A Continuous Gravitational Wave at 404.3 μHz

preprint

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Abstract

Continuous gravitational waves are identified by their signatures: The amplitude is constant and the frequency increases slowly. In addition, the Doppler effect as a consequence of the Earth's orbit causes a characteristic phase modulation at 31.69 nHz. The long-term records from eleven superconducting gravimeters in Europe contain a set of spectral lines at 404.3 μHz that meet all three criteria and are likely generated by a gravitational wave. The determined values of frequency deviation and the time of maximum frequency deviation allow the calculation of the ecliptic coordinates of the source of the CGW.

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1 Introduction

Following the assumptions of general relativity, some objects in distant galaxies emit gravitational waves (GW), which should be measurable here on Earth. The motion of the generating masses determines the frequency, which should be found in the range 10^{-9} Hz $< f_{GW} < 10^3$ Hz. So far, one searched in vain with different methods for continuous gravitational wave signals (CGW), which would open a new kind of observation window for astronomy. One reason is certainly that CGW are extremely weak signals with no frequency or amplitude modulation. As the receivers move with the Earth, the Doppler effect produces signatures that prove the extraterrestrial origin of the signals.

The poor signal-to-noise ratio of CGW can be improved by long-term integration of the signals. This requires receiving antennas that work reliably and without interference for many years. Superconducting gravimeters (SG) are particularly sensitive and long-term stable receiving antennas for gravitational signals from the largest possible test mass - the Earth - in the frequency range 1 μHz to 100 mHz. Some of them have already been recording data for decades, which are stored at the GFZ in Potsdam [1].

The SGs are (so far) primarily used for earthquake investigations. Therefore they are acoustically unfavorably mounted directly on the ground and react sensitively to vibrations. For investigations in the field of CGW, this type of mounting is in need of improvement. All measurements described below were performed in the frequency range around 400 μHz, where the noise of the SG is particularly low. The known natural resonances of the globe are usually much higher and do not influence the following investigations.

2 Signatures of CGW

The spectrum of SG records is characterized by different contributions:

1. Neighboring celestial bodies such as the Moon or Sun periodically deform the Earth. As a result, one measures strong spectral lines in the frequency range below 50 μHz [2].

2. Earthquakes excite natural resonances of the nearly spherical earth, which decay with half-lives of a few days. The initial amplitudes differ very much and in no case permanent signals are generated. The known frequencies are above 300 μHz (suspected oscillations of the solid core of the Earth could produce three spectral lines at about 70 μHz. The "Slichter triplet" has not been detected so far).

3. Noise generated by ocean surge, air movement, small displacements due to plate tectonics and turbulence in the liquid interior of the earth. These are not continuous signals with defined frequencies.

4. Inherent noise of the SG; the known resonances of the SG near 10^{-3} Hz do not interfere.

5. Strong earthquakes overdrive the SG, creating data gaps and nonlinearities. These short-term disturbances can be identified and cleared manually.

If the SGs react to CGW, it must be clarified which characteristic properties of these waves we expect here on earth:

CGW, generated for example by the motion of double stars, are neither frequency nor amplitude modulated. They radiate an unmodulated continuous signal during recording periods of several years. The energy radiation by CGW generates a slow increase of the frequency. One expects rates of change around 10^{-19} Hz/s.

Sensors near the earth move in the wave field of the CGW. The Doppler effect ensures that the received signal phase shifts periodically with two different frequencies:

The Earth rotates on its axis in 23.9345 hours. The modulation produces sidebands 11.606 μHz apart on either side of f_{GW}. Because of the low circumferential velocity ($v < v_{equator} = 463\ m/s$), the sidebands are very weak and disappear in the noise. The fast phase modulation would disappear in measurements near the north or south pole.

The earth orbits the sun in 365.256 days. Therefore, the sensors move at a maximum of ± 30 km/s through the CGW wave field. This value - about 1/10000 of the presumed propagation speed of CGW - produces a slow phase modulation of the recorded signals, resulting in spectral lines spaced $f_{MOD} = 31.69$ nHz on either side of f_{GW}. This signature allows identification of CGW because no Earth-based mechanism is known to produce a phase modulated oscillation with this frequency. The signature appears only when analyzing several years of data records and is particularly useful for detecting CGW whose sources are close to the ecliptic plane.

Briefly: As the binary system loses energy, the stars gradually draw closer to each other, and the orbital period decreases. The only characteristics of a CGW are

- a constant amplitude
- an almost constant frequency, increasing very slightly
- a phase modulation with 31.688 nHz.

3 Testing the amplitude constancy of the CGW

A spectrum does not contain any information whether and how the amplitude of a signal changes over time. A phase sensitive integrator can answer the question because it acts like an extremely narrow band filter of selectable frequency f. One possible method is based on the addition theorem of trigonometric functions and still works when the frequency changes slightly. The SG record is digitized and consists of a sequence of discrete values. Successive data points z_n and z_{n+1} are measured at time intervals T_s (sampling time). If one analyzes the oscillations of a fixed frequency f , the phase angle increases with each step by the value $\alpha = 2\pi f T_s$. Although each data point z_n results from the influences of many frequency components, a selective integration method can ignore interfering components and noise. The basis are the two formulas

$$x_{n+1} = cos(\alpha)x_n + sin(\alpha)y_n + z_n \tag{1}$$

$$y_{n+1} = cos(\alpha)y_n - sin(\alpha)x_n \tag{2}$$

The sequence of values x_n and y_n depends on the choice of parameters:

1) Without an injected signal and with the initial values $x_1 = 1$, $y_1 = 0$, the formulas calculate a table of values for $x = sin(2\pi tf)$ and $y = cos(2\pi tf)$ with constant amplitude.

2) Setting $x_1 = y_1 = 0$ and injecting a monochromatic signal z_n whose frequency matches f, the formulas calculate an oscillation whose amplitude increases in proportion to time.

3) If the programmed and the injected frequency differ or if the injected signal z_n varies in phase or amplitude, the output signal of the integrator changes and the linear increase is lost.

4) If noise is injected, the formulas calculate a low-bandwidth frequency mixture in the vicinity of f , whose amplitude fluctuates irregularly.

Choosing $f = 404.3$ μHz and injecting the signal mixture recorded by the SG, we obtain the result shown in Figure 1. The small deviations from the linear increase of the amplitude confirm that the injected time series contains a signal of frequency $f_{GW} = f$, whose amplitude and frequency are nearly constant during the entire period. Neither the tides with their much lower frequencies ($f < 100$ μHz), nor the known natural resonances of the globe, nor the numerous short-time disturbances due to earthquakes influence the result of the integrator.

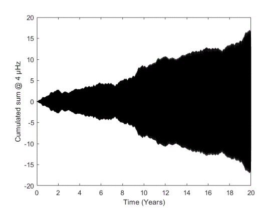

Figure 1: A synchronized oscillator integrates the CGW amplitudes

The frequency of the adjacent natural resonance ${}_2S_1$ (404.727 μHz according to Model PREM or 404.690 μHz according to Model 1066A) differs from f_{GW}. Natural resonances are always damped oscillations excited by earthquakes and decay after a few days [3]. In no case one observes a continuous signal.

All arguments speak for an extraterrestrial origin of the oscillation of frequency $f_{GW} = 404.3$ μHz. The systematic search in the range between 45 μHz and 420 μHz yielded further candidates with comparable linear increase of the integrated amplitude, whose investigation is not yet completed.

4 The signature of a phase modulated signal

Phase modulation is a standard method of communications engineering with known regularities. The spectrum of a signal phase modulated with a single sinusoidal frequency consists (theoretically) of an infinite number of spectral lines mutually spaced by the modulation frequency f_{MOD}. For practical applications, it is sufficient to evaluate only the strongest sidebands on either side of the carrier frequency. The modulation index β determines the amplitudes of all spectral lines. While in radio technology one can choose the value of β, in the case of CGW one has to determine the value β from the amplitudes of the measured spectral lines. A phase modulation always causes a frequency modulation as well, both methods can be converted into each other. The definition of the modulation index β is:

$$\beta = \frac{largest\ deviation\ from\ the\ average\ frequency}{(highest)\ modulation\ frequency}$$

Figure 2 shows that with a low modulation index $\beta < 1.5$, only a few sidebands in the immediate vicinity of the carrier frequency f_{GW} need to be considered. As the modulation index increases, so do the amplitudes of more and more sidebands, but the spectrum always remains symmetrical. The laws of phase modulation require that the carrier frequency is higher than the modulation frequency. In the present case, the following must apply: $f_{GW} > f_{MOD}$. However, a too large difference complicates the evaluation. Therefore, the following analysis uses the superhet principle to reduce the signal frequency f_{GW} without changing the modulation. The accuracy of the evaluation improves with increasing value of the frequency ratio f_{MOD}/f_{GW}.

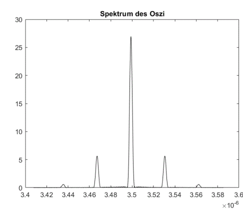

Figure 2: Spectrum of a phase modulated signal. $f_{MOD} = 32$ nHz

5 The spectral resolution

The very low modulation frequency of 32 nHz can only be detected with a spectrum of sufficient resolution. Küpfmüller's frequency-time uncertainty principle $df \times \Delta t > 1$ determines the minimum required observation period Δt: To achieve the desired frequency uncertainty of $df \leq 5$ nHz (compare Figure 2), a (minimum) six-year data record must be available. Although records spanning a ten-year period and with few data gaps were examined, no evidence of CGW could be found initially. The amplitudes of the CGW appear to be lower than the noise level of the SG. The following procedures improve the signal-to-noise ratio enough to detect some CGW:

1) Add the records of eleven SG installed in Europe: PE (Pecny), OS (Onsala), WE1+2 (Wettzell), VI (Vienna), ST (Strasbourg), MO1+2 (Moxa), BH (Bad Homburg) and BF1+2 (Black Forest). These gravimeters register approximately in-phase signals because the mutual distances are much smaller than the wavelength of the CGW, improving the signal-to-noise ratio.

2) The database Y = PE + OS + WE1 + WE2 + CO + ST + MO1 + MO2 + BH + BF1 + BF2 covers a time span of exactly ten years. Experiments have shown that doubling the file length by concatenation improves both signal-to-noise ratio and spectral resolving power. The resulting base Y+Y covers the

time span of 20 years. The frequency drift of basis Y must be determined before concatenation.

6 The search for gravitational waves

The CGW from sources near the ecliptic plane are always phase modulated and the spectrum must contain at least three lines at the characteristic spacing of the modulation frequency 31.69 nHz. The amplitudes of these lines are not decisive and, given the poor SNR, are not a good criterion for finding CGW. Additional signatures such as a slow frequency drift are not detectable at this resolution and only become apparent during the course of the analysis.

Not all of the 30 or so SGs operating worldwide produce low-noise data, and few SGs provide low-interruption files for a period of at least ten years. None of these spectra show prominent spectral lines that could be CGW.

Figure 3 shows an example of how to find candidates for more detailed analysis: One looks for combinations of three lines in the spectrum that satisfy the following conditions:

- The frequency spacing 31.69 nHz is precisely observed.
- The amplitudes of the two outer lines differ by a maximum of 20%.
- The amplitude of each of the three lines is almost constant during the entire measurement period of ten years (Check for a linear increase as in Figure 1).

If a triple fulfills all three conditions, a more detailed investigation is worthwhile. The systematic search in the frequency range 40 μHz to 420 μHz revealed many triples that could be CGW. One of them at $f_{GW} = 404.3$ μHz is analyzed below.

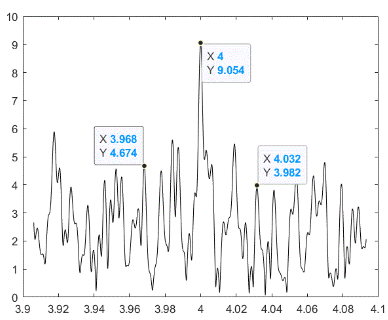

Figure 3: A combination of three spectral lines with correct distance $f_{Orbit} =$ 31.69 nHz. A phase-modulated signal includes spectral lines that are even further away, which usually disappear in the noise. The data analysis is limited to the marked lines. The unmarked lines in between have nothing to do with the phase modulated signal.

7 Data analysis of the triple at 404.3 μHz

The results of the preliminary investigation support the assumption that the three spectral lines at 404.3 μHz form a unit and are generated by a phase-modulated CGW. The objective is to determine missing information such as frequency deviation, frequency drift, and the direction of the source of the CGW.

The spectrum of a continuous signal of constant frequency phase modulated at a single frequency - the orbital frequency of the Earth - consists of an odd number of discrete lines mutually spaced at the modulation frequency 31.69 nHz. All intervening spectral components are noise and may be removed or ignored. As the construction of selective filters simplifies with decreasing frequency, the first step is to reduce the frequency of the CGW to 4 μHz (see Figure 4, intermediate frequency #1). The superhet method commonly used in radio engineering shifts the frequency without changing the modulation of the CGW signal. A frequency-shifting bandpass [5] is more suitable for this purpose than a simple mixer because no interfering mirror frequencies can occur. In the first version of the data analysis, we experimented with a matched filter with five passbands that allowed only the central carrier frequency f_{GW} and the first two sidebands $f_{GW} \pm f_{MOD}$ and $f_{GW} \pm 2 \times f_{MOD}$ to pass. After completing the evaluation, it was shown that a matched filter is unnecessary. Replacing it with a bandpass filter does not alter the results.

If the matched filter is used, the spectrum of the filtered signal shows the amplitudes of the sidebands and the amplitude of the carrier. If the modulation index β were known, the amplitudes A0, A1 and A2 of the spectral lines of a frequency modulated signal could be calculated with the following formulas (J_n are the Bessel functions of the first kind):

$A0 = J_0(\beta)$, $A1 = J_1(\beta)$ and (for $f_{GW} \pm 2 \times f_{MOD}$): $A2 = J_2(\beta)$

When searching for CGW, the reverse approach applies: one determines the amplitudes of carrier frequency and sidebands and uses the results to estimate the modulation index β of the CGW. The poor signal-to-noise ratio leads to an inaccurate result, which is improved by applying the superhet method once more: One changes the intermediate frequency #1 (about 4 μHz) again, but this time with a variable oscillator Osz #2 (about 3.5 μHz). The goal is to keep the resulting difference frequency constant after the second mixer (0.5 μHz). To achieve this, one iterates the FM-Osz #2 data so that the frequency drift and sidebands of the difference frequency #2 (after the lowpass) disappear.

The compensation procedure transfers the frequency drift and phase modulation characteristics of the CGW signal to the FM-Osz #2. Then, the direction to the source can be determined from the modulation index and phase. The amplitude of the intermediate frequency #2 increases during the iteration because the total energy of the signal, which was previously partially distributed among the sidebands, is now concentrated at the carrier frequency. This improves the signal-to-noise ratio. Figure 4 shows the signal flow of the data processing with the iteration to extract the required results.

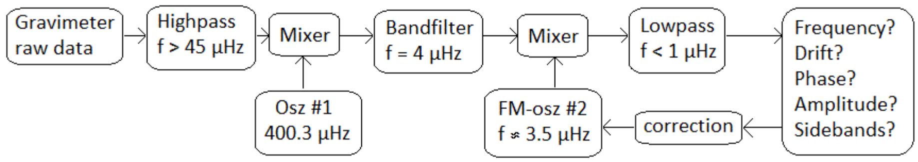

Figure 4: Signal flow in the receiver. While the frequency of Osz #1 is constant, all characteristics of FM-oscillator #2 are variable. These variables are iterated until the frequency of IF #2 behind the lowpass filter is constant and has no sidebands. It is easier to adjust the data of oscillator #2 than to demodulate IF #2 and analyze the noisy result.

Technical notes: In the frequency range 40 μHz to 420 μHz one can find quite a few triples which could be CGW according to the criteria defined above. A reliable exclusion criterion is whether the control loop reacts sensitively to phase changes of FM-Osz #2 (determining the direction to the source of the CGW). If a CGW generates the triple of three spectral lines, a forced phase change of, say, $\pm\pi/20$ must produce a strong correction signal. This property is a resilient indication of a CGW. If the control loop responds only weakly to a trial phase change, there is no reason to suspect a phase-modulated CGW.

8 Evaluation of the phase modulation

Figure 2 shows the spectrum of FM-Osz #2 ($f_{OSZ\#2} \approx 3.5$ μHz) after the end of the iterations. The very small frequency drift is not apparent at this resolution. Since the data processing consists of the succession of linear processes, the original GW of frequency $f_{GW} = 404.3$ μHz has an identical spectrum. However, this cannot be displayed with comparable precision because of the 115-fold higher frequency and the poor signal-to-noise ratio. From the amplitude ratio of the spectral lines and with the help of the Bessel functions of the first kind the modulation index $\beta = 0.41$ is calculated. This determines the frequency deviation $\Delta f = \pm 13$ nHz of the CGW signal around the mean f_{GW}. A direct measurement of the variable oscillation period of FM-Osz #2 confirms the result: One determines the time intervals between the zero crossings, calculates the frequency and represents the result as a function of the time.

Figure 5 shows the following preliminary information:

- The frequency of FM-Osz #2 oscillates sinusoidally around the mean value 3.5 μHz. The frequency deviation of $\Delta f = \pm 13$ nHz also applies to the original CGW of frequency $f_{GW} = 404.3$ μHz. The two-step reduction of the received frequency f_{GW} in mixers #1 and #2 to 0.5 μHz (see Figure 4) does not change Δf.
- Assuming a circular orbit of the Earth around the Sun, one receives the lowest frequency of CGW on about the 245st day of each year. During a

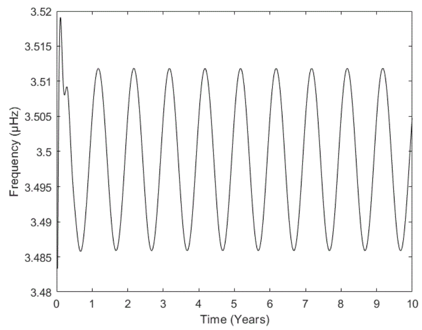

Figure 5: Periodic fluctuation of the frequency of FM-Osz # 2 around the average. The difference between the highest and lowest frequency is $2 \times \Delta f$. The small increase in the average frequency is hardly noticeable.

short observation period one can detect the almost constant instantaneous frequency $f_{GW} - \Delta f$ of the CGW in each year.

- The highest frequency is received on about the 65th day of each year. During a short period one measures the instantaneous frequency $f_{GW} + \Delta f$ of the CGW. These timings allow the determination of the ecliptic longitude of the source.
- The average frequency of the CGW increases by 35.6 pHz per year. The rate of change is $df/dt = 1.13 \times 10^{-18} s^{-2}$.

An error consideration is given below.

9 Calculation of the ecliptic latitude of the source of the CGW

Presumably the propagation speed of the GW is equal to the speed of light. The value of the frequency deviation of $\Delta f = \pm 13$ nHz is well below the limit value

$$\Delta f_{max} = f_{GW} \left(\sqrt{\frac{c + v_{Orbit}}{c - v_{Orbit}}} - 1 \right) = 40.43\ nHz \tag{3}$$

which is determined by the relativistic Doppler shift. The maximum value applies to signals whose source is in the plane of the ecliptic ($v_{Orbit} = 30$ km/s). As the ecliptic latitude of the CGW source increases, the frequency deviation decreases to zero. Using the formula of the relativistic Doppler effect

$$f_{GW} + \Delta f = \frac{f_{GW}}{\gamma(1 - cos(\varepsilon) v_{Orbit}/c)} \tag{4}$$

with

$$\frac{1}{\gamma} = \sqrt{1 - \left(\frac{v_{Orbit}}{c}\right)^2} \approx 1 \tag{5}$$

the ecliptic latitude ε of the CGW source can be calculated. Neglecting $\gamma \approx 1$, it follows from formula (4)

$$cos(\varepsilon) = \frac{c}{v_{Orbit}} \cdot \frac{\Delta f}{f_{GW} + \Delta f} \tag{6}$$

10 Supplementary measurements to determine the errors

All previous results are based on a single file. The base Y is the sum of the recordings of 11 European gravimeters: Y=PE + OS + WE1 + WE2 + CO + ST + MO1 + MO2 + BH + BF1 + BF2. The summation is necessary because the amplitude of the CGW is too small to be detected in the records of any individual SG. Nevertheless, in order to give error bounds, one varies the data base using the same method as the jackknife method of statistics. To do this, one repeats all analyses with subsets, in each of which the record of one SG is removed. This restriction to only ten SG at a time reduces the signal-to-noise ratio insignificantly. In the present study, the leave-one-out procedure led to the final exclusion of the records of one station (MC) because it contained too much interference. In the following table, the columns mean:

- SG used: Specification of the file that was removed from the sum Y in order to calculate the data in the table row.
- Frequency drift: Change rate of f_{GW} between the years 2009 and 2018.
- Frequency deviation Δf: Maximum deviation of f_{GW} from the average 404.3 μHz.
- Phase: Phase angle of the modulation frequency 31.69 nHz with respect to January 1 of each year.
- v(max): The day of each year when the instantaneous f_{GW} reaches its highest value (compare Figure 5).
- v(min): The day when the instantaneous frequency reaches its lowest value.
- $cos(\varepsilon)$: Cosine of the ecliptic latitude ε of the source of the CGW.

SG used	Drift (s^{-2}) $\times 10^{-18}$	Δf (nHz)	Phase	v(max) DoY	v(min) DoY	$\cos(\varepsilon)$
Y	1.13	12.99	5.2	64.9	245.4	0.3213
Y-PE	1.15	13.28	5.33	57.5	238	0.3284
Y-OS	0.894	15.65	5.6	39.7	222.4	0.3872
Y-WE2	1.17	17.49	4.81	87.2	268.5	0.4326
Y-WE1	1.08	21.36	4.81	86.99	268.2	0.5282
Y-CO	1.06	8.3	5.17	64.5	247	0.2053
Y-ST	1.18	9.82	5.56	42	224.5	0.2430
Y-MO1	1.33	12.52	5.54	43	225.7	0.3096
Y-MO2	1.32	9.82	5.97	17.9	200.7	0.2430
Y-BH	1.15	10.3	5.08	71.6	252.9	0.2547
Y-BF2	1.16	17.49	11.9	38.7	221.2	0.4326
Y-BF1	1.15	13.28	5.33	57.5	238.1	0.3284
Mean	1.148	13.53	5.858	55.96	237.72	0.3345
Error	±0.0328	±1.11	±0.558	±5.98	±5.81	±0.0276

The penultimate row of the table shows the mean values of each column; below, the error bars of each column were calculated with the Jackknife method.

11 Summary of the data analysis

A CGW of frequency $f_{GW} = 404.3$ μHz affects the gravitational data measured on Earth. The frequency of the CGW increases with $(1.148 \pm 0.033) \times 10^{-18} s^{-2}$. Because of the speed of the earth on its orbit, the measurable frequency varies by a maximum of $\Delta f = (13.53 \pm 1.11)$ nHz. Assuming an exactly circular orbit, the maximum value $f_{GW} + \Delta f$ is measured approximately on the 56th day of each year. The minimum value $f_{GW} - \Delta f$ follows about the 238th day of each year. From this, the ecliptic longitude of the source of the CGW can be determined. The ecliptic latitude of the source is $\pm 70.4° \pm 1.7°$.

The GFZ [1] stores and distributes the raw data of the superconducting gravimeters. The preparation consists of a few steps:

1. High-pass filtering ($f > 45$ μHz) of the original data to suppress the strong tidal signals generated by the Sun and the Moon [4].

2. Chaining short records to form coherent records that covers a period of at least ten years. To account for the influence of the variable air mass above the SG on the readings, add the value of the local air pressure at the SG location. Optimize the signal-to-noise ratio in the examined frequency range around 400 μHz by means of a suitable weighting.

3. Eliminate data gaps and jumps and delete obvious errors.

4. Reduce the sampling frequency to shorten file lengths and processing time.

MATLAB was used to analyze the processed data. The programs used for this purpose are available from the author.

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