The Gravitational Red-shift

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Abstract

It is shown that the derivation of the equation for the calculation of the Gravitational Red-Shift does not account the time during which the photon wavelength changes under static-weak gravitational field. If we solve this task in the frame of the idea of the possibility of using Newton's classical law of universal gravitation and the second law of Newton to explain the dynamic changes of the photon momentum even if its rest mass = 0, we get a new equation for the calculation of Gravitational Red-Shift that takes into account the time. This new mathematical model allows us to calculate gravitational red-shift of the electromagnetic radiation photons with no recourse to the theory of general relativity in the Newtonian limit, i.e. when r is sufficiently large compared to the Schwarzschild radius \( \sqrt{\frac{2GM}{c^2}} \). This raises grave doubts over the introduction of the measurement of the gravitational red-shift in the list of crucial tests of the theory of general relativity.
Sir Arthur Stanley Eddington was an English astronomer, physicist, and mathematician. He was also a philosopher of science and a populariser of science. The Eddington limit, the natural limit to the luminosity of stars, or the radiation generated by accretion onto a compact object, is named in his honour.

Einstein's theory of general relativity predicts that the wavelength of electromagnetic radiation photon will lengthen as it climbs out of a static gravitational well. Photons must expend energy to escape, but at the same time must always travel at the speed of light, so this energy must be lost through a change of frequency rather than a change in speed. If the energy of the photon $E = \frac{hc}{\lambda} = mc^2$ decreases, the frequency also decreases. This corresponds to an increase in the
wavelength of the electromagnetic radiation photon, or a shift to the red end of the electromagnetic spectrum – hence the name: **GRAVITATIONAL REDSHIFT**. This effect was confirmed in laboratory experiments conducted in the 1960s. For radiation photons emitted in a strong gravitational field, such as from the surface of a neutron star or close to the event horizon of a black hole, the gravitational redshift can be very large and is given by:

\[ 1 + z = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \]

In the Newtonian limit, i.e. when \( r \) is sufficiently large compared to the Schwarzschild radius \( \sqrt{\frac{2GM}{c^2}} \), the redshift can be approximated as:

\[ z = \frac{GM}{rc^2} \]

If a photon of mass \( m = \frac{h\nu}{c^2} \) is moving under the influence of a weak gravitational field generated by a massive central object of mass \( M \), Newton's law of gravitation shows that the force of gravitation experienced by the photon is given by \( \frac{GMm}{r^2} \), where \( G \) is Newton's universal constant of gravitation and \( r \) is the distance of the photon from the central massive object. The momentum of the photon \( (p = mc = \frac{h}{\lambda}) \) is poised to vary in the object's gravitational field while its velocity remains as a constant \( (c = 3 \times 10^8 \text{ m/s}) \). From this, there is still the possibility of using Newton's classical law of universal gravitation and the second law of Newton to explain the dynamic changes of the photon momentum even if its rest mass, \( m_0 = 0 \).

\[ -\frac{dp}{dt} = \frac{GMm}{r^2} \]
where \( m \) is the energy equivalent mass of the photon based on \( E = h\nu = mc^2 \), \( h \) represents the Planck constant. The \((-)\) sign for the change of momentum indicates the reduction of the photon momentum by the gravitational force as it moves away from the central point of a large mass.

Since: \( p = \frac{h}{\lambda} \). Therefore:

\[
\frac{p^2}{h} \frac{d\lambda}{dt} = \frac{GMm}{r^2}
\]

This equation can be rearranged to:

\[
d\ln\lambda = \frac{GM}{cr^2} \ dt
\]

On integration within the limits of \( \lambda_{\text{emit}} \) to \( \lambda_{\text{obs}} \) for wavelength of photon and 0 to \( t \) for time we get,

\[
\ln \left( \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} \right) = \frac{GM}{cr^2} \ t
\]

\[
\lambda_{\text{obs}} = \lambda_{\text{emit}} e^{\frac{GM}{cr^2} \ t}
\]

\[
\frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = e^{\frac{GM}{cr^2} \ t} - 1
\]

\[
1 + z = e^{\frac{GM}{cr^2} \ t}
\]

Thus, we have the formula for the calculation of the Gravitational Red-Shift, which takes into account the time during which the photon wavelength changes from \( \lambda_{\text{emit}} \) to \( \lambda_{\text{obs}} \) under
static-weak gravitational field. In the above derivation of the expression for the gravitational red-shift, no appeal has been made to any aspect of the theory of general relativity. Hence, the question will be raised as to how and why the measurement of the gravitational red-shift could ever be considered a real test of the **Einsteinian general theory of relativity**?

The equation \( 1 + z = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \) breaks when \( r \) is equal to \( \frac{2GM}{c^2} \). However, the equation:

\[
1 + z = e^{\frac{GM}{cr^2} t}
\]

is not going to break even when \( r \) is equal to \( \frac{2GM}{c^2} \).

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According to the approximation of the first order for a weak gravitational field (where \( r \) is inadequately large as opposed to the Schwarzschild radius \( \frac{2GM}{c^2} \)), the formula:

\[
1 + z = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}}
\]

for redshift emerges as,

\[
z = \frac{GM}{rc^2}
\]

\[
1 + z = e^{\frac{GM}{cr^2} t}
\]
\[ \ln (1 + z) = \frac{GM}{cr^2} t \]

\[ \frac{\ln (1 + z)}{z} = \frac{GM}{cr^2} \frac{t}{\frac{GM}{rc^2}} \]

\[ t = \frac{r}{c} \frac{\ln (1 + z)}{Z} \]

**Conclusion:**

As an example, take the white dwarf star Sirius B, with a gravitational field \( \sim 100,000 \) times as strong as the Earth's. Although it sounds extreme, this is still considered a relatively weak field, and the gravitational redshift can be approximated by:

\[ \ln (1 + z) = \frac{GM}{cr^2} t \]

where \( z \) is the gravitational redshift, \( G \) is Newton's gravitational constant, \( M \) is the mass of the object, \( r \) is the photon's starting distance from \( M \), and \( c \) is the speed of light. The above formula for redshift emerges as,

\[ z = \frac{GM}{rc^2} \]

only if
\[ t = \frac{r \ln (1 + z)}{c z} \]

Hence, the **time during which the photon wavelength** changes from \( \lambda_{\text{emit}} \) to \( \lambda_{\text{obs}} \) under static-weak gravitational field is given by:

\[ t = \frac{r \ln (1 + z)}{c z} \]

**Three Classical Tests of General Relativity:**

- Precession of Mercury's orbit
- Deflection of starlight (**gravitational lensing**)
- Gravitational Redshift

Albert Einstein (1915):

**Gravitation field \( \rightarrow \) curved spacetime**

Spacetime tells matter how to move  
Matter tells spacetime how to curve

If all of the galaxies are redshifted, then they are all moving apart from each other!!

*(This is the evidence that our Universe is expanding)*
- **Dark Matter** → Unseen mass which does not interact with the electromagnetic force.
- **Dark Energy** → Unseen energy causing the rate of expansion of our universe to accelerate over time.
- **Principle of equivalence** → Gravity and Inertia are indistinguishable.
- **Special Relativity** → There is no such thing as absolute space or time. Space and time are not wholly independent of each other, but are aspects of a single entity called spacetime.

**General Relativity**: Einstein described gravity as a warping of space-time around a massive object. The stronger the gravity, the more space-time is warped.

**General Relativity**: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.
Quantum physics
(behavior of very small things)

+ 

Relativity theory
(behavior of very large things)

A cluster of galaxies consists of 3 components:

- Galaxies
- Hot Gas
- Dark Matter

In one second, 4.5 million tons of rest mass is converted to radiant energy in the sun.

Because $E = mc^2$:

- A light bulb filament has more mass when it is energized with electricity than when it is turned off.
- A hot cup of tea has more mass than the same cup of tea when cold.

**General relativity** → gravity causes space to become curved and time to slow down.
A photon moving upwards in gravitational field is redshifted. Since
\[ \nu = \frac{1}{T} \]
the photon's period gets longer. Observer 1 will measure a longer period than Observer 2.
This gravitational time dilation effect is unnoticeable in our daily experience!
This is tiny in Earth's gravitational field, but large in a black hole's.

References:
- Gravitational Frequency Shifts for Photons and Particles by Jing-Gang Xie.
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