Transmission of a Single-Photon through a Polarizing Filter: An Analysis Using Wave-Particle Non-Duality

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The inner-product, $< \psi|\psi >$, between a state vector, $|\psi >$ and its dual, $< \psi|$, is thoroughly analyzed using the recently developed 'wave-particle non-dualistic interpretation of quantum mechanics'; here, $|\psi >$ is a solution of the Schrödinger wave equation. Using this analysis, *questions about what decides whether a photon is to go through or not and how it changes its direction of polarization when it does go through a polarizing filter* - a statement by Prof. Dirac - is unambiguously explained.

I. INTRODUCTION

Quantum mechanics (QM) is a theoretical description of Nature, which is extremely successful in both explaining and predicting the experimental data of the microcosm and also certain macroscopic physical phenomena. In principle, QM applies to all physical systems irrespective of whether they are microscopic or macroscopic and so far there are no experimental violations observed to the same. Nevertheless, what kind of physical reality is being revealed by its formalism is not at all straightforwardly clear like the case of classical mechanics and there is no consensus among physicists regarding this quantum reality. Hence, many interpretations to the quantum formalism was proposed to explain the same reality, i.e., "What's actually happening?" in the quantum world [1–15]. The present author also put forward a new interpretation namely, "wave-particle non-dualistic interpretation of quantum mechanics at a single-quantum level" (WPND) [16–26].

II. PHYSICAL REALITY OF THE INNER-PRODUCT

It's well-known that the inner-product,

$$< \psi|\psi >= \iiint_{\mathbf{R}^3} d^3\mathbf{r}| < \mathbf{r}|\psi > |^2 = 1, \tag{1}$$

is interpreted as the total probability of finding a particle - obviously equal to one - in the entire 3-Dimensional Euclidean space (3DES) spanned by the set of eigenvalues, $\{\mathbf{r}|\mathbf{r} \in \mathbf{R}^3\}$, of the position operator, $\hat{\mathbf{r}}$; here, $\mathbf{r}$, is the position eigenvalue where the particle is found upon an observation and $< \mathbf{r}|$ is the dual vector of the position eigenstate, $|\mathbf{r} >$. Infinitely large number of observations are necessary to construct the integrand, $| < \mathbf{r}|\psi > |^2$, which is interpreted as the probability density to find the particle in an infinitesimal volume around $\mathbf{r}$, because, $< \psi|\psi >= 1$; here, $|\psi >$ or equivalently its position basis representation, $< \mathbf{r}|\psi >$, is a solution of Schrödinger's wave equation representing a quantum state of the particle and $< \psi|$ is the dual-vector of $|\psi >$. All these details are well-known as the Born rule.

Many interpretations accept the Born rule as it is and explain the quantum phenomena by invoking some appealing physical mechanisms. And the remaining few arrive at the inner-product as a consequence of the physical process underlying the respective interpretation and then adopts Born's probabilistic interpretation for $| < \mathbf{r}|\psi > |^2$. On the contrary, WPND first derives the 'relative frequency of detection' (RFD) using the single-quantum events and then arrives at the Born rule as a limiting case of the RFD, which is exactly in one-to-one correspondence with the observations being carried out in any given quantum-experiment.

In the case of the inner-product, $<\psi|\psi>$, the three most fundamental questions asked by WPND in the context of a single-quantum event are as follows:

1. What is the physical nature of $|\psi>$ or equivalently the Schrödinger wave function, $<\mathbf{r}|\psi>$, and its relation to the observed particle (a single-quantum event or an observed eigenvalue) in any given experiment?

2. If $|\psi>$ is an ontological entity existing in the Nature, then where does its dual-state, $<\psi|$, can exists in the same Nature such that they can undergo an inner-product and where does such an inner-product actually occur?

3. Inside the inner-product, $<\psi|\psi>$, what's the actual physical role being played by the redundant overall phase, say ϕ, associated with $|\psi>$? i.e., if $|\psi>\longrightarrow e^{i\phi}|\psi>$, then $<\psi|\psi>\longrightarrow<\psi|\psi>$. Does this phase have any relevance to the observed particle nature?

And WPND answers the above questions as follows:

A. Answer to the 1st Question: Part-I
(Physical nature of the Schrödinger wave function)

Using a mathematical reasoning, WPND shows that the physical nature of the Schrödinger wave function is an 'instantaneous resonant spatial mode' [16–26].

Consider a particle emitted from a source such that it will be absorbed by a detector at some later time. In the classical scenario, its initial position is a particular unique value in the 3DES. But, the same can't be claimed for the quantum mechanical case, because, unlike the classical case, the particle is also associated with the de Broglie wave nature. Moreover, the space in which the quantum phenomena happen can't be the usual 3DES, because, quantum mechanics demands a complex vector space (CVS) for their happening due to the canonical commutation relations like, $[\hat{x}\ ,\ \hat{p}]=i\hbar$, though the observed eigenvalues, being real numbers, live in 3DES; here, $\hat{x}$ and $\hat{p}$ are position and momentum operators, respectively; $i=\sqrt{-1}$ and $\hbar$ is the reduced Plank's constant.

The 3DES is spanned by the eigenvalues of the position operator and therefore, the moment the particle appears at the source, its wave function appears instantaneously everywhere in the entire 3DES - which implies that the reverse is also true, i.e., the moment the particle disappears at some later time due to absorption by the detector, then the entire wave function also disappears instantaneously, resembling the 'wave function collapse' advocated in the Copenhagen interpretation [1–3]. As well-known from experiments, the collapse occurs at some particular eigenvalue [27–41]. Hence, even the appearance of particle at the source can be inferred to occur at some definite eigenvalue. From the initial position eigenvalue to the final position eigenvalue, the particle moves as if confined to the wave function which exists everywhere like a spatial mode. Notice that, "the appearance of wave function at the moment of particle's appearance and its disappearance at the moment of particle's disappearance" is like a resonance process, i.e., as if both the particle and wave natures are in resonance with each other! Hence, the physical nature of the Schrödinger wave function is concluded to be an 'instantaneous resonant spatial mode' (IRSM) in which a quantum flies akin to the case of a test particle moving along a geodesic in the curved space-time of the general theory of relativity [42].

This identification of wave function as an IRSM is consistent with Born's probabilistic interpretation (BPI) [1], "*The wave function determines only the probability that a particle - which brings with itself energy and momentum - takes a path; but no energy and no momentum pertains to the wave*", with an exemption to the notion of probability.

Only the physical nature of the Schrödinger wave function is explained here, which is a partial answer to the first question. The remaining answer, the actual relation between the wave function and its observed particle, is given in the subsection-II.D.

B. Answer to the 2nd Question

The intensity of s classical-wave is proportional to the square of its amplitude. But, according to the WPND, Schrödinger's wave function can't be claimed to have such an intensity, because, it's an IRSM and is unlike a propagating classical wave.

When a quantum particle hits a detector screen, then its state vector, $|\psi >$, induces a dual vector, $< \psi|$, in the same screen and interacts according to the inner-product, $< \psi|\psi >$. The scattering of $|\psi >$ into some other state, say $|\psi' >$, can be described by associating an operator, $\hat{O} = |\psi' >< \psi|$, to the detector screen:

$$\hat{O}|\psi >=< \psi|\psi > |\psi' > . \tag{2}$$

Notice that, $< \psi|$ is analogous to the image in a mirror, totally confined only to the detector screen, unlike $|\psi >$ (see Fig. (2)). Discording the scattered state , $|\psi' >$, in Eq. (2) implies that the particle must have interacted somewhere in the region of the inner-product, $< \psi|\psi >$.

C. Answer to the 3rd Question

Consider the toss of a coin in a CVS - which will be mapped later (in the section-III) into a spin-1 system. Hence, the eigenvalues $+1$ and -1 are chosen for the outcomes of head and tail, respectively.

Let $|H >$ and $|T >$ be the eigenstates for the head and tail, respectively (see Fig. (1)). $|n >$ is a vector normal to the head-surface passing through the center-of-mass of the coin and $|g >$ is a vector parallel to the gravitational field direction and perpendicular to the surface of the ground. Upon the outcome, $|n >$ will be pointing along either $|H >$ or $|T >$ which can also be regarded as anti-parallel vectors to $|g >$. Since, head and tail are mutually exclusive with respect to observation, one has $< T|H >= 0$. The vector space above the ground can be taken as a direct sum of $|H >$ and $|T >$. Let α and β be the phase-angles made by $|n >$ with $|H >$ and $|T >$, respectively, such that $|\alpha| + |\beta| = \pi$.

In any CVS of any dimensionality, one can always write $< a|b >= | < a|b > |.e^{i\theta}$ between any pair of vectors $|a >$ and $|b >$; where, $| < a|b > |$ is the absolute value of the complex number, $< a|b >$, and θ is the phase-angle between them:

$$< H|n >= | < H|n > |.e^{i\alpha} \ ; < T|n >= | < T|n > |.e^{i\beta} \ ; \ |\alpha| + |\beta| = \pi \ . \tag{3}$$

Let $\hat{C}$ be an observable of the coin:

$$\hat{C} = |H >< H| - |T >< T| \ ; \ \hat{C}|H >= |H > \ ; \ \hat{C}|T >= -|T >, \tag{4}$$

where, $< H|H >=< T|T >= 1$. Using the unit operator, $\hat{I} = |H >< H| + |T >< T|$ in the CVS above the ground-surface, $|n >$ can be expressed as,

$$\begin{aligned} |n > &= |H >< H|n > +|T >< T|n > \\ &= |H > .| < H|n > |.e^{i\alpha} + |T > .| < T|n > |.e^{i\beta}. \end{aligned} \tag{5}$$

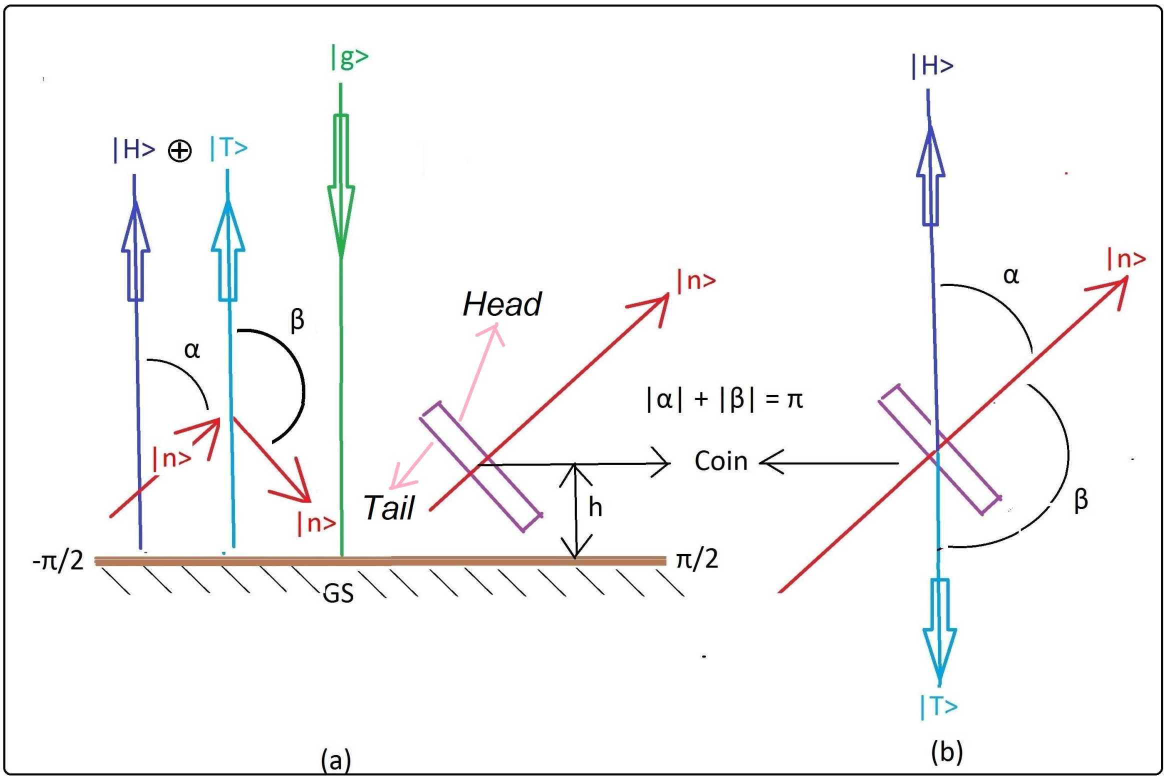

FIG. 1. **Schematic Diagram for the Toss of a Coin**: (a) h is the height of coin above the ground surface (GS) and is supposed to be less than the radius of coin. $|g>$ is a vector parallel to the gravitational field direction and perpendicular to the GS. $|n>$ is a vector normal to the head-surface passing through the center-of-mass of the coin. The outcomes, head and tail, are represented by the state vectors $|H>$ and $|T>$, respectively, which are taken to be anti-parallel to $|g>$. They are mutually exclusive with respect to the observation, i.e., $< T|H >= 0$ (in the space above the GS). (b) α and β are the phase-angles between $|H>$ &$|n>$ and $|T>$ &$|n>$, respectively; $|\alpha|+|\beta| = \pi$. If $|\alpha| < |\beta|$ ($|\beta| < |\alpha|$), then the coin enters into $|H>$ ($|T>$) - criterion of minimum phase.

As it can be easily seen from Fig. (1), if $|\alpha| < |\beta|$, then the coin enters into $|H>$ and if $|\alpha| > |\beta|$, then into $|T>$. Notice that, either α or β will be the minimum at a time, because, $|\alpha|+|\beta| = \pi$ (the case of $|\alpha| = |\beta|$ is ruled out because, $h < r$). Therefore, the coin will always be found in an eigenstate with minimum phase-angle. As an explicit example, consider $|\alpha| < |\beta|$; then, upon observation,

$$< n|n > \longrightarrow | < H|n > |^2 \; ; \text{(observation of the eigenvalue } +1) \, . \tag{6}$$

D. Answer to the 1st Question: Part-II
(Wave function and its relation to the observed particle)

In the case of an observable with continuous eigenvalues, there will always be an eigenstate whose phase with respect to $|\psi>$ will be the same as the initial phase of $|\psi>$ itself. Considering the position operator whose continuous eigenvalues span the 3DES:

$$|\psi>=\iiint d^3\mathbf{r}|\mathbf{r}><\mathbf{r}|\psi> . \tag{7}$$

The particle naturally enters into a position eigenstate, say $|\mathbf{r}_p><\mathbf{r}_p|\psi>$, such that phase$\{<\mathbf{r}_p|\psi>\}$ = phase$\{|\psi>\}$; here, the subscript p stands for 'particle'. Therefore, the interaction of $|\psi>$ with its induced dual in the detector screen is,

$$<\psi|\psi>=\iiint d^3\mathbf{r}<\psi|\mathbf{r}><\mathbf{r}|\psi>\longrightarrow|<\mathbf{r}_p|\psi>|^2, \tag{8}$$

because, except $|\mathbf{r}_p><\mathbf{r}_p|\psi>$, the remaining orthogonal states, $|\mathbf{r}><\mathbf{r}|\psi>$, are empty. The RFD in the limit of infinite number particles is,

$$<\psi|\psi>=\iiint d^3\mathbf{r}_p|<\mathbf{r}_p|\psi>|^2=1, \tag{9}$$

which is the Born rule. Notice the difference between the physical natures of Eqs. (1) and (9).

III. WHAT DECIDES WHETHER A PHOTON IS TO GO THROUGH OR NOT?

Prof. Feynmann said that the Young's double-slit experiment contains the central mystery of quantum mechanics [43]. Similarly, Prof. Dirac's statement [44], "*Questions about what decides whether the photon is to go through or not and how it changes its direction of polarization when it does go through can not be investigated by experiment and should be regarded as outside the domain of science*", contains the actual key to unlock the mystery of the origin of Born's rule. It's the actual inspiration behind the proposal of a relation between the initial/overall/global phase associated with the state vector and a particular eigenstate of an observable as given in the subsection-II.C.

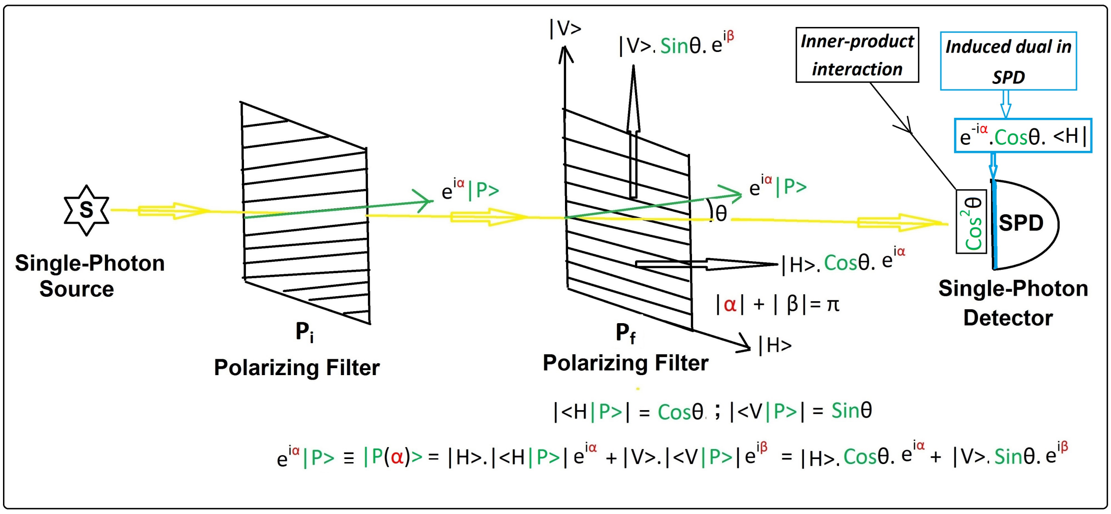

FIG. 2. **Schematic Diagram for a photon passing through the polarizing filters**: A single-photon source, S, emits individual photons one-at-a-time in such a way that each and every photon is emitted only after the registration of the previous one by a single-photon detector, SPD. The direction of polarization of any photon from S will have a random orientation. The polarizing filters, P_i and P_f prepare the initial and final polarization states, $|P(\alpha) >$ and $|H > .| < H|P > |.e^{i\alpha}$, respectively, for the photon; here, α is the phase-angle between $|P(\alpha) >$ and $|H >$ and similarly β between $|P(\alpha) >$ and $|V >$. α and β will be different for different photons and they occur randomly depending on the nature of the source, S, but they are always related to each other by the constraint equation $|\alpha| + |\beta| = \pi$. θ is the classical Cartesian angle between $|P(\alpha) >$ and $|H >$.

A. Mapping between the Tossed Coin in CVS and a Spin-1 Particle

A single-photon source, S, emits individual photons one-at-a-time in such a way that each and every photon is emitted only after the registration of the previous one by a single-photon detector, SPD as shown in Fig. (2). The direction of polarization of any photon from S will have an unknown, hence, a random orientation.

The toss of the coin in CVS as described in the subsection-II.C can be mapped into the polarization states of a photon as given below:

$$|\text{Head} >= |\text{H} > \longrightarrow |\text{H} >= |\text{Horizontal} > ; |\text{T} > \longrightarrow |\text{V} >, \tag{10}$$

$$\hat{C} \longrightarrow \hat{S} = |H >< H| - |V >< V|, \tag{11}$$

$$\hat{I} \longrightarrow \hat{I}_S = |H >< H| + |V >< V|, \tag{12}$$

where, $|H>$ and $|V>$ are the horizontal and vertical polarization states of the photon, respectively. $\hat{S}$ is the photon's spin operator and $\hat{I}_S$ is the unit vector in the CVS spanned by $|H>$ and $|V>$:

$$\begin{aligned} |n> \longrightarrow |P> &= |H><H|P> + |V><V|P> \\ &= |H>.|<H|P>|.e^{i\alpha} + |V>.|<V|P>|.e^{i\beta} \\ &= |H>.(\cos\theta).e^{i\alpha} + |V>.(\sin\theta).e^{i\beta} \\ &\equiv |P(\alpha)>, \end{aligned} \tag{13}$$

where, $|<H|P>| = \cos\theta$ and $|<V|P>| = \sin\theta$; here, θ is the classical Cartesian angle between $|P(\alpha)>$ and $|H>$. α and β are the phase-angles between $|H>$ & $|P>$ and $|V>$ & $|P>$, respectively, and they will be different for different photons, occurring randomly depending on the detailed nature of S. But they are always related to each other by the constraint equation $|\alpha| + |\beta| = \pi$. The CVS of P_i allows only the $|P(\alpha)>$ component of the polarization state, which is normalized to unity, to pass through and the rest are all absorbed.

Similar to P_i, P_f can be associated with a projector, $\hat{I}_{P_f} = |H><H|$, which, upon acting on $|P(\alpha)>$, allows only $|H>.(\cos\theta).e^{i\alpha}$ to pass through, i.e.,

$$\hat{I}_{P_f}|P(\alpha)>=|V><V|P(\alpha)>=|H>.(\cos\theta).e^{i\alpha}. \tag{14}$$

The inner-product interaction at SPD is given by,

$$e^{-i\alpha}.(\cos\theta).<H|H>.(\cos\theta).e^{i\alpha} = \cos^2\theta, \tag{15}$$

yielding the RFD of photons as $\cos^2\theta$. If $|\alpha| < |\beta|$, then the photon will be present in the component $|H>.(\cos\theta).e^{i\alpha}$ and will be detected by SPD. If $|\alpha| > |\beta|$, then the photon enters into $|V>.(\sin\theta).e^{i\beta}$ and gets absorbed by P_f, while, the ontological state $|H>.(\cos\theta).e^{i\alpha}$ remains empty until the absorption of the photon, making no contribution to SPD.

B. Phase-Hole Representation, Phase-Tube Geometry and the Born Rule

The set of all polarization states of photons, say P_H,

$$P_H = \{|P(\alpha)> |\alpha \in [0\,,\,2\pi]\}\,, \tag{16}$$

passing through the polarizing filter P_i (see Fig. (2)) can be plotted on a complex plane with a common origin as shown in Fig. (3). Here, $|P(\alpha)>$ is the polarization state of a

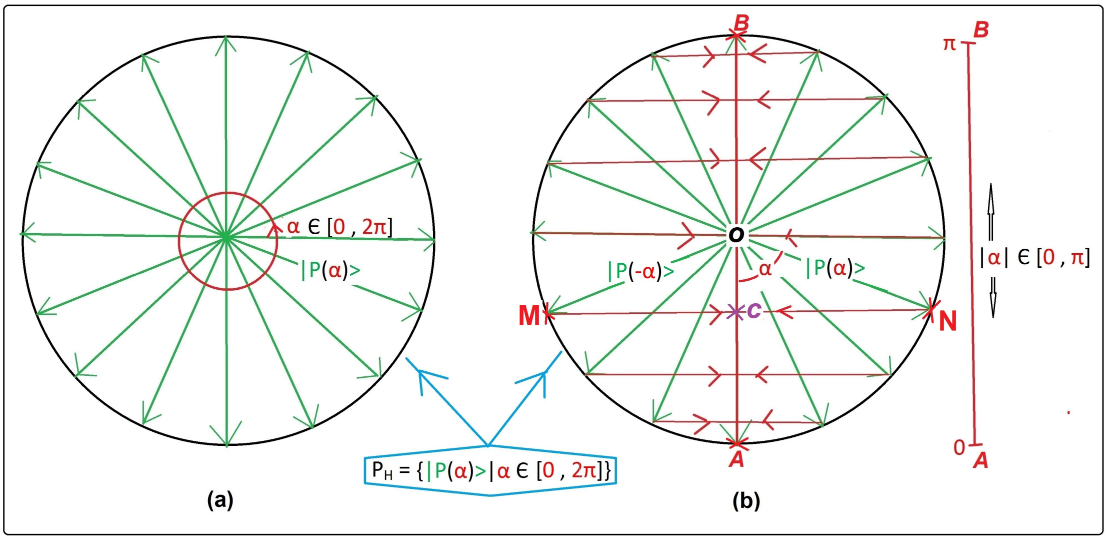

FIG. 3. **Phase-hole representation for all the initial states prepared in any given experiment**: **(a)** The set of all polarization states of photons, say P_H, passing through the polarizing filter P_i (see Fig. (2)) can be plotted on a complex plane with a common origin; here, $P_H = \{|P(\alpha) > | \alpha \in [0\,,\,2\pi]\}$ and $|P(\alpha)>$ is the polarization state of a particular photon with a global phase α. Notice that even though all the photons passing through P_i are identically prepared in the same state of polarization, each one of them is distinguishable with respect to α. The tips of all the vectors lie on a circle of unit radius - which is named as 'Phase-Hole' denoted by P_H. Therefore, from the photon's perspective, our perspective of a single polarization direction in P_i, i.e., $|P(\alpha)>$, actually appears as a hole, because, a photon with any α will always pass through P_H. **(b)** AOB can be any chosen diameter in P_H. Let's suppose that $|P(\alpha)>$ for a given value of α makes an angle α with respect to the radius AO, i.e., the angle AON is α. The point N on the circle is the tip of the vector $|P(\alpha)>=|ON>$, which is projected onto AOB at the point C such that the line CN is $\perp$ to AON. In the same manner, M can also be projected onto the same C. Now, the point C can be labeled by $|\alpha|$. Thus, a one-to-one correspondence between the two sets, $[A\,,\,B]$ and $[0\,,\,\pi]$, can be achieved and this will be helpful when $[0\,,\,\pi]$ splits into smaller intervals, because, $|P(\alpha)>$ can be expressed as a superposition - see Eq. (13) and Fig. (4).

particular photon with a global phase α and normalized to unity. Notice that, even though all the photons passing through P_i are identically prepared in the same state of polarization, each one of them is distinguishable with respect to α. The tips of all the vectors lie on a circle of unit radius - which is named as 'Phase-Hole', denoted by P_H. Therefore, from the photon's perspective, our perspective of single polarization direction in P_i, i.e., $|P(\alpha)>$, actually appears as a hole, because, a photon with any α will always pass through P_H. The P_H sweeps a phase-tube, say P_T, in the direction of photon's motion. P_T branches into an down-phase-tube, say P_{DT}, and a up-phase-tube, say P_{UT}, because, $|P(\alpha)>$ can be rewritten as a superposition as given in Eq. (13), i.e.,

$$|P(\alpha)>= |H> .(\cos\theta).e^{i\alpha} + |V> .(\sin\theta).e^{i\beta} \,. \tag{17}$$

Actually, akin to $|P(\alpha)>$, its $|H>$ and $|V>$ components form an down-phase-hole, P_{DH}, and a up-phase-hole, P_{UH}, which sweep P_{DT} and P_{UT} in the direction of photon's motion, respectively. Notice that, the radius of P_{DH} is $\cos\theta$ and that of P_{UH} is $\sin\theta$.

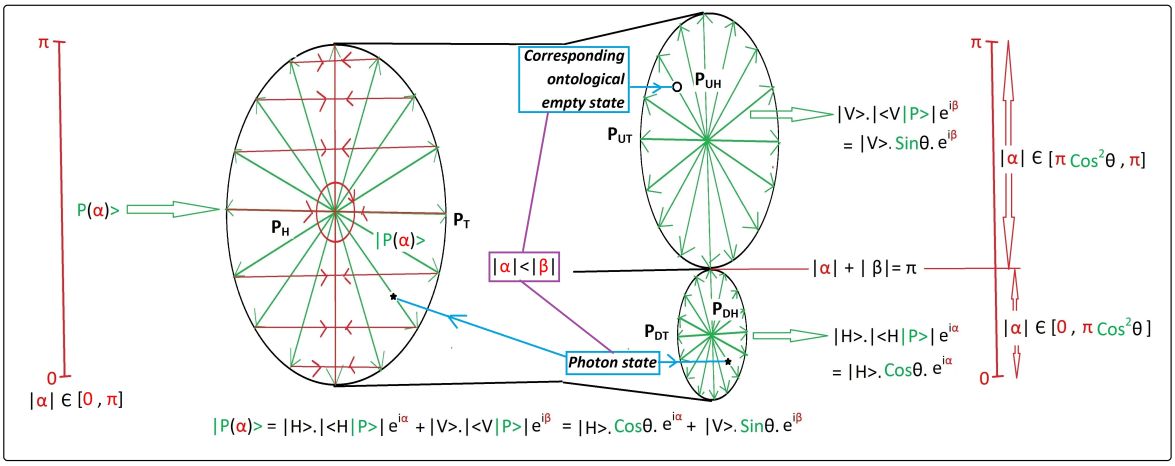

FIG. 4. **Schematic Phase-Tube Diagram for the Photon's Polarization State**: P_H sweeps a 'Phase-Tube', say P_T, in the direction of photon's motion. P_T branches into 'up-phase-tube', P_{UT}, and 'down-phase-tube', P_{DT}, because, any vector from P_{UH} is orthogonal to any vector in P_{DH}; here, P_{UH} and P_{DH} are up-phase-hole and down-phase-hole, respectively. For convenience, the state vectors are drawn symmetrically, which need not be true in reality. The global phase α will, in general, occur randomly due to the nature of single-photon source. See main text for the details of equations and further explanation.

When an extremely large number of photons, say N, enters P_T, then some of them, say N_D, moves through P_{DT} and the remaining, say N_U, through P_{UT}. Conservation of total number of photons implies $N = N_D + N_U$ and the geometry of phase-tube implies

$N_D = (A_D/A)N$ and $N_U = (A_U/A)N$ (for an incompressible photon fluid); here, A, A_D and A_U are the areas of cross-section of P_T, P_{DT} and P_{UT}, respectively, yielding,

$$\frac{N_D}{N} + \frac{N_U}{N} = \frac{A_D}{A} + \frac{A_U}{A} = 1 = R_D + R_U, \tag{18}$$

where, $R_i = N_i/N = A_i/A$, corresponds to the RFD or Born's probability; here, $i = D, U$. Therefore, it's clear that, the conservation of total number of photons implies the conservation of the total of area of cross-sections of the phase-tubes, which yields the Born rule in Eq. (18). Hence, one has,

$$A = A_D + A_U \implies \pi = \pi \cos^2\theta + \pi \sin^2\theta. \tag{19}$$

The above equation implies the splitting of the interval, $[0, \pi]$, as,

$$[0, \pi] = [0\,,\, \pi\cos^2\theta] \cup [\pi\cos^2\theta\,,\, \pi], \tag{20}$$

and the physical phenomenon in the interval, $[\pi, 2\pi]$, is exactly identical to the one in $[0, \pi]$ (see Fig. 3(b)). Therefore, depending on whether $|\alpha| \in [0\,,\, \pi\cos^2\theta]$ or $|\alpha| \in [\pi\cos^2\theta, \pi]$, the photon enters into either P_{DT} or P_{UT}, i.e., it will be found in either $|H>$ or $|V>$, respectively. Therefore, according to WPND, it's possible to theoretically predict that all the photons with $|\alpha| \in [0\,,\, \pi\cos^2\theta]$ will definitely pass through P_f and will be detected by SPD with the RFD equal to $\cos^2\theta$. Since, the information about α is unavailable in any experiment due to the inner-product interaction, Prof. Dirac's saying, "*what decides whether the photon is to go through or not and how it changes its direction of polarization when it does go through can not be investigated by experiment*", is indeed true with respect to WPND and hence, only the RFD becomes observable in any quantum mechanical experiment. Notice that, α is like a kind of hidden-variable already available in the quantum formalism.

IV. CONCLUSIONS

Inspired by Prof. Dirac's statement, a thorough investigation is carried out on the inner-product between a state vector and its dual and finally arrived at the true nature of the origin of Born's rule in quantum mechanics.

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