Mach's Principle and Gravitational Induction

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Abstract.- *We formulate Mach's principle and show that the inertial force is the gravitational induction force the Universe exerts on an accelerated body. We show that inertial mass depends on cosmic time, which has measurable consequences.*

The fundamental law of Newtonian mechanics states that in an inertial reference frame (hereinafter IRF; where the law of inertia is valid), the acceleration **a** of a body of inertial mass *m* when we apply a force **F** is $\mathbf{a} = \mathbf{F}/m$. The law must be modified when we measure the acceleration in a non-inertial reference frame (which is accelerated with respect to an IRF): $\mathbf{a} = \left(\mathbf{F} + \mathbf{F}_f\right)/m$. $\mathbf{F}_f$ are "fictitious forces", erroneously called inertial forces. $\mathbf{F}_f$ are not true forces because other bodies do not produce them; we call them forces because they are the product of a mass times an acceleration (Segura, 2019).

There is another way to express Newton's law: the principle of dynamic equilibrium or D'Alembert's principle. When a body has an acceleration **a** with respect to an IRF, it is subjected to the force $\mathbf{F}_i = -m\mathbf{a}$ called inertia force, which balances the applied force: $\mathbf{F} + \mathbf{F}_i = 0$. $\mathbf{F}_i$ is a true force (as simple experiments show) (Graneu & Graneu, 2006, p. 157-164). $\mathbf{F}_i$ is caused by interaction with other bodies. D'Alembert's principle applies to inertial and non-inertial frames; but in both, **a** is the acceleration with respect to an IRF.

Both formulations are very different, since D'Alembert's principle leads to Mach's principle, according to which the phenomenon of inertia (that is, the opposition of a body to changing its speed), the inertial mass, and the inertial force, originate from the gravitational action of the whole of the Universe, that is, they have a cosmic origin (Bondi & Samuel, 1969).

Newton's Universe is infinite and isotropic, and gravity is a radial force of action at a distance, then the total gravitational force that the whole of the Universe exerts on any mass is canceled, and the forces that produce the movement are those exerted by nearby bodies with an irregular spatial distribution. Therefore, in Newtonian vision, no cosmic action is exerted on the bodies.

In the Machian formulation there is a privileged reference frame; it is one that "on average the Universe is at rest" (which we can identify with the International Celestial Reference System; or with Newton's absolute space; or with "the fixed stars" as Mach said); IRFs are in uniform and rectilinear motion with respect to this cosmic frame.

In the gravitational field theory, in addition to the static force (such as Newton's), there are induction forces, that is, forces that have their origin in the motion of the source (active induction) or in the motion of the mass on which the force acts (passive induction). The sum of the induction forces exerted by the Universe on an accelerated body does not cancel out, resulting in a net force, which in the Machian vision is the inertial force.

To demonstrate Mach's principle, we use passive induction; that is, the Universe (which is the source of the field) is at rest, and the mass on which the force acts is in motion. It is not an interaction of the Universe on the moving particle, but an interaction with the field produced by the Universe, which explains the instantaneous action of the inertial force, without the need to use exotic theories (Woodward, 2013, p. 29 -55).

Calculating the cosmic induction force is complicated because the gravitational signals that reach the observer were produced at different times, from the beginning of the Universe to the present moment, by stars in the retarded position, not in the current position. Furthermore, to calculate the induction force, we need to choose a cosmological model.

We solve the mathematical problem with the Liénard-Wiechert potentials (Panofsky & Phillips, 1972, p. 240-245 and p. 326-327). Perfecting the pioneering work of Sciama (1953), we have calculated the cosmic induction force using the retarded potentials technique for the vector gravitational field derived from the linearized theory of General Relativity (Segura, 2013, p. 41-68) and applying it to simple cosmic models (Segura, 2018).

The results in the classical approximation are:

1) Terms that depend on the speed and the derivatives of the acceleration are canceled; only terms that depend on the acceleration remain; with this result, we deduce the law of inertia

2) The total induction force is proportional to the acceleration of the body with respect to the Universe and it has the opposite sense.

3) The proportionality coefficient between the induction force and the acceleration is proportional to the gravitational mass (mass appearing in Newton's law of gravitation); therefore, the inertial and gravitational mass are proportional; that is, from Mach's principle, we deduce the equivalence principle.

4) The quotient between inertial and gravitational mass (which we call the coefficient of inertia) is the same for all bodies and depends on cosmic time; that is, the inertial mass varies with time.

5) The calculations made with simplified cosmological models show that for a Universe with a density approximately $1/Gt^2$ (t is the age of the Universe and G is the universal gravitation constant), the coefficient of inertia is approximately one.

6) All the epochs of the Universe contribute to the inertial mass of a body. The effect of the early Universe is much more significant that the effect of the present Universe.

7) The inertial mass increases with cosmic time.

The variation of the inertial mass with time has observable effects:

1) There is a difference between the frequencies of the spectral lines of the distant galaxies and those of the nearby galaxies, since the mass of the electrons in distant galaxies is smaller because they emitted radiation at an earlier time. This shift in spectral lines adds to the redshift caused by cosmic expansion.
2) The energy associated with the inertial mass $\left(E = mc^2\right)$ has a cosmic origin, varying with time, altering the theory of nuclear processes in stars.
3) Galactic dynamics will also be affected, particularly the rotation of galaxies, faster in distant galaxies.

When we apply our theory to cosmological models with big bang, we find a singularity in our calculations, and the inertial mass would be infinite (Martin, Rañada & Tiemblo, 2007). We believe that this problem is because we cannot apply the linearized theory to the early Universe; furthermore, we would have to modify the cosmic parameters that we have used in the calculations; in particular, it is necessary to correct the Hubble constant.

References

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