Gupta-Feynman based Quantum Theory of Gravity and the Compressed Space

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Abstract

In this work we develop the quantum theory of gravity in the gravitational compressed space. The equivalence of spatial compression to the Lorentz contraction of special relativity, supported by the relative gravitational red-shift using the black hole clock leads to the brane potential and gives the minimum length at which the extra dimensions become dominant, comparable to that of the Schwarzschild radius. For Planck mass the minimum length is almost Planck's length. When doing the quantization of the theory, we find that those responsible for the evolution of time for luminous matter, graviton and for dark matter, the axion, have the property that in compressed gravitational space, naked and dressed propagators are equal and coincide with the corresponding naked propagators.

KEYWORDS:Cosmology; compressed space;Lorentz Contraction; Quantum Gravity

Contents

1 Introduction

With the red-shift observation, Hubble concluded that the universe is in continuous expansion, thus takes the credit of origin of the big bang theory. He discovered that velocity of a galaxy is proportional to its distance from the observer. The big bang theory enjoys the strongest support and confirmation from Arno Penzias and Robert Wilson's discovery of the cosmic background radiation in 1964 [1]. The special relativity says, gravitation alters space with a supporting argument, that space is distorted when a gravitational field bends light [2].
The Einstein's general theory of relativity says gravitational field alters time, and causes gravitational time dilation [3]. It considers gravity as a manifestation of the curvature of space-time. Massive objects distort time and space, and leaves the usual rules of geometry in doldrums with extreme severity near a black hole [4].
The whole history of human society remained puzzled with questions about time [5, 6]. Intervals of time, as they are measured by the recurrence of periodic events, are not impervious to everything. They instead depend upon the relative motion of the two systems in a gravitational field [2].

A lot of deliberations on the nature of time and space have been argued to challenge the relativity and the quantum theories [7],[8] [9],[10],[11]. In a pioneer work on time and relativity it is discussed that GR reintroduces the notion of temporal flow, where as Special Relativity does not support the notion. In an another argument the space-time of GR is considered just a timeless as the special relativistic one. The other concept argued is that the Big Bang should not be considered as the birth or beginning of the universe, but as one of the possible boundaries of a strongly deformed (timeless) space-time block [12].
In special relativity, the moving observers notice clock rate change by a Lorentz factor, the general theory of relativity justifies the change in clock rate as a response to the change in the gravitational potential from point to another, i.e., gravitational red-shift is a property of general covariance, which is global principle of general relativity. Thus the red-shift is characterized by the change in the gravitational potential, thus introducing a timeless state of gravity that was studied in details by one of the authors in [13, 14]. A catchy study on contraction of space and the gravitational field without going into the mathematical complexities has shown the equivalence of gravitationally influenced spatial compression and the Lorentz contraction of special relativity [15]. In another such work [16] the permitivity and permeability of free space are supposed to vary with gravity [17] and the Lorentz factor is not related to some inertial systems, it is shown to depend only on the variable density of the space. The impact of gravity on light is discussed from the perspective of variable density of the space. The gravitational waves are an outcome of ripples in space and gravitational lensing arises due to the gravity space fluctuations. Gravity originated from space fluctuations reduces the frequency of photons which is coined as gravitational red-shift [16].
In the present study we are using this equivalence and the gravitational red-shift to define the contracted distance between the two gravitationally interacting masses. The contracted distance in turn modifies the gravitational interaction and leads to the simple modification of Newtonian potential. The surprising result is none other than the brane potential [18, 19].
The relative gravitational red-shift between different points or slices of space indirectly supports the idea of space contraction. We will study it through the geometric prism of relative gravitational red-shift to show that the space contraction is not confined to only radial direction. Direction dependence of Spatial contraction is analyzed in this work using the clock postulate. The Newtonian potential has seen modifications at extreme scales, where the theory of gravity is not tested. The motivation behind these modifications is especially the discrepancy in the rotation curves with that of the theoretical predicted curves. Pertinent to mention few important theories in this direction are MOND [20, 21], MOG [22, 23], the non-commutative geometry [24, 25], the modification due to minimal length in quantum gravity [26, 27, 28], modification predicted in entropic gravity, the f(R) theories [29]. The brane world models draws its motivation from the existence of extra dimensions in string theory, and predicts the universe as a brane in a higher dimensional bulk [18, 19]. The Randall-Sundrum model permits the existence of infinite extra dimensions and stresses on the propagation of gravity into the bulk at large distances and thus modifies the Newtonian potential at such scales [30]. The Randall-Sundrum modified Newtonian potential is a power law deformation of the usual Newtonian potential [18, 19].
It has been discussed that Newton's force law pertains to only four non-compact dimensions, but in the presence of a non-factorizable background geometry the usual force law is not true. The hierarchy problem is dealt in higher dimensional mechanism where weak scale is obtained from the Planck scale through an exponential hierarchy. But it has been argued that the exponential has its roots in the background metric which consists of slice of AdS_5 space-time [18, 19]. In the context of black hole thermodynamics, Planck scale effect has been

analyzed [26]. Keeping into consideration the size of the observational universe, the need to see the effects at larger distance like Hubble's length is taken in a recent work [31]. The modifications in Newtonian potential has been addressed in three phases i.e. at short distance quantum effects due to $\hbar$, at intermediate distances classical mechanics is seen to work well and modifications are neglected and at very large distances the quantum effects reoccur. A recent work has been dedicated to study the clustering of galaxies by pursuing the non-local impact of gravity as the non-local extension of general relativity [32]. Exploring the chance of relevance of GR at small scales where quantum effects are dominant has experienced a drawback due to lack of experimental support, on the other hand examining quantum effects at large scales is turning into an interesting area of investigation because of the hopes pinned in future experimental supports [33]. The existence of the maximum length was raised in view of the cosmological particle horizon as the maximum measurable length in the Universe [34] The extension of the standard quantum field theory proposes that changes related to the multi-valued nature of the physical fields could be considered at large spatial scales [35]. The large distance logarithmic behavior of the gravitational potential could appear as an outcome of nontrivial properties of the vacuum state in Modified Field Theory (MOFT) [36].

The present letter is based on few important results. In one of the novel approaches, we use clock postulate to show that the gamma factor of special relativity arises from the gravitational red-shift, thus a geometrical or gravitational interpretation is assigned to the ratio $\frac{v^2}{c^2}$. The geometrical interpretation is further interpreted to be equivalent to the concept of space contraction in presence of a massive body. Considering the measurement at different distances being equivalent to the measurement at a single location but with a varying field, provides us a simple way to say that the distance between two gravitational masses contracts. Thus we replace the inter-particle distance by its contracted counterpart in the Newtonian potential which lead to the Randall-Sundrum modified potential. This provides us an opportunity to say that space contraction share its roots with extra dimension of the Brane world. The contracted inter-particle distance also paves a way to find the radius of black hole by assuming that once a massive body collapses to within its Schwarzschild radius a black hole is formed. This also gives us the idea of the strength of gravity near a black hole. We also study the clustering of galaxies in the contracted space by following the normal procedure of finding partition function and the thermodynamics of the system. Here we calculate the partition function between two defined limits with special emphasis on specific heat of the clustering in contracted space.

2 The clock measurements

As was showed in details in a paper by one of the authors [13, 14], the timeless state of gravity can be obtained for Rindler observer. In that timeless state, we can obtain gravitational or geometric interpretation of speed of light and mass. The idea of [13, 14] can be summarized as follows. One assumes an existence of Schwarzschild black hole with an event horizon. The gravitational red-shift which is a scalar function and is property of general covariance. The Rindler's can use this scalar function to set measurements on the nature of space around black hole through the relative values between any two points or slices of space-time. If we consider two points (A and R) in the gravitational field of black hole as shown in the figure 1, we notice that the two points form a triangle that follow a geodesics geometry of the considered black hole when connecting the two points with black hole center. If R and A are far enough from K, the triangle become approximately Euclidean triangle.

Between these two points A and R, there are two possible clock measurements that can be simply computed as follows:

1. Relative gravitational red-shift which is represented by the ratio at two different points

$$\frac{z_A}{z_R} = \frac{(1 - \frac{r_s}{r_A})^{-1/2} - 1}{(1 - \frac{r_s}{r_R})^{-1/2} - 1} \tag{2.1}$$

2. The difference in gravitational red-shift at two different points.

$$\Delta z = z_A - z_R = (1 - \frac{r_s}{r_A})^{-1/2} - (1 - \frac{r_s}{r_R})^{-1/2} \tag{2.2}$$

In the following section, we summarize the clock measurements in weak gravitational approximation

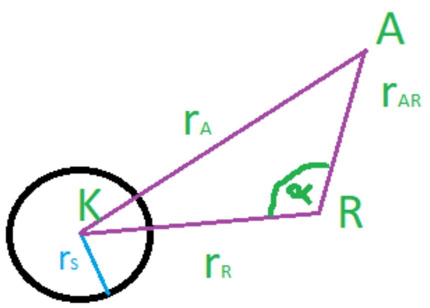

Figure 1: Black hole triangle of clock measurements

3 Weak Gravitational Approximation

3.1 Relative gravitational red-shift

We consider the weak gravitational approximations, $r_s << r_K$ and $r_s << r_R$. The gravitational red-shift for both A and R can be approximated as follows

$$z_A = (1 - \frac{r_s}{r_A})^{-1/2} - 1 \approx \frac{r_s}{2r_A} \tag{3.1}$$

$$z_R = (1 - \frac{r_s}{r_R})^{-1/2} - 1 \approx \frac{r_s}{2r_R}. \tag{3.2}$$

We compute the relative gravitational red-shift using Eq(2.1). We express it in terms of all lengths measured at R including the distance between A and R (r_{AR}).

$$\frac{z_R}{z_A} = \frac{1}{\sqrt{1 - \frac{r_{AR}^2}{r_A^2} + 2\frac{r_R r_{AR}}{r_A^2}\cos\alpha}} = \delta \tag{3.3}$$

As a consequence we impose:

$$\frac{v^2}{c^2} = \frac{r_{AR)^2}}{r_A^2} - \frac{2r_R r_{AR}}{r_A^2}\cos\alpha \geq 0 \tag{3.4}$$

From this last equation we can conclude that

$$\frac{r_{AR)^2}}{r_A^2} \geq \frac{2r_R r_{AR}}{r_A^2}\cos\alpha \tag{3.5}$$

Then we obtain:

$$\cos\alpha \leq \frac{r_{AR}}{2r_R} \tag{3.6}$$

From (3.4) we have:

$$\cos\alpha = \frac{r_{(AR)}}{2r_R} - \frac{r_A^2}{2r_R r_{AR}}\frac{v^2}{c^2} \tag{3.7}$$

Using now the cosine theorem we get:

$$r_A^2 = r_R^2 + r_{AR}^2 - 2r_R r_{AR}\left(\frac{r_{AR)}}{2r_R} - \frac{r_A^2}{2r_R r_{AR}}\frac{v^2}{c^2}\right) \tag{3.8}$$

From (3.8)we arrive to:

$$r_A^2\left(1 - \frac{v^2}{c^2}\right) = r_R^2 \tag{3.9}$$

Or equivalently:

$$r_R = r_A\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}} = r_A\left(1-\frac{v^2}{2c^2}\right) \tag{3.10}$$

And finally:

$$r_R = \frac{r_A}{1+\frac{v^2}{2c^2}} \tag{3.11}$$

On the other side, the measurement in local inertial frames are determined in terms of relative time dilation as follows

$$\frac{t_A}{t_R} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3.12}$$

where v is the relative speed between the two points A and R in the local inertial frames and c is the speed of light. We notice the relative gravitational red-shift or relative gravitational time dilation matches with the definition of time dilation in special relativity if the ratio r_{AR}/r_A can be replaced by ratio v^2/c^2. The match is legitimate and mathematically consistent since the relative gravitational red-shift introduces a local measurement which is the time dilation in local inertial frames. Therefore, for mathematical consistency of general relativity, its local measurement should be equivalent to measurement special relativity that hold only in local inertial frames. This means that the gamma factor of special relativity emerges as a ratio between the gravitational red-shift at A and R. The clock measurements depends only on **"one variable"**; the distance from the gravitational source, which is the reason for velocity ratios turned to be lengths ratios in this delta factor in Eq. (3.3). The ratio r_{AR}/r_A can be considered as a geometric or gravitational interpretation of the ratio v^2/c^2. This comparison can be written as

$$\frac{r_{AR}^2}{r_A^2} = \frac{2r_R r_{AR}}{r_A^2}\cos\alpha + \frac{v^2}{c^2} \tag{3.13}$$

For $\alpha = \frac{\pi}{2}$ we obtain

$$\frac{r_{AR}}{r_A} = \frac{v}{c} \tag{3.14}$$

This result is equivalent to the ratio of distance measured in absence of gravity to that of the distance measured in presence of gravity. We can interpret that taking measurements of different locations in presence of gravitational field is same as taking the measurement at a same location but with varying gravitational field. This obviously shows that space contracts in presence of gravity. Thus relative gravitational red-shift between different points or slices of space indirectly supports the idea of space contraction. The advantage of studying it through the prism/triangle of relative gravitational red-shift helps us to understand that, space contraction is not confined to only radial direction. Space contracts in different directions is clearly explained by the $\cos\alpha$. Space shows different contraction at different values of α. Thus if space is spherical in absence of gravity, it may deform into a different shape in presence of gravity as per the $\cos\alpha$ and thus giving a different value of eccentricity. A further interpretation can be given in terms of the shape of the orbit followed by a smaller mass around a bigger mass. This study needs a deeper insight in our future endeavors.

This would support the approach of time varying speed of light as a solution of cosmological puzzles that was suggested in [37]. It may support also the experimental findings of changing physical constants such as fine structure constant in gravitational field as shown recently in [38]. The result also supports the concept of compressed space in presence of gravitational field of massive objects [15]. In our case, the ratio v/c varies depending on the distance from the gravitational source. We note that time can be inserted easily in the previous equation as a "redundant variable" which suggest a possible timeless state which is consistent mathematically. The timeless state has been proposed in many contexts such as shape dynamics which introduce a gravitational origin of arrow of time [39, 40]. The timeless also emerged in Thermal time hypothesis which assume that time only flow in thermodynamics or statistical patterns [41]. It has been mathematically intuited as well that timeless universe is possible [42]

4 Space contraction

In relativity it is common procedure to discuss the variation in lengths of a theoretical meter stick, however it is important to understand that it is not a physical meter stick which shows contraction in length, it is space itself which experiences the compression. The radial compression of space due to massive body has been seen to yield same results as produced by curvature in space in general relativity. The same compression has been argued to be equivalent to the Lorentz spatial contraction of special relativity [15]. It has been discussed that in a gas of gravitational field the spatial compression is equivalent to the effective density of space under compression and is expressed in terms of refractive index of the gravitational field which acts as an optical medium $K\rho = \mu - 1$, with $\mu = e^{\frac{2Gm}{rc^2}}$ and K as the Gladstone-Dale constant which depends on the particular gas and slightly on the wavelength of the light [43].

$$\rho = \frac{1}{K}\left(e^{\frac{2GM}{c^2 r}} - 1\right) \tag{4.1}$$

The density has also been expressed as $\rho = \frac{x_0 - x}{x}$ in terms of the distance measurement x_0 in a region devoid of gravity and x the measurement made after considering the influence of gravitational spatial compression. This leads to

$$x = \frac{x_0}{\frac{1}{K}\left(e^{\frac{2Gm}{rc^2}} - 1\right) + 1} \tag{4.2}$$

The same expression yields gravitational red shift if distance is replaced by wavelength of light λ and is written as

$$\lambda = \frac{\lambda_0}{\frac{1}{K}\left(e^{\frac{2Gm}{rc^2}} - 1\right) + 1} \tag{4.3}$$

These concepts have been able to reproduce the gravitational red shift and gravitational lensing in the same form as predicted by general relativity [15].

5 Spatial Compression leads to modified gravity

The Newtonian potential ϕ is

$$\Phi(r) = -\frac{Gm^2}{r} \tag{5.1}$$

Where r is the distance between two masses. On considering the spatial compression, we can replace r by by compressed r under the influence of two gravitating masses such that

$$r_c = \frac{r}{\frac{1}{K}\left(e^{\frac{2Gm}{rc^2}} - 1\right) + 1} \tag{5.2}$$

This leads to the modified potential as

$$\Phi(r) = -\frac{Gm^2}{r}\left(\frac{1}{K}\left(e^{\frac{2Gm}{rc^2}} - 1\right) + 1\right) \tag{5.3}$$

Here we assume $\frac{2Gm}{Krc^2} << 1$ and take the first approximation and we obtain

$$r_c = \frac{r}{\frac{2Gm}{Kc^2 r} + 1} \tag{5.4}$$

This can be compared to the result provided by (3.11) and we can interpret $v^2 = \frac{4Gm}{Kr}$. It can further be written as

$$r_c = r\left(\frac{2Gm}{Kc^2 r} + 1\right)^{-1} \tag{5.5}$$

Thus the Newtonian potential in terms of the compressed space in its first approximation is

$$\Phi(r) = -\frac{Gm^2}{r}\left(\frac{2Gm}{Kc^2 r} + 1\right) \tag{5.6}$$

The same potential is obtained by considering the equivalence of special relativistic $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$ to that of the gravitational factor $\frac{1}{1+\frac{2Gm}{Kc^2 r}}$ and writing the mass equation as $m_c = m\left(1 + \frac{2Gm}{Kc^2 r}\right)$ in the Newtonian potential.

The modification of gravity due to contracted space and supported by relative gravitational red-shift is thus surprisingly equivalent to the brane world with large extra dimensions.

$$\Phi(r) = -\frac{Gm^2}{r}\left(\frac{k}{r}+1\right) \tag{5.7}$$

Thus comparing the two potentials, we can determine the value of $k = \frac{2Gm}{Kc^2}$ which is a typical length scale at which the correction due to the infinite extra dimensions become dominant. This indeed is an important finding. Here we can make a bold conclusion that the length scale at which extra dimensions are dominant depends on mass of the object, for heavier objects the length is more. For Planck mass the value of k is almost of the Planck size which again is a good result. For General relativity we can use $K = 2$ [15]. It is believed that once a massive body collapses to within its Schwarzschild radius i.e. $r_s = \frac{Gm}{c^2}$, a black hole is formed. Thus if we set $r_c = \frac{Gm}{c^2} = r_s$ get very interesting results which indicates that $r_s = 0.6r$ and the corresponding potential at this point is $\frac{10}{6}\frac{Gm^2}{r}$.

6 State Functions of the system

The cosmological many body partition function has been obtained by using the formalism of classical statistical mechanics for an ensemble of comoving cells containing gravitating galaxies in an expanding universe [44, 45, 46]. It was found that the size of the cells is important, so if it is smaller than the particle correlation length, then each member of the mentioned ensemble is correlated gravitationally with other cells, hence the extensivity is a good approximation [47, 44, 48]. In the case of a large system, consisting of an ensemble of cells with the identical volume V (radius R) and average density $\bar{\rho}$. As we know, the total energy and the number of galaxies vary among the mentioned cells, thus these are represented by grand canonical ensemble. The galaxies within the system have gravitational pairwise interaction. It has been further assumed that the distribution is statistically homogeneous over large regions.

The general partition function of a system of N particles of mass m interacting gravitationally with a potential energy ϕ, having momenta p_i and average temperature T given by [44],

$$Z_N(T,V) = \frac{1}{\Lambda^{3N}N!}\int\left[\exp\left(\frac{-\sum_{i=1}^{N}\frac{p_i^2}{2m}+\phi(\mathbf{r}_1,\mathbf{r}_2,....,\mathbf{r}_N)}{T}\right)\right]\mathrm{d}^{3N}\mathbf{p}\mathrm{d}^{3N}\mathbf{r} \tag{6.1}$$

It may be noted that the Newtonian potential gets modified from different approaches. The important to mention here are non-commutative geometry [24]; [49],minimal length in quantum gravity [26], f(R) gravity [50]. [51], ΛCDM [52] and the entropic force [53]. The impact of modified potentials on thermodynamics of galaxy clustering has been discussed in many instances [54], [55]; [56]. The effect of the cosmological constant on the gravitational partition function for galaxies in a universe and the influence on distribution of galaxies has been pursued.[57, 58] It is important to mention the phenomenological Tohline-Kuhn modified gravity approach to the problem of dark matter [59],[60]; [61].
For the potential in compressed space, the partition function is obtained by the usual procedure. Integration over momentum space yields

$$Z_N(T,V) = \frac{1}{N!}\big(\frac{2\pi mT}{\Lambda^2}\big)^{3N/2}Q_N(T,V) \tag{6.2}$$

where $Q_N(T,V)$ is the configurational integral and is given as:

$$Q_N(T,V) = \int....\int\prod_{1\le i<j\le N}\exp\left(\frac{Gm^2}{r}\left(\frac{Gm}{c^2r}+1\right)(r_1,r_2,\ldots,r_N)T^{-1}\right)]d^{3N}r \tag{6.3}$$

By using the two-particle function

$$f_{ij} = e^{-\Phi_{ij}/T}-1 \tag{6.4}$$

where the function f_{ij} is zero in absence of interactions and for the interacting system it is non-zero.
It has been shown that the configurational integral can be expressed as [44]

$$Q_N(T,V) = \int....\int\Big[(1+f_{12})(1+f_{13})(1+f_{23})\ldots(1+f_{N-1,N})\Big]d^3r_1\ldots d^3r_N \tag{6.5}$$

Following the earlier procedure we calculate the configurational integral between the limits R_0 and R and get

6.1 weak gravity

When the gravitational interaction is weak, we can take as a first approximation: $\exp\left[\frac{Gm^2}{rT}\left(\frac{2Gm}{c^2r}+1\right)\right] = 1+\frac{Gm^2}{rT}\left[\frac{Gm}{c^2r}+1\right]$ Thus, we obtain Q_1 the expression:

$$Q_1 = V = \frac{4}{3}\pi(R_1^3 - R_0^3) \tag{6.6}$$

For Q_2

$$Q_2 = V^2\left\{1+\frac{4\pi}{V}(R_1-R_0)\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]\right\} \tag{6.7}$$

And, in general, for Q_N

$$Q_N = V^N\left\{1+\frac{4\pi}{V}(R_1-R_0)\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]\right\}^{N-1} \tag{6.8}$$

Known Q_N we can then evaluate the free energy. Its expression is:

$$F = -T\ln Z_N(T,V) \tag{6.9}$$

Then:

$$F = -T\ln\left\{\frac{1}{N!}\left(\frac{2\pi mT}{\Lambda^2}\right)^{\frac{3N}{2}}V^N\left\{1+\frac{4\pi}{V}(R_1-R_0)\right.\right.$$
$$\left.\left.\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]\right\}^{N-1}\right\} \tag{6.10}$$

For entropy, starting from the definition:

$$S = -\left(\frac{\partial F}{\partial T}\right)_{N,V} \tag{6.11}$$

Then:

$$S = -\frac{F}{T}+\frac{3N}{2}-\frac{(N-1)T4\pi V(R_1-R_0)\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]}{1+4\pi V(R_1-R_0)\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]} \tag{6.12}$$

The average energy can then be calculated with the formula:

$$U = F + TS \tag{6.13}$$

We get now:

$$U = \frac{3NT}{2}-\frac{(N-1)4\pi V(R_1-R_0)\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]}{1+4\pi V(R_1-R_0)\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]} \tag{6.14}$$

Finally, we have for the specific heat:

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{N,V} \tag{6.15}$$

Thus:

$$C_V = \frac{3N}{2}-\frac{(N-1)4\pi V(R_1-R_0)\left[\frac{Gm^2}{2T^2}(R_1+R_0)+\frac{G^2m^3}{T^2c^2}\right]}{[1+4\pi V(R_1-R_0)\left[\frac{Gm^2}{2T}(R_1+R_0)+\frac{G^2m^3}{Tc^2}\right]]^2} \tag{6.16}$$

When: $T\to\infty$, the specific heat takes the value:

$$C_V = \frac{3N}{2} \tag{6.17}$$

And when $T\to 0$ we have:

$$C_V = \frac{3N}{2}-\frac{(N-1)}{[4\pi V(R_1-R_0)\left[\frac{Gm^2}{2}(R_1+R_0)+\frac{G^2m^3}{c^2}\right]]^2} \tag{6.18}$$

7 The Lagrangian of Einstein's QFT

The appropriate Lagrangian to quantize Einstein's gravity was obtained by the famous physicist Indu Suraj N. Gupta, who together with Konrad Bleuler were the ones who made the covariant quantization of the Electromagnetic Field. This quantization is known as the "Gupta-Bleuler" quantization of the electromagnetic field. The Lagrangian turns out to be [62]:

$$\mathcal{L}_G = \frac{1}{\kappa^2}\boldsymbol{R}\sqrt{|g|} - \frac{1}{2}\eta_{\mu v}\partial_\alpha h^{\mu\alpha}\partial_\beta h^{v\beta}, \tag{7.1}$$

In this case we select: $c = 1, \mu = 1, 2, 3$. In other words, we are working in a subspace of the complete space. If we consider the theory of the non-evolution of time, which is equivalent to the contraction of space in the vicinity of a black hole we obtain the expression for time:

$$t = r\left(\frac{r}{2Gm} - 1\right)^{\frac{1}{2}} \tag{7.2}$$

The expression of the arc element in this space is given by:

$$ds^2 = \left[1 - \frac{(3r - 4Gm)^2}{8Gmr - 16G^2m^2}\right] dr^2 \tag{7.3}$$

We now define the metric tensor in that subspace as:

$$h^{\mu v} = \lambda^{\mu v} + \kappa\psi^{\mu v}, \tag{7.4}$$

Where:

$$\lambda^{\mu v} = 1 - \frac{(3r - 4Gm)^2}{8Gmr - 16G^2m^2} \tag{7.5}$$

Equation (7.4) can be re-written as:

$$h^{\mu v} = \eta^{\mu v} + \kappa\phi^{\mu v}, \tag{7.6}$$

With:

$$\phi^{\mu v} = \psi^{\mu v} - \frac{(3r - 4Gm)^2}{\kappa(8Gmr - 16G^2m^2)} \tag{7.7}$$

and: $\eta^{\mu\nu} = diag(1, 1, 1,)$

$$h^{\mu v} = \eta^{\mu v} + \kappa\phi^{\mu v}, \tag{7.8}$$

Here κ^2 is the gravitation's constant and $\phi^{\mu v}$ the graviton field. We write

$$\mathcal{L}_G = \mathcal{L}_L + \mathcal{L}_I, \tag{7.9}$$

where

$$\mathcal{L}_L = -\frac{1}{4}[\partial_\lambda\phi_{\mu v}\partial^\lambda\phi^{\mu v} - 2\partial_\alpha\phi_{\mu\beta}\partial^\beta\phi^{\mu\alpha} + 2\partial^\alpha\phi_{\mu\alpha}\partial_\beta\phi^{\mu\beta}], \tag{7.10}$$

and, up to 2nd order, one has [62, 63, 64]:

$$\mathcal{L}_I = -\frac{1}{2}\kappa\phi^{\mu v}[\frac{1}{2}\partial_\mu\phi^{\lambda\rho}\partial_v\phi_{\lambda\rho} + \partial_\lambda\phi_{\mu\beta}\partial^\beta\phi_v^\lambda - \partial_\lambda\phi_{\mu\rho}\partial^\lambda\phi_v^\rho], \tag{7.11}$$

having made use of the constraint

$$\phi_\mu^\mu = 0. \tag{7.12}$$

This constraint is required in order to satisfy the Gupta-Bleuler quantization of the graviton field. For the graviton we have then

$$\triangle\,\phi_{\mu v}(x, t) = 0, \tag{7.13}$$

for $r > 2Gm$, Here we are postulating that the responsible for the creation and evolution of time is the graviton. The solution of the previous equation is:

$$\phi_{\mu v}(x, t) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} f_{\mu v l m}(t)\psi_{lm}(r, \theta, \theta_1) \tag{7.14}$$

With:

$$\psi_{lm}(r,\theta,\theta_1) = \frac{1}{\sqrt{2}}\left[\vartheta_{lm}(r,\theta,\theta_1) + \vartheta^*_{lm}(r,\theta,\theta_1)\right] \tag{7.15}$$

Where:

$$\vartheta_{lm}(r,\theta,\theta_1) = A_l r^{\frac{1}{2}}\left[r^{\frac{1}{2}\sqrt{1+4l(l+1)}} - (2Gm)^{\sqrt{1+4l(l+1)}} r^{-\frac{1}{2}\sqrt{1+4l(l+1)}}\right] Y_l^m(\theta,\theta_1) \tag{7.16}$$

here we have selected:

$$f_{\mu v lm}(t) = \frac{1}{(2\pi)^{\frac{1}{2}}}\int\left[\frac{a_{\mu v lm}(k)}{\sqrt{2|k|}}e^{-i|k|t} + \frac{a^+_{\mu v lm}(k)}{\sqrt{2|k|}}e^{i|k|}\right]dk \tag{7.17}$$

In other words, a Harmonic Oscillator for the temporary part. Also, it turns out to be

$$A_l = (2Gm)^{-1-\frac{1}{2}\sqrt{1+4l(l+1)}}\sqrt{\frac{4l(l+1)-3}{1+4l(l+1)}} \tag{7.18}$$

by demanding that $||\psi_{lm}(r,\theta,\theta_1)|| = 1$

8 The Quantization of the Theory

To quantize the theory we use the result given in [65] for the three-dimensional rotational invariant case:

$$[a_{\mu v lm}(k), a^{+\mu^{'}v^{'}l^{'}m^{'}}(k^{'})] = \left[\delta^{\mu^{'}}_{\mu}\delta^{v^{'}}_{v} + \delta^{\mu^{'}}_{v}\delta^{v^{'}}_{\mu}\right]\delta(k-k^{'})\delta_{l,l^{'}}\delta_{m,m^{'}} \tag{8.1}$$

As customary, the physical state $|\psi>$ of the theory is defined via the equation

$$\phi^{\mu}_{\mu}|\psi>=0. \tag{8.2}$$

We use now the the usual definition

$$\Delta^{\rho\lambda}_{\mu\nu}(x-y,t-t^{'}) =<0|T[\phi_{\mu\nu}(x,t)\phi^{\rho\lambda}(y,t^{'})]|0>. \tag{8.3}$$

The graviton's propagator then turns out to be

$$\Delta^{\rho\lambda}_{\mu\nu}(x-y,t-t^{'}) = \frac{1}{2\pi}\delta(x-y)(\delta^{\rho}_{\mu}\delta^{\lambda}_{v} + \delta^{\rho}_{v}\delta^{\lambda}_{\mu})\int Sgn(k)\frac{e^{-i|k|(t-t^{'})}}{k-i0}dk. \tag{8.4}$$

Where $Sgn(k)$ in the sign function.We should note now that:

$$\Delta^{\rho\lambda}_{\mu\nu}(x-y,t-t^{'}) \cdot \Delta^{\rho\lambda}_{\mu\nu}(x-y,t-t^{'}) = 0 \tag{8.5}$$

Since it has been shown in [66] that:

$$\delta^{(n)}(x)\cdot\delta^{(m)}(x) = 0 \tag{8.6}$$

9 The Self-Energy of the Graviton

We are now going to analyze the self-energy of the graviton. A typical term of it is:

$$\partial_{s_1}\partial_{s_2}\Delta^{\rho\lambda}_{\mu\nu}(x-y,t-t^{'}) \cdot \partial_{s_3}\partial_{s_4}\Delta^{\rho\lambda}_{\mu\nu}(x-y,t-t^{'}) = 0 \tag{9.1}$$

and, as we can see, it is null according to (8.6). This has a very important implication. It implies that the naked propagator is equal to the graviton dressed propagator, with which we conclude that in this case it is not necessary to do the complete perturbation theory in order to have the complete graviton propagator.

10 Including Axions into the picture

Axions are hypothetical elementary particles postulated by the Peccei–Quinn theory in 1977 to tackle the strong CP problem in quantum chromodynamics. If they exist and have low enough mass (within a certain range), they could be of interest as possible components of cold dark matter [67].

We include now a massive scalar field (axions) interacting with the graviton. The Lagrangian becomes

$$\mathcal{L}_{GM} = \frac{1}{\kappa^2}\boldsymbol{R}\sqrt{|g|} - \frac{1}{2}\eta_{\mu v}\partial_\alpha h^{\mu\alpha}\partial_\beta h^{v\beta} - \frac{1}{2}[h^{\mu v}\partial_\mu\phi\partial_v\phi + m^2\phi^2]. \tag{10.1}$$

We can now recast the Lagrangian in the fashion

$$\mathcal{L}_{GM} = \mathcal{L}_L + \mathcal{L}_I + \mathcal{L}_{LM} + \mathcal{L}_{IM}, \tag{10.2}$$

where

$$\mathcal{L}_{LM} = -\frac{1}{2}[\partial_\mu\phi\partial^\mu\phi + m^2\phi^2], \tag{10.3}$$

so that $\mathcal{L}_{IM}$ becomes the Lagrangian for the axion-graviton action

$$\mathcal{L}_{IM} = -\frac{1}{2}\kappa\phi^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{10.4}$$

The new term in the interaction Hamiltonian is

$$\mathcal{H}_{IM} = \frac{\partial\mathcal{L}_{IM}}{\partial\partial^0\phi}\partial^0\phi - \mathcal{L}_{IM}. \tag{10.5}$$

We now make the assumption that the creator of time for dark matter is the axion. Then the equation that satisfies the axion is:

$$(\triangle - m^2)\phi(x,t) = 0 \tag{10.6}$$

The solution of this equation with the condition $r > 2mg$ is given by

$$\phi_{\mu v}(x,t) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} f_{\mu vlm}(t)\psi_{lm}(r,\theta,\theta_1) \tag{10.7}$$

With:

$$\psi_{lm}(r,\theta,\theta_1) = \frac{1}{\sqrt{2}}\left[\vartheta_{lm}(r,\theta,\theta_1) + \vartheta^*_{lm}(r,\theta,\theta_1)\right] \tag{10.8}$$

Where:

$$\vartheta_{lm}(r,\theta,\theta_1) = A_l r^{\frac{1}{2}}\mathcal{K}_l(mr) \tag{10.9}$$

here we have selected:

$$f_{\mu vlm}(t) = \frac{1}{(2\pi)^{\frac{1}{2}}}\int\left[\frac{a_{\mu vlm}(k)}{\sqrt{2|k|}}e^{-i|k|t} + \frac{a^+_{\mu vlm}(k)}{\sqrt{2|k|}}e^{i|k|}\right]dk \tag{10.10}$$

In other words, a Harmonic Oscillator for the temporary part again. Also, it turns out to be

$$A_l = \left[2m^2G^2\mathcal{K}_l^{'2}(2m^2G) - \frac{1}{2}\left(4m^2G^2 + \frac{l^2}{m^2}\right)\mathcal{K}_l^2(2m^2G)\right]^{\frac{1}{2}} \tag{10.11}$$

by demanding that $||\psi_{lm}(r,\theta,\theta_1)|| = 1$, or equivalently

$$A_l^2\int\limits_{2mG}^{\infty} r\mathcal{K}_l^2(mr)dr = 1 \tag{10.12}$$

We use now the the usual definition

$$\Delta(x-y,t-t^{'}) =< 0|T[\phi(x,t)\phi(y,t^{'})]|0 > . \tag{10.13}$$

The axion's propagator then turns out to be

$$\Delta(x-y,t-t^{'}) = \frac{1}{2\pi}\delta(x-y)\int Sgn(k)\frac{e^{-i|k|(t-t^{'})}}{k-i0}dk. \tag{10.14}$$

Where $Sgn(k)$ in the sign function.We should note now that:

$$\Delta(x-y,t-t^{'})\cdot\Delta(x-y,t-t^{'}) = 0 \tag{10.15}$$

Since, as we saw, it has been shown in [66] that:

$$\delta^{(n)}(x)\cdot\delta^{(m)}(x) = 0 \tag{10.16}$$

11 The complete Self Energy of the Graviton

We analyze, you now a typical term of the contribution of axions to the self-energy of graviton. For him we find in this case that:

$$\partial_{s_1}\partial_{s_2}\Delta(x-y,t-t^{'})\cdot\partial_{s_3}\partial_{s_4}\Delta(x-y,t-t^{'}) = 0 \tag{11.1}$$

We then see that the contribution of the axions to the self-energy of the graviton is also zero, with which we can conclude that, even with the presence of axions, the naked propagator of the graviton is equal to the dressed propagator again.

12 Self Energy of the Axion

A typical term corresponding to the axion, in this theory it has the form

$$\partial_{s_1}\partial_{s_2}\Delta(x-y,t-t^{'})\cdot\Delta_{\mu\nu}^{\rho\lambda}(x-y,t-t^{'}) = 0 \tag{12.1}$$

and as we can see it is also null. Then for the axion, we also have that the naked propagator is equal to the dressed propagator.
We can conclude that those responsible for the temporal evolution of light matter and dark matter have similar behaviors.
We also see how simple it is to develop this theory in the compressed gravitational space. Even simpler than a renormalizable theory, since only one equation is needed to solve it: the equation (8.6).

13 Conclusions

In an accidental glance the simplicity of expressions of the work [15], caught our attention. We find it can be derived in more general way using the black hole clock in which we find that space contraction is not confined to only radial direction, but happens in different directions that can be shown using clock measurements was proposed in [13, 14]. The equivalence of contracted space under the influence of gravity to that of the Lorentz spatial contraction of special relativity devoid of any mathematical intricacy and keeping intact the basic principles of physics is the motivation behind this paper.
In one of the instances, we use clock postulate and the concept of timeless gravity to show that the gamma factor of special relativity arises from the gravitational red-shift, and assigned a geometrical or gravitational interpretation to the ratio $\frac{v^2}{c^2}$. The geometrical interpretation is further interpreted to be equivalent to the concept of space contraction in presence of a massive body.
We could express the distance between gravitating bodies as the contracted distance due to the gravitational influence. The contracted distance helped us to find the radius of black hole by assuming that once a massive body collapses to within its Schwarzschild radius a black hole is formed. This also gives us the idea of the strength of gravity near a black hole.
Simple replacement of distance by its contracted counterpart in the usual Newtonian potential surprisingly lead to the well celebrated brane potential of Randall-Sundrum [18, 19]. The potential thus obtained, resolved the choice of taking the minimum length where extra dimensions become dominant. we could see that the minimum

length scale as provided by brane potential happens to be $k = \frac{Gm}{c^2}$. k depends on the mass of the gravitating body and is almost of the order of Planck length for the body of Planck mass. This can also be comparable with that of the Schwarzschild radius. The same potential is obtained if the mass in the potential is replaced by the relativistic mass by using the the above mentioned equivalence. With these supportive arguments, we used the modified potential to study the partition function and the thermodynamic properties of clustering of galaxies. Further mathematical insight into the concept of timeless universe and the contracted space is needed, which we reserve for our upcoming endeavors.
We have found that with the use of a single mathematical equation we can quantize the theory in a simple way. A central result of this quantization is that the naked and dressed propagators of the graviton and the axion are equal to the corresponding naked propagator. This means that we have exactly solved the theory with the use of perturbation theory.

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