The submarine paradox from the perspective of length expansion

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01 August 2021

In this study, the submarine paradox proposed by Supplee was approached with the length expansion theory rather than the length contraction theory. I looked at how the density of submarines changes and whether this paradox can be solved without contradiction. In addition to what Supplee described, I have studied this problem by adding an observer to the sky. As a result, I looked to see if additional contradictions arise in this paradox or if the contradictions are resolved. Additionally, I studied the commonalities between this paradox and the paradox of the constancy of the speed of light.

I. Introduction

Since the submarine paradox was introduced by Supplee in 1989, several people have tried to solve this problem [1]. These attempts are complicated, and all attempts to solve this paradox under the premise that the length contraction is correct. Length contraction was proposed by Lorentz and Fitzgerald and accepted by Einstein, but to this day, its validity has not been experimentally proven. In this study, I will look at this paradox from the perspective of length expansion rather than length contraction. Then, this problem can be solved simply without introducing general relativity, complex formulas, and additional hypotheses.

II. Length expansion theory

Length contraction theory is a classical ether theory and does not originate from pure relativity. It was introduced temporarily to solve the difficulties of the null effect of the interference fringes appearing in the ether theory. It was later declared that there was no ether, but its influence remains in the theory of relativity to this day. Occasionally, some claim that the muon's reaching the sea level is evidence of length contraction, but this is only evidence of time dilation and cannot be evidence of length contraction. Hoffman hypothesized a pancake-shaped earth to solve the muon paradox with the length contraction theory, but this follows a larger logical contradiction [2, 5]. If length contraction theory is used, contradictions are found everywhere. The submarine paradox, which we will be discussing today, is no exception, and I believe that the fundamental cause of this paradox is not the submarine problem, but the length contraction theory.

Recently, there has been a lot of discussion about the correct length for relativistic judgment. The problem of length contraction has been pointed out by many people. Strel'tsov pointed out the problem of length contraction by taking the concept of radar length [3], and Kwak insisted that the correct relativistic length is not length contraction, but the opposite length expansion [4, 5]. Buenker insisted that length expansion, not length contraction, was found in GPS [6]. And Sato argued that if the length contraction was correct, GPS would not work [7]. In addition, Ashby said that they found the effect of time dilation in GPS, and he passed over the effect of length contraction [8]. I think because he could not find any length contraction effect in GPS. Some argue for partial length expansion [9]. Given the opinions of these various authors, it is reasonable to suspect that there is a problem with the relativistic length as we know it. First, we can simply prove the length expansion from the time dilation. (γ is Lorentz factor)

$$t = \gamma t_o \qquad \textit{time dilation} \tag{1}$$

$$ct = \gamma c t_o \qquad \textit{multiply both sides by } c \tag{2}$$

$$l = \gamma l_o \qquad \textit{length expansion} \tag{3}$$

There are many ways to prove length expansion, but here I will focus on solving this paradox under the premise that length expansion is correct [4, 5].

III. Solving the Submarine Paradox

There is a submarine that has neutral buoyancy and moves rapidly at relativistic speed in an infinitely wide ocean. Let the density of water is w_o and when the submarine is at rest, let the length of the submarine is l_o, the volume v_o, and the mass m_o. Then the density of the submarine is $d_o(= m_o/v_o)$. When a submarine floats in the sea in a state of neutral buoyancy, the equation is given by

$$d_o = w_o \tag{4}$$

And let l, v, m, d be the length, volume, mass, and density of this submarine when it moves quickly. If the

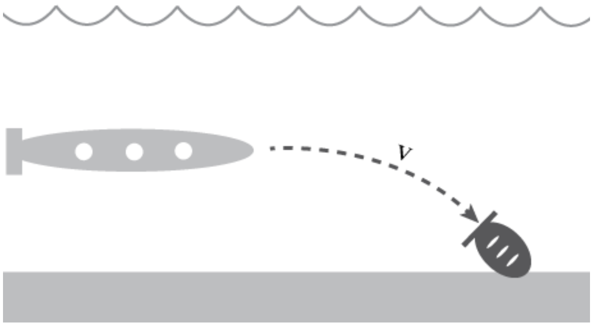

Fig. 1. Submarine moving in the sea

length contraction is correct, the length l of the submarine will contract to become $(1/\gamma)l_o$, the volume v will also contract to become $(1/\gamma)v_o$, and the mass m will increase to become γm_o. Then, the density d of the submarine is given by

$$d = \frac{m}{v} = \frac{\gamma m_o}{(1/\gamma)v_o} = \gamma^2 \frac{m_o}{v_o} = \gamma^2 d_o \tag{5}$$

As the volume decreases but the mass increases, the density increases rapidly, and the submarine will eventually sink to the bottom of the sea. However, if the crew members in the submarine see this, the opposite happens. As the density of water increases, the submarine must float. This is a contradiction, and we call it '*the submarine paradox*' [1].

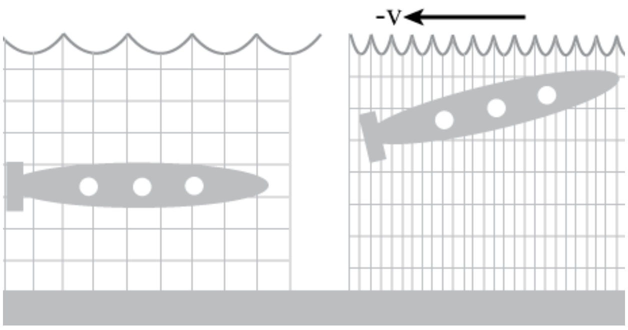

Fig. 2. Under the sea from the perspective of a submarine crew

Assuming that length contraction theory is correct, this contradiction arises, but length expansion theory can easily solve this problem. Length expansion theory states that when an object moves at a relativistic speed, its length expands rather than contracts. The table below shows the difference between the two theories.

Table 1. Changes in physical quantities according to the two theories

	Proper physical quantity	Length contraction	Length expansion
Length	l_o	$(1/\gamma)l_o$	γl_o
Volume	v_o	$(1/\gamma)v_o$	γv_o
Mass	m_o	γm_o	γm_o
Density	d_o	$\gamma^2 d_o$	d_o

According to the length expansion theory, if the submarine moves fast, the length becomes γl_o, so the volume becomes γv_o. Since the mass also becomes γm_o, the density becomes as follows.

$$d = \frac{d}{v} = \frac{\gamma m_o}{\gamma v_o} = \frac{m_o}{v_o} = d_o \tag{6}$$

$$\therefore d = d_o \tag{7}$$

If this problem is solved with the length expansion theory, the density of the submarine does not change because the mass also increases as the volume increases. The initial density of the submarine and the later density are the same. As the result, the submarine neither sinks nor rises. Therefore, a paradox is not formed from the beginning. If the crew of a submarine observes the sea, the water particles are moving in the opposite direction with relativistic speed, so the following process occurs (Figure 3). Prime indicates the physical quantity that the crew observed the sea.

$$d' = \frac{w'}{v'} = \frac{\gamma m_o'}{\gamma v_o'} = \frac{w_o'}{v_o'} = d_o' \tag{8}$$

$$\therefore\ d' = d_o' \tag{9}$$

Therefore, considering both the submarine crew and the underwater observer, the following relationship holds.

$$d = d_o = d' = d_o' = w = w_o = w' = w_o' \tag{10}$$

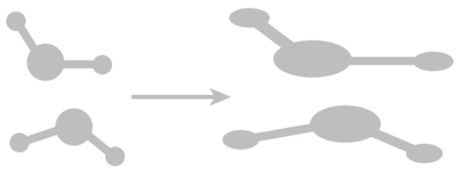

Fig. 3. Change in volume of water molecules

The length of the water molecule increases in the direction it moves, and the volume increases accordingly. However, the density of water does not change as the mass of water increases as the volume increases. Therefore, the submarine neither sinks nor rises, even if the crew sees. Even if the observer in the sea observes the submarine or the crew of the submarine observes the sea, the submarine is constantly moving with neutral buoyancy. No matter how you change the observer's point of view, the result is the same. This is the same reason why the speed of light is constant [5]. To test this, let us introduce an additional thought experiment. All the situations are the same as those suggested by Supplee. Additionally, suppose that there is an infinitely long zip line over the sea. Let us observe the submarine while moving at the same speed as the submarine and attach a camera to it. In this case, there are three people observing the submarine, since there is an observer in the sea, a crew member of the submarine, and an observer on the zip line. The submarine and the zip line's camera are moving at the same speed as each other. First, consider the length contraction theory. When an

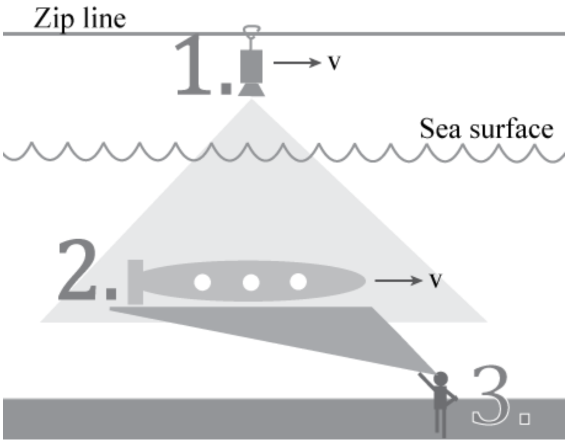

Fig. 4. Multiple submarine observers

observer in the sea sees a submarine, the submarine becomes denser and must fall to the bottom. However, the density did not increase in the eyes of the observer on the zip line. In fact, it is like observing a proper physical quantity in his own system because it is moving at the same speed. Therefore, the submarine must be moving while maintaining neutral buoyancy. An observer under the sea says that a submarine sinks, but an observer on a zip line says that the submarine maintains neutral buoyancy. Two arguments for one phenomenon. This is an obvious contradiction. If we consider these situations using the length expansion theory, we can see that there is no contradiction at all. An observer in the sea will measure the density of the submarine as d_o, an observer on the zip line will measure that density as d_o, and the crew aboard the submarine will measure the density as d_o. All three observers measure the same.

IV. The relationship between the speed of light paradox and the submarine paradox

In an inertial system, the speed of light is always constant regardless of the motion of the light source or the observer. Einstein made this the principle of the theory of relativity. However, we still do not know why the speed of light is always constant regardless of the motion of the light source or the observer. The speed of light paradox and the submarine paradox have something in common. Two paradoxes are related to length. Traditionally, we valued the dimensions of length, time, and mass. These three dimensions are transformed as follows.

$$l = \left({}^{1}/_{\gamma}\right) l_o \tag{11}$$

$$t = \gamma t_o \tag{12}$$

$$m = \gamma m_o \tag{13}$$

$$\therefore\ d = \frac{m}{v} = \frac{\gamma m_o}{(1/\gamma) v_o} = \gamma^2 \frac{m_o}{v_o} = \gamma^2 d_o \tag{14}$$

$$c = \frac{l}{t} = \frac{(1/\gamma) l_o}{\gamma t_o} = \frac{1}{\gamma^2} \frac{l_o}{t_o} = \frac{1}{\gamma^2} c_o \tag{15}$$

In this case, when time and length are combined, a paradox of invariant speed of light occurs, and when mass and length are combined, a submarine paradox occurs. A contradiction always arises when length contraction is involved. Here, those with subscripts '$_o$' are proper physical quantities, and those without subscripts are physical quantities observed by the other party. The notation l is the length, t is the time, and c is the speed of light.

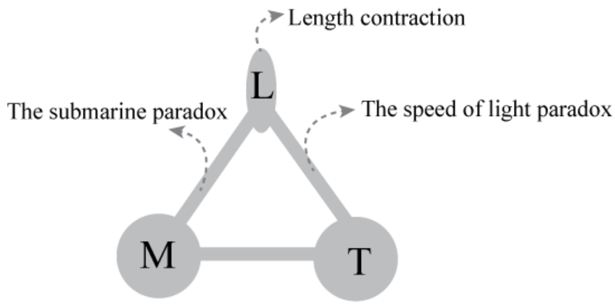

Fig. 5. Paradoxes caused by length contraction

Therefore, I believe that time, length, and mass should have the following relationship.

$$l = \gamma l_o \tag{16}$$

$$t = \gamma t_o \tag{17}$$

$$m = \gamma m_o \tag{18}$$

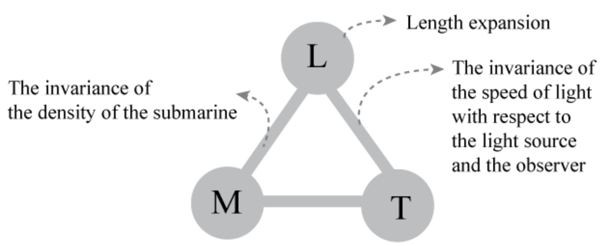

Fig. 6. Paradoxes solved by length expansion

In this case, when time and length are combined, the speed of light paradox is solved [5], and when mass and length are combined, the submarine paradox is solved.

$$d = \frac{d}{v} = \frac{\gamma m_o}{\gamma v_o} = \frac{m_o}{v_o} = d_o \tag{19}$$

$$c = \frac{l}{t} = \frac{\gamma l_o}{\gamma t_o} = \frac{l_o}{t_o} = c_o \tag{20}$$

In the above equation, $d = d_o$ (19) means that the density of the submarine is always the same, and $c = c_o$ (20) means that the speed of light is always the same. In other words, if the length expansion rather than the length contraction is correct, both the paradox of the constant speed of light and the paradox of the submarine are solved.

VI. Conclusion

If we want to explain the behavior of a submarine using the length contraction theory, many additional processes and complex equations are needed. However, this paradox is easily solved if explained by the length expansion theory. It is solved only by the basic assumptions of relativity. The submarine will always remain neutrally buoyant, even in additional circumstances beyond what Supplee described. Therefore, there is one additional reason why we should choose the length expansion theory rather than the length contraction theory.

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