Further observations of structure of a possible unification algebra

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Abstract. The trigintaduonion Cayley-Dickson algebra can be combined with the complexified geometric algebra of space-time to generate an algebra with features corresponding to the Clifford algebra of space-time, the complex doublet of the Higgs mechanism, three families of fundamental particles and anti-particles of the standard model, dark matter and dark energy.

Contents

1. Introduction

It seems reasonable to suppose that, in the tradition of the periodic table and Bohr's model of the atom, there might be a simplistic mathematical pattern with a structure similar to that of the standard model which would provide insights leading to a deeper understanding of the basis of reality. A pattern can be found for a combination of the trigintaduonion Cayley-Dickson algebra, $\mathbb{T}$ with the geometric algebra $Cl_4(C) \cong M_4(C)$ which may achieve this.

If a quaternionic subalgebra of the trigintaduonion algebra, $\mathbb{T}$, is associated with spatial bivectors for a space-time algebra so that spatial isotropy is exhibited by its subalgebras, a striking correspondence between part of the pattern of its sedenionic-type subalgebras and that of one family of fundamental particles of the standard model is found. The remaining part of the pattern may be associated with dark matter. This is described in section 3, using notation for $\mathbb{T}$ set out in section 2.

The algebra of complex 4×4 matrices, $M_4(C)$, is isomorphic to $Cl_4(C)$. When a quaternionic subalgebra of $M_4(C)$ is assigned to represent spatial bivectors for a spacetime Clifford algebra, there are six possible choices of sets of three unit elements of $M_4(C)$ which may be chosen to represent spatial vector unit elements. It is postulated that these choices generate three families of fundamental particles and their anti-particles. There are non-isotropic variations to these sets which may be associated with dark energy. This is described in section 5, using notation for $M_4(C)$ set out in section 4.

The combined algebra, $M_4(C) \otimes \mathbb{T}$, is the same size as $Cl_{1,9}$, the geometric spacetime algebra for the dimensionality used in string theories. It has subalgebras used for several models, including subalgebras isomorphic to $Cl_6(C)$ and $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$. This is discussed in section 6.

Parallels between Clifford algebras and Cayley-Dickson-type algebras are discussed in Appendix A.

A possible basis for the mexican hat potential for the Brout-Englert-Higgs mechanism[1][2][3] is described in Appendix B.

The Loops package[4] for GAP4[5] has been used to investigate isomorphisms and isotopisms for $\mathbb{T}$ and its subalgebras.

2. Notation for unit elements and subalgebras of $\mathbb{T}$

Greek letters with subscripts have been used to label unit elements of $\mathbb{T}$, as this assists in identifying spatial isotropy. The labels are shown in table 1 which maps those labels to the conventional labels. Their Cayley table is shown in table 2.

The structure of the trigintaduonion algebra, $\mathbb{T}$, has been described by Cawagas et al[6]. $\mathbb{T}$ contains an embedded loop T_L of order 64 generated by its 32 unit elements. T_L has 31 sedenion-type subloops of $\mathbb{T}$ of order 32, falling into four isomorphism classes, which Cawagas et al. designated $S_L, S_L^{\alpha}, S_L^{\beta}, S_L^{\gamma}$. This scheme has been modified by replacing the subscripts with numbers to index the subloops.

TABLE 1. Notation used to label positive unit elements for $\mathbb{T}$

e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ
e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ

TABLE 2. Labels and Cayley table for $\mathbb{T}$ unit elements

	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
	σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ
σ_o	$+\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$
σ_ι	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$
$\sigma_\jmath$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$
σ_κ	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$
λ_o	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$
λ_ι	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$
$\lambda_\jmath$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$
λ_κ	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$
μ_o	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$
μ_ι	$+\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\nu_\iota$	$+\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$
$\mu_\jmath$	$+\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\nu_o$	$+\nu_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$
μ_κ	$+\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$
ν_o	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$+\mu_o$	$-\mu_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$
ν_ι	$+\nu_\iota$	$-\nu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$
$\nu_\jmath$	$+\nu_\jmath$	$-\nu_\kappa$	$-\nu_o$	$+\nu_\iota$	$+\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$
ν_κ	$+\nu_\kappa$	$+\nu_\jmath$	$-\nu_\iota$	$-\nu_o$	$+\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$
α_o	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$
α_ι	$+\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$
$\alpha_\jmath$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$
α_κ	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$
β_o	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$
β_ι	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$
$\beta_\jmath$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$
β_κ	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$
γ_o	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$+\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$
γ_ι	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$+\alpha_\iota$	$+\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$
$\gamma_\jmath$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$
γ_κ	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$
δ_o	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$
δ_ι	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$+\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$
$\delta_\jmath$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$+\alpha_o$	$-\alpha_\iota$
δ_κ	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$+\alpha_o$

	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}	e_{25}	e_{26}	e_{27}	e_{28}	e_{29}	e_{30}	e_{31}
	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
σ_o	$+\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$
σ_ι	$+\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$	$-\beta_\iota$	$+\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$
$\sigma_\jmath$	$+\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$+\beta_o$	$+\beta_\iota$	$-\gamma_\jmath$	$-\gamma_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$
σ_κ	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$+\beta_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$
λ_o	$+\beta_o$	$+\beta_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$
λ_ι	$+\beta_\iota$	$-\beta_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$-\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$+\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$
$\lambda_\jmath$	$+\beta_\jmath$	$-\beta_\kappa$	$-\beta_o$	$+\beta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$-\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$+\gamma_o$	$-\gamma_\iota$
λ_κ	$+\beta_\kappa$	$+\beta_\jmath$	$-\beta_\iota$	$-\beta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$+\gamma_o$
μ_o	$+\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\delta_o$	$+\delta_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\alpha_o$	$-\alpha_\iota$	$-\alpha_\jmath$	$-\alpha_\kappa$	$-\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$
μ_ι	$+\gamma_\iota$	$-\gamma_o$	$+\gamma_\kappa$	$-\gamma_\jmath$	$+\delta_\iota$	$-\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$+\alpha_\iota$	$-\alpha_o$	$+\alpha_\kappa$	$-\alpha_\jmath$	$+\beta_\iota$	$-\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$
$\mu_\jmath$	$+\gamma_\jmath$	$-\gamma_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\delta_\jmath$	$+\delta_\kappa$	$-\delta_o$	$-\delta_\iota$	$+\alpha_\jmath$	$-\alpha_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$-\beta_o$	$-\beta_\iota$
μ_κ	$+\gamma_\kappa$	$+\gamma_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$-\delta_o$	$+\alpha_\kappa$	$+\alpha_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$-\beta_o$
ν_o	$+\delta_o$	$-\delta_\iota$	$-\delta_\jmath$	$-\delta_\kappa$	$-\gamma_o$	$+\gamma_\iota$	$+\gamma_\jmath$	$+\gamma_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\beta_\jmath$	$-\beta_\kappa$	$-\alpha_o$	$+\alpha_\iota$	$+\alpha_\jmath$	$+\alpha_\kappa$
ν_ι	$+\delta_\iota$	$+\delta_o$	$-\delta_\kappa$	$+\delta_\jmath$	$-\gamma_\iota$	$-\gamma_o$	$-\gamma_\kappa$	$+\gamma_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\beta_\kappa$	$+\beta_\jmath$	$-\alpha_\iota$	$-\alpha_o$	$-\alpha_\kappa$	$+\alpha_\jmath$
$\nu_\jmath$	$+\delta_\jmath$	$+\delta_\kappa$	$+\delta_o$	$-\delta_\iota$	$-\gamma_\jmath$	$+\gamma_\kappa$	$-\gamma_o$	$-\gamma_\iota$	$+\beta_\jmath$	$+\beta_\kappa$	$+\beta_o$	$-\beta_\iota$	$-\alpha_\jmath$	$+\alpha_\kappa$	$-\alpha_o$	$-\alpha_\iota$
ν_κ	$+\delta_\kappa$	$-\delta_\jmath$	$+\delta_\iota$	$+\delta_o$	$-\gamma_\kappa$	$-\gamma_\jmath$	$+\gamma_\iota$	$-\gamma_o$	$+\beta_\kappa$	$-\beta_\jmath$	$+\beta_\iota$	$+\beta_o$	$-\alpha_\kappa$	$-\alpha_\jmath$	$+\alpha_\iota$	$-\alpha_o$
α_o	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$
α_ι	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$
$\alpha_\jmath$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$+\lambda_o$	$+\lambda_\iota$	$-\mu_\jmath$	$-\mu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$
α_κ	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$+\lambda_o$	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$+\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$
β_o	$-\lambda_o$	$+\lambda_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\nu_o$	$-\nu_\iota$	$-\nu_\jmath$	$-\nu_\kappa$	$+\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$
β_ι	$-\lambda_\iota$	$-\lambda_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$+\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$
$\beta_\jmath$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\lambda_o$	$+\lambda_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$+\mu_o$	$-\mu_\iota$
β_κ	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\lambda_\iota$	$-\lambda_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$+\mu_o$
γ_o	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\sigma_o$	$-\sigma_\iota$	$-\sigma_\jmath$	$-\sigma_\kappa$	$-\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$
γ_ι	$-\mu_\iota$	$-\mu_o$	$+\mu_\kappa$	$-\mu_\jmath$	$+\nu_\iota$	$-\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$+\sigma_\iota$	$-\sigma_o$	$+\sigma_\kappa$	$-\sigma_\jmath$	$+\lambda_\iota$	$-\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$
$\gamma_\jmath$	$-\mu_\jmath$	$-\mu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\nu_o$	$-\nu_\iota$	$+\sigma_\jmath$	$-\sigma_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$-\lambda_o$	$-\lambda_\iota$
γ_κ	$-\mu_\kappa$	$+\mu_\jmath$	$-\mu_\iota$	$-\mu_o$	$+\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$-\nu_o$	$+\sigma_\kappa$	$+\sigma_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$-\lambda_o$
δ_o	$-\nu_o$	$+\nu_\iota$	$+\nu_\jmath$	$+\nu_\kappa$	$-\mu_o$	$+\mu_\iota$	$+\mu_\jmath$	$+\mu_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\lambda_\jmath$	$-\lambda_\kappa$	$-\sigma_o$	$+\sigma_\iota$	$+\sigma_\jmath$	$+\sigma_\kappa$
δ_ι	$-\nu_\iota$	$+\nu_o$	$-\nu_\kappa$	$+\nu_\jmath$	$-\mu_\iota$	$-\mu_o$	$-\mu_\kappa$	$+\mu_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\lambda_\kappa$	$+\lambda_\jmath$	$-\sigma_\iota$	$-\sigma_o$	$-\sigma_\kappa$	$+\sigma_\jmath$
$\delta_\jmath$	$-\nu_\jmath$	$+\nu_\kappa$	$+\nu_o$	$-\nu_\iota$	$-\mu_\jmath$	$+\mu_\kappa$	$-\mu_o$	$-\mu_\iota$	$+\lambda_\jmath$	$+\lambda_\kappa$	$+\lambda_o$	$-\lambda_\iota$	$-\sigma_\jmath$	$+\sigma_\kappa$	$-\sigma_o$	$-\sigma_\iota$
δ_κ	$-\nu_\kappa$	$-\nu_\jmath$	$+\nu_\iota$	$+\nu_o$	$-\mu_\kappa$	$-\mu_\jmath$	$+\mu_\iota$	$-\mu_o$	$+\lambda_\kappa$	$-\lambda_\jmath$	$+\lambda_\iota$	$+\lambda_o$	$-\sigma_\kappa$	$-\sigma_\jmath$	$+\sigma_\iota$	$-\sigma_o$

3. Sedenionic-type subalgebras of $\mathbb{T}$ and the standard model, dark matter

For an alignment of $\mathbb{T}$ to be spatially isotropic with respect to the geometric algebra of space-time, spatial rotation must not change the isomorphism type of any of its subalgebras. This property is found if $[\sigma_\iota, \sigma_\jmath, \sigma_\kappa]$ are chosen to be aligned with unit spatial bivectors, but not if $[\lambda_o, \mu_o, \nu_o]$ are chosen, as can be seen by inspection of unit element participation in sedenion-type subloops as shown in table 3.

Table 3. Unit elements for sedenion-type subloops

		σ_o	σ_ι	$\sigma_\jmath$	σ_κ	λ_o	λ_ι	$\lambda_\jmath$	λ_κ	μ_o	μ_ι	$\mu_\jmath$	μ_κ	ν_o	ν_ι	$\nu_\jmath$	ν_κ	α_o	α_ι	$\alpha_\jmath$	α_κ	β_o	β_ι	$\beta_\jmath$	β_κ	γ_o	γ_ι	$\gamma_\jmath$	γ_κ	δ_o	δ_ι	$\delta_\jmath$	δ_κ
1	S_0^γ	■	■	■	■	■	■	■	■																	■	■	■	■	■	■	■	■
2	S_0^α	■	■	■	■					■	■	■	■									■	■	■	■					■	■	■	■
3	S_0^β	■	■	■	■									■	■	■	■					■	■	■	■	■	■	■	■				
4	S_1^α	■	■			■	■			■	■			■	■					■	■			■	■			■	■			■	■
5	S_2^α	■		■		■		■		■		■		■		■			■		■		■		■		■		■		■		■
6	S_3^α	■			■	■			■	■			■	■			■		■	■			■	■			■	■			■	■	
7	S_1^β	■	■			■	■					■	■			■	■			■	■			■	■	■	■			■	■		
8	S_2^β	■		■		■		■			■		■		■		■		■		■		■		■	■		■		■		■	
9	S_3^β	■			■	■			■		■	■			■	■			■	■			■	■		■			■	■			■
10	S_4^α	■	■					■	■	■	■					■	■			■	■	■	■					■	■	■	■		
11	S_5^α	■		■			■		■	■		■			■		■		■		■	■		■			■		■	■		■	
12	S_6^α	■			■		■	■		■			■		■	■			■	■		■			■		■	■		■			■
13	S_4^β	■	■					■	■			■	■	■	■					■	■	■	■			■	■					■	■
14	S_5^β	■		■			■		■		■		■	■		■			■		■	■		■		■		■			■		■
15	S_6^β	■			■		■	■			■	■		■			■		■	■		■			■	■			■		■	■	
16	S_0	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■																
17	S_1	■	■	■	■	■	■	■	■									■	■	■	■	■	■	■	■								
18	S_2	■	■	■	■					■	■	■	■					■	■	■	■					■	■	■	■				
19	S_3	■	■	■	■									■	■	■	■	■	■	■	■									■	■	■	■
20	S_4	■	■			■	■			■	■			■	■			■	■			■	■			■	■			■	■		
21	S_5	■		■		■		■		■		■		■		■		■		■		■		■		■		■		■		■	
22	S_6	■			■	■			■	■			■	■			■	■			■	■			■	■			■	■			■
23	S_7	■	■			■	■					■	■			■	■	■	■			■	■					■	■			■	■
24	S_8	■		■		■		■			■		■		■		■	■		■		■		■			■		■		■		■
25	S_9	■			■	■			■		■	■			■	■		■			■	■			■		■	■			■	■	
26	S_{10}	■	■					■	■	■	■					■	■	■	■					■	■	■	■					■	■
27	S_{11}	■		■			■		■	■		■			■		■	■		■			■		■	■		■			■		■
28	S_{12}	■			■		■	■		■			■		■	■		■			■		■	■		■			■		■	■	
29	S_{13}	■	■					■	■			■	■	■	■			■	■					■	■			■	■	■	■		
30	S_{14}	■		■			■		■		■		■	■		■		■		■			■		■		■		■	■		■	
31	S_{15}	■			■		■	■			■	■		■			■	■			■		■	■			■	■		■			■

3.1. Correspondence with the standard model

The structure of $M_4(C)$ matches that of $\mathbb{T}$ in many respects. As noted by JWBales[7], the Cayley tables of both $M_4(C)$ and $\mathbb{T}$ can be assembled as normalised latin squares with elements ordered so that bit-wise 'exclusive or' (XOR) of binary representations of two unit elements' numbering generates the numbering of their product. As a result, if the sign of products is ignored, their Cayley tables are the same and, for subalgebras that include the negative of the identity, their subalgebra inventory is the same (refer to Appendix A). This enables them to be aligned with each other.

Spatial unit bivectors can be represented by unit quaternions, so may correspond to, or be aligned with unit elements for a quaternionic subalgebra of $\mathbb{T}$ such as $[\sigma_\iota\sigma_\jmath\sigma_\kappa]$. For fundamental particles, there is no special orientation in space, so an algebra representing particles that persist must be isotropic with respect to spatial bivectors. However, within confinement this requirement for isotropy could be relaxed to apply to combinations of particles.

This suggests the hypothesis that quantum fields with observable excitations are associated with sedenionic subalgebras of $\mathbb{T}$ that are either spatially equivalent, or can combined in sets of isomorphic subalgebras which would be collectively spatially equivalent, so that spatial reorientation results in a transformation for a sedenionic subalgebra that is either completely internal, or that is confined to a set of isomorphic subalgebras.

Analysis of table 3 reveals that:

1. One unit element of $\mathbb{T}$ (other than the identity), α_o , has unique status. As $M_4(C)$ also features an unique unit imaginary element, this suggests identification of these unit elements with the unit imaginary elements of the complex doublet for the Brout-Englert-Higgs mechanism[1][2][3].
2. If $[\sigma_\iota\sigma_\jmath\sigma_\kappa]$ are chosen as unit elements aligned with spatial unit bivectors, spatial equivalence for subalgebras of $\mathbb{T}$ is achieved, but not if $[\lambda_o, \mu_o, \nu_o]$ are chosen. This suggests alignment of $[\sigma_\iota\sigma_\jmath\sigma_\kappa]$ with spatial unit bivectors for a geometric space-time algebra. If $[\lambda_o\mu_o\nu_o]$ commute with all unit elements of the space-time geometric algebra, they would generate a scalar algebra. It is possible to combine a scalar quaternionic algebra with a spatial bivector quaternionic algebra in a way that generates a mexican hat potential, a feature of the Brout-Englert-Higgs mechanism (refer to Appendix A for details).
3. Subalgebras 1 to 15 exclude the unique imaginary element, α_o, and feature three isomorphism types. Subalgebras 16 to 31 include the unique imaginary element, α_o and feature one isomorphism types. This suggests assignment of subalgebras 1 to 15 to fermionic quantum fields and subalgebras 16 to 31 to bosonic fields.

4. Fermionic subalgebras 1, 2 and 3, feature internal spatially equivalent sets, suggesting that they correspond to leptons.
5. Fermionic subalgebras 4 to 15 do not feature internal spatial equivalence, but can be placed in sets of three which are collectively spatially equivalent, suggesting that each set of three corresponds to three colors of one chirality for one quark family.
6. For some subalgebras the $\lambda_{o\iota\jmath\kappa}$ content matches the $\mu_{o\iota\jmath\kappa}$ content, whereas for others the $\mu_{o\iota\jmath\kappa}$ content matches the $\nu_{o\iota\jmath\kappa}$ content or the $\nu_{o\iota\jmath\kappa}$ content matches the $\lambda_{o\iota\jmath\kappa}$ content. As sedenionic subalgebras are not isomorphic with respect to $[\lambda_{o\iota\jmath\kappa}\mu_{o\iota\jmath\kappa}\nu_{o\iota\jmath\kappa}]$ content, this could generate the chiral property of taking part in weak interactions for some subalgebras but not for others. There are three subalgebras identified as corresponding to leptons, so one of them would not have a counterpart with opposite chirality. The particle identifications in table 5 use the arbitrarily chosen criteria:
   Proportion of $I\lambda_{\iota\jmath\kappa}$ content present $\rightarrow$ charge.
   If $\lambda_{o\iota\jmath\kappa}$ content matches $\mu_{o\iota\jmath\kappa}$ content $\rightarrow$ isospin 0, if not $\rightarrow$ isospin 1/2.
7. Whilst they change isomorphism types for sedenionic type subalgebras, transformations such as $\lambda_{o\iota\jmath\kappa} \rightarrow \mu_{o\iota\jmath\kappa} \rightarrow \nu_{o\iota\jmath\kappa}$ do not affect spatial equivalence. This suggests the possibility of mixing.
8. The pattern for the bosonic subalgebras 16 to 31 resembles that of generators for $SU(4)$, suggesting a relationship to the Pati-Salaam $SU(4) \otimes SU(2) \otimes SU(2)$ approach to unification[8].
9. Bosonic subalgebras 16, 17, 18 and 19 are colorless with internal spatial equivalence so may relate to electro-weak fields.
10. Bosonic subalgebras 20, 21 and 22 do not feature internal spatial equivalence, but can be placed in sets of three which are collectively spatially equivalent, so each set of three may correspond to there being three colors for the strong nuclear force and relate to the strong nuclear field. Their $\lambda_{o\iota\jmath\kappa}$ content matches both their $\mu_{o\iota\jmath\kappa}$ and their $\nu_{o\iota\jmath\kappa}$ content, suggesting no interaction with electroweak forces.
11. Bosonic subalgebras 23 to 31 do not feature internal spatial equivalence, but can be placed in sets of three which are collectively spatially equivalent, so would also be subject to the strong nuclear force. However, they can also be placed in sets of three that are collectively equivalent with respect to $[\lambda_o\mu_o\nu_o]$ or $[\beta_o\gamma_o, \delta_o]$ transformations, suggesting that they would be subject to a further force, and could be associated with dark matter, and be responsible for anomalies in predicted flavor mixing. A fifth force has been proposed as being responsible for quark mixing and the related CP violation[9].
12. For each fermionic subalgebra, there is a bosonic subalgebra with the same $[\sigma_{o\iota\jmath\kappa}, \lambda_{o\iota\jmath\kappa}, \mu_{o\iota\jmath\kappa}, \nu_{o\iota\jmath\kappa}]$ content, but inverted $[\alpha_{o\iota\jmath\kappa}, \beta_{o\iota\jmath\kappa}, \gamma_{o\iota\jmath\kappa}, \delta_{o\iota\jmath\kappa}]$ content. This suggests a form of supersymmetry. The bosonic subalgebra $\mathbb{S}_0$ is left without a supersymmetric partner.

Re-labelling α_o, the unique unit element of $\mathbb{T}$ (postulated as being associated with a Higgs complex doublet) as I, and $[\beta_o, \gamma_o, \delta_o]$ as $[I\lambda_o, I\mu_o, I\nu_o]$, a possible identification of subalgebras with fundamental quantum fields based on these observations is shown in table 4.

TABLE 4. Sedenion-type subloops

Fermionic subloops							
		Composition	Type	Color	$I\lambda_{\iota\jmath\kappa}$ →Charge,Q	$\lambda \equiv \mu$ →Weak Isospin	Family
1	S_0^γ	$[\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ ] \otimes [\sigma_o\ \lambda_o\ I\mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_{\iota\jmath\kappa} \to$ w	No $I\lambda_{\iota\jmath\kappa} \to 0$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	$\nu_{e\mu\tau}$
2	S_0^α	$[\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ ] \otimes [\sigma_o\ I\lambda_o\ \mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_{\iota\jmath\kappa} \to$ w	All $I\lambda_{\iota\jmath\kappa} \to \pm 1$	$\lambda_{o\iota\jmath\kappa} \not\equiv \mu_{o\iota\jmath\kappa} \to 0$	e,μ, τ
3	S_0^β	$[\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ ] \otimes [\sigma_o\ I\lambda_o\ I\mu_o\ \nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_{\iota\jmath\kappa} \to$ w	All $I\lambda_{\iota\jmath\kappa} \to \pm 1$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	e,μ, τ
4	S_1^α	$[\sigma_o\ \sigma_\iota\ I\sigma_\jmath\ I\sigma_\kappa\ ] \otimes [\sigma_o\ \lambda_o\ \mu_o\ \nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\iota I\sigma_{\jmath\kappa} \to$ b	$I\lambda_{\jmath\kappa} \to \pm 2/3$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	quark$_{uct}$
5	S_2^α	$[\sigma_o\ \sigma_\jmath\ I\sigma_\iota\ I\sigma_\kappa\ ] \otimes [\sigma_o\ \lambda_o\ \mu_o\ \nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\jmath I\sigma_{\iota\kappa} \to$ g	$I\lambda_{\iota\kappa} \to \pm 2/3$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	quark$_{uct}$
6	S_3^α	$[\sigma_o\ \sigma_\kappa\ I\sigma_\iota\ I\sigma_\jmath\ ] \otimes [\sigma_o\ \lambda_o\ \mu_o\ \nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\kappa I\sigma_{\iota\jmath} \to$ r	$I\lambda_{\iota\jmath} \to \pm 2/3$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	quark$_{uct}$
7	S_1^β	$[\sigma_o\ \sigma_\iota\ I\sigma_\jmath\ I\sigma_\kappa\ ] \otimes [\sigma_o\ \lambda_o\ I\mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\iota I\sigma_{\jmath\kappa} \to$ b	$I\lambda_{\jmath\kappa} \to \pm 2/3$	$\lambda_{o\iota\jmath\kappa} \not\equiv \mu_{o\iota\jmath\kappa} \to 0$	quark$_{uct}$
8	S_2^β	$[\sigma_o\ \sigma_\jmath\ I\sigma_\iota\ I\sigma_\kappa\ ] \otimes [\sigma_o\ \lambda_o\ I\mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\jmath I\sigma_{\iota\kappa} \to$ g	$I\lambda_{\iota\kappa} \to \pm 2/3$	$\lambda_{o\iota\jmath\kappa} \not\equiv \mu_{o\iota\jmath\kappa} \to 0$	quark$_{uct}$
9	S_3^β	$[\sigma_o\ \sigma_\kappa\ I\sigma_\iota\ I\sigma_\jmath\ ] \otimes [\sigma_o\ \lambda_o\ I\mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\kappa I\sigma_{\iota\jmath} \to$ r	$I\lambda_{\iota\jmath} \to \pm 2/3$	$\lambda_{o\iota\jmath\kappa} \not\equiv \mu_{o\iota\jmath\kappa} \to 0$	quark$_{uct}$
10	S_4^α	$[\sigma_o\ \sigma_\iota\ I\sigma_\jmath\ I\sigma_\kappa\ ] \otimes [\sigma_o\ I\lambda_o\ \mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\iota I\sigma_{\jmath\kappa} \to$ b	$I\lambda_\iota \to \pm 1/3$	$\lambda_{o\iota\jmath\kappa} \not\equiv \mu_{o\iota\jmath\kappa} \to 0$	quark$_{dsb}$
11	S_5^α	$[\sigma_o\ \sigma_\jmath\ I\sigma_\iota\ I\sigma_\kappa\ ] \otimes [\sigma_o\ I\lambda_o\ \mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\jmath I\sigma_{\iota\kappa} \to$ g	$I\lambda_\jmath \to \pm 1/3$	$\lambda_{o\iota\jmath\kappa} \not\equiv \mu_{o\iota\jmath\kappa} \to 0$	quark$_{dsb}$
12	S_6^α	$[\sigma_o\ \sigma_\kappa\ I\sigma_\iota\ I\sigma_\jmath\ ] \otimes [\sigma_o\ I\lambda_o\ \mu_o\ I\nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\kappa I\sigma_{\iota\jmath} \to$ r	$I\lambda_\kappa \to \pm 1/3$	$\lambda_{o\iota\jmath\kappa} \not\equiv \mu_{o\iota\jmath\kappa} \to 0$	quark$_{dsb}$
13	S_4^β	$[\sigma_o\ \sigma_\iota\ I\sigma_\jmath\ I\sigma_\kappa\ ] \otimes [\sigma_o\ I\lambda_o\ I\mu_o\ \nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\iota I\sigma_{\jmath\kappa} \to$ b	$I\lambda_\iota \to \pm 1/3$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	quark$_{dsb}$
14	S_5^β	$[\sigma_o\ \sigma_\jmath\ I\sigma_\iota\ I\sigma_\kappa\ ] \otimes [\sigma_o\ I\lambda_o\ I\mu_o\ \nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\jmath I\sigma_{\iota\kappa} \to$ g	$I\lambda_\jmath \to \pm 1/3$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	quark$_{dsb}$
15	S_6^β	$[\sigma_o\ \sigma_\kappa\ I\sigma_\iota\ I\sigma_\jmath\ ] \otimes [\sigma_o\ I\lambda_o\ I\mu_o\ \nu_o\ ]$	No $I\sigma_o$ →Spin 1/2	$\sigma_\kappa I\sigma_{\iota\jmath} \to$ r	$I\lambda_\kappa \to \pm 1/3$	$\lambda_{o\iota\jmath\kappa} \equiv \mu_{o\iota\jmath\kappa} \to 1/2$	quark$_{dsb}$

Bosonic subloops				
		Composition	Attributes	Associated fields
16	$\mathbb{S}_0$	$[\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ ] \otimes [\sigma_o\ \lambda_o\ \mu_o\ \nu_o\ ]$	Uncolored, Unflavored	Electro-weak boson fields
17	$\mathbb{S}_1$	$[\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \lambda_o\ I\lambda_o\ ]$	Uncolored, Flavored	
18	$\mathbb{S}_2$	$[\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \mu_o\ I\mu_o\ ]$	Uncolored, Flavored	
19	$\mathbb{S}_3$	$[\sigma_o\ \sigma_\iota\ \sigma_\jmath\ \sigma_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \nu_o\ I\nu_o\ ]$	Uncolored, Flavored	
20	$\mathbb{S}_4$	$[\sigma_o\ \sigma_\iota\ I\sigma_o\ I\sigma_\iota\ ] \otimes [\sigma_o\ \lambda_o\ \mu_o\ \nu_o\ ]$	Colored, Unflavored	Strong nuclear color fields
21	$\mathbb{S}_5$	$[\sigma_o\ \sigma_\jmath\ I\sigma_o\ I\sigma_\jmath\ ] \otimes [\sigma_o\ \lambda_o\ \mu_o\ \nu_o\ ]$	Colored, Unflavored	
22	$\mathbb{S}_6$	$[\sigma_o\ \sigma_\kappa\ I\sigma_o\ I\sigma_\kappa\ ] \otimes [\sigma_o\ \lambda_o\ \mu_o\ \nu_o\ ]$	Colored, Unflavored	
23	$\mathbb{S}_7$	$[\sigma_o\ \sigma_\iota\ \mu_\jmath\ \mu_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \lambda_\iota\ I\lambda_\iota\ ]$	Colored, Flavored	New fields possibly associated with
24	$\mathbb{S}_8$	$[\sigma_o\ \sigma_\jmath\ \mu_\iota\ \mu_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \lambda_\jmath\ I\lambda_\jmath\ ]$	Colored, Flavored	flavor mixing mediated by
25	$\mathbb{S}_9$	$[\sigma_o\ \sigma_\kappa\ \mu_\iota\ \mu_\jmath\ ] \otimes [\sigma_o\ I\sigma_o\ \lambda_\kappa\ I\lambda_\kappa\ ]$	Colored, Flavored	Z-prime bosons or
26	$\mathbb{S}_{10}$	$[\sigma_o\ \sigma_\iota\ \nu_\jmath\ \nu_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \mu_\iota\ I\mu_\iota\ ]$	Colored, Flavored	Lepto-quark bosons
27	$\mathbb{S}_{11}$	$[\sigma_o\ \sigma_\jmath\ \nu_\iota\ \nu_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \mu_\jmath\ I\mu_\jmath\ ]$	Colored, Flavored	
28	$\mathbb{S}_{12}$	$[\sigma_o\ \sigma_\kappa\ \nu_\iota\ \nu_\jmath\ ] \otimes [\sigma_o\ I\sigma_o\ \mu_\kappa\ I\mu_\kappa\ ]$	Colored, Flavored	
29	$\mathbb{S}_{13}$	$[\sigma_o\ \sigma_\iota\ \lambda_\jmath\ \lambda_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \nu_\iota\ I\nu_\iota\ ]$	Colored, Flavored	
30	$\mathbb{S}_{14}$	$[\sigma_o\ \sigma_\jmath\ \lambda_\iota\ \lambda_\kappa\ ] \otimes [\sigma_o\ I\sigma_o\ \nu_\jmath\ I\nu_\jmath\ ]$	Colored, Flavored	
31	$\mathbb{S}_{15}$	$[\sigma_o\ \sigma_\kappa\ \lambda_\iota\ \lambda_\jmath\ ] \otimes [\sigma_o\ I\sigma_o\ \nu_\kappa\ I\nu_\kappa\ ]$	Colored, Flavored	

Although likely to be inaccurate, this analysis demonstrates the potential for the trigintaduonion algebra to account for many of the features of the standard model.

4. Notation for unit elements of $M_4(C)$

Capital roman letters are used to label real matrix unit elements of $M_4(C)$, as shown in table 4. These labels are combined with i to represent their imaginary counterparts which are used for imaginary matrix unit elements of $M_4(C)$.

These labels have been chosen so that, when unit elements of a quaternionic subalgebra, labelled $[LMN]$, are assigned to represent unit spatial bivectors for the Clifford algebra of space-time, other unit elements related by spatial rotation are also labelled using sets of three sequential letters:
$[X, Y, Z]$, $[D, E, F]$, $[P, Q, R]$ and $[iL, iM, iN]$, $[iX, iY, iZ]$, $[iD, iE, iF]$, $[iP, iQ, iR]$.

TABLE 5. Notation used to label 4×4 unit matrices

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad M = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Note: the forms of these matrices differ from those used in previous papers by this author[10][11]. Positive forms have been chosen to allow $[S, L, M, N]$ to represent unit elements for a right isoclinic quaternion algebra $\mathbb{H}_{\mathbb{R}}$, and $[S, T, U, V]$ to represent unit elements for a left isoclinic quaternion algebra $\mathbb{H}_{\mathbb{L}}$, as used by Van Elfrinkhof[12].

5. Space-time, three families of fundamental particles, dark energy

Based on the analysis in section 4, one family of fermions and a basis for the Brout-Englert-Higgs mechanism[1][2][3] can be found for an alignment of $\mathbb{T}$ with $M_4(C) \cong Cl_4(C)$. This suggests exploring the possibility of other alignments that might generate the observed three families of fermions. In section 4 the alignment has been defined in terms of aligning quaternionic unit elements of $\sigma_{\imath\jmath\kappa}$ of $\mathbb{T}$ with unit spatial bivectors for $Cl_4(C) \cong M_4(C)$.

For $[\sigma_\imath, \sigma_\jmath\sigma_\kappa]$ aligned with $[L, M, N]$ there are six sets of unit elements of $\mathbb{T}$ that may be aligned with spatial vector unit elements for the spacetime geometric algebra which result in spatial equivalence:
$[\lambda_\imath, \lambda_\jmath, \lambda_\kappa]$, $[\mu_\imath, \mu_\jmath, \mu_\kappa]$, $[\mu_\imath, \mu_\jmath, \mu_\kappa]$, $[I\lambda_\imath, I\lambda_\jmath, I\lambda_\kappa]$, $[I\mu_\imath, I\mu_\jmath, I\mu_\kappa]$ or $[I\mu_\imath, I\mu_\jmath, I\mu_\kappa]$.
These alignments can be assigned to three families of fundamental particles and their anti-particles. The alignments could have been identified as a choice of one of six sets of unit elements of $M_4(C)$ to be aligned with $[\sigma_\imath, \sigma_\jmath\sigma_\kappa]$:
$[X, Y, Z]$, $[D, E, F]$, $[P, Q, R]$, $[iX, iY, iZ]$, $[iD, iE, iF]$ or $[iP, iQ, iR]$.
The two ways of describing re-alignments could correspond to wave/particle duality. As re-orientation of alignment has similarities to the re-orientation in Minkowski space associated with the change in relativistic mass, this suggests that the families would differ in mass.

For each of these sets of isotropic spatial vector alignments there are two possible unit elements that could be aligned with a timelike unit vector. For $[\lambda_\imath, \lambda_\jmath, \lambda_\kappa]$, either μ_o or ν_o could be chosen. For sedenionic subalgebras S_0^γ, S_1^α, S_2^α, S_3^α, S_1^β, S_2^β and S_3^β, their $[\mu_{o\imath\jmath\kappa}]$ content matches their $[\nu_{o\imath\jmath\kappa}]$ content, an internal symmetry. For the other sedenionic subalgebras, their $[\mu_{o\imath\jmath\kappa}]$ content does not match their $[\nu_{o\imath\jmath\kappa}]$ content, but either their $[\nu_{o\imath\jmath\kappa}]$ content matches their $[\lambda_{o\imath\jmath\kappa}]$ content or their $[\lambda_{o\imath\jmath\kappa}]$ content matches their $[\mu_{o\imath\jmath\kappa}]$ content. This suggests that the availability of two possible unit elements that could be aligned with a timelike unit vector is associated with flavor mixing.

$M_4(C) \otimes \mathbb{T}$ is a larger algebra than required to generate these features. $M_2(C) \otimes \mathbb{T}$ could be used, with $[\sigma_\imath, \sigma_\jmath\sigma_\kappa]$ replacing $[L, M, N]$, but then spatial bivector unit elements would not be associative with respect to $\mathbb{T}$ making the smaller algebra less suitable. $M_4(C) \otimes \mathbb{T}$ is four times the size of $M_2(C) \otimes \mathbb{T}$, suggesting that three-quarters of it could describe something other than particles of the standard model. This is, approximately, the proportion of the universe's mass/energy attributed to dark energy. If the dark energy components of $M_4(C) \otimes \mathbb{T}$ are identified as corresponding to alignment of $[\lambda_\imath, \lambda_\jmath, \lambda_\kappa]$, $[\mu_\imath, \mu_\jmath, \mu_\kappa]$, $[\mu_\imath, \mu_\jmath, \mu_\kappa]$, $[I\lambda_\imath, I\lambda_\jmath, I\lambda_\kappa]$, $[I\mu_\imath, I\mu_\jmath, I\mu_\kappa]$ or $[I\mu_\imath, I\mu_\jmath, I\mu_\kappa]$ with one time-like and two spatial unit vector elements, this would indicate that the quantum fields for dark energy are tachyonic, as has been proposed for some models[13].

6. Comparison with other algebras used in unification models

For positive spatial signature the Clifford algebra basis of the original Kaluza-Klein model[14], which allows Maxwell's equations to be combined with general relativity[15], is $Cl_{4,1}$, isomorphic to $M_4(C)$. $M_4(C) \otimes \mathbb{T}$ has the same dimensionality as $Cl_{1,9}$, which is the geometric algebra for the dimensionality used in string theories[16][17]. When assembled as $Cl_{1,3} \otimes [\mathbb{T} \otimes \mathbb{C}]$ or as $Cl_{3,1} \otimes [\mathbb{T} \otimes \mathbb{C}]$, the Clifford algebra of space-time commutes with the $[\mathbb{T} \otimes \mathbb{C}]$ subalgebra, providing a logical basis for the dimensionality of spacetime.

$M_4(C) \otimes \mathbb{T}$ has $\mathbb{C} \otimes \mathbb{O}$ and $M_4(C) \otimes \mathbb{H}^{\sigma_{o\iota\jmath\kappa}} \cong Cl_6(C)$ as a subalgebras. Cohl Furey has postulated that minimal left ideals of a $Cl_6(C)$ algebra extracted from $\mathbb{C} \otimes \mathbb{O}$ correspond to one family of fundamental particles[18][19], and refers to others who have advocated the existence of a connection between non-associative algebras and particle theory[20] [21] [22] [23] [24] [25] [26] [27] [28] [29] [30] [31] [32] [33] [34] [35] [36] [37][38]. There have been approaches based on complexified sedenions[39]. $M_4(C) \otimes \mathbb{T}$ has sub-algebras on which all of these approaches are based. For instance, the octonion based approach proposed by G.Dixon[27][28] uses the tensor product $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ to account for one family of fundamental particles, which can be assembled with basis elements: $[SZQD]$ for three copies of $\mathbb{R}$, $[SiS]$ for $\mathbb{C}$, $[SLMN]$ for $\mathbb{H}$ and $[\sigma_o\sigma_\iota\sigma_\jmath\sigma_\kappa I\sigma_o I\sigma_o\iota I\sigma_\jmath I\sigma_\kappa]$ for $\mathbb{O}$.

For type IIB string theory based on AdS/CFT correspondence[40], $AdS_5 \times S^5$ is equivalent to $N = 4$ supersymmetric YangMills theory on the four-dimensional boundary. For this approach, the spacetime in which the gravitational theory embedded is effectively five-dimensional and there are five additional compact dimensions (encoded by the S^5). $Cl_{0,5}$ is isomorphic to $M_4(C)$.

Some twistor[41] approaches use quaternions to doubly complexify space-time. The observations made in this paper suggest extension of this to an "ultracomplexifying" of space-time using $\mathbb{T}$.

The geometric algebra of space-time is a graded algebra generated by anti-commuting vector unit elements, which may be represented by the matrices such as: $[iX, iY, iZ, iT]$. If a sedenion-type subalgebra of $\mathbb{T}$ is used to ultracomplexify it using, for instance, the S_0^γ subalgebra of $\mathbb{T}$, alignment of $[\lambda_\iota, \lambda_\jmath, \lambda_\kappa, \gamma_o]$ with those unit vectors, this would generate unit product elements: $[X\lambda_\iota, Y\lambda_\jmath, Z\lambda_\kappa, T\gamma_o]$. These unit elements commute and have opposite signature to the original vector unit elements. They can be used to generate a graded algebra. The Lie bracket for the original algebra becomes the Jordan bracket for this algebra, and vice-versa.

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Appendix A. Clifford algebras and Cayley-Dickson-type algebras

A.1. Cayley tables

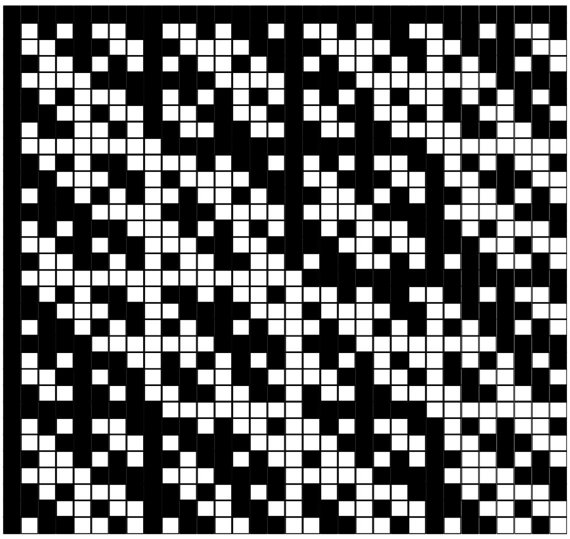
The Cayley tables of both $M_4(C)$ and $\mathbb{T}$ can be assembled as normalised latin squares with elements ordered so that bit-wise 'exclusive or' (XOR) of binary representations of two elements' numbering generates the numbering of their product. As a result, if the sign of products is ignored, their Cayley tables are the same and, for subalgebras that include the negative of the identity, their subalgebra inventory is the same.

A.2. The equivalent of isoclinic $\mathbb{C}$ subalgebras for $\mathbb{T}$

For $M_4(C)$ the unit imaginary has unique status, commuting with all other unit elements. For the subalgebra structure of $\mathbb{T}$, α_o has unique status.

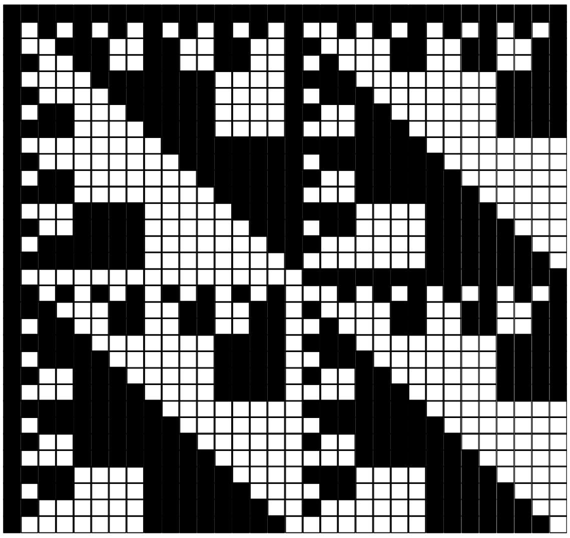
A.3. The equivalent of isoclinic $\mathbb{H}$ subalgebras for $\mathbb{T}$

A.3.1. Standard Cayley-Dickson construction for $\mathbb{T}$. Trigintaduonions are usually generated using the Cayley Dickson construction, with the product:
$(a, b)(c, d) = (ac - db^*, a^*d + cb)$
Its subalgebra structure has been analysed by Cawagas et al[6]. Cayley-Dickson type algebras have also been analysed by J.W.Bales[7]. He arranges Cayley tables for their unit elements as normalised latin squares with elements ordered so that the bit-wise 'exclusive or' (XOR) of binary representations of two element's numbering generate the numbering of their product. He uses "twist maps" to display the pattern of signs of products of unit elements. He designates $\mathbb{T}$ as the "ω_3 twisted Cayley-Dickson algebra for $\mathbb{A}_6$". Its twist map is:



A.3.2. An alternative construction of $\mathbb{T}$. A Cayley Dickson-type construction: $(a,b)(c,d) = (ac - b^*d, da^* + bc)$ is used by JW Bales to assemble the ω_2 algebra for $\mathbb{A}_5$. It contains an embedded S^β loop. It can be used to generate $\mathbb{T}$ from elements g and h in S^β using a procedure usually used to assemble Moufang loops from groups. A new element u, not in S^β, is defined. Then let $\mathbb{T} = S^\beta \cup (S^\beta u)$. Define the product in $\mathbb{T}$ as:
$(g, gu) \times (h, hu) = (g.h + gu.h + gu.h + gu.hu)$, where:
$g.h = (gh)$
$(gu)h = (gh^{-1})u$
$g(hu) = (hg)u$
$(gu)(hu) = h^{-1}g$

For its multiplication table arranged as a normalised latin square with elements ordered so that bit-wise 'exclusive or' (XOR) of binary representations of two element's numbering generate the numbering of their product, the twist map is:



The embedded loop of order 64 for this algebra is isotopic to the embedded loop T_L for the standard representation of $\mathbb{T}$. The isotopism is:
(2,25,21,19,26,45,39,12,22,43,14,31,32,8,20)(3,10,29,23,60,6,27,62,15,16,56,4,50,9,5)
(7,44,54,11,46,63,64,40,52,34,57,53,51,58,13)(18,41,37,35,42,61,55,28,38,59,30,47,48,24,36)
Applying the isotopism to $M_4(C)$ is equivalent to:
$\mathbb{H}_L \otimes \mathbb{H}_R \otimes \mathbb{C} \to \mathbb{H}_R \otimes \mathbb{H}_L \otimes \mathbb{C}$
together with swapping the labels of some unit matrices with their negatives.
This indicates $[\sigma_\iota, \sigma_\jmath, \sigma_\kappa]$ and $[\lambda_o, \mu_o, \nu_o]$ generate the equivalent of left and right handed quaterionic subalgebras of $\mathbb{T}$.

Appendix B. The Brout-Englert-Higgs mechanism

This appendix is part of a previous paper by this author[**?**].

The Brout-Englert-Higgs mechanism acts on a complex doublet and involves scalar fields. For $M_4C \otimes \mathbb{T}$ a scalar subalgebra can be assembled as the product: $[\sigma_oS, \sigma_oiS, \alpha_oS, \alpha_oiS] \otimes [\sigma_oS, \sigma_oT, \sigma_oV, \sigma_oU] \otimes [\sigma_oS, \lambda_oS, \mu_oS, \nu_oS]$.

$[\sigma_oS, \sigma_oT, \sigma_oV, \sigma_oU] \otimes [\sigma_oS, \lambda_oS, \mu_oS, \nu_oS]$ is isomorphic to $\mathbb{H} \otimes \mathbb{H}$ and to $M_4(R)$. Its unit elements can be relabeled as matrices from table 1 as follows:
$[\sigma_oS] \cong [S]$, $[\sigma_oT, \sigma_oV, \sigma_oU] \cong [TVU]$, $[\lambda_oS, \mu_oS, \nu_oS] \cong [LMN]$
$[\lambda_oT, \mu_oT, \nu_oT] \cong [PQR]$, $[\lambda_oV, \mu_oV, \nu_oV] \cong [DEF]$, $[\lambda_oU, \mu_oU, \nu_oU] \cong [XYZ]$

The Brout-Englert-Higgs mechanism is based on a scalar field with a mexican hat potential. It is possible to find subalgebras of $M_4(R)$, and thus of $[\sigma_oS, \sigma_oT, \sigma_oV, \sigma_oU] \otimes [\sigma_oS, \lambda_oS, \mu_oS, \nu_oS]$, with this property
Subalgebras of $M_4(R)$ for which the scalar component (unit matrix $[S]$), is associated with a mexican hat potential, can be found by considering unitary abelian subgroups of $M_4(R)$. Unitary abelian subgroups of $M_4(R)$ can be represented by diagonal 4×4 matrices.

$$\begin{bmatrix} e^{\theta_1} & 0 & 0 & 0 \\ 0 & e^{\theta_2} & 0 & 0 \\ 0 & 0 & e^{\theta_3} & 0 \\ 0 & 0 & 0 & e^{\theta_4} \end{bmatrix}$$

where $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 0$, allowing it to be rewritten:

$$\begin{bmatrix} e^{a} & 0 & 0 & 0 \\ 0 & e^{b} & 0 & 0 \\ 0 & 0 & e^{c} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The product of two elements of this type with parameters a, b, c and a', b', c' has parameters $a + a', b + b', c + c'$. A subgroup of the Heisenberg group H(5) shares this property:

$$\begin{bmatrix} 1 & a & b & c + ab \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix has determinant $= 1$, and the commuting products of the form:

$$\begin{bmatrix} 1 & a + a^{'} & b + b^{'} & c + c^{'} + (a + a^{'}) \times (b + b^{'}) \\ 0 & 1 & 0 & b + b^{'} \\ 0 & 0 & 1 & a + a^{'} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix can be written in terms of unit elements of $M_4(R)$ as:
$[S] + a/2[V + Y] + b/2[M + F] + (c + ab)/4[E + U + N + P]$.

There are other combinations of unit elements of $M_4(R)$ with similar properties. These can be found using a 6×6 array having anti-commuting basis matrices and the identity in each row/column:

$$\begin{bmatrix} S & V & T & X & Y & Z \\ V & S & U & P & Q & R \\ T & U & S & D & E & F \\ X & P & D & S & N & M \\ Y & Q & E & N & S & L \\ Z & R & F & M & L & S \end{bmatrix}$$

Interchanging rows and matching columns preserves group properties and commutation relationships with respect to position in the array. For example, rows and columns 1 and 2 can be interchanged to make the array:

$$\begin{bmatrix} S & V & U & P & Q & R \\ V & S & T & X & Y & Z \\ U & T & S & D & E & F \\ P & X & D & S & N & M \\ Q & Y & E & N & S & L \\ R & Z & F & M & L & S \end{bmatrix}$$

Inspecting this array to assign unit matrices for an equivalent H5 subgroup group, they would be:
$[S] + a/2[V + Q] + b/2[M + F] + (c + ab)/4[E + N + T + X]$
This combination has the same properties. Interchanging rows and columns 1 and 2 has not changed the signatures of the matrices allocated to each position.

If a further interchange is made that does affect the signatures, e.g interchanging rows and columns 1 and 4, to generate:

$$\begin{bmatrix} S & Q & U & P & V & R \\ Q & S & E & N & Y & L \\ U & E & S & D & T & F \\ P & N & D & S & X & M \\ V & Y & E & N & S & L \\ R & L & F & M & Z & S \end{bmatrix}$$

For the combination:
$[S] + a/2[Y + Q] + b/2[M + F] + (c + ab)/4[P + U + T + X]$
The determinant is no longer 1. To make this combination generate a unitary matrix, a factor has to be applied to $[S]$. That factor is $\sqrt{(} \pm 1 \pm 2(a/2)^2)$, provided that the factor is real and not imaginary.

For the resulting matrix, there are four plus/minus permutations, for which the possible components for [S] are :

$$\begin{bmatrix} \sqrt(1+a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(1+a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(1+a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(1+a^2/2) \end{bmatrix}$$

Which always has real entries, and determinant $= 1 + a^2 + a^4/4$

$$\begin{bmatrix} \sqrt(-1-a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(-1-a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(-1-a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(-1-a^2/2) \end{bmatrix}$$

Which never has real entries, and determinant $= 1 + a^2 + a^4/4$

$$\begin{bmatrix} \sqrt(1-a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(1-a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(1-a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(1-a^2/2) \end{bmatrix}$$

Which has real entries for $a^2/2 \leq 1$, and determinant $= 1 - a^2 + a^4/4$

$$\begin{bmatrix} \sqrt(-1+a^2/2) & 0 & 0 & 0 \\ 0 & \sqrt(-1+a^2/2) & 0 & 0 \\ 0 & 0 & \sqrt(-1+a^2/2) & 0 \\ 0 & 0 & 0 & \sqrt(-1+a^2/2) \end{bmatrix}$$

Which has real entries for $a^2/2 \geq 1$, and determinant $= 1 - a^2 + a^4/4$

The function $f(a) = 1 - a^2 + a^4/4$ has the form of a mexican hat potential.

For the assignment of unit elements of $\mathbb{U}$ to matrices:

$[\sigma_o S] = [S]$, $[\sigma_o T, \sigma_o V, \sigma_o U] = [TVU]$, $[\lambda_o S, \mu_o S, \nu_o S] = [LMN]$
$[\lambda_o T, \mu_o T, \nu_o T] = [PQR]$, $[\lambda_o V, \mu_o V, \nu_o V] = [DEF]$, $[\lambda_o U, \mu_o U, \nu_o U] = [XYZ]$

The group represented by a plus/minus choice for:

$$\sqrt(\pm 1 \pm a^2/2)[S] + a/2[Y+Q] + b/2[M+F] + (c+ab)/4[P+U+T+X]$$

is isomorphic to that for the same plus/minus choice for:

$$\sqrt(\pm 1 \pm a^2/2)[\sigma_o S] + a/2[\mu_o U + \mu_o T] + b/2[\mu_o S + \nu_o V]$$
$$+ (c+ab)/4[\lambda_o T + \sigma_o U + \sigma_o T + \lambda_o U]$$

for which $[TVU]$ symmetry is broken.