To Options

Original ABC Conjecture Proved on a Single Page

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Abstract

The author proves the original ABC conjecture which states that if A, B and C are three coprime positive integers such that $A + B = C$, and d is the product of the distinct prime factors of A, B and C, then d is usually not much smaller than C. The author will adhere to the wording of the original conjecture and not to any equivalent conjecture, since if one proves an equivalent conjecture, logically, one would also have to prove the equivalency, otherwise, the proof of the original conjecture would be incomplete. The author assumes that the statement " d is usually not much smaller than C" means the difference between C and d is usually less than a small positive number, say , ε. Then, one obtains $|C - d| < \varepsilon$, which would be the conclusion. If $A + B - C = 0$, then for a very small positive number δ, $0 < \delta$, one can write $|A + B - C| < \delta$. From above, the hypothesis, would be $|A + B - C| < \delta$, and the conclusion would be $|C - d| < \varepsilon$. The author has proved that if $|A + B - C| < \delta$ then $|C - d| < \varepsilon$.

To First Page

Options

Back to Options

Option 1
Introduction

The ABC conjecture states that if A, B and C are three coprime positive integers such that $A + B = C$, and d is the product of the distinct prime factors of A, B and C, then d is usually not much smaller than c. In sticking to the wording of the original conjecture, the statement " d is usually not much smaller than C" would be guided by the following analogy: If B's mass is usually not much less than A's mass, the implication is that A's mass minus B's mass is usually small. Applying this analogy to the above conjecture, the difference between C and d is less than a small positive number, say , ε. Then, one obtains $|C - d| < \varepsilon$,which would be the conclusion. When the author read " d is not much smaller than C, the author thought about epsilon-delta proofs, but the other statement $A + B = C$, is an equation. However if $A + B - C = 0$, $|A + B - C| = |0| = 0$. For a very small positive number δ, $0 < \delta$, one can write $|A + B - C| < \delta$ From above, the hypothesis would be, $|A + B - C| < \delta$, and the conclusion would be $|C - d| < \varepsilon$.

Back to Options

Back to Options

Option 2

Original ABC Conjecture Proved on a Single Page

Given: 1. $A + B = C$, where A, B and C are positive integers. with A, B and C being coprime.
2. d = product of the distinct prime factors of A, B and C.

Required: To prove that d is not much smaller than C, or simply, $C - d < \varepsilon$, where ε is any positive number. That is prove that $|C - d| < \varepsilon$; (1)

Plan: Let r, s and t be prime factors of A, B and C .respectively, such that $A = Dr$, $B = Es$, $C = Ft$ where D, E and F are positive integers; $\boxed{r \neq s \neq t}$, with the product $d = rst$

Then, the equation $A + B = C$ becomes $Dr + Es = Ft$

One will prove that if $|A + B - C| < \delta$ $(\delta > 0)$, then $|C - rst| < \varepsilon$. ($\varepsilon > 0$

Proof: One will apply the continued inequality (condensed) method to handle the inequalities.

$|A + B - C| < \delta$ (hypothesis) (2)

$|C - rst| < \varepsilon.$ (conclusion) (3)

The conclusion (3) is equivalent to

$-\varepsilon < C - rst < \varepsilon$

$-\varepsilon < Ft - rst < \varepsilon$ (conclusion) ($C = Ft$)

$-\delta < Dr + Es - Ft < \delta$ (hypothesis) (4) ($A = Dr$, $B = Es$, $C = Ft$),

$-\varepsilon < Dr + Es - rst < \varepsilon$; (conclusion) (5) ($Dr + Es = Ft$)

Make the middle terms of (4) and (5) the same..

Then, (4) becomes $-\delta + Ft < Dr + Es < \delta + Ft$ (hypothesis) (6)

and (5) becomes $-\varepsilon + rst < Dr + Es < \varepsilon + rst$ (conclusion) (7)

Since (6) and 7) have the same middle terms, equate the left sides to each other and equate the right sides to each other. Then one obtains

$-\delta + Ft = -\varepsilon + rst$, and solving for δ, $\boxed{\delta = \varepsilon + Ft - rst, \text{ say } \delta_1}$, and for the right sides,

$\delta + Ft = \varepsilon + rst$, and solving for δ, $\boxed{\delta = \varepsilon - Ft + rst, \text{ say } \delta_2}$

$|A + B - C| < \delta$, implies that $-\delta_1 \leq -\delta < Dr + Es - Ft < \delta \leq \delta_2$ (hypothesis)

For $\varepsilon > 0$, choose $\delta = \min(\delta_1, \delta_2)$.

$-\delta < Dr + Es - Ft < \delta$ (hypothesis) implies

$-\delta_1 \leq -\delta < Dr + Es - Ft < \delta \leq \delta_2$ (hypothesis) (8)

Replace the left and right sides of (8) by $\boxed{\delta = \varepsilon + Ft - rst, \text{ say } \delta_1}$ and $\boxed{\delta = \varepsilon - Ft + rst, \text{ say } \delta_2}$, from above, respectively to obtain

$-\varepsilon - Ft + rst < Dr + Es - Ft < \varepsilon - Ft + rst$ (hypothesis) (9)

Break up (9 into two simple inequalities and solve each one for $-\varepsilon$ and ε , respectively.

$\boxed{-\varepsilon - Ft + rst < Dr + Es - Ft}$	$\boxed{Dr + Es - Ft < \varepsilon - Ft + rst}$
$\boxed{-\varepsilon < Dr + Es - rst}$	$\boxed{Dr + Es - rstt < \varepsilon}$

$-\varepsilon < Dr + Es - rst$ **and** $Dr + Es - rstt < \varepsilon$ is equivalent to

$|Dr + Es - rst| < \varepsilon$

$|Ft - rst| < \varepsilon$ ($Dr + Es = Ft$)

$|C - rst| < \varepsilon$ ($Ft = C$)

Since $d = rst$, $|C - d| < \varepsilon$

Therefore, if $|A + B - C| < \delta$ $(\delta > 0)$ or $A + B = C$, $|C - d| < \varepsilon$ and d is not much smaller than C, and the proof is complete.

Back to Options

Back to Options

Option 3
Discussion

The author is sure of the procedure from the hypothesis, $|A+B-C|<\delta$ to the conclusion, $|C-d|<\varepsilon$ The author would like to convince the reader about how from the wording of the conjecture, one obtained $|A+B-C|<\delta$ and $|C-d|<\varepsilon$.,

The original ABC conjecture which states that if A,B and C are three coprime positive integers such that $A+B=C$, and d is the product of the distinct prime factors of A,B and C, then d is usually not much smaller than C

1. From $\boxed{A+B=C}$ to $\boxed{|A+B-C|<\delta}$

$A+B=C$; $A+B-C=0$

Since $A+B-C=0$, $|A+B-C|=|0|=0$

If $\delta>0$, $\boxed{|A+B-C|<\delta}$ $(0<\delta$ and $|A+B-C|=|0|)$

2. From "d is usually not much smaller than C" to $\boxed{|C-d|<\varepsilon}$

Since $(C-d)>0$, $|C-d|>0$

If for a small posive number, $\varepsilon>0$,

$\boxed{|C-d|<\varepsilon}$ (d is usually not much smaller than C}

Back to Options

Note: An interesting observation is that for the difference between C and d, the prime factor t of C is a common factor of C and d. $(|C-d|=|C-rst|=|Ft-rst|)$

Option 4
Conclusion

From the hypothesis, $|A+B-C|<\delta$, it was proved that $|C-d|<\varepsilon$, the conclusion. The author assumed that the statement " d is usually not much smaller than C" meant the difference between C and d is usually less than a small positive number, say ,ε, where d is the product of the distinct factors of A, B and C. The author adhered to the wording of the original conjecture and not to any equivalent conjecture, since if one proved an equivalent conjecture, logically, one would also have to prove the equivalency, otherwise, the proof of the original conjecture would be incomplete.

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Back to Options