Simulation of observable properties of a quantum object on a classical computer

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Abstract

Properties such as the expectation values of energy and momentum of a quantum object can be calculated exactly in quantum theory, but they cannot be simulated on a classical computer. This is in part due to the fact that the physical nature of quantum objects is not yet understood (ontology problem).
In this paper it is shown that it is possible to simulate observable properties of a quantum object on a classical computer. For this purpose, the wave describing the quantum object is considered as a physical element with a constant amplitude of a quarter of the Planck constant (Ψ_{max}=h/4=const.).
As a result, the expectation values of energy and momentum, as well as the de Broglie wavelength, can be simulated *without the aid of further parameters*.
This is expected to give new ideas to ontological issues.

Content

1 Introduction

Quantum theory is comprised of a well-proven mathematical set of rules, however, their physical interpretation is still controversial [1-5, 9]. Related to this is the unsolved problem of simulating observable properties of individual quantum objects on a classical computer [6].
In this paper, two simple assumptions about the nature of a quantum object are made and it is shown that properties such as energy, momentum and de Broglie wavelength of a single quantum object can be simulated on its basis.

2 Assumptions

In the major interpretations of quantum mechanics, such as the Copenhagen Interpretation and Quantum Bayesianism, the wave representing the quantum object is considered to be a mathematical entity for which there is no equivalent in physical reality [7, 8].
For the purposes of the simulations presented in this paper, the following assumptions are made:
- The function values of the wave have the unit of action (corresponds to the unit of angular momentum kg·m²/s).
- The amplitude of the wave is constant and has the value of a quarter of the Planck constant *h* (figure 1):

$$\left|\Psi_{max}\right| = \frac{h}{4} = const. \approx 1.65 \cdot 10^{-34} kg \cdot m \cdot m/s \qquad (1)$$

Ψ_{max}: Amplitude of the quantum wave

- The wave can be spread over a wide area of space. If the absolute function values of all maxima (wave crest or wave trough) are added over all its locations (figure 2) a constant value of *h/4* results.

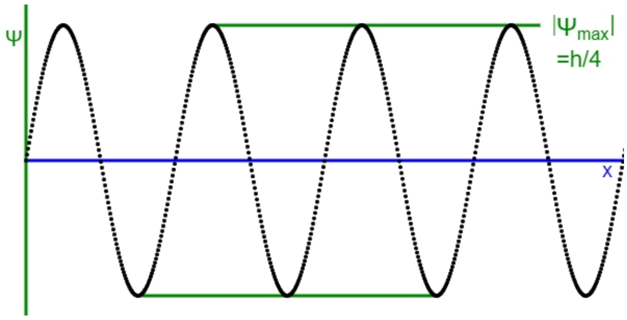

Figure 1: Schematic representation of a quantum wave with a constant amplitude of *h/4* in one spatial dimension.

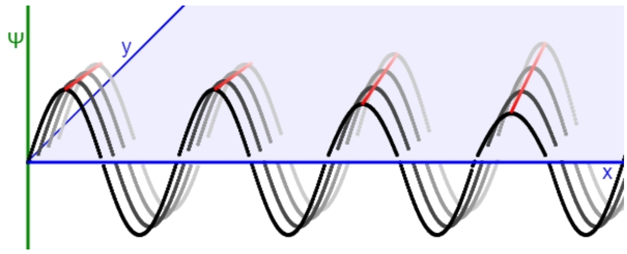

Figure 2: Schematic representation of a quantum wave in two spatial dimensions. The sum of the absolute values within the range of an amplitude (red lines) has a constant value of *h/4*.

An important question is whether it is possible to reproduce observable properties of a quantum object by simulating it on a classical computer based on these assumptions?

3 Simulations

3.1 Energy

In quantum mechanics, energy is calculated by the energy operator

$$\hat{E} = i\hbar \frac{\partial}{\partial t} \qquad (2)$$

ħ: reduced Planck constant ħ = h/2π
i: imaginary unit

which acts on the wave function *Ψ*. In other words, it examines the development of *Ψ* over time.
In the context of this simulation, the wave function *Ψ* describing the object is considered not as an abstract mathematical construct, but as a wave with a constant amplitude of *h/4* (equation 1). Based on equation 2, the change in the functional values of such a wave is simulated *over time*, in order to get the expectation value of the energy:

$$\langle E \rangle = \left| \overline{\frac{\partial}{\partial t} \Psi(t)} \right| \qquad (3)$$

Additional parameters are not required! The change in the wave over time provides the expectation value of the energy *directly*.
As an example, we use a photon as our quantum object. The speed of propagation of this object corresponds to the speed of light. In this example, the change over time of the wave representing the photon for different wavelengths will be simulated (table 1). Only absolute values will be considered.
For comparison, the values calculated according to the equation *E=h·f* are also given in table 1. Deviations from the simulated results are possible due to the limited number of simulation steps.
You can carry out the simulation for other wavelengths by yourself on the author's website:
https://www.quanten-krimi.de/pop/02/?ch0040?en

Conclusion: If a photon is described as a wave with a constant amplitude of *h/4*, the energy of the photon results *directly* from the mean time change of this wave.

3.2 Momentum

In quantum mechanics, the momentum is calculated using the momentum operator

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x} \qquad (4)$$

which acts on the wave function *Ψ*. In this case the development of *Ψ* over space is considered. We limit ourselves also to one spatial dimension *x*.
Based on equation 4, the *local* change in the functional values of a wave with constant amplitude of *h/4* can be used to get the expectation value of the momentum:

$$\langle p_x \rangle = \left| \overline{\frac{\partial}{\partial x} \Psi(x)} \right| \qquad (5)$$

The results of the simulations using a photon can also be found in table 1. In the last column, the values calculated using the equation *p=h/λ* are given for comparison. You can test it by yourself:
https://www.quanten-krimi.de/pop/02/?ch0070?en

Conclusion: If a photon is described as a wave with a constant amplitude of *h/4*, the momentum of the photon can be directly derived from the mean local change of this wave.

	λ [nm]	Energy [kg·m·m/s·s]		Momentum [kg·m/s]	
		simulated mean temporal change of the quantum wave	calculated E=h·f	simulated mean local change in the quantum wave	calculated p=h/λ
Red light	700	$2,838 \cdot 10^{-19}$	$2,838 \cdot 10^{-19}$	$9,466 \cdot 10^{-28}$	$9,466 \cdot 10^{-28}$
Blue light	450	$4,414 \cdot 10^{-19}$	$4,414 \cdot 10^{-19}$	$1,472 \cdot 10^{-27}$	$1,472 \cdot 10^{-27}$
UV	300	$6,622 \cdot 10^{-19}$	$6,621 \cdot 10^{-19}$	$2,209 \cdot 10^{-27}$	$2,209 \cdot 10^{-27}$

Table 1: Energy and momentum of photons of different wavelengths λ. The values simulated on the basis of a quantum wave of constant amplitude are highlighted in yellow. The calculated values are given for comparison.

3.3 De Broglie wavelength of an electron

In this section the quantum nature of an electron is considered. To do this, we build on the model of a photon used for the simulation as a quantum wave with a constant amplitude of *h/4*.

An electron, together with a positron, can be created from a photon by pair production. We investigate whether observable properties of an electron can also be simulated within the framework of the assumptions made in equation 1.

The simplified model used for the purpose of the simulation represents an electron as a superposition of a backwards and forwards propagating light wave. The wavelength of these "inner" waves (the two upper waves in figure 3) is $2.4 \cdot 10^{-12}$m in the case of an electron that is not moving (Compton wavelength of the electron). This corresponds to about twice the wavelength of the photon required for pair formation.

When this object moves relative to an observer, the optical Doppler effect occurs: The wavelength of the partial wave *in* the direction of movement decreases:

$$\lambda_F = \lambda_0 \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \tag{6}$$

λ_0: Wavelength of the inner wave when the object is at rest
λ_F: Wavelength of the inner wave in the direction of movement
v: Speed of the object
c: Speed of light

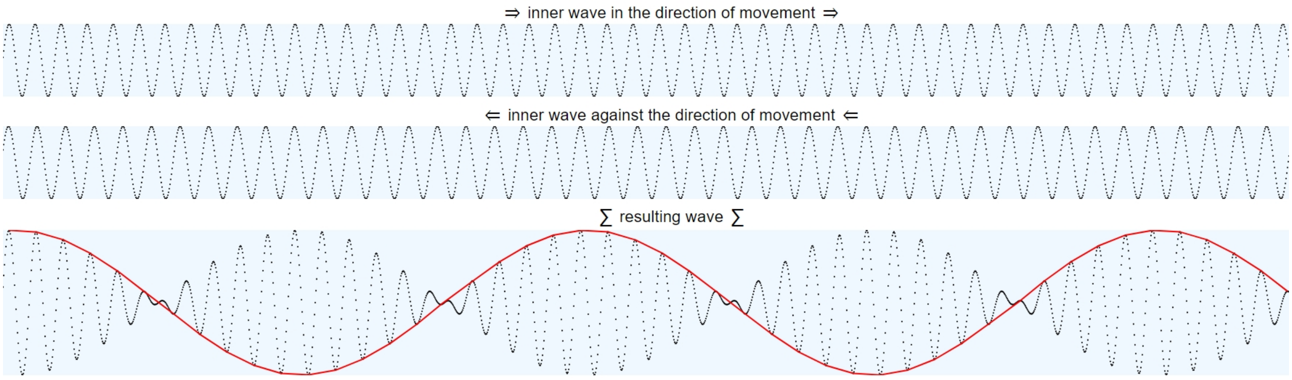

Figure 3: Superposition of two waves of different wavelengths and opposite directions of propagation. The enveloping wave (envelope) is shown in red.

speed of the electron [m/s]	Wavelength of the inner wave in the direction of movement [m]	Wavelength of the inner wave opposite to the direction of movement [m]	Wavelength of enveloping wave [m]	De Broglie wavelength of an electron according to λ=h/p [m]
10000	$2,426229 \cdot 10^{-12}$	$2,426391 \cdot 10^{-12}$	$7.274 \cdot 10^{-8}$	$7.274 \cdot 10^{-8}$
15000	$2,426189 \cdot 10^{-12}$	$2,426432 \cdot 10^{-12}$	$4.849 \cdot 10^{-8}$	$4.849 \cdot 10^{-8}$
80000	$2,425663 \cdot 10^{-12}$	$2,426958 \cdot 10^{-12}$	$9.092 \cdot 10^{-9}$	$9.092 \cdot 10^{-9}$

Table 2: De Broglie wavelength of an electron for different speeds. Values determined by superimposing two waves (yellow) and results calculated according to λ=h/p (last column).

The wavelength of the partial wave *against* the direction of movement increases:

$$\lambda_b=\lambda_0\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \qquad (7)$$

λ_b: Wavelength of the inner wave against the direction of movement

The frequencies f of the inner waves are obtained by means of the relationship $f=c/\lambda$ (c: speed of light).
As a result of the superposition of the partial waves, an envelope wave forms (figure 3 below). The frequency f_E of which results from:

$$f_E=\frac{f_F-f_B}{2} \qquad (8)$$

Table 2 shows the wavelengths of the envelope obtained by simulation for various electron velocities. For comparison the values calculated using $\lambda=h/p$ are shown.
You can also carry out this simulation by yourself:
https://www.quanten-krimi.de/pop/04/?ch0040?en

4 Motivation of some formulas

Now there are completely different options available to determine the energy and momentum of a quantum object:
- by calculation using the equations $E=h\cdot f$ and $p=h/\lambda$.
- by simulating the mean temporal or local change of a wave with a constant amplitude of $h/4$.

The question is: Is there any connection?

4.1 E=h·f

According to the simulation in chapter 3.1, the expectation value of the energy of a quantum wave results from the mean change of the wave over time. In the following, this will be formulated mathematically. We only consider absolute values.
The wave function for a harmonic wave that only depends on time t is:

$$\Psi(t)=\Psi_{max}\cdot\sin\left(2\pi\cdot\frac{t}{T}\right) \qquad (9)$$

Ψ_{max} = Maximum value (amplitude) of Ψ, T = period duration

The 1st derivative with respect to the time t gives:

$$\Psi(t)'=\Psi_{max}\cdot\frac{2\pi}{T}\cdot\cos\left(2\pi\cdot\frac{t}{T}\right) \qquad (10)$$

In order to determine the *expectation value* of the energy, we require the mean value $\overline{\Psi(t)'}$ of this function. The mean value of an aligned cosine function can be determined from its maximum value:

$$\overline{\Psi'}=\frac{2}{\pi}\cdot\Psi'_{max} \qquad (11)$$

For the sake of clarity, we just omit the absolute symbols.

We require the maximum value Ψ'_{max}. In equation 10, Ψ' is greatest when the cosine has its maximum possible value of 1:

$$\Psi(t)'_{max}=\Psi_{max}\cdot\frac{2\pi}{T}\cdot 1 \qquad (12)$$

$$\Psi(t)'_{max}=\Psi_{max}\cdot\frac{2\pi}{T} \qquad (13)$$

Inserting equation 13 into equation 11, we have:

$$\overline{\Psi(t)'}=\frac{2}{\pi}\cdot\Psi_{max}\cdot\frac{2\pi}{T} \qquad (14)$$

$$\overline{\Psi(t)'}=\Psi_{max}\cdot\frac{4}{T} \qquad (15)$$

The period T corresponds to the reciprocal of the frequency:

$$T=\frac{1}{f} \qquad (16)$$

With this we replace T in equation 15:

$$\overline{\Psi(t)'}=\Psi_{max}\cdot 4\cdot f \qquad (17)$$

According to the assumption in equation 1, the amplitude of a quantum wave corresponds to a quarter of the Planck constant h. By substituting this in place of Ψ_{max} in equation 17 we obtain:

$$\overline{\Psi(t)'}=\frac{h}{4}\cdot 4\cdot f \qquad (18)$$

$$\overline{\Psi(t)'}=h\cdot f \qquad (19)$$

The mean time dependent change of a quantum wave can therefore be calculated from the product $h\cdot f$. According to the simulation in chapter 3.1, this change over time corresponds to the energy E of a quantum wave:

$$\overline{\Psi(t)'}=h\cdot f=E \qquad (20)$$

Conclusion: The equation $E=h\cdot f$ results from the assumption that a quantum object is described as a wave with a constant amplitude of $h/4$. Therefore, the energy of the quantum object corresponds directly to the mean temporal change of the wave.

4.2 p=h/λ

According to the simulation in chapter 3.2, the expectation value of the momentum of a quantum wave results from the mean local change of the wave. In the following this will be formulated mathematically.
The wave function for a harmonic wave that only depends on one spatial dimension x is:

$$\Psi(x)=\Psi_{max}\cdot\sin\left(2\pi\cdot\frac{x}{\lambda}\right) \qquad (21)$$

Ψ_{max} = Maximum value (amplitude) of Ψ, λ = wavelength

The 1st derivative of equation 22 with respect to the location x results in:

$$\Psi(x)' = \Psi_{max} \cdot \frac{2\pi}{\lambda} \cdot \cos\left(2\pi \cdot \frac{x}{\lambda}\right) \qquad (22)$$

In order to determine the expectation value of the momentum, we require the mean value $\overline{\Psi(x)'}$ of this function (equation 11). For this, we have to determine the maximum value of the 1st derivative Ψ'_{max}. In equation 22, Ψ' is greatest when the cosine has its maximum possible value of *1*:

$$\Psi(x)'_{max} = \Psi_{max} \cdot \frac{2\pi}{\lambda} \cdot 1 \qquad (23)$$

$$\Psi(x)'_{max} = \Psi_{max} \cdot \frac{2\pi}{\lambda} \qquad (24)$$

We insert equation 24 into equation 11:

$$\overline{\Psi(x)'} = \frac{2}{\pi} \cdot \Psi_{max} \cdot \frac{2\pi}{\lambda} \qquad (25)$$

$$\overline{\Psi(x)'} = \Psi_{max} \cdot \frac{4}{\lambda} \qquad (26)$$

According to the assumption in equation 1, the amplitude of a quantum wave corresponds to a quarter of the Planck constant *h*. By substituting this in the place of Ψ_{max} in equation 26 we obtain:

$$\overline{\Psi(x)'} = \frac{h}{4} \cdot \frac{4}{\lambda} \qquad (27)$$

$$\overline{\Psi(x)'} = \frac{h}{\lambda} \qquad (28)$$

The mean local change of a quantum wave can therefore be calculated from the quotient h/λ. According to the simulation in chapter 3.2, this local change corresponds to the momentum *p* of a quantum wave:

$$\overline{\Psi(x)'} = p = \frac{h}{\lambda} \qquad (29)$$

Conclusion: The equation $p=h/\lambda$ results from the assumption that a quantum object is considered to be a wave with a constant amplitude of *h/4*. Therefore, the momentum of the quantum object can be directly obtained from the mean local change of the wave.

5 Summary

In order to simulate observable properties of a quantum object on a classical computer, an assumption was made that a quantum object can be described as a wave with a constant amplitude of *h/4* (*h* = Planck constant). On this basis the expectation value of the energy of the quantum object results directly from the mean temporal change of the wave, the expectation value of the momentum from its mean local change. If a model in the form of oppositely propagating light waves is used for quantum objects with a rest mass, the speed-dependent wavelength of the enveloping wave corresponds to the de Broglie wavelength observed.
The assumptions made here are expected to allow a deeper understanding of the nature of quantum objects.

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