Dirac Equation in Cosmological Inertial Frame

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ABSTRACT
Dirac equation is a one order-wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Dirac Equation from wave function, Type A in cosmological inertial frame. The Dirac equation satisfy Klein-Gordon equation in cosmological inertial frame.

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1. Introduction
Dirac equation is in special relativity theory,

\[(ih\gamma^\mu \partial_\mu - mcI)\psi = 0\]

\[I \text{ is } 4 \times 4 \text{ unit matrix}.
\]

\[
\gamma^0 = \begin{pmatrix} 0 & I' \\ I' & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

\[I' \text{ is } 2 \times 2 \text{ unit matrix}, \quad \sigma^i \text{ is Pauli’s matrix}.
\]

(1)

2. Dirac Equation from Wave Function-Type A in Cosmological Inertial Frame
Dirac equation is the wave equation. Therefore, Dirac equation is in cosmological inertial frame,[2]

Wave function Type A:

\[r \rightarrow r\sqrt{\Omega(t_0)}, \quad t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}},
\]

t_0 \text{ is the cosmological time. } \Omega(t_0) \text{ is the expanding ratio of universe in the cosmological time } t_0.

\[(ih\sqrt{\Omega(t_0)}\gamma^0 \partial_0 + ih\frac{1}{\sqrt{\Omega(t_0)}}\gamma^i \partial_i - mcI)\phi = 0
\]

(2)

If \( \partial_\mu \) is

\[\partial_\mu = (\sqrt{\Omega(t_0)}\partial_0, \frac{1}{\sqrt{\Omega(t_0)}}\partial_i)
\]

(3)

Dirac equation is in cosmological inertial frame,

\[(ih\gamma^\mu \partial_\mu - mcI)\phi = 0
\]

(4)

Eq(4) multiply \( ih\gamma^\nu \partial_\nu \), hence

\[-h^2(\gamma^\nu \partial_\nu)(\gamma^\mu \partial_\mu) - ih(\gamma^\nu \partial_\nu)mcI)\phi = 0
\]

(5)

In this time,

\[ih\gamma^\nu \partial_\nu \phi = mcI\phi
\]

(6)

Hence, Eq(5) is

\[-h^2\gamma^\nu \gamma^\mu \partial_\mu \partial_\nu - m^2c^2I)\phi = 0
\]

(7)

In this time, matrix \( \gamma^\mu \) is
\[ \frac{1}{2} (\gamma^\nu \gamma_\mu + \gamma_\mu \gamma^\nu) = \frac{1}{2} \{\gamma^\mu, \gamma_\nu\} = \eta^{\mu\nu} I \]  

(8)

Therefore, \([1],[3]\)

\[ \frac{1}{2} (\gamma^\nu \gamma_\mu + \gamma_\mu \gamma^\nu) \partial_\mu \partial_\nu \phi + \frac{m^2 c^2}{\hbar^2} I \phi \]

\[ = (\eta^{\mu\nu} \partial_\mu \partial_\nu + \frac{m^2 c^2}{\hbar^2}) I \phi = 0 \]  

(9)

Rq(9) is the matrix equation of Klein-Gordon.

Dirac spinor \( \phi \) is \( \phi = (\phi_1, \phi_2, \phi_3, \phi_4) \). \( \phi \)'s hermitian conjugate \( \phi^* = (\phi_1^*, \phi_2^*, \phi_3^*, \phi_4^*) \).

Hence, \( \phi \)'s adjoint spinor \( \bar{\phi} \) is

\[ \bar{\phi} = \phi^* \gamma^0, \quad \bar{\phi} (i \gamma^\mu \partial_\mu + mc I) = 0 \]  

(10)

Hence, positive probability density \( j_0^0 \) is

\[ j_0^0 = \bar{\phi} \gamma^0 \phi = \phi^* \phi = |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 \]  

(11)

3. Conclusion

We found Dirac equation from Wave Function-Type A in cosmological special theory of relativity. The wave function uses as a probability amplitude.

References


