Measurement Unification

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Abstract: *In many measurement experiments and thought experiments, unexplained results appear. As example, entangled particles appear to interact instantly across distances. A 2018 Measurement paper proposed that the calibration of a measuring apparatus to a reference is required in both quantum measurement models and empirical measurement results. Measuring apparatus intervals are equalized by calibration. Current quantum measurement models assume equal units, but do not formally include the equalization required. Not recognizing this equalization in quantum mechanics creates the unexplained results. Including calibration explains the results of the particle spin experiments, quantum teleportation experiments, and Mach-Zehnder interferometer experiments, as well as Mermin's device, Schrödinger's Cat, and other thought experiments.*

Introduction

In 2018, the Measurement paper, Relative Measurement Theory (RMT)[1] formally developed and verified that quantum measurement models and empirical measurement results have equal uncertainty when the effect of calibrating the measuring apparatus[i] intervals (MAI) is treated. Calibration in RMT includes the equalization of each MAI with a standard of reference. This paper explains how the equalization of each MAI (often referred to as a unit of measurement) to a standard of reference (the definition of a unit of measurement) is not recognized in either metrology or quantum measurement theory. When this equalization by calibration is recognized the unexplained results that appear when equalization occurs (e.g., instantaneous action at a distance) have empirical explanations.

J. C. Maxwell defined a quantity (e.g., a measurement result) as including two components: a numerical value and a unit (e.g., gram, second, metre, etc. or any smaller or larger multiple[2]), where all the units of comparable measurements are equal.[3] An example of his definition is *3 grams* where 3 is the numerical value and gram is the unit of measurement. In his definition, each MAI, which has a numerical value, is equal to the gram standard of reference and also independent of the numerical value of the quantity. To implement Maxwell's definition, a calibration process is required to quantify the numerical value of each measuring apparatus interval (MAI) relative to the numerical value of a standard of reference. It is true, the numerical value of any standard of reference is arbitrary. However, the equalization of each MAI using the standard of reference is required to compare measurement results. The way calibration is defined in the International Vocabulary of Measurement (VIM)[4] correlates a mean measuring instrument interval to the standard of reference.[5] The significance of this difference between Maxwell's definition of a measurement result and the VIM defined implementation of calibration is explained further in Appendix A.

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[i] The term measuring apparatus identifies both formal measurement models and empirical measuring instruments.

Physics in general, and quantum mechanics in specific, currently treats a quantity as a numerical value and the MAI calibration to a reference as only empirical.[6] Physics currently does not recognize the requirement to equalize each MAI for measurement results to be compared. In formal quantum measurements currently, the function of equalizing formal units to each other is usually accomplished by assigning each to be one.[7] Assigning each unit to be one does not formally include the required equalization in the measurement process and also does not recognize the quantized nature (i.e., uncertainty) of a standard of reference.

A comparison of two numerical values of two independent quantities (a ratio), when the units are equal, is invariant. That is, the ratio of two independent quantities is dimensionless (invariant to changes in the numerical value of a unit), as a ratio of equal units cancels. Measurement results are only invariant when all the intervals/units are equal. Invariant comparisons are how all measurements results are compared.[8,9] Thus each unit and interval equalization to a reference is a requirement for all measurement comparisons.

In the current usage of the quantum measurement model (von Neumann's Process 1[10]), the unit of a quantity is not considered. Process 1, as currently applied, treats unit equalization to a reference as an empirical process. In metrology calibration correlates the mean MAI to a standard of reference. Then neither physics discipline equalizes each of the intervals/units included in a quantity. Although this equalization process is not recognized in physics, it still must occur in all measurement comparisons and its effects appear as entanglement in Bell state measurements (the quantum states of two qubits representing maximal entanglement).

When equalization by calibration is identified as the cause of entanglement effects in Bell state measurements, then entangled is not a superposition. Therefore calibration does not impact the double slit experiments or wave/particle phenomena. However, the transformation of a superposition of numerical values into a measurement result occurs when the undefined unit of a superposition is equalized to a reference, i.e., defined.

Various experiments and thought experiments, which are characterized by quantities consisting of simple numerical values (e.g., +/-) and undefined units, provide examples of the effects of not including equalization by calibration in quantum measurement comparisons.

Interferometer experiments

Maxwell's definition of a quantity consists of two independent values: the numerical value and the numerical value of the unit. This definition of a quantity is confirmed in the Mach-Zehnder interferometer experiments.[11] These experiments identify that the unit is the neutron spin state, which is shown to be independent from its numerical value, i.e., +/-. When a measurement result is not recognized as two components, the separation of the unit from the numerical value has been considered a paradoxical phenomenon and described as a Cheshire cat effect, where the smile (numerical value) is independent of the cat (unit).

Particle spin experiments

N. D. Mermin's 1981 paper,[12] which R. Feynman[13] referred to as a "...beautiful paper in physics...", recognized that physics does not provide a complete description of physical reality without a physical reason for the apparently instantaneous interactions of entangled particles. This paper by Mermin and papers by others[14] describe different physical experiments that appear to evidence instantaneous action at a distance.[15] Instantaneous action at a distance describes a system (e.g., two atomic particles) where one particle appears to interact with the other instantly. Such instantaneous interactions conflict with the well verified understandings of physics.

Bell state measurements also suggest quantum teleportation,[16] another conception of instantaneous action at a distance. Mermin's paper explores the operation of the Bell state measurement system shown in Fig.1. How do the two magnet sets change the particles so that correlated flashes appear on the two screens in three of the configurations (Mermin's case a) and uncorrelated flashes appear in the other six configurations (Mermin's case b)?

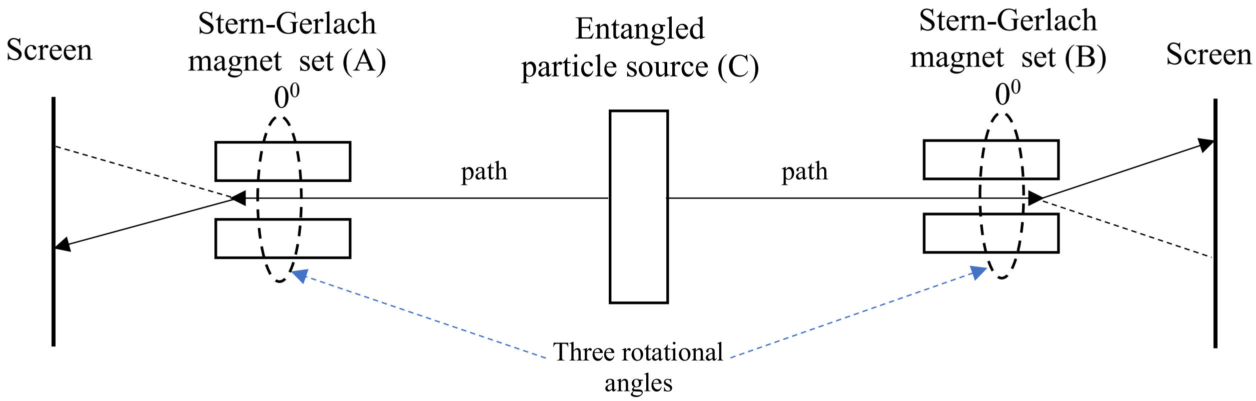

Fig. 1. Entanglement experiment
(using Mermin's notation)

Mermin's experiment

Each of the Stern-Gerlach magnet sets, shown in Fig. 1, is at 0^0. Each magnet set may be rotated +/- 120^0 about the particle path from 0^0 into three possible positions, following each dashed oval shown. When the two magnet sets are correlated to each other (same rotational angle), the two flashes on each screen will always be reverse correlated, i.e., up/down spin states. Further, when the A & B magnets sets are rotated 120^0 apart, the flashes on the two screens are no longer correlated, i.e., the four spin indications (up/down, up/up, down/up and down/down) each appear 1/4 of the time over many runs.

In Fig. 1, each Stern-Gerlach magnets set (A & B), with their nearby screen, is a measuring apparatus which compares the particle spin direction (up/down) of a particle passing through it to the other measuring apparatus. Each of the 3 x 3 = 9 configurations of the A & B magnet set rotations is one configuration of the comparison system. These configurations produce one of four combinations of flashes (indications) on the two screens.

A single run (one comparison of two measurements) is where the entangled particle source C produces two particles. One of the particles passes through each magnet set (this configuration is noted) and passes to the two screens. Then the two spin indications (up or down) are compared. In Mermin's results, when the experiment is run over all nine configurations many times, the distribution of the different indications is uniform. That is, in 4.5/9 (average ratio of the distribution of comparisons) of the configurations, the comparison indications are up/down or down/up (equally) and in 4.5/9 of the configurations the comparison indications are up/up or down/down (equally).

This uniform distribution of indications at first suggests the indications are random. But Mermin's experimental data identifies a correlation in 3/9 of the distributions of the runs between configurations and indications when the two magnet sets are configured in the same rotational orientation (Mermin's case a). The other 6/9 distributions are not correlated (Mermin's case b). In case a the correlation appears when the two flashes are always 180^0 apart (e.g., up/down or down/up) and the up/down and down/up flashes appear equally. In the current (very well verified) understanding of quantum measurements, the flash (up or down) of the particle spin is not determined until each particle is measured. How can one particle always have the reverse spin of the other particle when each particle collides with each screen some distance apart (case a)?

Mermin recognized the two particles ejected from C are always spin unit reverse-equalized to each other (i.e., entangled). This is correct, but still does not explain case b. What occurs to change this reverse-equalization? Mermin presented his challenge: how do the magnet sets A & B physically operate on the particles?

What Mermin did not recognize (as the physics of measurement does not) is that rotating the A & B magnet sets to the same position is the unit equalization of magnet set A to magnet set B and a part of the measurement process. Perhaps he considered these rotational configurations part of the set-up of the experiment. Rotational configuration equalization is the only part of calibration in this experiment as numerical value correlation does not apply when a measurement result (i.e., a numerical value) is one of two spin directions. The two particles' spin only appears entangled when the two measuring apparatus are rotationally equalized. In case b, when the two magnet sets are not equalized (120^0 apart), the measuring apparatus rotational units are not equalized to each other and comparisons of the two particles' spin indications are uncorrelated. A more detailed analysis of case b is provided in Appendix B.

In the following thought experiment, an empirical measurement experiment is shown to be logically consistant with a particle spin experiment.

Creating and comparing measurement results

This thought experiment steps through how two classic measurement results may be established and compared. Fig. 2 illustrates the thought experiment: a blindfolded carpenter wishes to make two unknown lengths of wooden dowels (the same quantities) equal (perfectly correlated) and independently verify his work. This thought experiment is described in five steps:

1. Set-up of the measuring system
2. Unit equalization
3. Quantity equalization
4. Measurement result generation (numerical value of a sum[ii] of different units).
5. Measurement result comparison.

First, the blindfolded carpenter takes the two dowels, aligns one end of both of the two dowels by feel (zero setting, 1st step) and makes a saw cut across both dowels. The saw cut, which correlates each dowel length to the other, is a unit dowel equalization (2nd step). This equalization of the two dowels has a similar effect to the two entangled particles that emerge from C in the particle spin comparisons.

[ii] Only additive scale measurements are addressed in this paper.

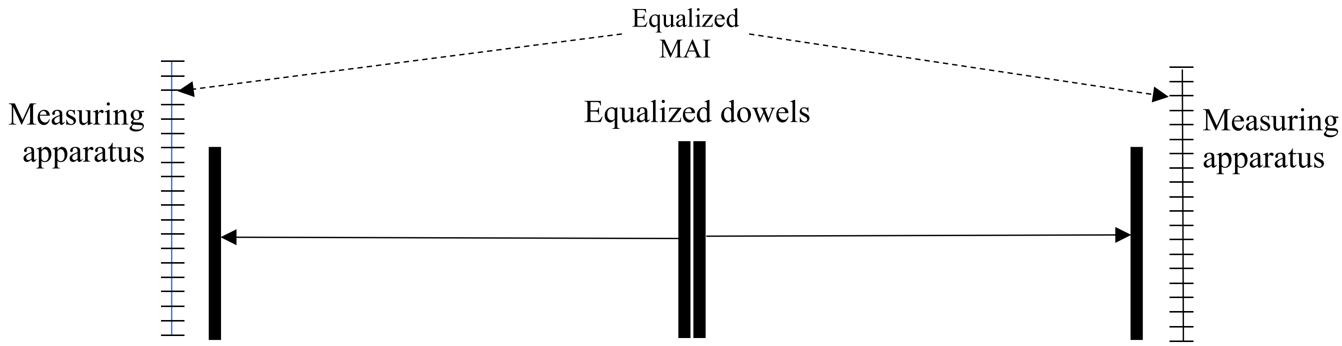

Fig. 2. Blindfolded carpenter thought experiment

Next, the two equalized dowels are moved to two independent measuring apparatus. The MAI of both independent measuring apparatus are equalized to each other (3rd step). This 3rd step may be seen as a VIM calibration process without an independent standard of reference. In the 4th step each measuring apparatus indicates each dowel's numerical value, i.e., a numerical value in different units than the dowel. The dowel lengths, which were previously unknown, when compared (5th step) are equal. This is equivalent to Mermin's case a in the particle spin experiment. Mermin's case b (uncorrelated) would occur if the 2nd step or 3rd step did not occur.

It is clear that the two measurement processes (4th step) did not require any instantaneous interaction. It is the equalization of the two unit dowels by the blindfolded carpenter, along with the VIM calibration of the two measuring apparatus (similar effect to rotational unit equalization), that establish the equal comparison result. Entangled systems do have a logically consistent empirical description.

Schrödinger's Cat

Schrödinger only provided the following information on his thought experiment.[17] "One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, *so* small, that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The ψ-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts."

The operation of Schrödinger's experiment consists of many runs, destroying one cat in each run. After each run the diabolical device is reset and another unfortunate cat penned up for the next run. Schrödinger stated the mean probability of an atom's decay is one hour. His experiment first appears to correlate the probabilistic time distribution of an atom's state with each cat's state (alive or dead) by correlating an atom's decay time with each cat's time of death. In fact, by virtue of the diabolical device, the actual time of each cat's death is a short fixed time after the time of an atom's decay, i.e., an atom's decay distribution (first probability distribution) and a cat's time of death distribution (second probability distribution) are fixed to each other. The parallel lines in Fig. 3 show this relationship.

Considering an atom's decay occurs with 0.5 probability at one hour from the start, the maximum range of the first and second probability distributions is estimated to be 2 hours. Assuming the range of the atom's decay probability distribution to be longer or shorter than 2 hours does not change the understanding of the experiment.

Next, Schrödinger indicates two human observations of each cat's state, the first when a cat is penned up into the diabolical device (time zero) and the second after one hour. These two observations are a third probability distribution, the observed or measured states of each cat in time. These measurements may be compared with each cat's actual state. Each cat's actual time of death occurs over about a 2 hour range. The second human observation in each run is at one hour, near the middle of this range which makes the results more confusing.

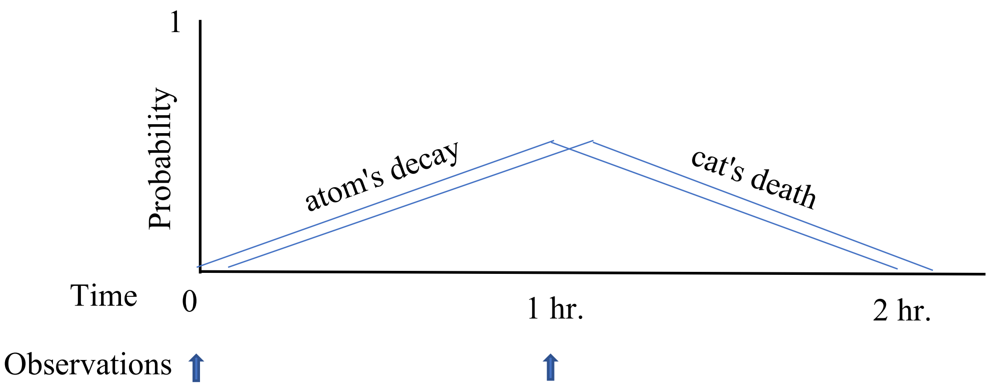

Fig. 3. An atom's decay and a cat's death probability vs time distribution.

Fig. 3 presents the range of an atom's probability of decay and a cat's probability of death vs time distribution. Two parallel and inverted V shaped lines are shown for simplicity. Any two parallel curve shapes where an atom's probability of decay was 0.5 at 1 hr. would apply. At the bottom of Fig. 3 is shown (upward bold arrows) the two human observations of a cat's state in each run.

In Fig. 3 these observations of the cat's state are unit time measurements. The two time measurements, one hour apart, (one = a numerical value, hour = a unit of time) may be seen as a time measuring apparatus with just one unit (e.g., a timer which sounds at one hour). Schrödinger's thought experiment compares the recognized distribution of an atom's decay to the not-as-well-recognized variation of the measurement results of a measuring apparatus with only two units (before or after 1 hour), not to the certainty of each cat's demise caused by the diabolical device.

In each run of the experiment, an alive cat is penned up at time zero and a second human observation of the cat occurs one hour in time from time zero. The actual time of death of each cat (a range of about two hours) is rarely exactly one hour. But probabilistically, in half the runs the human's second observation of a cat is alive and in half the runs the cat is dead. If there were more observations in each run, more accurate measurement results of a cat's time of death could occur.

As example, the timer and diabolical device could be set and reset every 10 minutes. Then the human's observations occur every 10 minutes (10 = numerical value, minute = different unit of time). In this example the accuracy of the observed time of death is within +/-10 minutes of the actual time of death. As the measuring apparatus units become smaller, the accuracy of the cat's measured time of death versus the actual time of death improves. Schrödinger's thought experiment has been confusing because the unit correlation (i.e., calibration) to a time reference, which determines the accuracy of the human's observations, is not recognized as included in the physics of measurement.

The calibration by the timer defines the time units. The necessity for calibration is identified by the change in measurement result accuracy when the time between the human's observations is changed from a 1 hour unit to a 10 minute unit. Each measurement result unit must be correlated to the other (i.e., 1 hour = 60 minutes) to be compared. Such a correlation has been treated as part of the measurement set-up. When the times are stated as 1 and 10 numerical values these numerical values have only local meaning. Making the smallest unit common (numerical values of 60 and 10) allows the comparison of the measurement results.

The atom's decay and the cat's actual state are near-perfectly correlated, which means there is one ψ function for both. But there is also a unit correlation to a standard of reference (calibration), which defines the time between observations. The human's observation of physical reality, a measurement result, can occur only when the second function is applied. Before the unit correlation to a standard of reference, an atom's mixture is probabilistic and there is no measurement result. All that appears of the atom (or other physical entity) is its ψ function - a superposition of numerical values.

Bell's non-local variable and Lorentz transformations

J. S. Bell was close to recognizing unit equalization to a standard of reference: "...there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote."[18] Further, Bell, when discussing the particle spin experiments states, "The above argument relies very much on the perfection of the correlation (or rather anticorrelation) when the two magnets are aligned..."[19] In the particle spin experiments, unit correlation to a standard of reference appears to Bell to be the set-up of the measuring system. Bell's "non-local structure"[20] is provided by a standard of reference which must be treated as part of the measurement process or the results will violate physics.

In special relativity, the Lorentz factor is applied in the Lorentz transform[21] equations. The Lorentz factor, gamma, provides the calibration of each unit to the velocity of light, which is the standard of reference. The Lorentz factor is how the calibration of units to a reference appears in relativity.

Relative Measurement Theory (ref. 1) provides a formal measurement equation which includes the effects of accuracy and precision, mitigated by calibration. When the different implementations of calibration are recognized, the unification of measurement results in metrology, quantum mechanics and relativity is accomplished.

Conclusion

In 1935, the EPR paper[22] recognized that the quantum mechanical understanding of physical reality is incomplete. An EPR pair (i.e., entangled) is created by a calibration process in which two previously separate and independent systems (each measuring apparatus) become entangled. Equalization by calibration is the missing process that resolves the discontinuities seen between quantum and empirical measurements.

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Appendix A
Comparison of Different Calibrations

Different implementations of calibration provide different assurances. An example of VIM calibration and correction: The four digit indication (numerical value) of a voltmeter is adjusted to *00.00* (zero setting). A known voltage source of *1.000* volt (volt = unit) is applied and a voltage gain adjustment on the voltmeter calibrates and corrects the displayed numerical value to *01.00* volt. An assumption is that the voltage gain of the voltmeter is equal across each *00.01* MAI over the voltmeter's range, i.e., each 00.01 MAI of the instrument is equal.

Another implementation of a calibration and correction process: each *00.01* volt unit of the voltmeter is calibrated to a reference *00.010* volt and the variation of each *00.01* volt measuring apparatus unit is corrected. This implementation of calibration may be seen as formal and not empirical, as any voltage gain non-linearity is corrected (i.e., the *00.01* volt intervals are equalized), but the VIM calibration process is far more practical. The significance of a formal implementation of calibration is verified in RMT.

Appendix B
Analysis of Mermin's case b

In Mermin's experimental data the sum of the 3/9 (correlated) and 6/9 (uncorrelated) distributions appears uniform, but does have correlation. Subtracting the equal units 3/9 ratio of distributions of the runs (case a, up/down and down/up runs) from the 4.5/9 distributions of the runs (all the up/down and down/up runs), leaves 1.5/9 distributions of the runs where the indications, although up/down and down/up, have occurred randomly. The other 4.5/9 of the distributions of indications (up/up and down/down runs) also appear randomly.

These two uncorrelated distributions (1.5/9 and 4.5/9) sum to a 6/9 distribution which matches Mermin's run data (case b) and the formal development which Mermin provides: "It is a well-known elementary result that when the orientations of the magnets differ by an angle θ, then the probability of spin measurements of each particle yielding opposite values is $\cos^2(\frac{1}{2}\theta)$. This probability is 1 when $\theta = 0°$ [case a] and 1/4 when $\theta = 120°$ [case b]." When there are only two spin indications, numerical value correlation is not required. Then the measurement comparisons in case a are unit equalized (the same rotational unit is compared by each measuring apparatus) and the measurement comparisons in case b are random which fully supports this paper's proposal.

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