Electromagnetic Field in Curvilinear Coordinates

A. V. Parfyonov[0000-0003-3256-5105]

Ulyanovsk state technical university
Ulyanovsk, repin street 37-197, 432026, RU
e-mail: avp@ulstu.ru.

Abstract: Having considered the electromagnetic field in a curvilinear coordinate system, a theory has been created that expands our understanding of the electromagnetic field, the nature of quarks, the nature of strong interaction, and the connection between strong interaction and electromagnetic interaction.

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1 Introduction

Classical theories, such as Newtonian Mechanics, Maxwell's Electrodynamics are theories that do not have complete generality. So, Classical Mechanics cannot describe mechanical systems in the entire range of speeds with which these systems can move. It describes mechanical systems that move at speeds, the magnitude of which is so much less than the speed of light that the speed of light can be considered an infinitely large value. As you know, mechanics, which has complete generality, since it describes mechanical systems over the entire range of speeds with which these systems can move, are called relativistic mechanics, and was created by Einstein. Classical Electrodynamics does not have complete generality, since it cannot describe the electromagnetic field in the entire four-dimensional space. It becomes an internally inconsistent theory in the field surrounding a point elementary charged particle. Indeed, when tending to the point at which a point charged particle is located, the electric field according to Coulomb's Law will tend to infinity. Consequently, the field energy, and hence the mass corresponding to this energy, will also tend to infinity. The physical meaninglessness of this result is the essence of this contradiction. This immediately implies the need to create electrodynamics with complete commonality. However, before creating such electrodynamics, one should get rid of the contradiction, which can be done only by refusing to consider elementary particles as point particles. Moreover, we now know that elementary particles are not so elementary; they have a very complex internal structure. So, protons, neutrons, and a number of other particles consist of quarks, then, if they are considered point particles, not only do we neglect their size, but also their complex internal structure.

Refusing to consider elementary particles to be point particles, we must consider them particles having finite sizes. But if we consider them particles having finite sizes, then we must know the law by which the shape of the surface of the particles will change, because we cannot consider particles to be absolutely solid bodies, which is prohibited by the basic principles of the theory of relativity, working for electrodynamics. And we will know this law if we know the nature of the mass of elementary particles. To reveal the nature of the mass of elementary particles, we will use a hint. During the interaction of a particle and its antiparticle that is during the annihilation reaction, the particle and antiparticle disappear and gamma quanta appear, which are electromagnetic waves. Electromagnetic waves, in their turn, are vibrations of electric and magnetic fields. Therefore, it is natural to assume that the nature of the mass of an elementary particle and its antiparticle also has an electromagnetic character. Having accepted this idea that mass is a specially formed electromagnetic field, we can begin to create electrodynamics with complete generality, which can describe the electromagnetic field in the entire four-dimensional space.

2 Method

Obviously, such electrodynamics should be created using curvilinear coordinates. But here we have a problem of how to connect the electromagnetic field with some curvilinear coordinate system. Unlike the gravitational field, which is directly related to the space-time metric, the electromagnetic field does not have such a direct connection. To overcome this problem, we will use one more hint. We know that electric and magnetic fields can be represented in the form of force lines, and if we direct the coordinate axes of a curvilinear coordinate system along the force lines of an electromagnetic field, then this problem can be solved. But this is only an idea; to make it work, it is necessary to find a mathematical expression of this idea. And here we have a clue – we know that if a vector field is specified in three-dimensional space, then the equations describing the lines of a given vector field can be found as follows: taking the vector of a given vector field at an arbitrary point of this field, and multiplying it vectorially by the radius vector element and equating the result to zero, we obtain a system of equations describing the lines of this vector field. Moving on to four-dimensional space, if we consider electromagnetic fields in four-dimensional space, and if the magnitude of the electromagnetic field is determined by the second-rank antisymmetric tensor and, using the analogy with three-dimensional space, we must therefore find another antisymmetric second-rank tensor in four-dimensional space that would describe some geometric object defined in this space. And we do have such an antisymmetric tensor of the second rank which describes a two-dimensional surface defined in four-dimensional space. Based on these two antisymmetric second-order tensors, a number of quantities can be compiled, starting from a scalar, that is a zero-rank tensor, and ending with two second-rank tensors. Considering these two second-rank tensors in rectangular coordinates (in four-dimensional non-curved space they are called Galilean Coordinates), we see that each of these two tensors can be represented as the sum of a symmetric and antisymmetric tensor. The importance of this result is that the symmetric tensor for each of these two tensors of the second rank is the metric tensor of the four-dimensional non-curved space. Thus, we have found the connection of the electromagnetic field with the space-time metric using two tensors of the second rank compiled on the basis of two antisymmetric tensors of the second rank, one of which describes the electromagnetic field, while the second describes a two-dimensional surface.

3.1 Harmonized electromagnetic field

The trace of the stress-energy tensor of the electromagnetic field is zero: $T_i^i = 0$, therefore, the scalar curvature of space-time R in the presence of a single electromagnetic field is also zero [1]. Thus, it may be concluded that the electromagnetic field has no connection with the space-time metric, in contrast to the gravitational field, where the metric tensor g_{ik} plays the role of 'potentials'. Therefore, to describe the electromagnetic field in curvilinear coordinates, we must first match the electromagnetic field with a system of curvilinear coordinates. Coordination is an operation that resembles the introduction operation for a vector field **F**, defined in three-dimensional space, of vector lines using differential equations describing these same vector lines: $\mathbf{F} \times d\mathbf{r} = 0$, where $\mathbf{r}$ is a radius vector. Moving to a four-dimensional space and having an antisymmetric tensor of the second rank F_{ik}, describing an electromagnetic field, we take an antisymmetric tensor of the second rank

$$df^{ik} = dx^i dx'^k - dx^k dx'^i, \tag{1.1}$$

describing an infinitesimal element of a two-dimensional surface $x^i = x^i(u, v)$, where u and v will be considered as curvilinear coordinates on the specified surface. We choose these coordinates so that the four-dimensional vectors dx^i and dx'^i are tangent vectors to the coordinate lines u and v, respectively. This allows writing expression (1.1) as follows: $df^{ik} = f^{ik} du dv$, where

$$f^{ik} = \frac{\partial x^i}{\partial u}\frac{\partial x^k}{\partial v} - \frac{\partial x^k}{\partial u}\frac{\partial x^i}{\partial v}.$$

Using the tensors F_{ik} and f^{ik}, we construct two tensors of the second rank A^{ik} and B^{ik}:

$$A^{ik} = F_l^i f^{kl} - F_l^{*i} f^{*kl}, \tag{1.2}$$

$$B^{ik} = F_l^i f^{*kl} + F_l^{*i} f^{kl}, \tag{1.3}$$

where the pseudo-tensors F^{*ik}, f^{*ik} and accordingly the tensors F^{ik} and f^{ik} are dual to each other. We show that tensors (1.2) and (1.3) can be written as the sum of a symmetric tensor and an antisymmetric tensor. To do so, we write them in Galilean coordinates. The quantities considered in Galilean coordinates will be distinguished by the index Γ. Thus, in the Galilean coordinates we have:

$$A_\Gamma^{ik} = \frac{1}{4} A_\Gamma g_\Gamma^{ik} + a_\Gamma^{ik}, \tag{1.4}$$

$$B_\Gamma^{ik} = \frac{1}{4} B_\Gamma g_\Gamma^{ik} + a_\Gamma^{*ik}, \tag{1.5}$$

where $A_\Gamma = A_{\Gamma i}^{i}$, $B_\Gamma = B_{\Gamma i}^{i}$. The tensor a_Γ^{ik} and pseudo-tensor a_Γ^{*ik} are dual to each other. The correctness of the equalities (1.4) and (1.5) can be verified by direct calculation, which gives the values to which are included in these equalities:

$$A_\Gamma = 4(\mathbf{E}_\Gamma \mathbf{f}_\Gamma - \mathbf{H}_\Gamma \mathbf{s}_\Gamma), \tag{1.6}$$

$$B_\Gamma = 4(\mathbf{E}_\Gamma \mathbf{s}_\Gamma + \mathbf{H}_\Gamma \mathbf{f}_\Gamma), \tag{1.7}$$

where $\mathbf{E}_\Gamma$ and $\mathbf{H}_\Gamma$ are electric and magnetic field tension vectors,

$$\mathbf{f}_\Gamma = (\mathrm{f}^{01}, \mathrm{f}^{02}, \mathrm{f}^{03}), \tag{1.8}$$

$$\mathbf{s}_\Gamma = (\mathrm{f}^{23}, \mathrm{f}^{31}, \mathrm{f}^{12}), \tag{1.9}$$

where, for instance,

$$\mathrm{f}^{01} = \frac{\partial ct}{\partial u}\frac{\partial x}{\partial v} - \frac{\partial x}{\partial u}\frac{\partial ct}{\partial v}, \tag{1.10}$$

and so on; ct, x Cartesian coordinates.

The components of the antisymmetric tensor of the second rank a_Γ^{ik} are components of the two vectors:

$$\mathbf{a} = \mathbf{E}_\Gamma \times \mathbf{s}_\Gamma + \mathbf{H}_\Gamma \times \mathbf{f}_\Gamma, \tag{1.11}$$

$$\mathbf{b} = \mathbf{E}_\Gamma \times \mathbf{f}_\Gamma - \mathbf{H}_\Gamma \times \mathbf{s}_\Gamma, \tag{1.12}$$

where

$$a_\Gamma^{ik} = \begin{pmatrix} 0 & a_x & a_y & a_z \\ -a_x & 0 & -b_z & b_y \\ -a_y & b_z & 0 & -b_x \\ -a_z & -b_y & b_x & 0 \end{pmatrix}.$$

The connection of the tensor component A^{ik} written in curvilinear coordinates x^i with the tensor component A_Γ^{ik} written in Galilean coordinates is given by the law of transformation:

$$A^{ik} = \frac{\partial x^i}{\partial x_\Gamma^l}\frac{\partial x^k}{\partial x_\Gamma^m} A_\Gamma^{lm}. \tag{1.13}$$

Substituting the right side of the equation (1.4) instead of the tensor A_Γ^{lm}, we take into account that the components of the tensors g^{ik} and g_Γ^{ik}, a^{ik} and a_Γ^{ik} are also connected by the same transformation law (1.13) as the components of the tensors A^{ik} and A_Γ^{ik}. Thus, after the substitution, we obtain that the tensor A^{ik} can be represented as a sum of symmetric and antisymmetric tensors:

$$A^{ik} = \frac{1}{4} A_\Gamma g^{ik} + a^{ik}.$$

Simplifying this equation and taking into account the antisymmetric nature of the tensor a^{ik}, we find: $A_i^i = A_\Gamma$. Denoting $A = A_i^i$, we have a relation stating that the value of A remains unchanged in any coordinate system: $A = A_\Gamma$. From here we finally obtain the following for the equation considered:

$$A^{ik} = \tfrac{1}{4} A g^{ik} + a^{ik}. \tag{1.14}$$

Similarly, we find:

$$B^{ik} = \tfrac{1}{4} B g^{ik} + a^{*ik}, \tag{1.15}$$

where

$$B = B_i^i = B_\Gamma.$$

3.2 Equations of motion

Starting to find the equations to which the values under consideration are subjected, we pay attention to the antisymmetric character of the tensors a^{ik} and a^{*ik}. It implies the equation to zero of the double covariant derivatives of the indicated tensors:

$$a^{ik}_{;i;k} = 0, \tag{1.16}$$

$$a^{*ik}_{;i;k} = 0. \tag{1.17}$$

In this article, we consider only the electromagnetic field, which, as mentioned above, is not related to the space-time metric, therefore, any coordinate transformations considered in the article should not change the space-time metric. Such infinitesimal coordinate transformations are determined by the so-called Killing equations [1] $\xi^{i;k} + \xi^{k;i} = 0$, where ξ^i are small values that describe the transformation from the coordinates x^i to coordinates $x'^i = x^i + \xi^i$. Killing equations mean that with the specified coordinate transformations the variation of the metric tensor is zero: $\delta g^{ik} = 0$. From here it is easy to get that the Jacobians of such coordinate transformations are equal to one. To do so, we consider the indicated transformation from the Galilean coordinates x_Γ^i to the curvilinear coordinates $x^i = x_\Gamma^i + \xi^i$. With this coordinate transformation, the components of the metric tensor are transformed according to the law:

$$g^{ik} = \frac{\partial x^i}{\partial x_\Gamma^l} \frac{\partial x^k}{\partial x_\Gamma^m} g_\Gamma^{lm}.$$

We first find the determinant from the left and right side of this transformation law, which leads to the following relation:

$$\frac{1}{\sqrt{-g}} = \left| \frac{\partial x^i}{\partial x_\Gamma^l} \right| \approx 1 + \xi^i_{,i}\,,$$

where $g = |g_{ik}|$ is a determinant of the metric tensor g_{ik}. Killing's equations in Galilean coordinates are as follows: $\xi^{i,k} + \xi^{k,i} = 0$. Simplifying them, we get the following: $\xi^i_{,i} = 0$.

Thus, in Galilean coordinates we have $\sqrt{-g} = 1$, as it should be. It will be proved below that this equality holds not only in Galilean coordinates, but also in curvilinear coordinates describing spherically symmetric systems (1.62).

Taking into account this condition, twice covariantly differentiating between the left and right parts of equations (1.14) and (1.15) and considering equations (1.16) and (1.17), we obtain equations resulting from the matching of the electromagnetic field and the curvilinear coordinate system (x^0, x^1, x^2, x^3):

$$\frac{\partial}{\partial x^i}\left(g^{ik}\frac{\partial A}{\partial x^k}\right) = 4A^{ik}_{;i;k}\,, \tag{1.18}$$

$$\frac{\partial}{\partial x^i}\left(g^{ik}\frac{\partial B}{\partial x^k}\right) = 4B^{ik}_{;i;k}\,. \tag{1.19}$$

In electrodynamics, considered in curvilinear coordinates, the equations (1.18) and (1.19) play the role of equations of motion.

3.3 Variational problem

Considering an electromagnetic field in a four-dimensional space-time, limited neither in space nor in time.

We write the law of transformation connecting the components of the tensor $F^i_l f^{kl}$, given in the curvilinear coordinates x^i, and the components of the tensor $F^i_{\Gamma l} f^{kl}_{\Gamma}$, given in the Galilean coordinates x^i_{Γ}. We write the law of transformation connecting the components of the tensor $F^i_{(1)l} f^{kl}_{(1)}$, given in the curvilinear coordinates $x^i_{(1)}$, and the components of the same tensor $F^i_{\Gamma l} f^{kl}_{\Gamma}$, given in the Galilean coordinates x^i_{Γ}. Since the left sides of the relations obtained are equal, we equate the right sides of these relations and then, simplifying them, we get [2]:

$$F_{(1)ik} f^{ik}_{(1)} = F_{ik} f^{ik}. \tag{1.20}$$

We multiply the left side and the right side of equation (1.20) by the value *dudv*. Then, integrating over an arbitrary domain *S* lying on a two-dimensional surface $x^i(u.v)$, we get the following equation:

$$\iint\limits_S F_{(1)ik} f^{ik}_{(1)} dudv = \iint\limits_S F_{ik} f^{ik} dudv. \tag{1.21}$$

We transform from the curvilinear coordinates x^i to the curvilinear coordinates $x^i_{(1)} = x^i + \xi^i$, where ξ^i means small values. Substituting $x^i_{(1)} = x^i + \xi^i$ in left part of the equation (1.21) and decomposing the integrand in a series of powers ξ^i, we get after reduction:

$$\delta \iint\limits_S F_{ik} f^{ik}\, dudv = 0. \tag{1.22}$$

When matching the electromagnetic field with a curvilinear coordinate system, the components of the tensor F_{ik} should be considered as functions of the coordinates x^i: $F_{ik} = F_{ik}(x^i)$. Thus, from the equation (1.22) we get the following variational problem:

$$\delta \iint\limits_S \Lambda(x^i, x^i_{,u}, x^i_{,v})\, dudv = 0, \tag{1.23}$$

where

$$\Lambda = \frac{1}{2} F_{ik} f^{ik} = F_{ik} x^i_{,u} x^k_{,v}\,, \qquad x^i_{,u} \equiv \frac{\partial x^i}{\partial u}, \qquad x^i_{,v} \equiv \frac{\partial x^i}{\partial v}.$$

Performing the variation in the left-hand side of the equation (1.23), we arrive at the Euler equation and the natural boundary conditions

$$\frac{\partial \Lambda}{\partial x^i} - \frac{\partial^2 \Lambda}{\partial u \partial x^i_{,u}} - \frac{\partial^2 \Lambda}{\partial v \partial x^i_{,v}} = 0, \tag{1.24}$$

$$\iint\limits_S \left[\frac{\partial}{\partial u}\left(\frac{\partial \Lambda}{\partial x^i_{,u}} \delta x^i\right) + \frac{\partial}{\partial v}\left(\frac{\partial \Lambda}{\partial x^i_{,v}} \delta x^i\right)\right] dudv = 0. \tag{1.25}$$

Substituting the value Λ in the Euler equation and performing differentiation, we find:

$$F_{ik;l} + F_{kl;i} + F_{li;k} = 0.$$

This is the first pair of Maxwell's equations. It follows that the Euler equation is carried out automatically.

We consider the natural boundary conditions for an infinitely small section ΔS of a two-dimensional surface. Its area will tend to zero, therefore, in the first approximation, this area can be considered flat and we can apply the Green formula to the integral (1.25), written for the section ΔS, we get:

$$\oint_{\Delta C} \left(\frac{\partial \Lambda}{\partial x^i_{,u}} \delta x^i dv - \frac{\partial \Lambda}{\partial x^i_{,v}} \delta x^i du\right) = 0 \tag{1.26}$$

where ΔC is a closed loop covering ΔS. We introduce another system of curvilinear coordinates x'^0, x'^1, x'^2, x'^3, the first two coordinates of which are coordinates on the two-dimensional surface under consideration $x'^0 = u,\ x'^1 = v$. The two remaining coordinates will be denoted as $x'^2 = w,\ x'^3 = n$. The (') sign was used only once in the formula (1.1), so its new use should not cause any confusion. The tangent vectors to the coordinate lines *w* and *n* are denoted by:

$$x^i_{,w} \equiv \frac{\partial x^i}{\partial w},\ x^i_{,n} \equiv \frac{\partial x^i}{\partial n}.$$

We imagine the variation δx^i as a sum:

$$\delta x^i = \alpha x^i_{,u} + \beta \partial x^i_{,v} + \gamma \partial x^i_{,w} + \theta \partial x^i_{,n}, \tag{1.27}$$

where: $\alpha = \frac{\partial u}{\partial x^k} \delta x^k$; $\beta = \frac{\partial v}{\partial x^k} \delta x^k$; $\gamma = \frac{\partial w}{\partial x^k} \delta x^k$; $\theta = \frac{\partial n}{\partial x^k} \delta x^k$.

Substituting these values into (1.27), we obtain:

$$\delta x^i = \frac{\partial x^i}{\partial x'^l}\frac{\partial x'^l}{\partial x^k}\delta x^k = \delta^i_k \delta x^k = \delta x^i.$$

Substituting the value Λ in the integral over the closed contour ΔC (1.26) and the right-hand side of the equation (1.27) for the variation δx^i, we get:

$$\oint_{\Delta C}\left[\Lambda(\alpha dv - \beta du) + F_{ik}(\gamma x^i_{,w}x^k_{,v}dv + \theta x^i_{,n}x^k_{,v}dv - \gamma x^k_{,w}x^i_{,u}du - \theta x^k_{,n}x^i_{,u}du)\right] = 0. \quad (1.28)$$

Since $\alpha, \beta, \gamma, \theta$ are arbitrary values, the integral (1.28) can be divided into four integrals and be considered independently of each other [2]. From here we get:

$$\alpha = \beta = 0 \quad (1.29)$$

and

$$F_{ik}x^i_{,w}x^k_{,u} = F_{ik}x^i_{,w}x^k_{,v} = F_{ik}x^i_{,n}x^k_{,u} = F_{ik}x^i_{,n}x^k_{,v} = 0. \quad (1.30)$$

The closed contour ΔC, like C, has an arbitrary shape, so when considering the four integrals (1.28), one should consider options for which the main contribution to the integral can be summed either by the coordinate u, or by the coordinate v. Condition (1.30) can be rewritten as follows:

$$F'_{02} = F'_{03} = F'_{12} = F'_{13} = 0, \quad (1.31)$$

if we consider that the components of the electromagnetic field tensors F_{ik} and F'_{ik}, considered in the curvilinear coordinates x^i and x'^i, are related by the following transformation law:

$$F_{ik}\frac{\partial x^i}{\partial x'^l}\frac{\partial x^k}{\partial x'^m} = F'_{lm}. \quad (1.32)$$

But α and β describe that 'part' of the variation that lies in the tangent plane to the two-dimensional surface $x^i(u, v)$. Therefore, the condition (1.29) implies that the variation along the two-dimensional surface is zero. This means that the variation does not change the distance between any two points on the surface $x^i(u, v)$, i.e., the surface behaves like an incompressible and inextensible film. Such changes that occur with the surface are called deformations in mathematics.

Thus (1.31), in the curvilinear coordinates x'^i, the electromagnetic field tensor only two components F'_{01} and F'_{23} are nonzero. For the component F'_{01}, the transformation law (1.32) can be written as follows:

$$\frac{1}{2}F_{ik}f^{ik} = F'_{01} \text{ or } \frac{1}{2}F_{\Gamma ik}f^{ik}_{\Gamma} = F'_{01}, \quad (1.33)$$

if we write the transformation law (1.32) connecting the components of the tensors $F_{\Gamma ik}$ and F'_{ik}, considered in the Galilean coordinates x^i_Γ and the curvilinear coordinates x'^i, respectively. From the obtained equations we find:

$$F'_{01} = \frac{1}{4}A. \tag{1.34}$$

Now we consider the value $\frac{1}{4}B_\Gamma$. Since $B = B^i_i = B_\Gamma$, we will do all calculations in Galilean coordinates. It is easy to verify that

$$F_{\Gamma ik} f_\Gamma^{*ik} = F^*_{\Gamma ik} f_\Gamma^{ik}, \tag{1.35}$$

but

$$\frac{1}{2} F^*_{\Gamma ik} f_\Gamma^{ik} = F^*_{\Gamma ik} \frac{\partial x_\Gamma^i}{\partial x'^0} \frac{\partial x_\Gamma^k}{\partial x'^1} = F'^*_{01}. \tag{1.36}$$

Thus, we get:

$$F'^*_{01} = \frac{1}{4}B. \tag{1.37}$$

3.4 Two-dimensional spaces

We return to the condition (1.29) and its corollary: the surface $x^i(u, v)$ is an incompressible and inextensible film, and the changes that occur with the surface when it is varied are deformations. All this suggests that the surface $x^i(u, v)$ can be considered a two-dimensional space, which has certain properties and preserves them with variation. Indeed, with variation, the distances between any two points of the surface, and hence the two-dimensional space, remain constant. When bending, the Gaussian curvature at each point of the surface $x^i(u, v)$, and therefore at every point of two-dimensional space, remains unchanged. Additional confirmation of the above can be obtained by considering the following calculations. We write the first pair of Maxwell's equations in curvilinear coordinates x'^i taking into account the condition (1.31):

$$F'_{01,2} = F'_{01,3} = F'_{23,0} = F'_{23,1} = 0.$$

It follows there from that $F'_{01} = F'_{01}(\, x'^0,\, x'^1)$, i.e. this component is a function of the coordinates $x'^0 = u$ and $x'^1 = v$, and $F'_{23} = F'_{23}(\, x'^2,\, x'^3)$, i.e. this component is a function of the coordinates $x'^2 = w$ and $x'^3 = n$. Thus, we find that in the curvilinear coordinates x'^i each of the two nonzero components of the electromagnetic field tensor depends on a strictly individual set of coordinates consisting of only two curvilinear coordinates. This fact is another confirmation of the fact that we are dealing with two two-dimensional spaces. One of them is formed by a two-dimensional surface $x^i(u, v)$; the second two-dimensional space is formed by a two-dimensional surface $x^i(w, n)$. Since these surfaces are coordinate surfaces of four-dimensional curvilinear coordinate system (u, v, w, n), therefore, their geometry, and hence, the geometry of two-dimensional spaces, is determined by metric tensors [3]:

$$g'_{ab} = \frac{\partial x_\Gamma^i}{\partial x'^a} \frac{\partial x_\Gamma^k}{\partial x'^b} g_{\Gamma ik}, \tag{1.38}$$

$$g'_{\hat{a}\hat{b}} = \frac{\partial x_\Gamma^i}{\partial x'^{\hat{a}}} \frac{\partial x_\Gamma^k}{\partial x'^{\hat{b}}} g_{\Gamma ik}, \tag{1.39}$$

where a, b, ... = 0,1; $\hat{a}, \hat{b}$, ... = 2,3.

Each of these tensors is obviously connected with the metric tensor of a curvilinear coordinate system (*u*, *v*, *w*, *n*):

$$g'_{ik} = \frac{\partial x^l_\Gamma}{\partial x'^i}\frac{\partial x^m_\Gamma}{\partial x'^k} g_{\Gamma lm}.$$

Using the calculation of Riemannian spaces [3], it is arguable that the surface $x^i(u,v)$ is a two-dimensional space with a metric tensor (1.38). It is clear that all this can be repeated for a two-dimensional space with the metric tensor $g'_{\hat{a}\hat{b}}$. In each of these two-dimensional spaces, respectively, one can enter the tensor of the electromagnetic field:

$$F'_{ab} = \begin{pmatrix} 0 & F'_{01} \\ -F'_{01} & 0 \end{pmatrix},\ F'_{\hat{a}\hat{b}} = \begin{pmatrix} 0 & F'_{23} \\ -F'_{23} & 0 \end{pmatrix}$$

and write accordingly the following tensor equation:

$$F'_{ab} = g'_{ac}g'_{bd}F'^{cd} = \frac{1}{2}(g'_{ac}g'_{bd} - g'_{ad}g'_{bc})F'^{cd},$$

$$F'_{\hat{a}\hat{b}} = g'_{\hat{a}\hat{c}}g'_{\hat{b}\hat{d}}F'^{\hat{c}\hat{d}} = \frac{1}{2}(g'_{\hat{a}\hat{c}}g'_{\hat{b}\hat{d}} - g'_{\hat{a}\hat{d}}g'_{\hat{b}\hat{c}})F'^{\hat{c}\hat{d}}.$$

From here we get:

$$F'_{01} = qF'^{01}, \tag{1.40}$$

$$F'_{23} = \hat{q}F'^{23}, \tag{1.41}$$

where

$$q = g'_{00}g'_{11} - g'^2_{01} = \det[g'_{ab}], \tag{1.42}$$

$$\hat{q} = g'_{22}g'_{33} - g'^2_{23} = \det\left[g'_{\hat{a}\hat{b}}\right]. \tag{1.43}$$

The formulas (1.40) and (1.41) establish a connection between the covariant and contravariant components of the electromagnetic field in the corresponding two-dimensional space. We note that if two-dimensional surfaces are represented as planes and viewed in Galilean coordinates, then for the values (1.42) and (1.43) we will have the following values: $q = -1$ and $\hat{q} = 1$. Substituting these values in (1.40) and (1.41) we arrive at a well-known connection between the various types of components of the tensor of the electromagnetic field, given in Galilean coordinates.

3.5 The law of stress-energy tensors equality

For further calculations, we consider the stress-energy tensor of the electromagnetic field, and then we write it in Galilean coordinates as follows:

$$4\pi T_{\Gamma ik} = -F_{\Gamma il}F_{\Gamma k}^{\ l} + \frac{1}{4}g_{\Gamma ik}F_{\Gamma lm}F_\Gamma^{lm}. \tag{1.44}$$

We prove that for a given tensor the equation is true:

$$T_{\Gamma ik} = T^{(*)}_{\Gamma ik}\,, \tag{1.45}$$

where

$$4\pi T^{(*)}_{\Gamma ik} = -F^{*}_{\Gamma il}F^{*l}_{\Gamma k} + \frac{1}{4}g_{\Gamma ik}F^{*}_{\Gamma lm}F^{*lm}_{\Gamma}. \qquad (1.46)$$

To do this, we substitute the right-hand sides of the calculations (1.44) and (1.46) in the equation (1.45), after multiplying the left and right sides of equation (1.45) by 4π. Considering that

$$-\frac{1}{2}F^{*}_{\Gamma lm}F^{*lm}_{\Gamma} = \frac{1}{2}F_{\Gamma lm}F^{lm}_{\Gamma} = \mathbf{H}^{2}_{\Gamma} - \mathbf{E}^{2}_{\Gamma}\ ,$$

we get:

$$F_{\Gamma il}F_{\Gamma k}^{\ \ l} - F^{*}_{\Gamma il}F^{*l}_{\Gamma k} = \left(\mathbf{H}^{2}_{\Gamma} - \mathbf{E}^{2}_{\Gamma}\right)g_{\Gamma ik}. \qquad (1.47)$$

The validity of tensor equation (1.47) can be checked directly for each of its components. That proves the validity of the equation (1.45). Next, applying the transformation law connecting the components of the tensor $T_{\Gamma lm}$, given in the Galilean coordinates x^{i}_{Γ}, with the components of the tensor T_{ik}, given in curvilinear coordinates x^{i}, and applying the same law respectively for the tensors $T^{(*)}_{\Gamma lm}$ and $T^{(*)}_{ik}$, to the left and right sides of equation (1.45), we obtain:

$$T_{ik} = T^{(*)}_{ik}. \qquad (1.48)$$

The validity of this equation follows from the validity of equation (1.45). The form of the stress-energy tensors of the electromagnetic field, which are in the equation (1.48), can be established by using the laws of transformation given above. Substituting in their right-hand side, respectively, the values of $T_{\Gamma ik}$ or $T^{(*)}_{\Gamma ik}$, found from the calculations (1.44) and (1.46), and taking into account that their values are related by the same transformation laws, we get:

$$4\pi T_{ik} = -F_{il}F^{l}_{k} + \frac{1}{4}g_{ik}F_{\Gamma lm}F^{lm}_{\Gamma},\ \ 4\pi T^{(*)}_{ik} = -F^{*}_{il}F^{*l}_{k} + \frac{1}{4}g_{ik}F^{*}_{\Gamma lm}F^{*lm}_{\Gamma}.$$

Simplifying these calculations and taking into account that the trace of the stress-energy tensor of the electromagnetic field is zero, and from (1.48) it follows that $T^{(*)i}_{i} = 0$, we find:

$$F_{ik}F^{ik} = F_{\Gamma lm}F^{lm}_{\Gamma},\ \ F^{*}_{ik}F^{*ik} = F^{*}_{\Gamma lm}F^{*lm}_{\Gamma}.$$

Considering these equalities, we can write down the calculations of the stress-energy tensors of the electromagnetic field in the system of curvilinear coordinates x^{i} in the following form:

$$4\pi T_{ik} = -F_{il}F^{l}_{k} + \frac{1}{4}g_{ik}F_{lm}F^{lm},\ \ 4\pi T^{(*)}_{ik} = -F^{*}_{il}F^{*l}_{k} + \frac{1}{4}g_{ik}F^{*}_{lm}F^{*lm},$$

where all members of these calculations are expressed in the same coordinate system x^{i}. Once again applying the transformation law now to the left and right side of the tensor equation (1.47), we get:

$$F_{il}F^{l}_{k} - F^{*}_{il}F^{*l}_{k} = \left(\mathbf{H}^{2}_{\Gamma} - \mathbf{E}^{2}_{\Gamma}\right)g_{ik}. \qquad (1.49)$$

The validity of this formula follows from the validity of formulas (1.47) and (1.48). Note that formula (1.49) can also be obtained by substituting the calculations of the stress-energy tensors of the electromagnetic field in the equation (1.48), considered in the curvilinear coordinates x^i. The equation (1.48) extends our understanding of the properties of electromagnetic fields, therefore, to emphasize this, it can be called the law of stress-energy tensor equality, composed of the electromagnetic field tensors dual to each other.

3.6 Field in two-dimensional spaces

Since the formula (1.49) is another form of writing the law the law of stress-energy tensor equality of an electromagnetic field, therefore, writing down the formula (1.49) in curvilinear coordinates x'^i and taking into account

$$g'^{il}g'_{kl} = \delta^i_k\,, \tag{1.50}$$

we write the formulas (1.49) and (1.50) in the components. In order not to give all thirty-two equations, which are obtained by writing the formulas (1.49) and (1.50) in the components, we restrict ourselves to the minimum number of equations necessary to demonstrate the method of calculations. From (1.49) we have in curvilinear coordinates x'^i:

$$g'^{00}(F'^2_{01} - F'^{*2}_{01}) = \left(\mathbf{H}^{\mathbf{2}}_{\Gamma} - \mathbf{E}^{\mathbf{2}}_{\Gamma}\right)g'_{11},\ g'^{01}(F'^2_{01} - F'^{*2}_{01}) = -\left(\mathbf{H}^{\mathbf{2}}_{\Gamma} - \mathbf{E}^{\mathbf{2}}_{\Gamma}\right)g'_{01},$$

$$g'^{02}(F'_{01}F'_{23} - F'^{*}_{01}F'^{*}_{23}) = \left(\mathbf{H}^{\mathbf{2}}_{\Gamma} - \mathbf{E}^{\mathbf{2}}_{\Gamma}\right)g'_{13},$$

$$g'^{03}(F'_{01}F'_{23} - F'^{*}_{01}F'^{*}_{23}) = -\left(\mathbf{H}^{\mathbf{2}}_{\Gamma} - \mathbf{E}^{\mathbf{2}}_{\Gamma}\right)g'_{12}.$$

Multiply the first equation by g'_{00}, the second equation by g'_{01}, the third equation by g'_{02} and the fourth equation by the value g'_{03}. From (1.50) we have:

$$g'_{00}g'^{00} + g'_{01}g'^{01} + g'_{02}g'^{02} + g'_{03}g'^{03} = 1.$$

We substitute here the values of the components from the left-hand side of this equation, which can be found from the four equations obtained after multiplying by the components of the metric tensor. Performing similar calculations for the other components of the formulas (1.49) and (1.50) and taking into account the calculations (1.42) and (1.43), we arrive at the following equation:

$$\hat{q}(F'^2_{01} - F'^{*2}_{01}) = q(F'^2_{23} - F'^{*2}_{23}), \tag{1.51}$$

Considering the condition $\sqrt{-g'} = 1$, we can write for a pseudo-tensor given in curvilinear coordinates, $F'^{*}_{ik} = \frac{1}{2}e_{iklm}F'^{lm}$. Hence, using the equations (1.40) and (1.41), we find:

$$F'^{*}_{01} = -\frac{F'_{23}}{\hat{q}}\,, \tag{1.52}$$

$$F'^{*}_{23} = -\frac{F'_{01}}{q}\,. \tag{1.53}$$

Substituting (1.52) and (1.53) in (1.51), we finally get:

$$\frac{F_{01}'^2}{q} = \frac{F_{23}'^2}{\hat{q}}. \tag{1.54}$$

From (1.34) it follows that

$$F_{ik}' = \begin{pmatrix} 0 & F_{01}' & 0 & 0 \\ -F_{01}' & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{23}' \\ 0 & 0 & -F_{23}' & 0 \end{pmatrix}. \tag{1.55}$$

The components of the tensor (1.55) $F_{a\hat{b}}' = 0$ that are equal to zero are connected with the components $F'^{a\hat{b}}$of the tensor F'^{ik} by the relation:

$$F_{a\hat{b}}' = g_{ac}' g_{\hat{b}\hat{d}}' F'^{c\hat{d}} = 0,$$

from which it follows that $F'^{a\hat{b}} = 0$, therefore

$$F'^{ik} = \begin{pmatrix} 0 & q^{-1}F_{01}' & 0 & 0 \\ -q^{-1}F_{01}' & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{q}^{-1}F_{23}' \\ 0 & 0 & -\hat{q}^{-1}F_{23}' & 0 \end{pmatrix}. \tag{1.56}$$

From (1.54), (1.55) and (1.56) we find:

$$F_{01}' - \frac{1}{2}\sqrt{F_{ik}'F'^{ik}}\sqrt{q} = 0, \tag{1.57}$$

$$F_{23}' - \frac{1}{2}\sqrt{F_{ik}'F'^{ik}}\sqrt{\hat{q}} = 0. \tag{1.58}$$

These equations determine the electromagnetic field in two-dimensional spaces. The equations (1.57) defines an electromagnetic field in a two-dimensional space (*u*, *v*), and the equation (1.58) defines a field in a two-dimensional space (*w*, *n*), and establishes a relationship between the electromagnetic field and determinants of the metric tensors (1.38) and (1.39) that define the two-dimensional spaces.

3.7 Spherically symmetric systems

We show that the formula (1.57) is the Coulomb law written in curvilinear coordinates. To do this, we write the formula (1.57) in three-dimensional space in orthogonal coordinates. Using the formulas [1]:

$$\gamma_{\alpha\beta} = -g_{\alpha\beta}' + \frac{g_{0\alpha}' g_{0\beta}'}{g_{00}'}, \qquad \alpha, \beta = 1,2,3, \tag{1.59}$$

$$-g' = g_{00}'\gamma\,, \tag{1.60}$$

where $\gamma = \det[\gamma_{\alpha\beta}]$ is a determinant of the metric tensor $\gamma_{\alpha\beta}$, and taking into account that in orthogonal coordinates $\gamma = \gamma_{11}\gamma_{22}\gamma_{33}$, as well as the equation $\sqrt{-g'} = 1$, we obtain for the determinant (1.42):

$$q = -\frac{1}{\gamma_{22}\gamma_{33}}. \tag{1.61}$$

The electric field, which is considered in the Coulomb law, is spherically symmetric. Such a field is most conveniently viewed in spherical coordinates. Therefore, to determine γ_{22} and γ_{33}, we write the square of the element of length in spherical coordinates

$$dS^2 = dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2.$$

But in this form it is impossible to use this equation to determine the components of the three-dimensional metric tensor. The fact is that the spherical coordinates ϑ and φ enter it non-symmetrically and, moreover, they are dimensionless.

To eliminate these shortcomings, one should consider an infinitely small neighborhood of a point with the spherical coordinates (ρ_0 ,ϑ_0,φ_0). Then we draw through this point a tangent plane to a sphere of the radius ρ_0. Let us introduce on this plane a rectangular coordinate system ($\tilde{x}$,$\tilde{y}$) with the origin at a point (ρ_0,ϑ_0,φ_0) so that the coordinate axis $\tilde{x}$ is tangent to the coordinate line ϑ, and the coordinate axis $\tilde{y}$ is tangent to the coordinate line φ. In an infinitely small neighborhood of the point, we have:

$$d\tilde{x} \approx \rho_0 d\vartheta; d\tilde{y} \approx \rho_0 sin\vartheta_0 d\varphi \approx \rho_0 sin\vartheta d\varphi.$$

From here, we get:

$$dS^2 = dr^2 + \frac{r^2}{\rho_0^2} d\tilde{x}^2 + \frac{r^2}{\rho_0^2} d\tilde{y}^2. \tag{1.62}$$

From (1.62), we have the following values for the components of the three-dimensional metric tensor in an infinitely small neighborhood of the point (ρ_0 ,ϑ_0,φ_0):

$$\gamma_{22} = \gamma_{33} = \frac{r^2}{\rho_0^2}. \tag{1.63}$$

For the transformations $x'^i = x^i + \xi^i$ considered in the article, from (1.27), (1.29) we have $\delta x^0 = \delta x^1 = 0$. Therefore, for the variation of the metric tensor, we obtain:

$$\delta g_{ik} = \frac{\partial g_{ik}}{\partial x^l} \delta x^l = \frac{\partial g_{ik}}{\partial x^2} \delta x^2 + \frac{\partial g_{ik}}{\partial x^3} \delta x^3.$$

Hence, if the components g_{ik} depend only on the coordinates x^0, x^1, for example, as in the spherically symmetric system (1.62), then in this system $\delta g_{ik} = 0$. Let us construct a tensor g_{ik}, satisfying the above conditions. We find the component g_{00} from (1.60). The components $g_{\alpha\beta}$ are determined from (1.59). They will depend on $\frac{r^2}{\rho_0^2}$ (1.63) and on the components $g_{0\alpha} = g_{0\alpha}(x^0, x^1)$. It is easy to check that the determinant of this tensor is -1. Let us find the values of the diagonal components of the metric tensor g_{ik} given in curvilinear coordinates x^i. We neglect the terms $\frac{r^4}{\rho_0^4} g_{0\alpha}^2$, which have a higher order of smallness. Using $x_\Gamma^i = x^i - \xi^i$, we get:

$$g_{00} = 1 - 2\xi^0_{,0};\ g_{11} = -1 + 2\xi^1_{,1};\ g_{22} = -1 + 2\xi^2_{,2};\ g_{33} = -1 + 2\xi^3_{,3}.$$

In Galilean coordinates, the values of the diagonal components of the metric tensor on the left side of these equalities will be as follows: $1, -1, -1, -1$. Hence, we obtain that the derivatives on the right-hand side of these equalities will be equal to zero. This is one more proof that the equality $\xi^i_{,i} = 0$ holds in Galilean coordinates. Now let us consider the curvilinear coordinates x^i describing a spherically symmetric system. As mentioned above, such a system should be considered in the tangent plane to a sphere of radius ρ_0 in an infinitesimal neighborhood of the point of tangency $(\rho_0, \vartheta_0, \varphi_0)$ of the plane with the sphere. In an infinitely small neighborhood of this point, we can write $\frac{r}{\rho_0} \approx 1 + \delta$, where δ is a small quantity, therefore, for the diagonal components of the metric tensor of a spherically symmetric system, we have:

$$g_{00} = 1 - 4\delta;\ g_{11} = -1;\ g_{22} = g_{33} = -1 - 2\delta.$$

Comparing these values with the previously obtained ones, we find:

$$\xi^0_{,0} = 2\delta;\ \xi^1_{,1} = 0;\ \xi^2_{,2} = \xi^3_{,3} = -\delta.$$

From this, we see that for a spherically symmetric system $\xi^i_{,i} = 0$.

Substituting (1.63) into (1.61) and the result of this substitution into (1.57), we arrive at the formula:

$$F'_{01} = \frac{1}{2}\sqrt{-F'_{ik}F'^{ik}}\frac{\rho_0^2}{r^2}. \tag{1.64}$$

Considering that $A = A_\Gamma$, the equation (1.36) and the calculation (1.6), we find $F'_{01} = \mathbf{E}_\Gamma\mathbf{f}_\Gamma - \mathbf{H}_\Gamma\mathbf{s}_\Gamma$. But we consider only the electric field, therefore $\mathbf{H}_\Gamma = 0$. In the absence of any movement and change, time remains unchanged, therefore $u \equiv x'^0 = x^0_\Gamma \equiv ct$. Thus, everything comes down to the transformation of spatial coordinates: the rectangular Cartesian coordinates x, y, z and the curvilinear coordinates v, w, n, which naturally should be taken as spherical coordinates. So, for instance, $v = r$, and for the electric field we have $E_r = E$; $E_\vartheta = E_\varphi = 0$. It follows that $\mathbf{E}_\Gamma = E(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$. From (1.8), (1.10), etc., we obtain: $\mathbf{f}_\Gamma = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$. Considering the above, we arrive at this value $F'_{01} = \mathbf{E}_\Gamma\mathbf{f}_\Gamma = E$. Now the formula (1.64) can be written as follows:

$$E = \frac{1}{2}\sqrt{-F'_{ik}F'^{ik}}\frac{\rho_0^2}{r^2}.$$

From this formula it follows that a physical value equal to

$$2\pi\varepsilon_0\rho_0^2\sqrt{-F'_{ik}F'^{ik}},$$

is an electric charge e, where ε_0 – electric constant. Thus, we get the formula $E = \frac{e}{4\pi\varepsilon_0 r^2}$, which completely coincides with Coulomb's law.

Now we will consider the formula (1.58) in a three-dimensional space in spherical coordinates. To do this, we again use the formulas (1.59), (1.60) and again we take into account that $\sqrt{-g'} = 1$. Thus, after the transformation, we obtain the determinant (1.43):

$$\hat{q} = \gamma_{22}\gamma_{33} - \frac{g_2^2}{\gamma_{11}\gamma_{22}} - \frac{g_3^2}{\gamma_{11}\gamma_{33}}, \tag{1.65}$$

where $g_\alpha = -\frac{g'_{0\alpha}}{g'_{00}}$ [1]. Substituting in this expression the values of the components of the three-dimensional metric tensor (1.63), as well as $\gamma_{11} = 1$, see (1.62), we arrive at the following formula:

$$\hat{q} = \frac{r^4}{\rho_0^4} - \frac{\rho_0^2}{r^2}(g_2^2 + g_3^2). \tag{1.66}$$

We multiply the left side and the right side of the formula (1.66) by the value $\frac{r^2}{\rho_0^2}$. Then, denoting $\chi = \frac{r^2}{\rho_0^2}$, we represent (1.66) as a cubic equation

$$\chi^3 - \hat{q}\chi - g_2^2 - g_3^2 = 0. \tag{1.67}$$

Its solution is three roots:

$$\chi_1 = \frac{r_1^2}{\rho_0^2}; \chi_2 = \frac{r_2^2}{\rho_0^2}; \chi_3 = \frac{r_3^2}{\rho_0^2}.$$

These roots satisfy the following relations:

$$\chi_1 + \chi_2 + \chi_3 = 0. \tag{1.68}$$

$$\chi_1\chi_2 + \chi_2\chi_3 + \chi_3\chi_1 = -\hat{q}. \tag{1.69}$$

$$\chi_1\chi_2\chi_3 = g_2^2 + g_3^2. \tag{1.70}$$

Raising the left side of the equation (1.68) to the square and taking into account (1.69), we get:

$$\hat{q} = \frac{r_1^4 + r_2^4 + r_3^4}{2\rho_0^4}. \tag{1.71}$$

We divide the left side of the equation (1.69) by the left side of the equation (1.70) and, accordingly, the right side of the equation (1.69) by the right side of the equation (1.70), and thus, we find:

$$\hat{q} = -\rho_0^2\left(\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2}\right)(g_2^2 + g_3^2). \tag{1.72}$$

From (1.72) it follows that the value $\hat{q}$ is formed by three separate 'particles' with relative charges as follows:

$$\frac{1}{\chi_1} = \frac{\rho_0^2}{r_1^2}; \frac{1}{\chi_2} = \frac{\rho_0^2}{r_2^2}; \frac{1}{\chi_3} = \frac{\rho_0^2}{r_3^2}.$$

Their total relative charge is equal to the relative charge of the proton. Taking the relative charge of the proton equal to unity, from (1.72) we obtain the relationship between the coefficient and the free term of equation (1.67): $\hat{q} = -g_2^2 - g_3^2$. Obviously, only quarks can be such 'particles'.

It is easy to verify as for quarks forming a proton and having charges of $\frac{2}{3}; \frac{2}{3}; -\frac{1}{3}$ the relation (1.68) is really fulfilled: $\frac{3}{2}+\frac{3}{2}-\frac{3}{1}=0$. From these simple considerations, it follows that the two-dimensional space (w, n) has finite dimensions and it, in fact, is what we call an elementary particle, for example, a proton.

Here is one more proof of the correctness of the theoretical calculations and the conclusions made on their basis. From (1.57) and (1.58), we find:

$$\frac{F'_{23}}{F'_{01}} = \sqrt{\frac{\hat{q}}{q}}.$$

For the proton $\hat{q} = -g_2^2 - g_3^2$, and from (1.60) and (1.61) we have $q = -g'_{00}$. If we use these values, by means of (1.66), we obtain:

$$\frac{F'_{23}}{F'_{01}} = \frac{\frac{r^5}{\rho_0^5}}{\sqrt{1-\frac{r^2}{\rho_0^2}}}.$$

At $\frac{r^2}{\rho_0^2} \to 0$, $\frac{F'_{23}}{F'_{01}} \to 0$; at $\frac{r^2}{\rho_0^2} \to 1$, $\frac{F'_{23}}{F'_{01}} \to \infty$. This result proves that formula (1.58) describes a strong interaction acting in a finite region of space, the magnitude of which is determined by the radius ρ_0. And in this region of space, the magnitude of the strong interaction grows with the increasing radius r.

3.8 Evidence

The solution to equation (1.67) was obtained for $\hat{q} > 0$. This inequality is fulfilled in the region of four-dimensional space, which in spherical coordinates is defined as follows: $\rho_0 < r \leq \infty$. It is in this region that the quark nature of an elementary particle is manifested. This can be explained by the fact that two invariants $\hat{q}$ and $F'_{ik}F'^{ik}$ (their invariance follows from the equality to unity of the Jacobian transformation, since $\sqrt{-g} = 1$) in Galilean coordinates decompose into three invariants of Lorentz transformations [2]. For example,

$$H_{\Gamma x}^2 - E_{\Gamma x}^2; H_{\Gamma z}^2 - E_{\Gamma y}^2; H_{\Gamma y}^2 - E_{\Gamma z}^2$$

which behave like independent entities. But in curvilinear coordinates, these three invariants will no longer be invariants. Therefore, their complete independence is impossible. Because of this, quarks are not particles in the usual sense. In the absence of a magnetic field

$$F'_{ik}F'^{ik} = -2\mathbf{E}_\Gamma^2 < 0,$$

and in the indicated region of space, a complex quantity appears in equality (1.58), which is unacceptable. Therefore, Equality (1.58) is inapplicable in this region of four-dimensional space. Equality (1.58) will consist of real values for $\hat{q} < 0$. This inequality holds in the region defined as $0 \leq r < \rho_0$. This is easy to prove if we notice that it is in this region of the four-dimensional space that Equation (1.67) has one more solution. Substituting the equal value $\hat{q} = -g_2^2 - g_3^2$ into Equation (1.67) instead of the free term, we find

$$\hat{q} = \frac{\chi^3}{\chi - 1}.$$

Hence it follows that for $\chi \leq 1$ we have $\hat{q} \leq -\infty$. It can be seen that the indicated solution is obtained for $\chi \sim 1$, when $|\hat{q}| \gg \chi^3$, therefore χ^3 in the equation can be neglected. Note, that in the region $0 \leq r < \rho_0$ the electromagnetic field radically changes its dependence on the spatial coordinates (1.58) and completely coincides with the dependence that is observed for the strong interaction.

4 Conclusions

Summing up, it must be said that in electrodynamics, considered in curvilinear coordinates, the second pair of Maxwell's equations can be obtained using the antisymmetric character of the electromagnetic field tensor. From this antisymmetry it follows: $F^{ik}_{;i;k} = 0$. If we mark

$$-\frac{c}{4\pi} F^{ik}_{;k}$$

as a four-dimensional vector of current density, we obtain the second pair of Maxwell's equations in a known form, and from the equation $F^{ik}_{;i;k} = 0$, taking into account the introduced notation, we get the continuity equation. So, classical electrodynamics which neglects the internal structure of elementary particles can be called a macroscopic theory that considers electromagnetic fields on the scale of the macro-world.

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