Schrodinger Equation in Cosmological Inertial Frame

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ABSTRACT

Schrodinger equation is a wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Schrodinger from Klein-Gordon free particle's wave function in cosmological special theory of relativity.

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1. Introduction

At first, Klein-Gordon equation is for free particle field ϕ in cosmological inertial frame.[1]

$$\frac{m^2c^2}{\hbar^2}\phi + \Omega(t_0)\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\phi = 0$$

m is free particle's mass , $\Omega(t_0)$ is the ratio of universe's expansion in cosmological time t_0 (1)

If we write wave function as solution of Klein-Gordon equation for free particle,[1]

$$\phi = A_0 \exp[i(\frac{\omega t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

A_0 is amplitude, ω is angular frequency, $k = |\vec{k}|$ is wave number (2)

Energy and momentum is in inertial frame,[1]

$$E = \hbar\omega, \vec{p} = \hbar\vec{k} / \Omega(t_0) \tag{3}$$

Hence, energy-momentum relation is[1]

$$E^2 = \hbar^2\omega^2 = \Omega^2(t_0)p^2c^2 + m^2c^4 = \hbar^2k^2c^2 + m^2c^4 \tag{4}$$

Or angular frequency- wave number relation is

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2c^2}{\hbar^2} \tag{5}$$

Hence, wave function is

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\hbar\frac{\omega t}{\sqrt{\Omega(t_0)}} - \hbar\vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

$$= A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)})] \tag{6}$$

2. Schrodinger Equation from Klein-Gordon Free Particle Field in Cosmological Inertial Frame

Because, Schrodinger equation is made from Klein-Gordon free particle's wave function in cosmological special theory of relativity,

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)})] \tag{7}$$

If we calculate the derivation of Schrodinger equation,

$$\sum_i (\frac{\partial}{\partial x^i})^2 \phi = -\sum_i \frac{(p^i)^2}{\hbar^2} \Omega^3(t_0)\phi = -\frac{p^2}{\hbar^2}\Omega^3(t_0)\phi \tag{8}$$

$$\frac{\partial \phi}{\partial t} = -\frac{i}{\hbar} E \frac{\phi}{\sqrt{\Omega(t_0)}} \tag{9}$$

Energy E is

$$E = \frac{p^2}{2m}\Omega^2(t_0) + V , \quad V \text{ is the potential energy} \tag{10}$$

Hence,

$$E\phi = \frac{p^2}{2m}\Omega^2(t_0)\phi + V\phi \quad , \quad V \text{ is the potential energy} \tag{11}$$

Therefore, by Eq(8),Eq(9)

$$E\phi = i\hbar \frac{\partial \phi}{\partial t}\sqrt{\Omega(t_0)} , \ \Omega^2(t_0) p^2 \phi = -\hbar^2 \nabla^2 \phi \frac{1}{\Omega(t_0)} \tag{12}$$

Therefore, Schrodinger equation in cosmological inertial frame,

$$E\phi = i\hbar \frac{\partial \phi}{\partial t}\sqrt{\Omega(t_0)} = -\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\phi + V\phi \tag{13}$$

If the energy E is not concerned by time t,

$$\frac{\partial E}{\partial t} = 0 \tag{14}$$

$$\phi = A_0 \exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)})]$$

$$= \varphi \exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}})] \tag{15}$$

Hence, stationary state of Schrodinger equation is in cosmological inertial frame,

$$E\varphi \exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}]$$

$$= [-\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi + V\varphi]\exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}] \tag{16}$$

Hence, stationary state of Schrodinger equation is

$$E\varphi = -\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi + V\varphi \tag{17}$$

Or,

$$\frac{1}{\Omega(t_0)}\nabla^2\varphi + \frac{2m}{\hbar^2}(E - V) = 0 \tag{18}$$

3. Conclusion

We found Schrodinger equation from Klein-Gordon's free particle equation in cosmological special theory of relativity. The wave function uses as a probability amplitude.

References

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