Dirac Equation in Cosmological Inertial Frame

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ABSTRACT
Dirac equation is a one order-wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Dirac Equation from wave function, Type A in cosmological inertial frame. The Dirac equation satisfy Klein-Gordon equation in cosmological inertial frame.

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1. Introduction

Dirac equation is in special relativity theory,[8]

\[(ih\gamma^\mu \partial_\mu - mcI)\psi = 0,\]

\(I\) is a 4 × 4 unit matrix,

\[
\begin{pmatrix} 1' & 0 \\ 0 & -1' \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & \sigma^0 \\ -\sigma^0 & 0 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

(1)

2. Dirac Equation from Wave Function-Type A in Cosmological Inertial Frame

Dirac equation is the wave equation. Therefore, Dirac equation is in cosmological inertial frame,[2]

Wave function Type A:

\[r \rightarrow r\sqrt{\Omega(t_0)}, \quad t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}},\]

\(t_0\) is the cosmological time, \(\Omega(t_0)\) is the expanding ratio of universe in the cosmological time \(t_0\).

\[(ih\sqrt{\Omega(t_0)}\gamma^0 \partial_0 + ih \frac{1}{\sqrt{\Omega(t_0)}}\gamma^i \partial_i - mcI)\phi = 0\]  

(2)

If \(\bar{\partial}_\mu\) is

\[
\bar{\partial}_\mu = \left(\sqrt{\Omega(t_0)}\partial_0, \frac{1}{\sqrt{\Omega(t_0)}}\partial_i\right)
\]

(3)

Dirac equation is in cosmological inertial frame,

\[(ih\gamma^\mu \bar{\partial}_\mu - mcI)\phi = 0\]

(4)

Eq(4) multiply \(ih\gamma^\mu \bar{\partial}_\mu\), hence

\[(-\hbar^2 (\gamma^\mu \bar{\partial}_\mu)(\gamma^\nu \bar{\partial}_\nu) - ih(\gamma^\nu \bar{\partial}_\nu)mcI)\phi = 0\]

(5)

In this time,

\[ih\gamma^\mu \bar{\partial}_\mu \phi = mcI\phi\]

(6)

Hence, Eq(5) is

\[(-\hbar^2 \gamma^\mu \gamma^\nu \bar{\partial}_\mu \bar{\partial}_\nu - m^2 c^2 I)\phi = 0\]

(7)

In this time, matrix \(\gamma^\mu\) is
\[ \frac{1}{2} \left( \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu \right) = \frac{1}{2} \{ \gamma^\mu , \gamma^\nu \} = \eta^{\mu\nu} I \] (8)

Therefore, [1], [3]

\[
\frac{1}{2} \left( \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu \right) \partial_\mu \bar{\phi} \phi + \frac{m^2 c^2}{h^2} I \phi = (\eta^{\mu\nu} \partial_\mu \bar{\phi} \phi + \frac{m^2 c^2}{h^2})I \phi = 0
\] (9)

Eq(9) is the matrix equation of Klein-Gordon.

Dirac spinor \( \phi \) is \( \phi = (\phi_1, \phi_2, \phi_3, \phi_4) \). \( \phi \)'s hermitian conjugate \( \phi^* = (\phi_1^*, \phi_2^*, \phi_3^*, \phi_4^*) \).

Hence, \( \phi \)'s adjoint spinor \( \bar{\phi} \) is

\[
\bar{\phi} = \phi^* \gamma^0, \quad \bar{\phi} (i \gamma^\mu \partial_\mu + mc I) = 0
\] (10)

Hence, positive probability density \( j^0 \) is

\[
j^0 = \bar{\phi} \gamma^0 \phi = |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2
\] (11)

3. Conclusion

We found Dirac equation from Wave Function-Type A in cosmological special theory of relativity. The wave function uses as a probability amplitude.

References