies assume a traditional or classical perspective of fractal growth, popularised by the view of the Romanesco broccoli. This is a 'forward-looking' *evolving* perspective into the set and is best demonstrated with the Koch snowflake (Figure 2 a). The convergent 'snowflake' fractal structure emerges in and around 7 ± 2 iterations from the addition of new but diminishing sized bits (blue 1 and green 2) to the initial constant in size (thatched red '0') triangle bit.

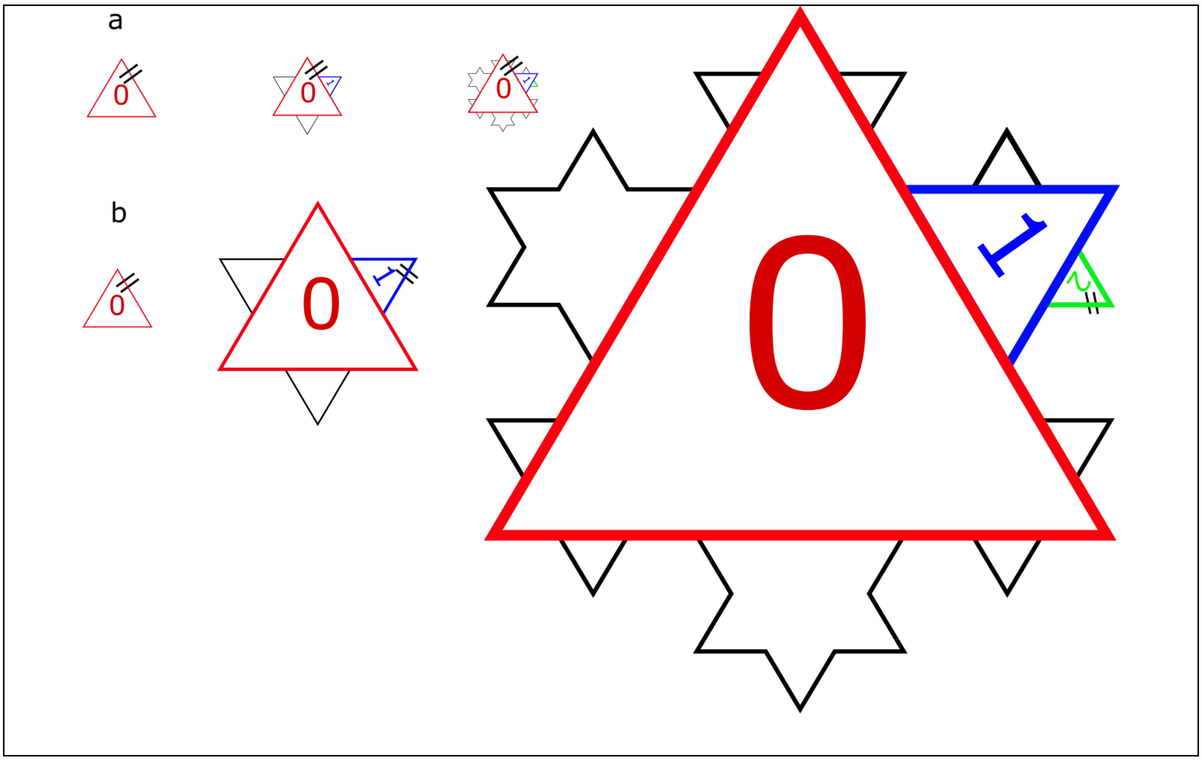

Figure 2. Dual Perspectives of (Koch Snowflake) Fractal Growth. The schematics above demonstrate fractal development by (A) the (classical) forward or evolving Snowflake perspective, where the standard sized thatched (iteration '0') is the focus, and the following triangles diminish in size from colour red iteration 0 to colour green iteration 2; and (B) the inverted retrospective perspective where the new (thatched) triangle is the focus and held at standard size while the original red iteration 0 triangle expands in area — as the fractal iterates.

However, the fractal can be—simultaneously—modelled from an alternative retrospective or backwards-looking perspective (Figure 2 b). To model this retrospective perspective the fractal was 'inverted' where bits grow rather than diminish. The new 'thatched' bit sizes remain constant in size (the same size as the original bit size '0') while the older generations of bit sizes grow with iteration-time as demonstrated with colours red (the original size) blue (the 1st iteration), green (the 2nd). With iteration-time, the size of the initial red iteration 0 triangle expands relative to the size of the new blue triangle.

The closest analogy or practical example of this fractal model may be to think of the growth of a tree—an example of a 'natural' fractal. This is not saying the universe is as a tree, but that a tree is a fractal, and fractals come in many forms. Is it that we are observing the universe as if in a tree structure — from a position of the outer branches? Surrounded by other similar sized branches, we see, looking back and down, larger branches then boughs and finally the — once seedling sized trig — trunk. All new branches on average start as a similar size to the seedling size and expand with time. On a mature tree, there will be many young seedling sized branches on its outer. To complement this tree analogy, in a recent paper it was found all trees accelerate in size with age [20]. Observing Figure 3 A and B show this seedling versus outer branch assumption on a real tree. Figure 3 A-A shows the leaf size as a seedling, and Figure 3 A-B the size of a leaf of a fully grown/developed tree. Figure 3 B shows the seedling held in a hand alongside the outer branches leading down to the boughs and the truck behind.

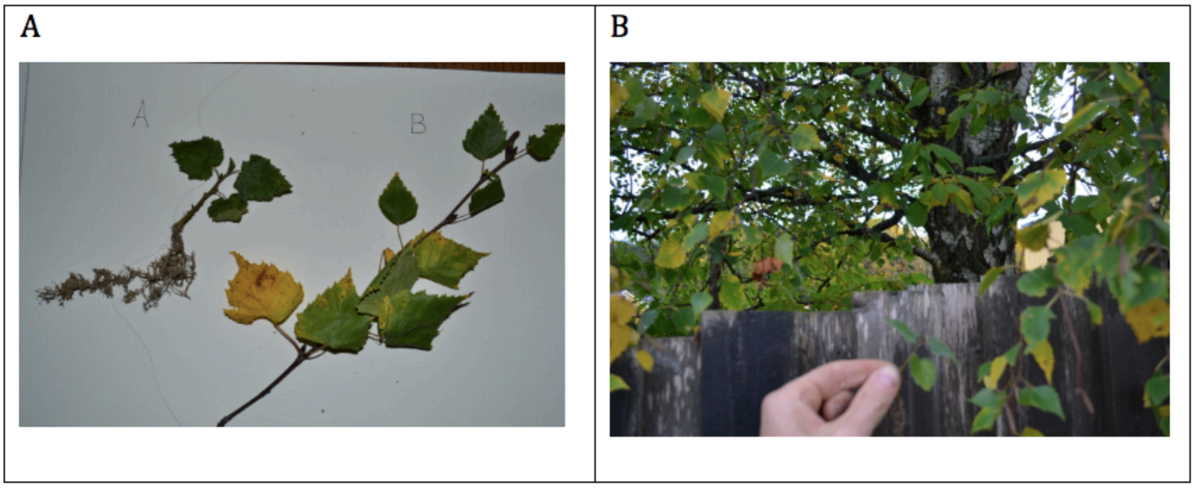

Figure 3. Fractal Tree Growth from a Constant Leaf Size. Figure 3A shows the one constant on an iterating tree fractal, the leaf size: A-A as a seedling size, and A-B the leaf size at the outer branches of the fully-grown tree. Figure 3B shows (held) the same seedling as in A-A alongside the outer branch size of a fully developed tree. The trunk and boughs of the same tree and branches are behind. The trunk of the tree, it can be deduced, was once the same size as the seedling in hand.

Notwithstanding the fractal-cosmologies successes and recent rebuttals, the retrospective perspective of the fractal has not been directly modelled and tested to explain the current observations and conjectures. This was the goal of this paper. If the universe does behave as a fractal, the focus should be on how the fractal grows and how it would be observed from within one as demonstrated.

To model this, Figure 4 shows an observer, represented by an 'eye image', at iteration-time 4 ($t4$) within the growing Koch snowflake fractal. The blue lines show the displacement between the observer and the centre points of the equilibrium triangles of earlier ages ($t0$,$t1$,$t2$,$t3$).

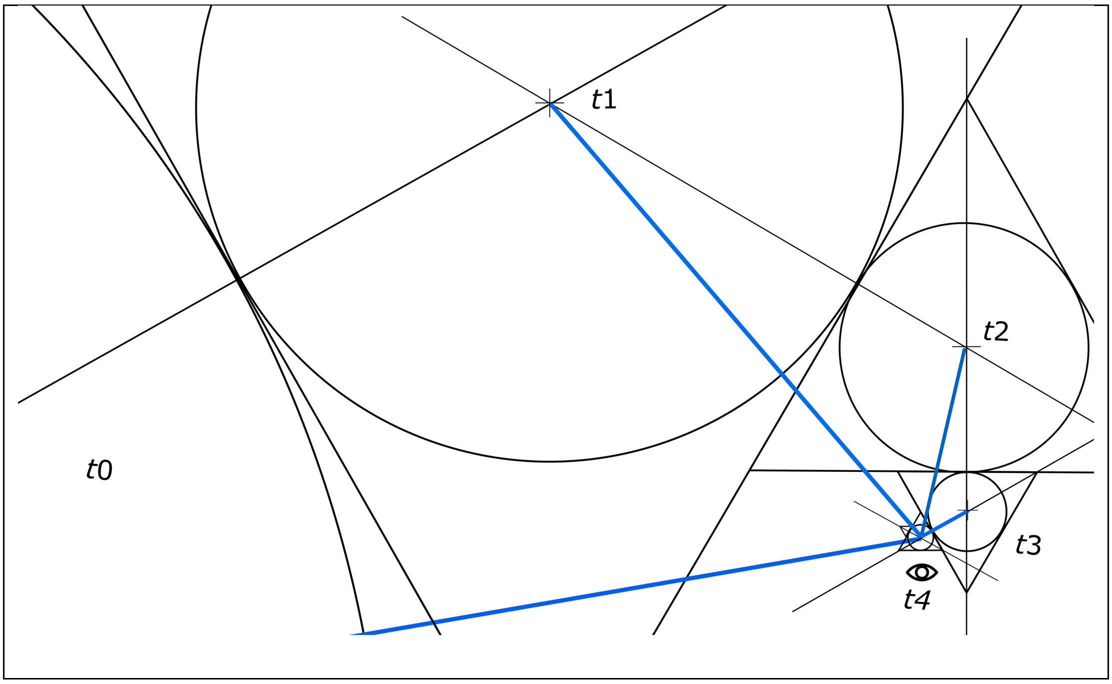

Figure 4 Displacement Measurements within an Iterating Fractal. Showing the displacement between a fixed observation position (eye *t*4) with the iterating (Koch snowflake) fractal

In this investigation, the following questions were asked about the retrospective iterating fractal from the perspective of an in-situ observer within this fractal.

1. Does the fractal demonstrate accelerated expansion?
2. Does the fractal expand from a single point Planck area to an arbitrary size of 1 in a time comparable to the conjectured inflation epoch?
3. Can a Hubble-Lemaitre diagram be produced from the observing perspective of an arbitrary location within the set?
4. Concerning arbitrary centre points of component fractal bits (triangles in the Koch snowflake), is the distribution of these bits change dependent on the location of the observer?

It was hypothesised that current observations of the universe are all what one would expect to see if inside a growing fractal that began at a single point. Specifically, the retrospective fractal will demonstrate:

1. a ‘singularity’ (Big Bang) beginning; the presence, and dominance of a ‘uniform’ Cosmic Microwave Background [21] like origin;
2. an inflation epoch expansion;

3. accelerating (exponential) 'dark energy' expansion;
4. a cosmological constant;
5. a Hubble-Lemaitre Law of expansion[22],[23];
6. the transition in galaxy distribution from rough to smooth on small to large time scales as made in Figure 1 A and B; and,
7. the retrospective fractal model will contravene the two assumptions of the cosmological principle.

To test these questions a model of the fractal was developed measuring the change in the area, and displacement with each iteration from a fixed position of observation. Results were positive.

2 METHODS

To answer the questions a spreadsheet model [24] was developed to trace area expansion of the retrospective fractal by iteration-time(***t***). The classical Koch Snowflake area equations were adapted to account for this perspective. A quantitative data series was made ready for analysis. The scope of this investigation was limited to two-dimensional—as a demonstration; three-dimensional space or volume can be inferred from this initial assumption. Changes in the areas of triangles and distances between points in the fractal set were measured and analysed to determine whether the fractal area and distance between points expand.

2.1 Area Expansion of the Total Inverted Fractal with Iteration-time

To answer question 1, the following tables were produced. A data table was produced (Table 1) to calculate the area growth at each iteration-time for a single triangle. The area (***A***) of a single triangle was calculated from the following equation (1) measured in standard (arbitrary) centimetres (***cm***)

$$A = \frac{l^2\sqrt{3}}{4} \tag{1}$$

where ***l*** is the triangle's base length. ***l*** was placed in Table 1 and was set to 1.51967128766173 ***cm*** so that the area of the first triangle (t_0) approximated an arbitrary area of 1 ***cm***^{2}. To expand the triangle with iteration-time, the base length was multiplied by a factor of 3. The iteration-time number was placed in a column, followed by the base length of the equilateral triangle, and in the final column the formula to calculate the area of the triangle. Calculations were made to the 10th iteration, and the results were graphed.

With iteration, new triangles are (in discrete quantities) introduced into the set. While the areas of new triangles remain constant, the earlier triangles expand, and by this, the total fractal set expands. To calculate the area change of a total inverted fractal (as it iterated), the area of the single triangle (at each iteration-time) was multiplied by its corresponding quantity of triangles (at each iteration-time).

Two data tables (tables 3 and 4 in the spreadsheet file) were developed. Table 3 columns were filled with the calculated triangle areas at each of the corresponding iteration-time — beginning with the birth of the triangle and continuing to iteration ten. Table 4 triangle areas of table 3 were multiplied by the number of triangles in the series corresponding with their iteration-time. Values calculated in Tables 3 and 4 were totalled and analysed in a new table (table 5). Analysed were: total area expansion per iteration, expansion ratio, expansion velocity, expansion acceleration, and expansion acceleration ratio. Calculations in the columns used kinematic equations developed below.

To answer question 2 the total area expansion per iteration equation was set to calculate the iteration-time taken to expand from one size to another. The time taken was calculated by setting the initial triangle area (the Planck area) using the Planck length constant ($1.61619926 \times 10^{-35}$) and the final area was set to an arbitrary area size of 1 (cm).

2.2 Acceleration

Acceleration (a) was calculated by the following equation

$$a = \frac{\Delta \boldsymbol{v}}{\Delta \mathbf{t}} \tag{2}$$

Acceleration is measured in standard units per iteration ***cm***$^{-1t-2}$ and ***cm***$^{-2t-2}$.
Using the same methods as used to develop the Hubble diagram (as described above in

2.3) an 'acceleration vs. distance' diagram was created, regressed, and an expansion constant derived. Ratios of displacement expansion and acceleration were calculated by dividing the outcome of t_1 by the outcome of t_0.

The same method of ratio calculation was used to determine the change or expansion of the area.

2.3 Distance and Displacement, Hubble-Lemaitre Diagram

To answer question 3, distances between an arbitrary observation position and points on retrospective triangle bits were measured and analysed. Calculations were made on a second data table (table 2) on the spreadsheet.

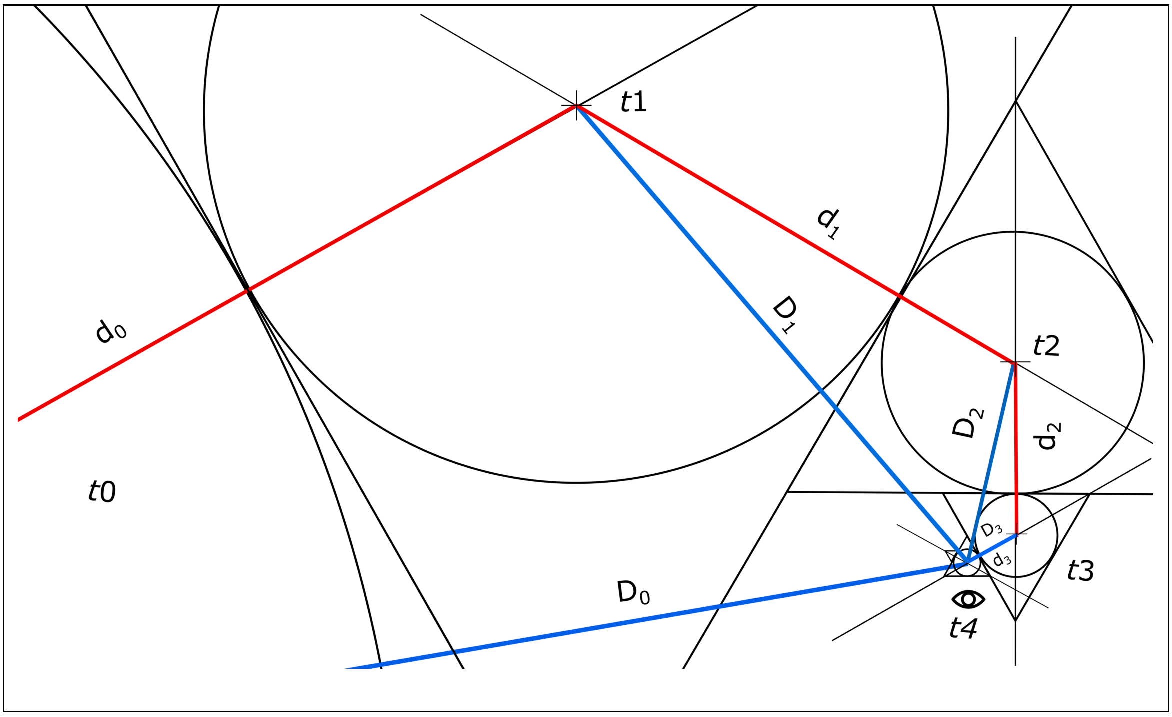

Figure 5. Measurement of Displacement and Distance Inside Fractal. Measuring the blue line displacement (*D*) and the red line distance (d) from an observer (*t*4) to triangle centre points inside an iterating Koch Snowflake fractal. *t* = iteration-time.

Triangle geometric centre points were chosen as the points of measurement as shown in Figure 5. The blue and red lines trace the displacement (D) and distance (d) respectively between an arbitrary observation point (*t*4) and triangle centre points. To calculate the displacement was out of the scope of this study; distances were calculated instead. Distances were measured by calculating the inscribed radius for each equilateral triangle by the equation (3) below. The total distance between points was calculated by adding the inscribed radius of the first triangle, for example, from t_0 to the inscribed radius of the next expanded triangle t_1.

$$r = \frac{\sqrt{3}}{6} l \tag{3}$$

From the radius distance measurements; total distance, distance expansion ratio, velocity, acceleration, and expansion acceleration ratio for every iteration-time were calculated using classical mechanics equations. Velocity (v) was calculated by the following equation

$$\boldsymbol{v} = \frac{\Delta \boldsymbol{d}}{\Delta \boldsymbol{t}} \tag{4}$$

where distance($\boldsymbol{d}$). Velocity is measured in standard units per iteration cm^{-1t-1} for receding points and cm^{-2t-1} for the increasing area.

To test for Hubble's Law, a Hubble (like) scatter graph titled 'The Fractal/Hubble diagram' was constructed from the results of the recession velocity and distance calculations (in table 2 of inverted fractal spreadsheet file). On the x-axis was the displacement (total distance) of triangle centre points at each iteration-time from t_0 and on the y-axis the expansion velocity at each iteration-time. A best-fitting linear regression line was calculated and a Hubble's Law equation (5) was derived

$$\boldsymbol{v} = \boldsymbol{H_0 d} \tag{5}$$

where $\boldsymbol{H_0}$ is the (present) Hubble constant (the gradient).

2.3.1 Test Measuring Real Displacement

The propagation of triangles in the (inverted) Koch Snowflake fractal, is not linear but in the form of a logarithmic spiral—as shown in Figure 2 B (above), and Appendix Figure 15. The method thus far assumes and calculates the linear circumference of this spiral and not the true displacement (the radius). This method was justified by arguing the required radius (or displacement) of the logarithmic spiral calculation was too complex to calculate, (and beyond the scope of this investigation), and that expansion inferences from inverted fractal could be made from the linear circumference alone. A spiral model was created independently, and radii were measured to test whether spiral results were consistent with the linear results in the investigation. Measurements were made using geometric software (see Appendix I Figure 15). Displacements and the derived Hubble diagram from this radius model were expected to show significantly lower values than the above (calculated) circumference non-vector method but share the same (exponential) behaviour. Appendix Figure 1 shows the distance between centre points, and blue, the displacement. See Appendix Figure 16, and Figure 17, and Table 1 for results.

2.4 Small Scale Long Scale Point Distribution Analysis

To address question 5: the number of triangle sizes per total distance increment on the fractal-Hubble diagram was calculated by counting the number of triangle sizes (in distance column in table 2) and dividing this by the distance increments measured in the sample. See Table 2a of the spreadsheet model. The number of triangles at each increment was calculated by totalling the number of triangles (from table 4) for each respective iteration-distance. An amended Fractal-Hubble diagram was created combining (recessional) velocity with the number of triangles at every distance. See table 7 of the spreadsheet model.

3 Results

3.1 Accelerated Area Expansion

The area of the initial triangle of the inverted Koch Snowflake fractal increased exponentially.

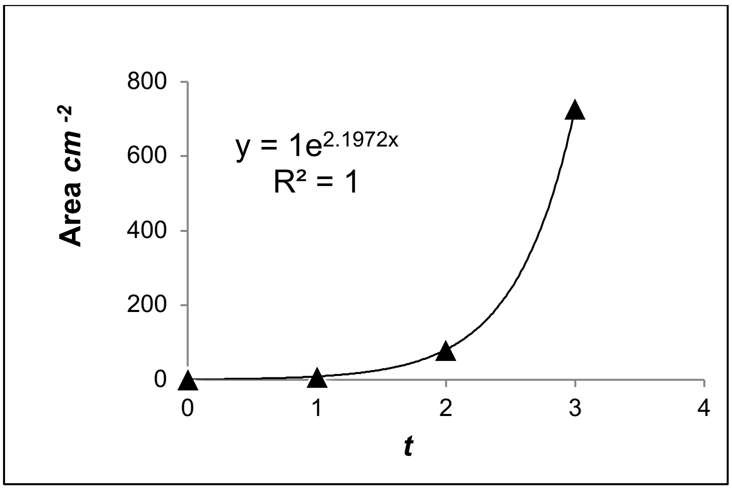

Figure 6. Initial triangle exponential area expansion. The area of the initial triangle bite on the inverted Koch Snowflake fractal increases exponentially with iteration-time. cm = centimetres.t= iteration-time.

This expansion with respect to iteration-time is written as

$$\boldsymbol{A = 1e^{2.197t}} \qquad (6)$$

The area of the total area of the fractal (Figure 7 A) and the distance between centre points (Figure 7 B) on the fractal increased exponentially.

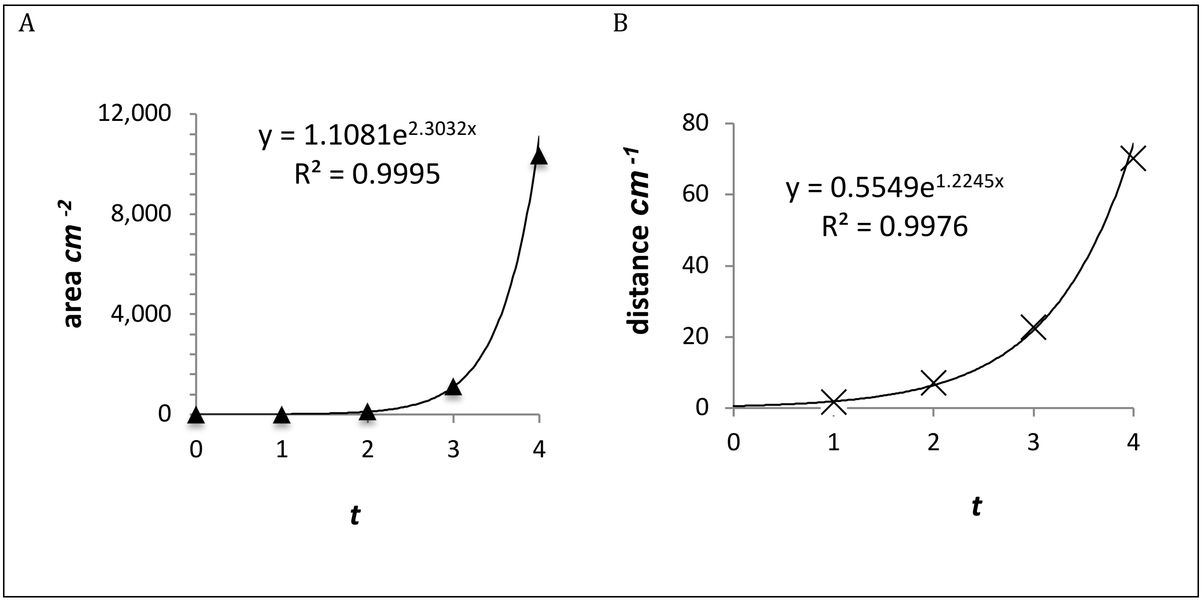

Figure 7. Area/distance expansion per iteration-time on the Inverted Koch Snowflake fractal. (A) total area expansion and (B) distance between points. cm = centimetres.*t*= iteration-time.

The expansion of the total area ($\boldsymbol{A^T}$) is described as

$$\boldsymbol{A^T = 1.1081e^{2.3032t}} \qquad (7)$$

The expansion of distance between points (***d)*** is described by the equation

$$\boldsymbol{d = 0.5549e^{1.2245t}} \qquad (8)$$

The (recession) velocities for both total area and distance between points (

Figure 8 A and B respectively) increased exponentially per iteration-time.

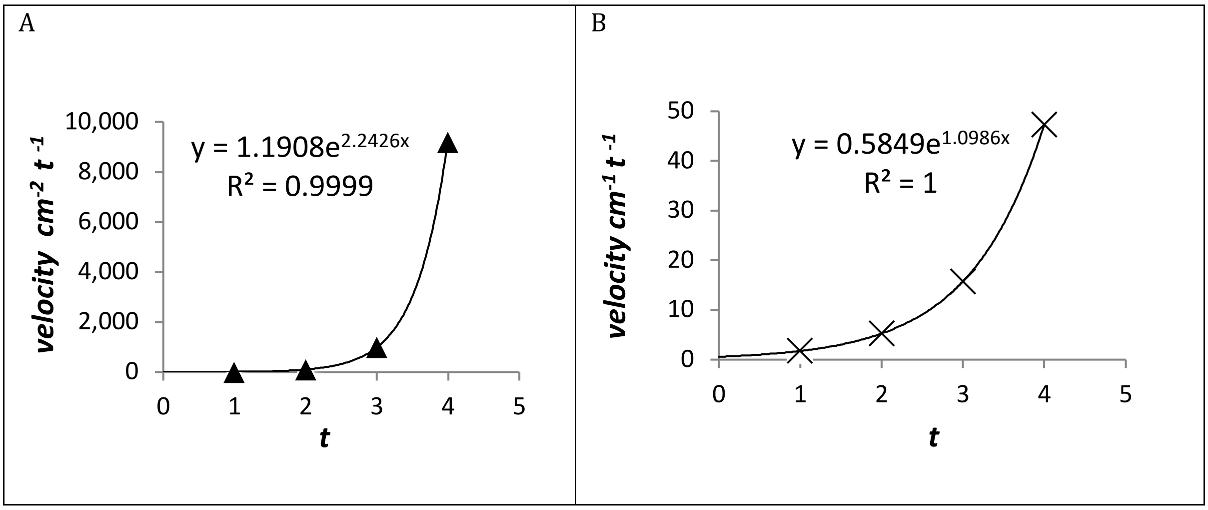

Figure 8. (Expansion) velocity of Area and Points on the Inverted Koch Snowflake Fractal. Expansion velocity of the inverted fractal at each corresponding iteration-time (*i*): (A) expansion of the total area, and (B) distance between points. cm = centimetres, *t* = iteration-time.

Velocity is described by the following equations respectively

$$\boldsymbol{v} = \mathbf{1.1908}\boldsymbol{e}^{\mathbf{2.2426}\boldsymbol{t}} \quad (9)$$

$$\boldsymbol{v}^{\boldsymbol{T}} = \mathbf{0.5849}\boldsymbol{e}^{\mathbf{1.0986}\boldsymbol{t}} \quad (10)$$

where $\boldsymbol{v}^{\boldsymbol{T}}$ is the (recession) velocity of the total area; and $\boldsymbol{v}$ the (recession) velocity of the distance between points.

The accelerations for both total area and (recession) distance between points (Figure 9A and B respectively) increased exponentially per iteration-time.

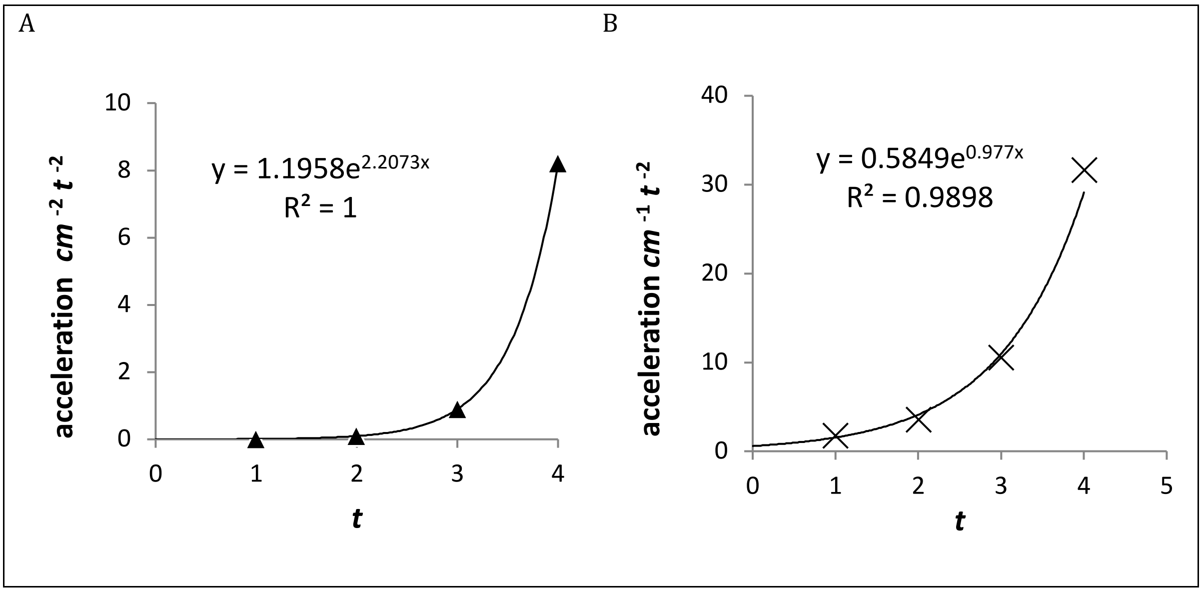

Figure 9. (Expansion) Acceleration of Area and Points on the Inverted Koch Snowflake Fractal. Acceleration of the inverted fractal at each corresponding iteration-time (i): (A) expansion of the total area, and (B) distance between points. cm = centimetres.*t*= iteration-time.

Acceleration is described by the following equations respectively

$$\boldsymbol{a^T} = \mathbf{1.1958}\boldsymbol{e}^{2.2073t} \tag{11}$$

$$\boldsymbol{a} = \mathbf{0.5849}\boldsymbol{e}^{0.977t} \tag{12}$$

where $\boldsymbol{a^T}$ is the (recession) acceleration of the total area, and **a**, the (recession) acceleration of distance between points.

3.2 Inflation Epoch Expansion

From equation (11) the development of the fractal takes 72.59 (2s.f.) iteration-times to expand from this arbitrary small area to the arbitrary large area of 1 $\boldsymbol{cm^{-2}}$. a

$$\boldsymbol{t} = \frac{\mathbf{1}}{\mathbf{2.2073}} \ln\left(\mathbf{2.61223} \times \mathbf{10^{70}}\right) \tag{13}$$

3.3 The Fractal/ Hubble-Lemaitre Diagram

As the distance between centre points increased (with each corresponding iteration-time) the recession velocity of the points also increased — as shown in Figure 10 below.

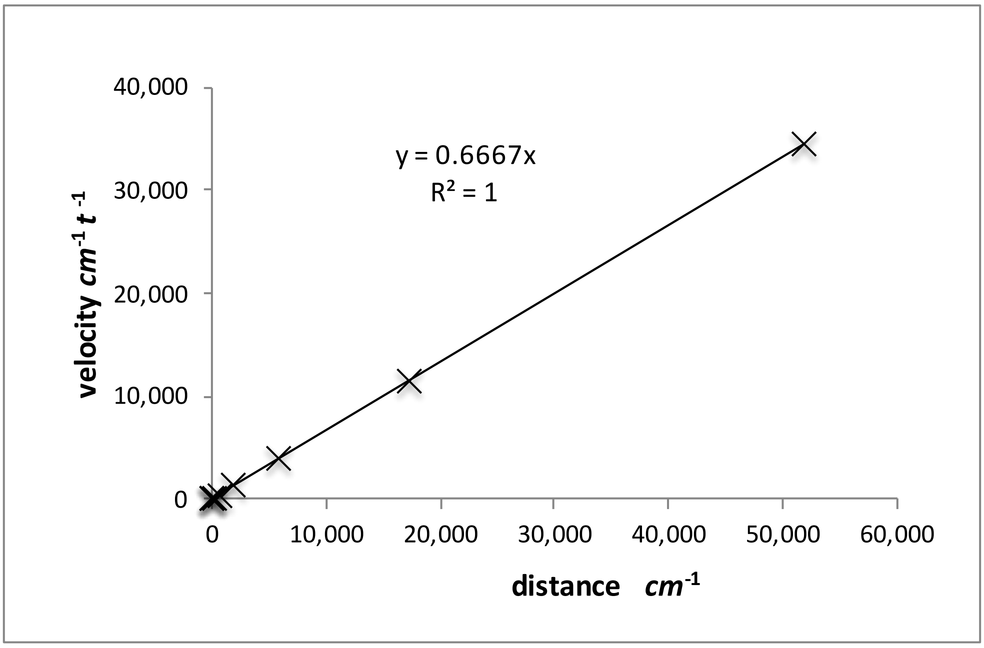

Figure 10. The Fractal Hubble-Lemaitre diagram. As the (exponential) distance between triangle geometric centres increases with iteration-time, the recession velocity of the points increases. cm = centimetres.t= iteration-time.

Recession velocity vs. distance of the fractal is described by the equation

$$\boldsymbol{v = 0.6667d} \tag{14}$$

where the constant factor is measured in units of $\boldsymbol{cm^{-1t^{-1}}\ cm^{-1}}$.

The spiral radius distance (***d***) velocity by experiment (see Appendix Figure 16 and Appendix Table 1 for details) resulted in a Fractal-Hubble equation of

$$\boldsymbol{v = 0.6581d} \tag{15}$$

In terms of acceleration vs. distance from the observer; the recession of points accelerated away with increasing distance — as shown in Figure 11 below.

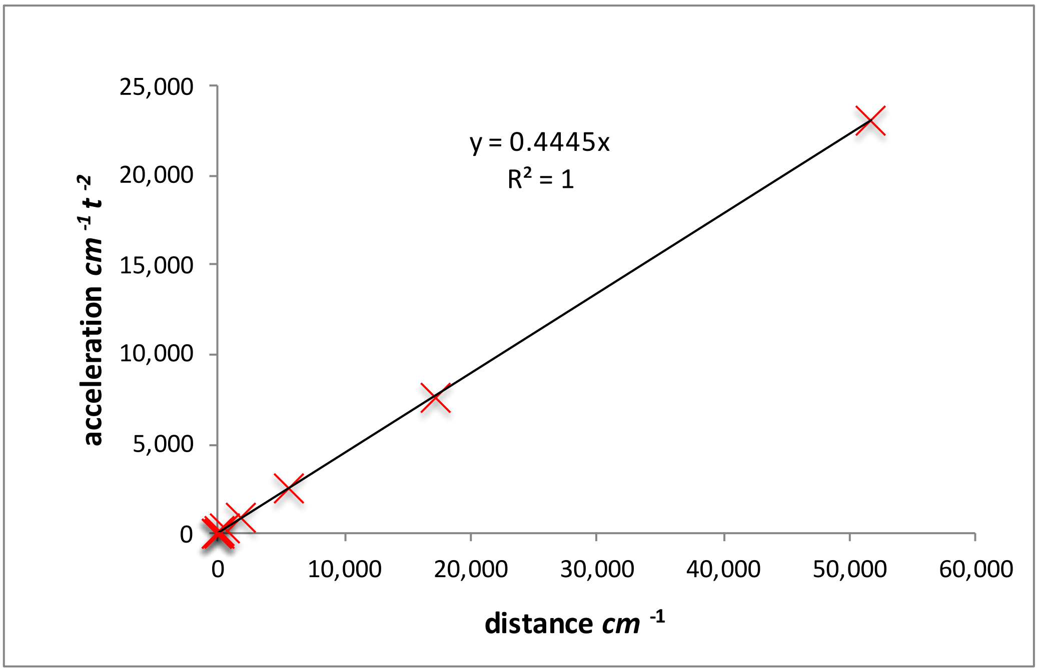

Figure 11. Recessional Acceleration vs. distance on the Inverted Koch Snowflake Fractal. As the distance between triangle geometric centres increases with iteration, the recession acceleration of the points increases. cm = centimetres, *t* = iteration-time.

The recession acceleration of points at each iteration-time at differing distances on the inverted fractal is described by the equation

$$\boldsymbol{a = 0.4445d} \tag{16}$$

where the constant factor is measured in units of cm^{-1t-2}. a = acceleration; d = distance.

The spiral radius displacement (D) acceleration by experiment (see Appendix Figure 17 and Appendix Table 1 for details), was described by the equation:

$$a = 0.4295D. \tag{17}$$

3.4 Distribution of Points and Triangles with Iteration-Time

Eight of the ten measurement points are located inside the first (1.20E+4cm^{-1}) increment distance. The remaining 2 measurement points are outside this range.

Figure 12 below shows the number of triangles by distance — between geometric centres from the observer. The number of triangles decreased exponentially from 7.86E+05, at the observation point, iteration-distance 0, to a quantity of 1 at distance 51800cm^{-1} (iteration-10).

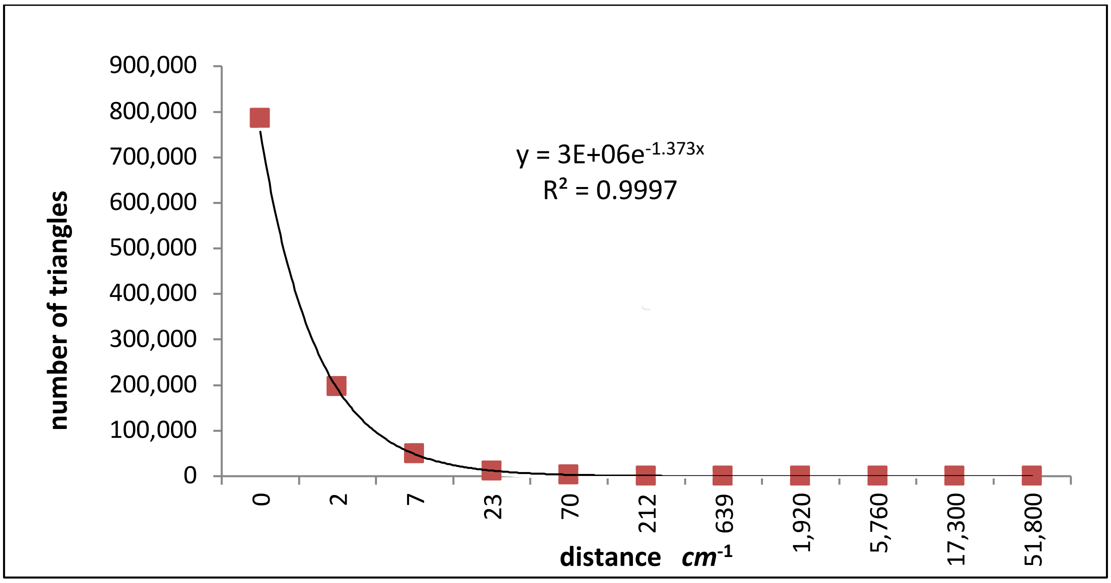

Figure 12. Number of Triangles at each Distance (Point) from the Observer on the Inverted Koch Snowflake Fractal first 10 iteration-times. As the distance between triangle geometric centres increases (exponentially) with iteration, and so increasing the distance from the observer, the quantity of triangles per iteration decreases exponentially to one — at time 0. cm = centimetre.

Combing the fractal-Hubble diagram (Figure 10) with the number of triangles at each distance point (Figure 12) produces a fractal Hubble point distribution diagram (Figure 13). The diagram reveals the relationship between the clustering of measurement points close to the (low recessional velocity) origin, and the smooth distribution (high recessional velocity) at large distances — towards the origin of the set.

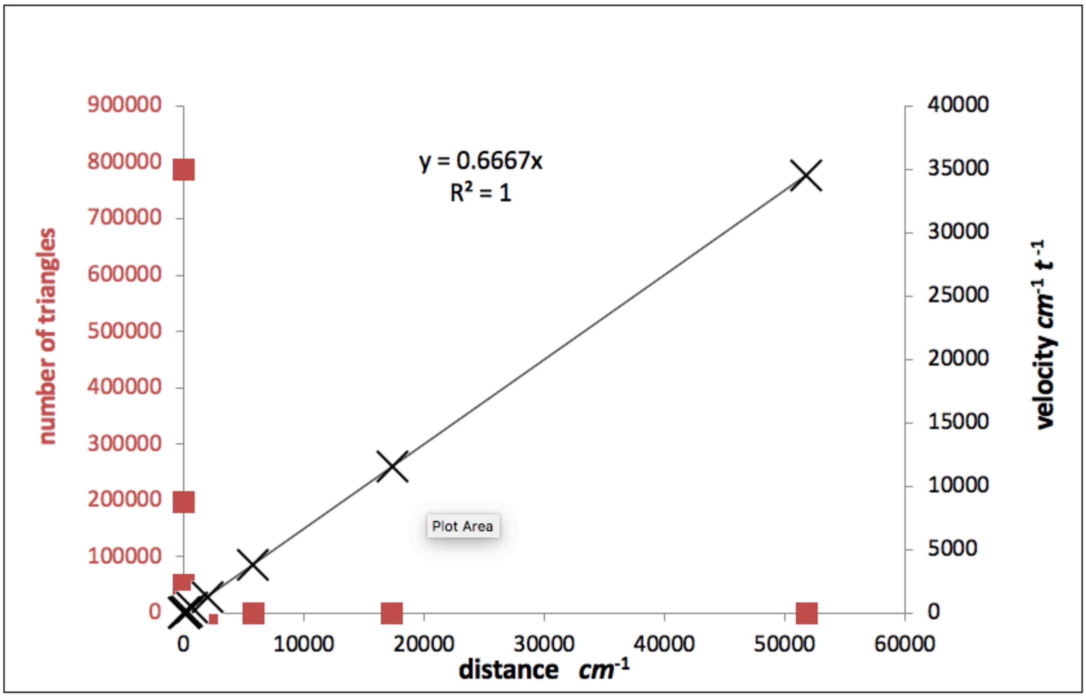

Figure 13. Fractal-Hubble Point Distribution Diagram. As the distance increases from the observer with respect to iteration-time: the recession velocity of the distance between geometric points increases; while the number of triangles at each distance decreases. cm = centimetre.

4 DISCUSSIONS

4.1 Singularity Beginning

The expansion of the first single triangle bit (

Figure 6) in this model demonstrates a singularity 'Big Bang' beginning. Its area begins from an arbitrarily small size, and may be set to the size value of the Planck area. More on this below.

This simplest of demonstrations is consistent with the observed very cool cosmic microwave background (CMB). It is not an explosion: it is an infinite exponential expansion of area — consistent with descriptions that 'space itself that is expanding'. The fractal in isolation is expanding into 'nothing', just as space is claimed to be. It has a frontier; however, any position beyond this is unattainable. To an observer anywhere in the set, this initial triangle (t_0) will dominate the extreme horizon, but it will not be seen by all observers. If an observer is more than 7 ± 2 iterations distant from (triangle) bit t^0 and observing without any form of technology — to 'zoom' back in iteration-time, the said bit will not be seen: this is the fractal distance. The 7 ± 2 is derived from the classical emergent development of the fractal as shown in Figure 2 may be termed the equilibrium iteration count or observable fractal distance.

The fractal model implies the beginning was a specific place, as opposed to the standard model where the 'Big Bang' is everywhere. The space we on Earth inherit is emergent, is created and is newer than the original space.

4.2 Accelerating Expansion of the Fractal Explains 'Dark Energy' Conjecture

The acceleration property of the fractal—Figure 9—is consistent with the 1998 astronomical discovery (by observation) of the accelerating expanding universe and conjectures surrounding the term 'dark energy' and the cosmological constant (lambda). It can be inferred (from this inverted fractal model) that the accelerating expansion of the universe with respect to distance (Figure 11) is a property of fractal geometry, and can be described by the equation

$$\boldsymbol{a} = \boldsymbol{F_a d} \tag{18}$$

where $\boldsymbol{F_a}$ is the fractal (cosmological) recession acceleration constant measured in units of $\boldsymbol{cm^{-1t-2}}$ $\boldsymbol{cm^{-1}}$. The constant $\boldsymbol{F_a}$ in equation (18) may be interpreted as a fractal a 'cosmological constant' — lambda — with respect to point acceleration and distance.

The acceleration between points with respect to time (from equation (12) is described as

$$\boldsymbol{a} = \boldsymbol{a_0 e^{F\lambda i}} \tag{19}$$

where the constant $\boldsymbol{F\lambda}$ may be interpreted as a fractal 'Cosmological Constant' Lambda with respect to point acceleration and iteration-time.

With continual entry (or birth) of new triangles into the fractal set, the total fractal area (Figure 11 above) of the total universe, grows exponentially. The total area expansion with respect to time is described by the function

$$A^T = A_0 e^{F\Lambda t} \quad (20)$$

where $F\Lambda$ is a fractal constant with respect to total area expansion and time.

4.3 Fractal Growth Consistent with Inflation Epoch Expansion

The expansion rate of the isolated (unbounded) fractal demonstrates the conjected early universe expansion rate [4]. The opportunity is open to further modelling. Papers on the 'universe ticking' of light' have conjectured the 'ticking' be at around 10^{-33} per second[25],[26]. If the production of triangle bits by iteration of the fractal is set to correspond to this oscillation frequency or 'ticking' of photons, this 72.59 iteration-times to expand from the Planck area size to a size of one may be found to be consistent with conjectured inflationary epoch speeds.

4.4 Hubble-Lemaitre Law

The shape of the fractal-Hubble curve (Figure 10) has direct significance Georges Lemaitre's conjecture surrounding the expanding universe [23] and Edwin Hubble and Humason's 1929 concurring observations [27]. The fractal model (Figure 10) demonstrates the Hubble-Lemaitre Law where from any observation point within a fractal the recession speed of points increases with distance, without any talk of 'rising raisin bread' or 'rubber sheets' as used as demonstrations by experts today.

When velocity (v) is plotted against the distance of points (D) (Figure 10, and Appendix Figure 16) the inverted fractal demonstrates Hubble's Law described by the equation

$$v = F_v D \quad (21)$$

where (F_v) is the slope of the line of best fit — where the fractal (Hubble) recession velocity is constant. The scale invariance of the Fractal-Hubble diagram concurs with the historical development of the Hubble diagram through the ages. From its 1929 original to the improved 1931 to its most recent, the shape of the diagram remains constant, just as with the fractal model.

4.5 Galaxy Distribution — Clustering of Measurement Points — Explains Small Scale Fractality

Figure 10 also shows the distance between measurement points on the fractal Hubble curve is not linear but increases in what appears to be exponential. Of course, this is a result of the increasing size of the triangles with growth. This observation has significance on the changing concentration of points—or galaxies—within these triangles. This further supports the decreasing distribution — the smoothness — of galaxies looking back in time.

The claimed 'small-cosmic scale fractality of galaxy distribution' (Figure 1A) as identified and concluded by the WiggleZ survey Figure 1B is, from the retrospective fractal model, what one would expect to observe if one were observing within a greater fractal.

Figure 12 shows that from the origin on the Fractal-Hubble diagram a quantity of 786,432 triangles is first observed, all of which are the same size as the observer's triangle viewing position. This quantity of bits also corresponds to the clustering of the measurement points near the origin of the diagram and this is due to the location the observer is within the emergent (inverted) fractal and the relative size of these triangle bits near the observer. The observer is 'in the branches' so to speak. Indeed, the best analogy fractal to visualise this geometric perspective is the tree plant fractal. It is as if the observer is on a branch of the tree (see section 4.6.11) surrounded by branches of similar age and size and is able to look back — down — to the trunk of the tree, which was the origin of the tree and has now expanded.

The observer will not see all these triangles, how many they will see is beyond the scope of this investigation, but it will be many. As we view further out, the quantity of triangles decreases and while the area of the respective triangles increases. This property of clustering near the origin is scale-invariant: no matter the distance, this pattern of clustering near the origin will remain.

Using a tree as a metaphor to model the fractal universe is not to say the universe is a fractal tree structure; it is to say, that just as a tree is a fractal structure, the universe is a fractal structure.

4.5.1 LQGs and Large-scale Structure Observations Bough Branches

The 'very large', 'thin' and old large-scale survey structures in the assumed smooth universe concur totally with my model of the fractal. They are the 4 billion light-years in sized Huge 'Large Quasar Group'[18] and the 10 billion light-years sized Hercules—Corona Borealis Great Wall [19]. They represent the large 'bough branches' away from the CMB 'trunk' of the fractal structure. To support this claim the structures are very large, they are also old in age — being composed of quasars; and are rather thinly distributed, compared to the small-scale clustered region.

4.6 Raised Questions

4.6.1 Which Fractal Shape?

This investigation also does not in any way suggest the universe has the shape of a tree or a snowflake: fractal-expansion could have equally been demonstrated using the Sierpinski triangle. The universe shares a feature special to fractals: fractals come in many forms, what that form is beyond the scope of this paper.

4.6.2 Accelerating Growth and the Development of the Fractal Tree

The growing tree is the perfect example of a fractal and stands as the perfect real-life metaphor of the inverted fractal model, they have similar properties. If the retrospective observation from deep within a snowflake fractal is substituted with an observation from high within a common branching tree, the clustering of points on the Fractal-Hubble diagram would equally correspond to the clustering of self-similar (sized) branches — in the tree — surrounding the observer. If the observer were to look down, inwards from the outer branches — towards the trunk of the tree — the branch (nodes) quantity would decrease, the volume of the single branches would increase, and the branch 'clustering' would smooth out. In a recent publication, it was found trees were found to be growing at an accelerating rate [20],[32]. The study measured up to 80 years of tree growth, on more than 600,000 trees, over 6 continents and found that the growth of 97 per cent of the trees was accelerating with age. This accelerated growth rate with time is a mystery to biologists.

Trees and all plants are the perfect examples of fractals. A tree's growth is generally described as being of 'natural' fractal geometry (or L systems). This phenomenon of acceleration of plant growth may be explained by the plant's growth being fractal. If the productive leafy stem of the emergent tree (Figure 3A-A) becomes the focus of the tree's growth and is held constant in size — just as with the standard triangle size is to the inverted Koch snowflake — then the older branches and the load-bearing trunk of the tree will grow exponentially with iteration-time — again just as the snowflake did.

4.6.3 Expansion in Excess of Light Speed and the Cosmological Principle

Following from the above (4.3 and 4.6.4), the modelled *retrospective* inverted fractal-expansion demonstrates — and is consistent with — space's ability to expand 'extremely fast'. If we think about the production of the fractal from the classical fractal perspective (Figure 2 A) and that this production has a speed, a rate of production that is propagated akin to the propagation of a light photon, then if we compare this speed with the inverted expanding area behind the fractal the complete model makes sense and the claim 'space expands faster than the speed of light' as proposed by Albert Einstein in his General Theory of Relativity and as conjectured by inflation theory.

Arbitrary points on the surface of the original — iteration 0 — triangle may be assumed to be close enough to assumed to have 'causal contact'; however, with the exponential expansion of the fractal object, this contact will not remain and the points will exponentially expand apart at a rate demonstrated from this experiment (4.3) Concerning the speed of light; the fractal has a constant propagation speed, this speed can be assumed, in principle, to be able to be surpassed by the (accelerating) area expansion 'speed' of the fractal itself. This fractal expansion speed claim is also consistent with and addresses issues surrounding the particle horizon problem and the cosmological principle (axiom) as discussed in 4.6.9.

4.6.4 The Fractal and the Speed of Light.

From the 'classical view' of the fractal Figure 2A, there may be strong insights gained on the nature and behaviour of light — it seems to point towards that. If this is so, this may help understand why the universe expands and behaves the way it does and also help unify the large-scale universe with the quantum nature of the universe. One question that may need addressing for a fractal understanding is that light may not be constant. If light is by nature following a fractal geometry, then it may mean light is not constant at large scales. Current experiments I am running on the fractal and its light characteristics are not — so far — pointing to any concept of constant 'light speed', but the fractal fits many of its other properties. This may have implications on the age we perceive the universe to be: why is it so young — relative to the age of our solar system? Could it be there is a distortion to how we receive the light information?

4.6.5 Multiverse,

With some trepidation, fractal-expansion and the fractal itself is consistent with conjectures surrounding a multiverse as it demonstrates multiple beginnings. An isolated fractal, by definition, has no arbitrary single beginnings and is an infinity of beginnings.

4.6.6 Emergent History and the Big Bang

A fractal universe would imply an emergent structure — the whole is made of many parts — just as the tree is made of many branches. It may force us to question the initial conditions of the Big Bang beginning. Namely, whether all mass (in the universe) was together in one place and at one time. It could now be argued — from the principles of fractal emergence — the universe developed/evolved mass from the bottom up, with the passing of time. It started small, from a seedling and developed structure. However, this does not explain the extreme temperatures claimed. There is a begging question from the hot dense 'Big Bang'; how can there be dense and heat before the time of — at least — photons? Was it emergent all the way?

4.6.7 Decreasing Fractal Dimension looking Back

Recent studies have shown fractal dimension decreases with increased z values [29]. This complements my model and claims as the complexity of the fractal system 'develops' with iteration-time.

4.6.8 Addressing Dark Flow, the Great Attractor and Dissenting 'Dark Energy' Papers

At the time of this update, there have been papers published [30] — based on the existence of so-called 'dark flow' and the Great Attractor which appears to be 'flowing' in the opposite direction as to 'dark energy accelerating observations — that challenge the observations pointing to an accelerating universe (and thus the existence of dark energy).

I believe the fractal model can address these rebuttals as being part of the fractal system. If an observer is assumed to be within the fractal set (the universe), which I am assuming we are in my model; then a flow in the opposite direction to the early and older parts of the fractal — as claimed in the paper — is to be expected, even predicted as part of the continued growth of the system. To use my analogy of the fractal tree, the former and older branches — even trunk — are expanding and accelerating behind us, while in front of us new branches are forming. There will appear to be a flow in the opposite direction, and it would be reasonable to think that the 'flow' of growth points back to the observer. Of course—again—I am not suggesting the universe is a tree, but I am suggesting it acts like to geometry of a tree.

4.6.9 Dark Matter Halo Trees and the Evolution of Stars and Galaxies

Something that is rather beyond the scope of the investigation but important enough to mention as it is seen by the author to be inextricable to the fractal model is the evolution of galaxy demographics and distribution in the form of Dark Matter halo trees. From a presentation given by Sandra Faber on this subject, a fractal interpretation of the universe would give rise to this 'fractal' tree structure; again, from smooth and thin at far and early distances, to rough and clustered nearby.

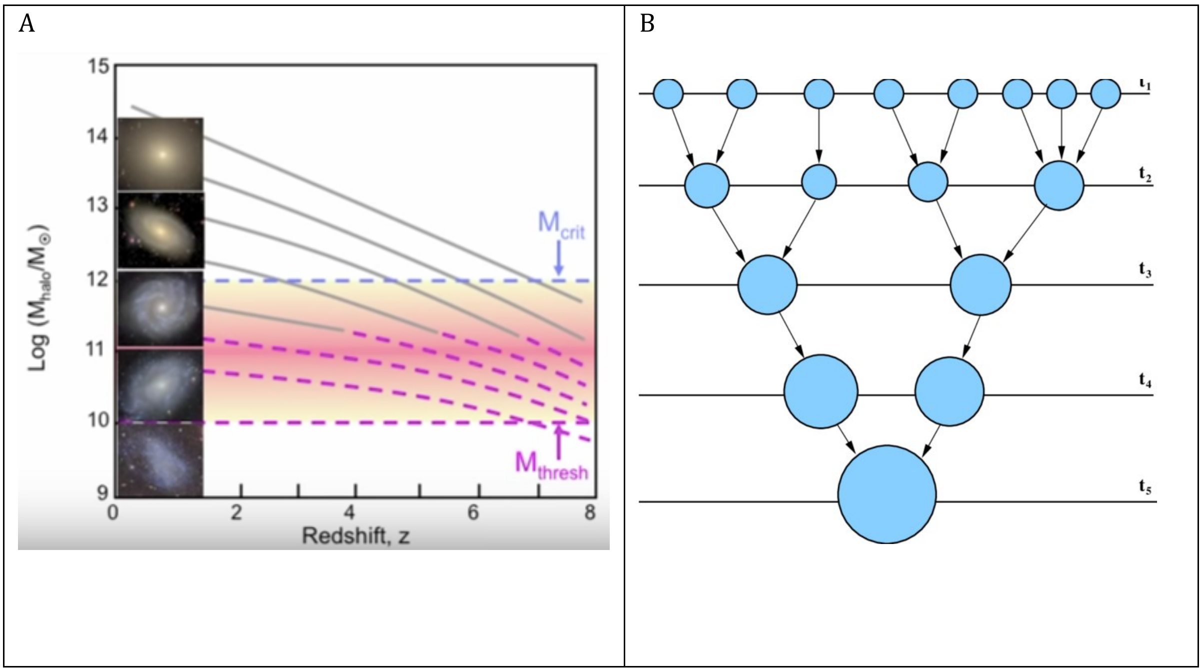

Figure 14. Fractal Dark Matter Halo Trees and the Evolution of Atoms, Stars and Galaxies. 'A' Diagram showing the age and size structure of the galaxies — we (Earth) are surrounded by large and old galaxy clusters. 'B', the classic Dark Matter Halo tree — evolving from early t_1 (top) to large clusters t_5 (bottom).

The significance of these merger halo structures is that they concur and correspond with an evolving emerging fractal model universe as revealed in this study. Halo trees are what one would expect to see if observing within a fractal space.

4.6.10 The Fractal Refutes the — Homogeneity and Isotropy — Cosmological Principle

Observations concurring with an in-situ fractal perspective reveals the Universe to be neither homogenous nor isotropic. The standard model of cosmology's key assumption

— the cosmological principle — maybe, as it stands today, be a mere illusion, a false paradigm.

Before explaining how this fractal model does not conform to the status quo — something that has been continually explained throughout this paper — it should be made clear it is already claimed and granted by cosmologists in their explanations of the cosmological principle that based on modern observation it only holds on large scales — scales larger a redshift z factor of .25 (about 4 billion light-years) — and that on small cosmic scales it does not. The recent discovery of — thinner and older — large quasar groups (LQGs) and the Hercules—Corona Borealis Great Wall (4.5.1) that are beyond this z factor distance (beyond a z factor of 4) add strength to the large scale rebuttal [18],[31].

The following deals with this cosmological principle rebuttal.

1. On homogeneity: from this fractal experiment, distributions is not the same in all directions but rather the galaxy distribution will diminish with distance and time as explained in section 4.5. Smoothness will be observed on large — older — scales (towards the trunk), and clustered fractal activity on small — newer — scales (the branches); just as observed looking out from the Earth's position towards the singularity CMB smoothness. More on this in section 4.5. Also, as we look back in time, the fractal model concurs with observations and claims made about the evolution of galaxies — evidenced by dark matter halo merger trees structures (section 4.6.9). This corresponds to the 'old' LQGs discovery also.
2. On isotropic: in a fractal, observations will not be the same in all directions; points will be very different from different locations. As with a fractal tree modelled here, there is an obvious trunk to the structure and there are obvious clusters of branches, and these will not be observed isotopically in all directions. The view will be different if viewed from the perspective of the trunk, and if viewed from within the branches. In this fractal model, it remains true everything is receding away from any observer, but the view will be different — depending on the position of the observer — and thus not necessarily the same in all directions. There is a 'strange' fractal edge that has grown since the fractal's origin, and this edge appears — by the model — to also be 'the centre',

> though this has expanded and is viewed today — in part and consistent with the standard model — as the CMB. All space between this 'edge' and Earth observation is newer, and this is again supported by the evolution of galaxies.

But still, the cosmological principle persists. It is as if, and this claim has been made by some commentators, that even with the many said most modern observational facts, the cosmological principle must be saved; saved to save General Relativity.

4.6.11 Vacuum Catastrophe

Continuing from the above (4.6.3) the 'vacuum catastrophe discrepancy' may also be resolved by understanding the universe as a fractal and that we, the observer, are in one. As described in the introduction, the fractal shares a duality of perspectives from an observer in one; the classical (forward) view and the expanding (back) view, together they are different aspects of the one. This investigation focused on the expansion and has claimed this to be the dark energy cosmological constant. The classical aspect—outside this investigation—can be shown to behave as the quantum problem. The classical fractal demonstrates wave-like spiralling, smaller and smaller (wavelengths), higher and higher frequencies; while the expansion (behind) is in terms of exponentials.

In detail, focusing on the unit used to calculate the total area of the inverted fractal set at any iteration-time. If the standard fixed area size (the area of iteration 0 triangle) is used to calculate the total area of the set, the result will be a very large number; however, if the total area of the inverted fractal set is divided by the area sizes of the expanded triangles (allowing for their expansion at each iteration-time) the number will equate to a lower and more realistic number. The total area will equate to the total number of triangles propagated in the set. In principle, all triangles are as identical as the iteration 0 standard triangle, and only differ in scale due to the fractal expansion.

4.6.12 Fractal Age of a Universe.

The tree grows in terms of iteration-time, and not solar time. As trees grow, they lay down tree rings, these rings do not show exponential growth. Trees can generally—by counting the tree rings — age several hundreds of years old, but in terms of fractal age may only be some 4 to 7 iteration-times old. Could it be the universe is a similar paradox in that what we observe is not the true age of the universe, it is older?

4.6.13 General Relativity

What a fractal universe means for the future of General Relativity theory is unclear and beyond the scope of the author — though it is conceivable it may have to be adapted to take into account the geometry of the fractal. Work has already begun in this area: from noted theorist Laurent Nottale [33],[34] and others [35]. It should be made clear; this fractal model does not point to anything to any new insight to do with gravity.

4.6.14 On Blackholes and Galaxy Formation

Outside the scope of this investigation; spiral galaxy formation and black holes at their centres may be explained by the fractal as posited by Lori Gardi [36].

4.6.15 Quantum Mechanics (Like) Properties of the Fractal

Viewed from an (arbitrary) position outside the set a fractal will grow at a decreasing rate to form the classical fractal shape — a snowflake as shown in Figure 2A. But from the perspective of an observer within the fractal set the same expansion will appear to expand. This assumption of observation from within the set, looking forward from a fixed position has an uncanny resemblance to properties and problems shared with objects described only by the quantum mechanics and the electromagnetic spectrum.

When isolated, the iterating (snowflake) fractal is an infinitely of discrete triangles (bits). The snowflake is a superposition of all triangles, in one place, at one time. The production of new triangles propagates in the geometry of a spiral: rotating in an arbitrary direction to form — when viewed from a side elevation — a logarithmic sinusoidal wave, comparable to the described electromagnetic spectrum. This spiralling wave-like propagation is illustrated below in Figure 2B and Appendix Figure 15.

Location or position within this infinite set is only known when observed or measured; otherwise, all positions are possible — at the same time. These quantum-like features of the fractal are essential background to this investigation — one that will not be taken further in this publication, but cannot be overlooked and will be the topic of my next paper. Together the dual perspectives will make sense of the universe.

5 CONCLUSIONS

The in-site fractal model is an exquisite and inextricable fit to what is observed and conjectured in the cosmos. This fractal model demonstrates and addresses problems directly associated with the ΛCMB model. From a fixed (but arbitrary) location within a (Koch snowflake) fractal set — and from its beginning — the areas of triangles bits expand exponentially and marked points (on triangles) recession velocity from 'the observers' perspective also increased exponentially as a function of distance and time. This exponential expansion is a property shared by all irregular/chaotic fractal objects. The fractal model explains the conjectured 'dark energy' accelerated expansion of the universe. The model produced a cosmological constant. The fractal offers a geometric mechanism that explains the presence of the CMB and corresponds directly with conjectured expansion times of inflationary epoch expansion of the universe. There is an opportunity to further test and tie the fractal to the speed and nature of light and this (inflationary) expansion. The model demonstrated the expansion of space and reveals directly both a Hubble-Lemaitre Law and diagram. Both observations and the fractal model refute the cosmological principle. The fractal model explains and concurs with the distribution and demographics of galaxies in the observable universe — from the granted 'rough and fractal' on small cosmic scales to the old large and thin LQGs structures on large-cosmic-scales. As a result of the former, the model is in total violation of the cosmological principle. We do observe homogeneity, nor should we expect to in observing with a growing fractal, and the universe is therefore not isotropic. The fractal model offers a direct solution to the cosmological catastrophe, that the quantum and the cosmos are different sides of the one emergent geometry. From the former, the model offers the opportunity to further our understanding of foundational quantum mechanics. The mechanism of fractal development, growth and emergence points to how quantum mechanics—the wave-particle duality of light and matter—is described by experts and this demands further exploration. I have found that looking 'back' — as I have with this investigation — models the cosmos; I hypothesis that looking 'forward' into the fractal from an in-situ observation point — models 'the quantum'. Together, fractal geometry will complete the gap of knowledge. The fractal opens the door to a quantum unification, so-called quantum gravity. The model does not take away from what has already been achieved — namely General Relativity — it complements it and is a simple geometric.

The universe, by observation, is behaving exactly how like a growing fractal. If we had no cosmological observations, and only fractal geometry to work from to make predictions on what the structure and evolution of our universe might be — derived from its apparent universal ubiquity — it would match. This is not the first time a geometry has solved observational discrepancies or paradoxes; one only has to look at how circles and later ellipses explained ended the paradigm of epicycles.

6 APPENDIX

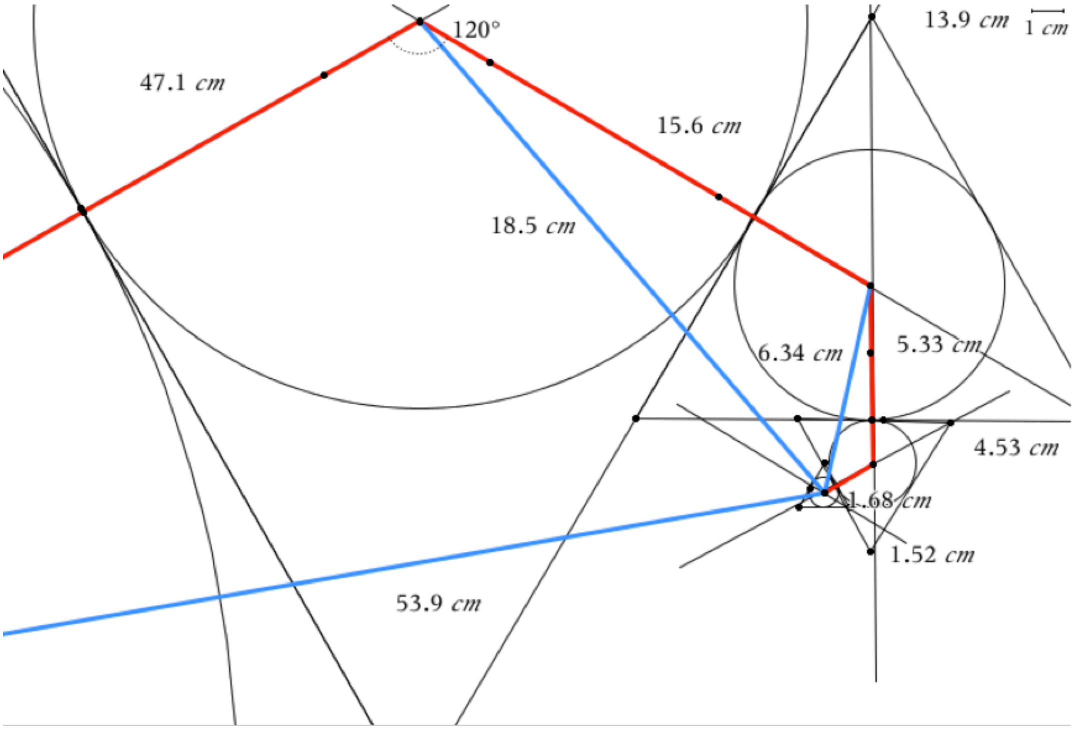

Figure 15. Displacement measurements from radii on the iterating Koch Snowflake created with TI-Nspire ™ software. Displacement is measured between (discrete) triangle centres and used in the calculation of the fractal/Hubble constant. The red line traces the circumference (the distance) of the fractal spiral, and the blue line the displacement of the fractal spiral from an arbitrary centre of observation. cm = centimetres.

Table 1. Displacement taken from radius measurements and calculations from the iterating Koch Snowflake fractal spiral (Appendix Figure 15).

t	Displacement: cm	Total Displacement: (***D***) cm	Expansion Ratio	Velocity: $cm\ i^{-1}$	Acceleration: $cm\ i^{-2}$	Acceleration Ratio
0			-			
1	1.68	1.68	-	1.68	1.68	
2	4.66	6.34	3.77	4.66	2.98	1.773809524
3	12.16	18.5	2.92	12.16	7.50	2.516778523
4	35.4	53.9	2.91	35.40	23.24	3.098666667

cm = centimetres.t= iteration-time.

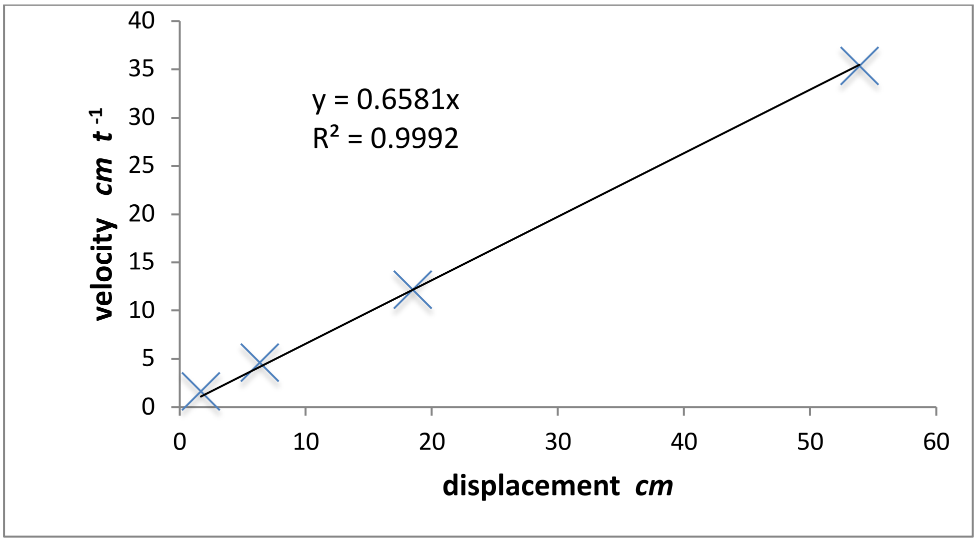

Figure 16. The Hubble Fractal Diagram (recessional velocity vs. distance) from radius measurements (Appendix Figure 15). From an arbitrary observation point on the inverted (Koch Snowflake) fractal: as the distance between triangle geometric centres points increases, the recession velocity of the points receding away increases. cm = centimetres.t= iteration-time.

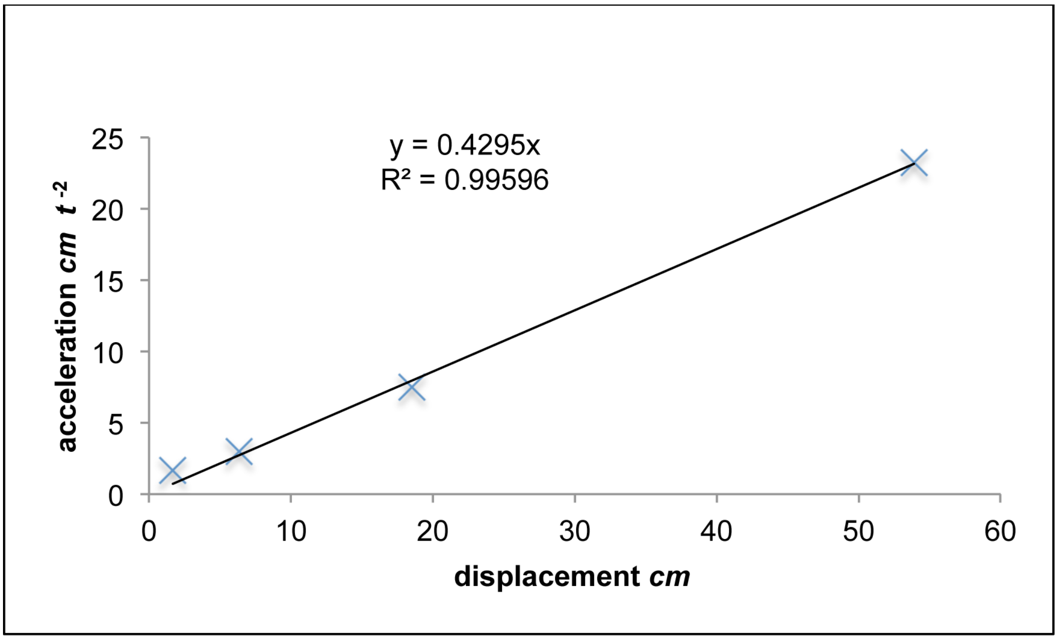

Figure 17. Recessional acceleration with distance on the inverted Koch Snowflake fractal. from a fixed central observation point. Using radius measurements (Appendix Figure 1): as the distance between triangle geometric centres points increases, the recession acceleration of the points receding away increases. cm= centimetres. t = iteration-time.

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